DEPARTMENT OF MECHANICAL ENGINEERING
(ACADEMIC YEAR: 2020-2021)

CE 8395 -STRENGTH OF MATERIALS FOR MECHANICAL ENGINEERS

(Regulation 2017)
Semester - IV

NAME :

REG NO :

MOHAMED SATHAK
A J COLLEGE OF ENGINEERING
SIRUSERI IT PARK, OMR, CHENNAI - 603103

UNIT -1

STRESS, STRAIN AND DEFORMATION OF SOLIDS

1.1 Rigid and deformable bodies

Rigid body motion theory is a fundamental and well-established part of physics. It is based on the approximation that for stiff materials, any force applied to a body produces a negligible deformation. Thus, the only change a force can produce is change in the center of mass motion and change in the rotational motion. This means that simulation of even complex bodies is relatively simple, and thus this method has become popular in the computer simulation field.

Given the forces acting on the body, the motion can be determined using ?? ??for translational motion, and a similar relation for rotational motion .

The rigid body motion model has traditionally been applied in range analysis in CAD and for computer animation where deformation is not required. If the deformation is not negligible, then the approximation does not hold, and we need to start over and come up with some other model. There exists many different models, but the two models which have emerged to become the most widely used in practice are: mass-spring models and statics models solved using the Finite Element Method (FEM).

Mass-spring models represent bodies as discrete mass-elements, and the forces between them are transmitted using explicit spring connections ("spring" is a historical term, and is not limited to pure Hooke interactions). Given the forces acting on an element, we can determine its motion using . The motion of the entire body is then implicitly described by the motion of its elements.

Mass-spring models have traditionally been applied mostly for cloth simulation. Statics models are based on equilibrium relations, and thus make the approximation that the effect of dynamics are negligible. Relations between the strain and stress fields of a body are set up, and through specifying known values of these fields, through for example specifying forces acting on the body, the unknown parts can be determined. These relations form differential equations, and the known values are boundary values. The FEM is an effective method for solving boundary value problems, and has thus given its name to these types of problems. Statics models have traditionally been applied in stress and displacement analysis systems in CAD.

1.2 General Concepts and Definitions

- **Strength** The ability to sustain load.
- **Stiffness** Push per move; the ratio of deformation to associated load level.
- **Stability** The ability of a structure to maintain position and geometry. Instability involves collapse that is not initiated by material failure. External stability concerns the ability of a structure's supports to keep the structure in place; internal stability concerns a structure's ability to maintain its shape.
- **Ductility** The amount of inelastic deformation before failure, often expressed relative to the amount of elastic deformation.

Strength Material strength is measured by a stress level at which there is a permanent and significant change in the material's load carrying ability. For example, the yield stress, or the ultimate stress.

Stiffness Material stiffness is most commonly expressed in terms of the modulus of elasticity: the ratio of stress to strain in the linear elastic range of material behavior.

Stability As it is most commonly defined, the concept of stability applies to structural elements and systems, but does not apply to materials, since instability is defined as a loss of load carrying ability that is not initiated by material failure.

Ductility Material ductility can be measured by the amount of inelastic strain before failure compared to the amount of elastic strain. It is commonly expressed as a ratio of the maximum strain at failure divided by the yield strain.

1.3 Mechanical properties of materials

A tensile test is generally conducted on a standard specimen to obtain the relationship between the stress and the strain which is an important characteristic of the material. In the test, the uniaxial load is applied to the specimen and increased gradually. The corresponding deformations are recorded throughout the loading. Stress-strain diagrams of materials vary widely depending upon whether the material is ductile or brittle in nature. If the material undergoes a large deformation before failure, it is referred to as ductile material or else brittle material.Stress-strain diagram of a structural steel, which is a ductile material, is given.

Initial part of the loading indicates a linear relationship between stress and strain, and the deformation is completely recoverable in this region for both ductile and brittle materials. This linear relationship, i.e., stress is directly proportional to strain, is popularly known as Hooke's law.

s = Ee

The co-efficient E is called the modulus of elasticity or Young's modulus.

