MOHAMED SATHAK A J COLLEGE OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

ME 8692 FINITE ELEMENT ANALYSIS

QUESTION BANK

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Prepared By

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UNIT-I

PART-A

1 What is meant by finite element?

A small units having definite shape of geometry and nodes is called finite element.

2 What is meant by node or joint?

Each kind of finite element has a specific structural shape and is inter- connected with the adjacent element by nodal point or nodes. At the nodes, degrees of freedom are located. The forces will act only at nodes at any others place in the element.

3 What is the basic of finite element method?

Discretization is the basis of finite element method. The art of subdividing a structure in to convenient number of smaller components is known as discretization.

4 What are the types of boundary conditions?

Primary boundary conditions

Secondary boundary conditions

5 State the methods of engineering analysis?

Experimental methods

Analytical methods

Numerical methods or approximate methods

6 What are the types of element?

1D element

2D element

3D element

7 State the three phases of finite element method.

Preprocessing

Analysis

Post Processing

8 What is structural problem?

Displacement at each nodal point is obtained. By these displacements solution stress and strain in each element can be calculated.

9 What is non structural problem?

Displacement at each nodal point is obtained. By these displacements solution stress and strain in each element can be calculated.

10 What is non structural problem?

Temperature or fluid pressure at each nodal point is obtained. By using these values properties such as heat flow fluid flow for each element can be calculated.

11 What are the methods are generally associated with the finite element analysis?

Force method

Displacement or stiffness method.

12 Explain stiffness method.

Displacement or stiffness method, displacement of the nodes is considered as the unknown of the problem. Among them two approaches, displacement method is desirable.

13 What is meant by post processing?

Analysis and evaluation of the solution result is referred to as post processing. Postprocessor computer program help the user to interpret the result by displaying them in graphical form.

14 Name the variation methods.

Ritz method.

Ray-Leigh Ritz method.

15 What is meant by degrees of freedom?

When the force or reaction act at nodal point node is subjected to deformation. The deformation includes displacement rotation, and or strains. These are collectively known as degrees of freedom

16 What is meant by discretization and assemblage?

The art of subdividing a structure in to convenient number of smaller components is known as discretization. These smaller components are then put together. The process of uniting the various elements together is called assemblage.

17 What is Rayleigh-Ritz method?

It is integral approach method which is useful for solving complex structural problem, encountered in finite element analysis. This method is possible only if a suitable function is available.

18 What is Aspect ratio?

It is defined as the ratio of the largest dimension of the element to the smallest dimension. In many cases, as the aspect ratio increases the in accuracy of the solution increases. The conclusion of many researches is that the aspect ratio should be close to unity as possible.

19 What is truss element?

The truss elements are the part of a truss structure linked together by point joint This transmits only axial force to the element.

20 What are the h and p versions of finite element method?

It is used to improve the accuracy of the finite element method. In h version, the order of polynomial approximation for all elements is kept constant and the numbers of elements are increased. In p version, the numbers of elements are maintained constant and the order of polynomial approximation of element is increased.

21 Name the weighted residual method.

Point collocation method

Sub domain collocation method

Lest squares method

Galerkins method.

22 List the two advantages of post processing.

Required result can be obtained in graphical form. Contour diagrams can be used to understand the solution easily and quickly.

23 During discretization, mention the places where it is necessary to place a node?

Concentrated load acting point

Cross-section changing point

Different material interjections point

Sudden change in point load

24 What is the difference between static and dynamic analysis?

Static analysis: The solution of the problem does not vary with time is known as static analysis

Example: stress analysis on a beam

Dynamic analysis: The solution of the problem varies with time is known as dynamic analysis

Example: vibration analysis problem.

25 Name any four FEA software's.

ANSYS

NASTRAN

COSMOS

26 Differentiate between global and local axes.

Local axes are established in an element. Since it is in the element level, they change with the change in orientation of the element. The direction differs from element to element.

Global axes are defined for the entire system. They are same in direction for all the elements even though the elements are differently oriented.

UNIT-II

PART-A

1. What are the types of loading acting on the structure?

   Body force (f) Traction force (T) Point load (P)

2. Define the body force

   A body force is distributed force acting on every elemental volume of the body

   Unit: Force per unit volume.

   Example: Self weight due to gravity

3. Define traction force

   Traction force is defined as distributed force acting on the surface of the body.