Most of the engineering structures are designed to function within their linear elastic region only.After the stress reaches a critical value, the deformation becomes irrecoverable. The corresponding stress is called the yield stress or yield strength of the material beyond which the material is said to start yielding.

In some of the ductile materials like low carbon steels, as the material reaches the yield strength it starts yielding continuously even though there is no increment in external load/stress. This flat curve in stress strain diagram is referred as perfectly plastic region.

The load required to yield the material beyond its yield strength increases appreciably and this is referred to strain hardening of the material. In other ductile materials like aluminum alloys, the strain hardening occurs immediately after the linear elastic region without perfectly elastic region.

After the stress in the specimen reaches a maximum value, called ultimate strength, upon further tretching, the diameter of the specimen starts decreasing fast due to local instability and this p henomenon is called necking.

The load required for further elongation of the material in the necking region decreases with decrease in diameter and the stress value at which the material fails is called the breaking strength. In case of brittle materials like cast iron and concrete, the material experiences smaller deformation before rupture and there is no necking.

1.4 True stress and true strain

In drawing the stress-strain diagram as shown in figure 1.13, the stress was calculated by dividing the load P by the initial cross section of the specimen. But it is clear that as the specimen elongates its diameter decreases and the decrease in cross section is apparent during necking phase. Hence, the actual stress which is obtained by dividing the load by the actual cross sectional area in the deformed specimen is different from that of the engineering stress that is obtained using undeformed cross sectional area as in equation 1.1 Though the difference between the true stress and the engineering stress is negligible for smaller loads, the former is always higher than the latter for larger loads.

Similarly, if the initial length of the specimen is used to calculate the strain, it is called engineering strain as obtained in equation 1.9

But some engineering applications like metal forming process involve large deformations and they require actual or true strains that are obtained using the successive recorded lengths to calculate the strain. True strain is also called as actual strain or natural strain and it plays

1.5 TYPES OF STRESSES :

Only two basic stresses exists : (1) normal stress and (2) shear shear stress. Other stresses either are similar to these basic stresses or are a combination of these e.g. bending stress is a combination tensile, compressive and shear stresses. Torsional stress, as encountered in twisting of a shaft is a shearing stress.

Let us define the normal stresses and shear stresses in the following sections.

Normal stresses : We have defined stress as force per unit area. If the stresses are normal to the areas concerned, then these are termed as normal stresses. The normal stresses are generally denoted by a Greek letter (s)

This is also known as uniaxial state of stress, because the stresses acts only in one direction however, such a state rarely exists, therefore we have biaxial and triaxial state of stresses where either the two mutually perpendicular normal stresses acts or three mutually perpendicular normal stresses acts as shown in the figures below :

Tensile or compressive stresses :

The normal stresses can be either tensile or compressive whether the stresses acts out of the area or into the area

Bearing Stress : When one object presses against another, it is referred to a bearing stress (They are in fact the compressive stresses).

Shear stresses :

Let us consider now the situation, where the cross – sectional area of a block of material is subject to a distribution of forces which are parallel, rather than normal, to the area concerned. Such forces are associated with a shearing of the material, and are referred to as shear forces. The resulting force interistes are known as shear stresses.

The resulting force intensities are known as shear stresses, the mean shear stress being equal to

Where P is the total force and A the area over which it acts.

Stress is defined as the force per unit area. Thus, the formula for calculating stress is:

s= F/A

Where s denotes stress, F is load and A is the cross sectional area. The most commonly used units for stress are the SI units, or Pascals (or N/m^2), although other units like psi (pounds per square inch) are sometimes used.

Forces may be applied in different directions such as:

- Tensile or stretching
- Compressive or squashing/crushing
- Shear or tearing/cutting
- Torsional or twisting

This gives rise to numerous corresponding types of stresses and hence measure/quoted strengths. While data sheets often quote values for strength (e.g compressive strength), these values are purely uniaxial, and it should be noted that in real life several different stresses may be acting.

Tensile Strength

The tensile strength is defined as the maximum tensile load a body can withstand before failure divided by its cross sectional area. This property is also sometimes referred to Ultimate Tensile Stress or UTS.

Typically, ceramics perform poorly in tension, while metals are quite good. Fibres such as glass, Kevlar and carbon fibre are often added polymeric materials in the direction of the tensile force to reinforce or improve their tensile strength.

Compressive Strength

Compressive strength is defined as the maximum compressive load a body can bear prior to failure, divided by its cross sectional area.

Ceramics typically have good tensile strengths and are used under compression e.g. concrete.

Shear Strength

Shear strength is the maximum shear load a body can withstand before failure occurs divided by its cross sectional area.

This property is relevant to adhesives and fasteners as well as in operations like the guillotining of sheet metals.

Torsional Strength

Torsional strength is the maximum amount of torsional stress a body can withstand before it fails, divided by its cross sectional area.

This property is relevant for components such as shafts.

Yield Strength

Yield strength is defined as the stress at which a material changes from elastic deformation to plastic deformation. Once the this point, known as the yield point is exceeded, the materials will no longer return to its original dimensions after the removal of the stress.

Stress is defined as the force per unit area. Thus, the formula for calculating stress is:

Where s denotes stress, F is load and A is the cross sectional area. The most commonly used units for stress are the SI units, or Pascals (or N/m^2), although other units like psi (pounds per square inch) are sometimes used.

Forces may be applied in different directions such as:

- Tensile or stretching
- Compressive or squashing/crushing
- Shear or tearing/cutting
- Torsion or twisting

This gives rise to numerous corresponding types of stresses and hence measure/quoted strengths. While data sheets often quote values for strength (e.g compressive strength), these values are purely uni axial, and it should be noted that in real life several different stresses may be acting

Deformation of simple bars under axial load Deformation of bodies

Concept of strain : if a bar is subjected to a direct load, and hence a stress the bar will change in length. If the bar has an original length L and changes by an amount dL, the strain produce is defined as follows:

Strain is thus, a measure of the deformation of the material and is a non dimensional Quantity i.e. it has no units. It is simply a ratio of two quantities with the same unit.

Shear strain: As we know that the shear stresses acts along the surface. The action of the stresses is to produce or being about the deformation in the body consider the distortion produced b shear sheer stress on an element or rectangular block This shear strain or slide is f and can be defined as the change in right angle. or The angle of deformation g is then termed as the shear strain. Shear strain is measured in radians & hence is non – dimensional i.e. it has no unit .So we have two types of strain i.e. normal stress & shear stresses.

Hook's Law :

A material is said to be elastic if it returns to its original, unloaded dimensions when load is removed.

Hook's law therefore states that Stress (s) a strain(Î)

Modulus of elasticity : Within the elastic limits of materials i.e. within the limits in which Hook's law applies, it has been shown that

Stress / strain = constant

This constant is given by the symbol E and is termed as the modulus of elasticity or Young's modulus of elasticity Thus ,The value of Young's modulus E is generally assumed to be the same in tension or compression and for most engineering material has high, numerical value of the order of 200 GPa

Poisson's ratio: If a bar is subjected to a longitudinal stress there will be a strain in this direction equal to s / E . There will also be a strain in all directions at right angles to s . The final shape being shown by the dotted lines.

It has been observed that for an elastic materials, the lateral strain is proportional to the longitudinal strain. The ratio of the lateral strain to longitudinal strain is known as the poison's ratio .

Poison's ratio (m) = - lateral strain / longitudinal strain

For most engineering materials the value of m his between 0.25 and 0.33.

Deformation of compound bars under axial load

For a prismatic bar loaded in tension by an axial force P, the elongation of the bar can be determined as Suppose the bar is loaded at one or more intermediate positions, then equation (1) can be readily adapted to handle this situation, i.e. we can determine the axial force in each part of the bar i.e. parts AB, BC, CD, and calculate the elongation or shortening of each part separately, finally, these changes in lengths can be added algebraically to obtain the total charge in length of the entire bar.