   Unit: Force per unit area.

   Example: Frictional resistance, viscous drag, surface shear

4. What is point load?

   Point load is force acting at a particular point which causes displacement.

5. What are the basic steps involved in the finite element modeling

   Discretization of structure.

   Numbering of nodes.

6. What is discretization?

   The art of subdividing a structure in to a convenient number of smaller components is known as discretization.

7. What are the classifications of coordinates?

   Global coordinates

   Local coordinates

   Natural coordinates

8. What is Global coordinates?

   The points in the entire structure are defined using coordinates system is known as global coordinate system.

9. What is natural coordinates?

   A natural coordinate system is used to define any point inside the element by a set of dimensionless number whose magnitude never exceeds unity. This system is very useful in assembling of stiffness matrices.

10. Define shape function.

    In finite element method, field variables within an element are generally expressed by the following approximate relation:

    $\Phi (x,y) = N_1(x,y)\, \Phi_1 + N_2 (x,y)\, \Phi_2 + N_3(x,y)\, \Phi_3$ where Φ_1 Φ_2 Φ_3 Φ_4 are the values of the field variable at the nodes and N_1 N_2 N_3 N_4 are interpolation function. N_1 N_2, N_3, N_4 are called shape functions because they are used to express the geometry or shape of the element.

11. What are the characteristics of shape function?

The characteristics of the shape functions are follows:

1. The shape function has unit value at one nodal point and zero value at the other nodes.
2. The sum of the shape function is equal to one.

12. Why polynomials are generally used as shape function?

Polynomials are generally used as shape functions due to the following reasons:

1. Differentiation and integration of polynomials are quite easy.
2. The accuracy of the results can be improved by increasing the order of the polynomial.
3. It is easy to formulate and computerize the finite element equations.

13. Give the expression for element stiffness matrix.

Stiffness matrix [K] = $\int [B]^T [D][B] d\,v$

Where, [B] matrix is a strain displacement matrix

[D] matrix is stress, strain relationship matrix

14. State the properties of a stiffness matrix.

The properties of the stiffness matrix [K] are,

1. It is a symmetric matrix
2. The sum of the elements in any column must be equal to zero.
3. It is an unstable element, so the determinant is equal to zero.

15. Write down the general finite element equation.

General finite element equation is,

{F} = [K] {u}

Where, {F} is a force vector

[K] is the stiffness matrix

{u} is the degrees of freedom

16. State the assumptions made in the case of truss element.

The following assumptions are made in the case of truss element,

1. All the members are pin jointed.
2. The truss is loaded only at the joints
3. The self weight of the members are neglected unless stated.

17. State the principle of minimum potential energy.

The total potential energy Π of an elastic body is defined as the sum of total strain energy U and the potential energy of the external forces, (W)

18. Distinguish between essential boundary condition and natural boundary condition. There are two types of boundary conditions. They are,

1. Primary boundary condition (or) essential boundary condition:

The boundary condition which in terms of the field variables is known as primary boundary condition

2. Secondary boundary condition or natural boundary condition:

The boundary conditions which are in the differential form of field variables is known as secondary boundary condition.

19. What are the difference between boundary value problem and initial value problem?

The solution of differential equation obtained for physical problems which satisfies some Specified conditions known as boundary conditions. If the solution of differential equation is obtained together with initial conditions then it is known as initial value problem. If the solution of differential equation is obtained together with boundary conditions then it is known as boundary value problem.

UNIT-III

PART-A

1. How do you define two dimensional elements?

   Two dimensional elements are defined by three or nodes in a two dimensional plane (ie., x,y plane). The basic element useful for two dimensional analysis is the triangular element.

2. What is a CST element?

   Three nodded triangular element is known as constant strain triangular element. It has 6 unknown degrees of freedom called u1, v1, u2, v2, u3, v3. The element is called CST because it has constant strain throughout it.

3. What is LST element?

   Six nodded triangular element is known as Linear Strain Triangular element. It has 12 unknown displacement degrees of freedom. The displacement function for the element are quadratic instead of linear as in the CST.

4. What is a QST element?

   Ten nodded triangular element is known as Quadratic Strain Triangle.

5. What is meant by plane stress analysis?

   Plane stress is defined as a state of stress in which the normal stress (σ) and the shear stress (τ) directed perpendiculars to the plane are zero.