When either the axial force or the cross – sectional area varies continuosly along the axis of the bar, then equation (1) is no longer suitable. Instead, the elongation can be found by considering a deferential element of a bar and then the equation (1) becomes i.e. the axial force Pxand area of the cross – section Ax must be expressed as functions of x. If the expressions for Pxand Ax are not too complicated, the integral can be evaluated analytically, otherwise Numerical methods or techniques can be used to evaluate these integrals.

Relation between E, G and u :

Let us establish a relation among the elastic constants E,G and u. Consider a cube of material of side „a' subjected to the action of the shear and complementary shear stresses as shown in the figure and producing the strained shape as shown in the figure below.

Assuming that the strains are small and the angle A C B may be taken as 450.

Therefore strain on the diagonal OA

= Change in length / original length

Since angle between OA and OB is very small hence OA @ OB therefore BC, is the change in the length of the diagonal OA

Now this shear stress system is equivalent or can be replaced by a system of direct stresses at 450 as shown below. One set will be compressive, the other tensile, and both will be equal in value to the applied shear strain.

Thus, for the direct state of stress system which applies along the diagonals:

We have introduced a total of four elastic constants, i.e E, G, K and g. It turns out that not all of these are independent of the others. Infact given any two of then, the other two can be found.

irrespective of the stresses i.e, the material is incompressible.

When g = 0.5 Value of k is infinite, rather than a zero value of E and volumetric strain is zero, or in other words, the material is incompressible.

Relation between E, K and u :

Consider a cube subjected to three equal stresses s as shown in the figure below

The total strain in one direction or along one edge due to the application of hydrostatic stress or volumetric stress s is given as

Relation between E, G and K :

The relationship between E, G and K can be easily determained by eliminating u from the already derived relations

E = 2 G (1 + u) and E = 3 K (1 - u)

Thus, the following relationship may be obtained

Relation between E, K and g :

From the already derived relations, E can be eliminated

Engineering Brief about the elastic constants :

We have introduced a total of four elastic constants i.e E, G, K and u. It may be seen that not all of these are independent of the others. Infact given any two of them, the other two can be determined. Further, it may be noted that

hence if u = 0.5, the value of K becomes infinite, rather than a zero value of E and the volumetric strain is zero or in other words, the material becomes incompressible

Further, it may be noted that under condition of simple tension and simple shear, all real materials tend to experience displacements in the directions of the applied forces and Under hydrostatic loading they tend to increase in volume. In other words the value of the elastic constants E, G and K cannot be negative

Therefore, the relations

E = 2 G (1 + u)

E = 3 K (1 - u)

Yields

In actual practice no real material has value of Poisson's ratio negative . Thus, the value of u cannot be greater than 0.5, if however u > 0.5 than Îv = -ve, which is physically unlikely because when the material is stretched its volume would always increase.

Elastic constant - problems

1. The Young's modulus and the Shear modulus of material are 120 GPa and 45 GPa respectively. What is its Bulk modulus?

2. A 20 mm diameter bar was subjected to an axial pull of 40 KN and change in diameter was found to be 0.003822 mm. Find the Poisson's ratio, modulus of elasticity and Bulk modulus if the shear modulus of material of the bar is 76.923 GPa.

3. A steel plate 300 mm long, 60 mm wide and 30 mm deep is acted upon by the forces shown in Fig. Determine the change in volume Take E = 200 KN/mm^2 and Poisson's ratio = 0.3.

4. A bar of 30 mm x 30 mm x 250 mm long was subjected to a pull of 90 KN in the direction of its length. Then extension of the bar was found to be 0.125 mm, while the decrease in each lateral dimension was found to be 0.00375 mm. Find the Young's modulus, Poisson's ratio and rigidity modulus of the bar.

Unit II

TRANSVERSE LOADING ON BEAMS AND STRESSES IN BEAM

2.1 Beams- classification

Classification of Beams:

Beams are classified on the basis of their geometry and the manner in which they are supported.

Classification I: The classification based on the basis of geometry normally includes features such as the shape of the X-section and whether the beam is straight or curved.

Classification II: Beams are classified into several groups, depending primarily on the kind of supports used. But it must be clearly understood why do we need supports. The supports are required to provide constrainment to the movement of the beams or simply the supports resists the movements either in particular direction or in rotational direction or both. As a consequence of this, the reaction comes into picture whereas to resist rotational movements the moment comes into picture. On the basis of the support, the beams may be classified as follows:

Cantilever Beam: A beam which is supported on the fixed support is termed as a cantilever beam: Now let us understand the meaning of a fixed support. Such a support is obtained by building a beam into a brick wall, casting it into concrete or welding the end of the beam. Such a support provides both the translational and rotational constrainment to the beam, therefore the reaction as well as the moments appears, as shown in the figure below

Simply Supported Beam: The beams are said to be simply supported if their supports creates only the translational constraints.

Some times the translational movement may be allowed in one direction with the help of rollers and can be represented like this

Statically Determinate or Statically Indeterminate Beams:

The beams can also be categorized as statically determinate or else it can be referred as statically indeterminate. If all the external forces and moments acting on it can be determined from the equilibrium conditions alone then. It would be referred as a statically determinate beam, whereas in the statically indeterminate beams one has to consider deformation i.e. deflections to solve the problem.

Supports and Loads

2.2 Types of beams: Supports and Loads

In many engineering structures members are required to resist forces that are applied laterally or transversely to their axes. These type of members are termed as beam. There are various ways to define the beams such as

Definition I: A beam is a laterally loaded member, whose cross-sectional dimensions are small as compared to its length.

Definition II: A beam is nothing simply a bar which is subjected to forces or couples that lie in a plane containing the longitudnal axis of the bar. The forces are understood to act perpendicular to the longitudnal axis of the bar.

Definition III: A bar working under bending is generally termed as a beam.

2.3 Materials for Beam:

The beams may be made from several usable engineering materials such commonly among them are as follows:

Metal

Wood

Concrete

Plastic

Issues Regarding Beam:

Designer would be interested to know the answers to following issues while dealing with beams in practical engineering application

• At what load will it fail

• How much deflection occurs under the application of loads.

Cantilever Beam: A beam which is supported on the fixed support is termed as a cantilever beam: Now let us understand the meaning of a fixed support. Such a support is obtained by building a beam into a brick wall, casting it into concrete or welding the end of the beam. Such a support provides both the translational and rotational constrainment to the beam, therefore the reaction as well as the moments appears, as shown in the figure below

Simply Supported Beam: The beams are said to be simply supported if their supports creates only the translational constraints.

Some times the translational movement may be allowed in one direction with the help of rollers and can be represented like this

Statically Determinate or Statically Indeterminate Beams:

The beams can also be categorized as statically determinate or else it can be referred as statically indeterminate. If all the external forces and moments acting on it can be determined from the equilibrium conditions alone then. It would be referred as a statically determinate beam, whereas in the statically indeterminate beams one has to consider deformation i.e. deflections to solve the problem.

Types of loads acting on beams:

A beam is normally horizontal where as the external loads acting on the beams is generally in the vertical directions. In order to study the behaviors of beams under flexural loads. It becomes pertinent that one must be familiar with the various types of loads acting on the beams as well as their physical manifestations.

A. Concentrated Load: It is a kind of load which is considered to act at a point. By this we mean that the length of beam over which the force acts is so small in comparison to its total length that one can model the force as though applied at a point in two dimensional view of beam. Here in this case, force or load may be made to act on a beam by a hanger or though other means

B. Distributed Load: The distributed load is a kind of load which is made to spread over a entire span of beam or over a particular portion of the beam in some specific manner

In the above figure, the rate of loading „q' is a function of x i.e. span of the beam, hence this is a non uniformly distributed load.

The rate of loading „q' over the length of the beam may be uniform over the entire span of beam, then we cell this as a uniformly distributed load (U.D.L). The U.D.L may be represented in either of the way on the beams

some times the load acting on the beams may be the uniformly varying as in the case of dams or on inclind wall of a vessel containing liquid, then this may be represented on the beam as below:

The U.D.L can be easily realized by making idealization of the ware house load, where the bags of grains are placed over a beam.