6. Define plane strain.

   Plane strain is defined to be a state of strain in which the strain normal to the xy plane and the shear strains are assumed to be zero.

7. Write the shape function for a CST element.

   For CST element,

   Shape function,
   $$N_1 = \frac{p_1 + q_1 x + r_1 y}{2A}$$
   $$N_2 = \frac{p_2 + q_2 x + r_2 y}{2A}$$
   $$N_3 = \frac{p_3 + q_3 x + r_3 y}{2A}$$

   Where,
   $$p_1 = x_2 y_3 - x_3 y_2$$
   $$p_2 = x_3 y_1 - x_1 y_3$$
   $$p_3 = x_1 y_2 - x_2 y_1$$

8. Write a displacement function equation for CST element.

Displacement function u = $\begin{Bmatrix} u(x,y) \\ v(x,y) \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v \\ u_3 \\ v_3 \end{Bmatrix}$ where N_1 N_2 N_3 are shape functions.

9. write a strain-displacement matrix for CST element

$$[B] = \frac{1}{2A}\begin{bmatrix} q_1 & 0 & q_2 & 0 & q_3 & 0 \\ 0 & r_1 & 0 & r_2 & 0 & r_3 \\ r_1 & q_1 & r_2 & q_2 & r_3 & q_3 \end{bmatrix}$$

Where, A = Area of the element

q_1 = y2-y3 q_2= y3-y1 q_3= y1-y2

r_1 = x3-x2 r_2= x1-x3 r_3= x2-x1

10. Write down the stiffness matrix equation for two dimensional CST elements.

Stiffness matrix [K] = $[B]^T [D][B]\ dt$

Where, [B] is the strain displacement matrix

[D] is the stress-strain matrix

A is the area of the element

't' is the thickness of the element

11. Write down the stress-strain relationship matrix for plane stress condition.

For plane stress problems, stress-strain relationship matrix is,

$$[D] = \frac{E}{1-\upsilon^2}\begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{1-\upsilon}{2} \end{bmatrix}$$

Where, E = young's modulus

V = Poisson's ratio

12. Write down the stress-strain relationship matrix for plain strain condition.

For plain strain problems, stress-strain relationship matrix is,

$$[D] = \frac{E}{(1+\upsilon)(1-2\upsilon)}\begin{bmatrix} (1-\upsilon) & \upsilon & 0 \\ 0 & (1-\upsilon) & 0 \\ 0 & 0 & \frac{(1-2\upsilon)}{2} \end{bmatrix}$$

13. Define heat transfer.

Heat transfer can be defined as the transmission of energy from one region to another region due to temperature difference.

UNIT-IV

PART-A

1. Define Quasi static response.

   When the excitations are varying slowly with time then it is called quasi static response.

2. Write down the displacement equation for an axisymmetric triangular element.

   Displacement function, u(r,z) = $\begin{Bmatrix} u(r,z) \\ w(r,z) \end{Bmatrix} = \begin{bmatrix} N1 & 0 & N2 & 0 & N3 & 0 \\ 0 & N1 & 0 & N2 & 0 & N3 \end{bmatrix} \begin{Bmatrix} u1 \\ w1 \\ u2 \\ w2 \\ u3 \\ w3 \end{Bmatrix}$

3. What are the conditions for a problem to axi symmetric?

   1. The problem domain must be symmetric about the axis of rotation.
   2. All the boundary conditions must be symmetric about the axis of rotation.
   3. All loading conditions must be symmetric about the axis of rotation.

4. What are the ways in which a three dimensional problem can be reduced to a two dimensional approach.

   1. Plane Stress: on dimension is too small when compared to other two dimensions

   Example: Gear — thickness is small

   2. Plane Strain: one dimension is too large when compared to other two dimensions.

   Examples: Long Pipe (length is long compared to diameter)