2.3 Shear force and Bending Moment in beams

Concept of Shear Force and Bending moment in beams:

When the beam is loaded in some arbitrarily manner, the internal forces and moments are developed and the terms shear force and bending moments come into pictures which are helpful to analyze the beams further. Let us define these terms

Now let us consider the beam as shown in fig 1(a) which is supporting the loads P1, P2, P3 and is simply supported at two points creating the reactions R1 and R2respectively. Now let us assume that the beam is to divided into or imagined to be cut into two portions at a section AA. Now let us assume that the resultant of loads and reactions to the left of AA is „F' vertically upwards, and since the entire beam is to remain in equilibrium, thus the resultant of forces to the right of AA must also be F, acting downwards. This forces „F' is as a shear force. The shearing force at any x-section of a beam represents the tendency for the portion of the beam to one side of the section to slide or shear laterally relative to the other portion.

Therefore, now we are in a position to define the shear force „F' to as follows:

At any x-section of a beam, the shear force „F' is the algebraic sum of all the lateral components of the forces acting on either side of the x-section.

Sign Convention for Shear Force:

The usual sign conventions to be followed for the shear forces have been illustrated in figures 2 and 3.

Bending Moment:

Let us again consider the beam which is simply supported at the two prints, carrying loads P1, P2 and P3 and having the reactions R1 and R2 at the supports Fig 4. Now, let us imagine that the beam is cut into two potions at the x-section AA. In a similar manner, as done for the case of shear force, if we say that the resultant moment about the section AA of all the loads and reactions to the left of the x-section at AA is M in C.W direction, then moment of forces to the right of x-section AA must be „M' in C.C.W. Then „M' is called as the Bending moment and is abbreviated as B.M. Now one can define the bending moment to be simply as the algebraic sum of the moments about an x-section of all the forces acting on either side of the section

Sign Conventions for the Bending Moment:

For the bending moment, following sign conventions may be adopted as indicated in Fig 5 and Fig 6.

Some times, the terms „Sagging' and Hogging are generally used for the positive and negative bending moments respectively.

Bending Moment and Shear Force Diagrams:

The diagrams which illustrate the variations in B.M and S.F values along the length of the beam for any fixed loading conditions would be helpful to analyze the beam further.

Thus, a shear force diagram is a graphical plot, which depicts how the internal shear force „F' varies along the length of beam. If x dentotes the length of the beam, then F is function x i.e. F(x).

Similarly a bending moment diagram is a graphical plot which depicts how the internal bending moment „M' varies along the length of the beam. Again M is a function x i.e. M(x).

Basic Relationship Between The Rate of Loading, Shear Force and Bending Moment:

The construction of the shear force diagram and bending moment diagrams is greatly simplified if the relationship among load, shear force and bending moment is established.

Let us consider a simply supported beam AB carrying a uniformly distributed load w/length. Let us imagine to cut a short slice of length dx cut out from this loaded beam at distance „x' from the origin „0'.

Let us detach this portion of the beam and draw its free body diagram.

The forces acting on the free body diagram of the detached portion of this loaded beam are the following

• The shearing force F and F+ **d**F at the section x and x + **d**x respectively.

• The bending moment at the sections x and x + **d**x be M and M + dM respectively.

• Force due to external loading, if „w' is the mean rate of loading per unit length then the total loading on this slice of length **d**x is w. **d**x, which is approximately acting through the centre „c'. If the loading is assumed to be uniformly distributed then it would pass exactly through the centre „c'.

This small element must be in equilibrium under the action of these forces and couples.

Now let us take the moments at the point „c'. Such that

Conclusions: From the above relations,the following important conclusions may be drawn

• From Equation (1), the area of the shear force diagram between any two points, from the basic calculus is the bending moment diagram

• The slope of bending moment diagram is the shear force,thus

Thus, if F=0; the slope of the bending moment diagram is zero and the bending moment is therefore constant.'