   3. Axisymmetric: Geometry is symmetric about the axis.

   Example: cooling tower

5. Give the stiffness matrix equation for an axisymmetric triangular element.

   Stiffness matrix, $[K] = 2\pi rA[B]^T[D]\,[B]$

   Where, co-ordinate $r = \dfrac{r1+r2+r3}{3}$

6. Write down the shape functions for an axisymmetric triangular element.

   Shape function,

   $N_1 = \dfrac{\alpha 1+\beta 1r+\gamma 1z}{2A}$

   $N_2 = \dfrac{\alpha 2+\beta 2r+\gamma 2z}{2A}$

$$N_3 = \frac{\alpha 3+\beta 3r+\gamma 3z}{2A}$$

where,

$\alpha_1 = r_2z_3 - r_3z_2$

$\alpha_2 = r_3z_1 - r_1z_3$

$\alpha_3 = r_1z_2 - r_2z_1$

$\beta_1 = z_2 - z_3$

$\beta_1 = z_3 - z_1$

$\beta_1 = z_1 - z_2$

$\gamma_1 = r_3 - r_2$

$\gamma_1 = r_1 - r_3$

$\gamma_1 = r_2 - r_1$

7. Give the stress-strain matrix equation for an axisymmetric triangular element.

Stress –strain relationship matrix, [D] =

$$\frac{E}{(1+\nu)(1-2\nu)}\begin{pmatrix} 1-\nu & \nu & \nu & 0 \\ \nu & 1-\nu & \nu & 0 \\ \nu & \nu & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2} \end{pmatrix}$$

Where, E = young's modulus

ν = Poisson's ratio

8. Give the strain displacement matrix equation for an axisymmetric triangular element.

Strain displacement matrix,

$$[B] = 1/2A \begin{pmatrix} \beta_1 & 0 & \beta_2 & 0 & \beta_3 & 0 \\ \frac{\alpha_1}{r}+\beta_1+\frac{\gamma_1 z}{r} & 0 & \frac{\alpha_2}{r}+\beta_2+\frac{\gamma_2 z}{r} & 0 & \frac{\alpha_3}{r}+\beta_3+\frac{\gamma_3 z}{r} & 0 \\ 0 & \gamma_1 & 0 & \gamma_2 & 0 & \gamma_3 \\ \gamma_1 & \beta_1 & \gamma_1 & \beta_1 & \gamma_1 & \beta_1 \end{pmatrix}$$

UNIT-V

PART-A

1 What is the purpose of Iso parametric elements?

It is difficult to represent the curved boundaries by straight edges finite elements. A large number of finite elements may be used to obtain reasonable resemblance between original body and assemblage. In order to overcome this drawback, iso parametric elements are used i.e for problems involving curved boundaries, a family of elements “isoparametric elements” are used

2 Write down the shape functions for 4 noded rectangular element using natural cordinate system.

Shape functions:

$N_1 = \frac{1}{4}(1-\varepsilon)(1-\eta)$

$N_2 = \frac{1}{4}(1+\varepsilon)(1-\eta)$

$N_3 = \frac{1}{4}(1+\varepsilon)(1+\eta)$

$N_4 = \frac{1}{4}(1-\varepsilon)(1+\eta)$ where, ε and η are natural co-ordinates.

3 Write down the jacobian matrix for four noded quadrilateral element.

Jacobian Matrix,[J] = $\begin{bmatrix} J11 & J12 \\ J21 & J22 \end{bmatrix}$

Where,

$J_{11} = \frac{1}{4}[-(1-\eta)x_1+(1-\eta)x_2+(1+\eta)x_3-(1+\eta)x_4]$

$J_{12} = \frac{1}{4}[-(1-\eta)y_1+(1-\eta)y_2+(1+\eta)y_3-(1+\eta)y_4]$

$J_{21} = \frac{1}{4}[-(1-\varepsilon)x_1-(1+\varepsilon)x_2+(1+\varepsilon)x_3+(1-\varepsilon)x_4]$

$J_{22} = \frac{1}{4}[-(1-\varepsilon)y_1-(1+\varepsilon)y_2+(1+\varepsilon)y_3+(1-\varepsilon)y_4]$

4 Write down the stiffness matrix equation for four noded isoparametric quadrilateral elements.

Stiffness matrix, [K] = t $\int_{-1}^{1}[D][B][B]^T * |J| * \delta\varepsilon * \delta\eta$

Where,

t = thickness of the element

$|J|$ = Determinant of the jacobian

ε,η = Natural co-ordinates

[B] = strain-displacement matrix

[D] = stress-strain relationship matrix

5 Write down the element force vector equation for four noded quadrilateral element.

Force vector, $\{F\}_e = [N]^T \begin{Bmatrix} F_x \\ F_y \end{Bmatrix}$

Where, N is the shape function.

F_x is a load or force on x direction.

Fy is a load on y direction.

6 Write down the Gaussian quadrature expression for numerical integration.

Gaussian quadrature expression,

$\int_{-1}^{1} f(x)dx = \sum_{n=1}^{n} (w_i f(x_i))$

Where, w_i = weight function

$F(x_i)$ = values of the function at pre-determined sampling points.

7 Define super parametric element.

If the number of nodes for defining the geometry is more than the number of nodes used for defining the displacements is known as super parametric element.

8 What is meant by sub parametric element?

If the number of nodes used for defining the geometry is less than the number of nodded used for defining the displacements is known as sub parametric element.

9 What is meant by isoparametric element?

If the number of nodes used for defining the geometry is same as number of nodes used for defining the displacements then it is called iso parametric element.

10 Is beam element an isoparametric element?

Beam element is not an isoparametric element since geometry and displacements are defined by different interpolation functions.

11 What are the types of non-linearity?

(a) Non-linearity in material behaviour from point to point
(b) Non-linearity in loading deformation relation
(c) Geometric non-linearity
(d) Change in boundary condition for different loading.

12 Give examples for essential and non-essential boundary conditions.

The geometric boundary conditions are displacement, slope, etc. the natural boundary conditions are bending moment, shear force, etc.

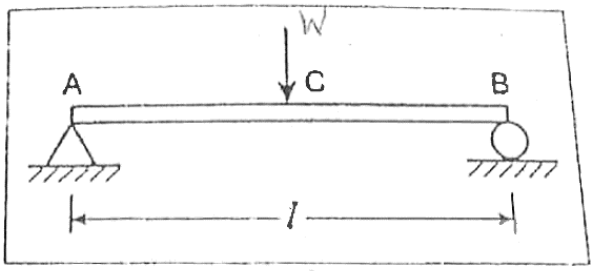
PART B

UNIT – 1

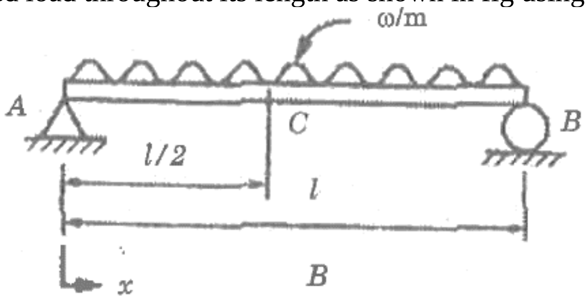
1. Explain the various steps involved in the Finite Element Formulations

2. Using Collocation method, find the solution of given governing equation
   $\frac{d^2y}{dx^2} + y + x = 0; \quad 0 \le x \le 1$ subject to boundary conditions y(0)=y(1)=0
   Use x=1/4 and x=1/2 as collocation points.

3. The differential equation for a physical problem is given by $\frac{d^2y}{dx^2} + 500x^2 = 0;$
   $0 \le x \le 1$, boundary conditions as y (0) = 0 and y (1) = 0. Find the approximate solution for any classical technique using trial solution as $y(x)=C_1x(1-x) + C_2x^2(1-x)$.
4. A beam AB of span length 'l' simply supported at ends and carrying a concentrated load W at the center 'C' as shown in fig. Determine the deflection at midspan by using Raleigh Ritz Method and compare with exact solution.

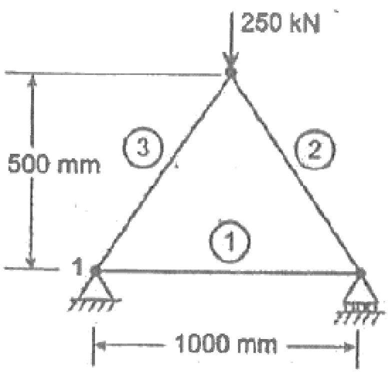


5. Find the deflection at the Centre for the simply supported beam of the span length 'l' subjected to uniformly distributed load throughout its length as shown in fig using Raleigh Ritz Method.



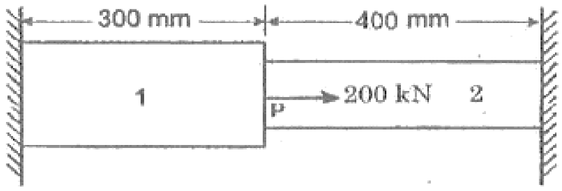
UNIT – 2

6. Consider three bar trusses as shown in fig. Calculate the nodal displacement. Take $E=2\times10^5$ N/mm^2, $A_1=2000$ mm^2, $A_2=2500$ mm^2, $A_3=2500$ mm^2.

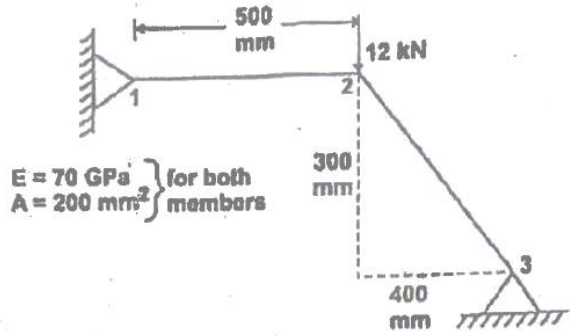


7. Using two finite elements, find the stress distribution in a uniformly tapering bar of cross-sectional area 300 mm^2 and 200 mm^2 at their ends, length 100 mm, subjected to an axial tensile load of 50N at smaller end and fixed at larger end. Take $E=2\times10^5$ N/mm^2.

8. Derive the expression for Stiffness Matrix of one-dimensional liner bar element.
9. Consider a bar element as shown in fig. An axial load of 200kN is applied at point P. Take A_1=2400mm^2, E_1=70×10^3 N/mm^2, A_2=600 mm^2, E_2=200×10^3 N/mm^2. Calculate the following,
   i) Nodal displacement
   ii) Stresses in each element
   iii) Reaction forces

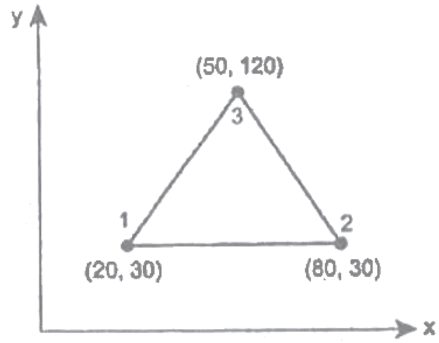


10. For the two-bar truss as shown in fig. determine the displacement at node 2 and stresses in both the elements.

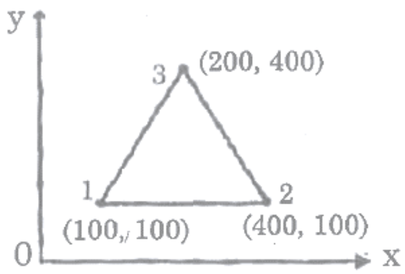


UNIT – 3

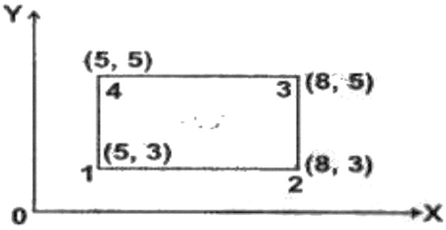
11. Derive the shape functions for Two-dimensional CST element.
12. Determine the stiffness matrix for the CST element shown in fig. The Coordinates are given in mm. Assume Plane strain conditions. E=210 GPa, μ=0.2 and t=10 mm.



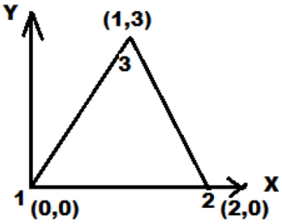
13. For a plane stress element as shown in fig, the nodal displacements ((u_1, v_1), (u_2, v_2), (u_3, v_3)) are ((2,1), (1, 1.5), (2.5,0.5)) respectively. Determine the element stresses. Assume E=200GN/m^2, μ=0.3 and t=10 mm, all co-ordinates are in mm.



14. For a 4 noded rectangular element shown in fig. estimate the temperature point (7,4). The nodal values of the temperatures are T_1=42°C, T_2=54°C, T_3=56°C and T_4=46°C. Also determine the 3 Point on the 50°C contour line.

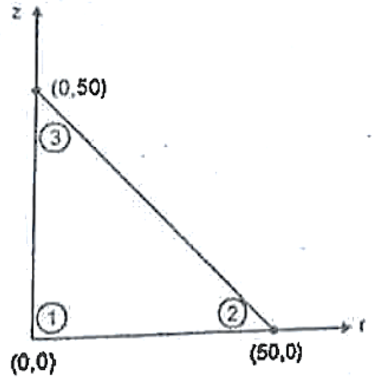


15. Calculate the element stiffness matrix and temperature force vector for the plane stress element shown in fig. The element experiences a 20°C increase in temperature. Assume α= 6×10^{-6} C. Take E= 2×10^5 N/mm^2, μ=0.25, t= 5mm.

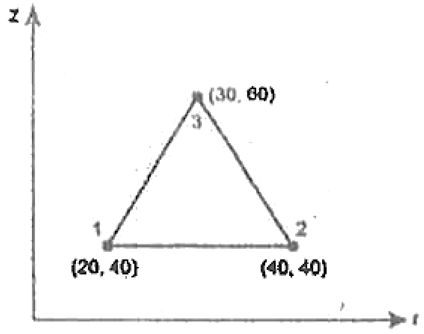


UNIT – 4

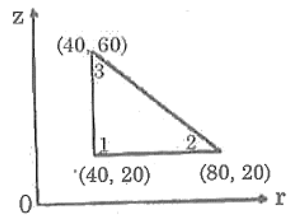
16. Derive the shape functions for Two-dimensional axisymmetric triangular element.
17. For the axisymmetric element shown in the fig, determine the stiffness matrix. Let E= 2.1×10^5 N/mm^2 and μ=0.3. The coordinates are in mm.



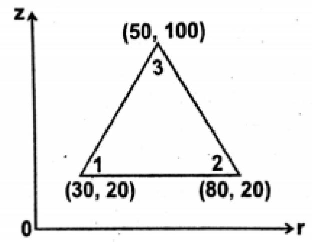
18. The nodal coordinates for an axisymmetric triangular element are given in fig. Evaluate the Strain Displacement Matrix and Constitutive Matrix. Take E= 2.1 x 10^6 N/mm^2 and Poisson's ratio as 0.3.



19. Calculate the element stresses for the axisymmetric element shown in the fig. The nodal displacements are u_1= 0.02mm, u_2 = 0.01mm, u_3 = 0.04mm, w_1 = 0.03mm, w_2 = 0.06mm and w_3 = 0.01mm. Take E=210GPa and μ=0.25.

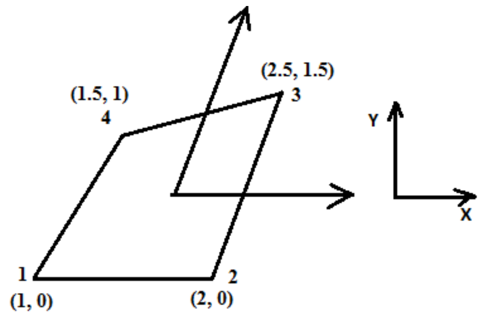


20. For an axisymmetric triangular element as shown in fig. Evaluate the stiffness matrix. Take modulus of elasticity E = 210 GPa. Poisson's ratio = 0.25. The coordinates are given in millimetres.

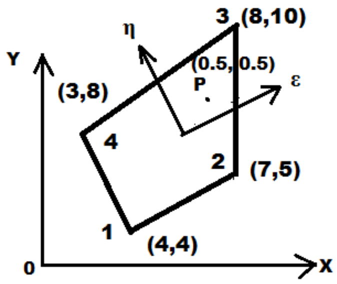


UNIT – 5

21. Evaluate the Jacobian matrix for the iso parametric quadrilateral element shown in the figure



22. Develop the shape function for 4 noded iso parametric quadrilateral element.
23. Evaluate the Jacobian matrix at the local coordinates ε=η= 0.5 for the linear quadrilateral element with its global coordinates as shown in fig. Also evaluate the strain-displacement matrix



24. Evaluate the integral by two-point Gaussian Quadrature, Gauss points are +0.57735 and -0.57735 each of weight 1.0000.

$$I=\int_{-1}^{1}\int_{-1}^{1}(2x^2+3xy+4y^2)\,dx\,dy$$

25. Evaluate the following by applying 3-point Gaussian quadrature.

$$\int_{-1}^{1}\left(x^4+x^2\right)dx$$