• The maximum or minimum Bending moment occurs where

The slope of the shear force diagram is equal to the magnitude of the intensity of the distributed loading at any position along the beam. The –ve sign is as a consequence of our particular choice of sign conventions

Procedure for drawing shear force and bending moment diagram:

Preamble:

The advantage of plotting a variation of shear force F and bending moment M in a beam as a function of „x' measured from one end of the beam is that it becomes easier to determine the maximum absolute value of shear force and bending moment.

Further, the determination of value of M as a function of „x' becomes of paramount importance so as to determine the value of deflection of beam subjected to a given loading.

Construction of shear force and bending moment diagrams:

A shear force diagram can be constructed from the loading diagram of the beam. In order to draw this, first the reactions must be determined always. Then the vertical components of forces and reactions are successively summed from the left end of the beam to preserve the mathematical sign conventions adopted. The shear at a section is simply equal to the sum of all the vertical forces to the left of the section.

When the successive summation process is used, the shear force diagram should end up with the previously calculated shear (reaction at right end of the beam. No shear force acts through the beam just beyond the last vertical force or reaction. If the shear force diagram closes in this fashion, then it gives an important check on mathematical calculations.

The bending moment diagram is obtained by proceeding continuously along the length of beam from the left hand end and summing up the areas of shear force diagrams giving due regard to sign. The process of obtaining the moment diagram from the shear force diagram by summation is exactly the same as that for drawing shear force diagram from load diagram.

It may also be observed that a constant shear force produces a uniform change in the bending moment, resulting in straight line in the moment diagram. If no shear force exists along a certain portion of a beam, then it indicates that there is no change in moment takes place. It may also further observe that dm/dx= F therefore, from the fundamental theorem of calculus the maximum or minimum moment occurs where the shear is zero. In order to check the validity of the bending moment diagram, the terminal conditions for the moment must be satisfied. If the end is free or pinned, the computed sum must be equal to zero. If the end is built in, the moment computed by the summation must be equal to the one calculated initially for the reaction. These conditions must always be satisfied.

Cantilever beams - problems

Cantilever with a point load at the free end:

$$M_x = - w.x$$

W.K.T $\quad M = EI. \frac{d^2}{dx^2}$

$$EI. \frac{d^2y}{dx^2} = - w.x$$

on integrating we get

$$\frac{\quad}{dx} = \frac{-wx^2}{2} + c_1$$

Integrating again

$$EI.y = -\frac{wx^3}{6} + c_1x + c_2$$

Boundary conditions

i) when x = L , slope dy/dx = 0

ii) when x = L, deflection y = 0

Applying the first B.C to eqn (1)

$$0 = -\underline{wl^2} + c_1 \qquad c_1 = \underline{wl^2}$$

Applying the second B.C to eqn (2)

$$0 = \frac{-wl^3}{6} + c_1l + c_2$$

$$C_2 = \frac{-wl3}{3}$$

Sub c_1,c_2 values in slope eqn we get

$$EI.\frac{dy}{dx} = \frac{-wx^2}{2} + \frac{wl^2}{2}$$

Max. slope eqn can be obtained by x = 0

$$EI.\frac{dy}{dx} = 0 + \frac{wl^2}{2} \qquad ?_B = \frac{wl^2}{2EI}$$

Sub c_1,c_2 values in deflection eqn we get

$$EI.y = \frac{-wx^3}{2} + \frac{wl^2}{2}.x - \frac{wl^3}{6}$$

Max. deflection can be obtained by x = 0

$$EI.y_B = 0 - 0 - \frac{wl^3}{3} \qquad y_B = \frac{wl^3}{3EI}$$

Cantilever with a point load at a distance of ‘a’ from free end:

$$?_B = ?_c = \frac{w(l-a)^2}{2EI}$$

$$y_B = \frac{w(l-a)^3}{3EI} + \frac{w(l-a)^2.a}{2EI} \qquad y_c = \frac{w(l-a)^3}{3EI}$$

When the load acts at mid span:

$$y_B = \frac{5wl^3}{48EI}$$

Cantilever with UDL:

$$?_B = \frac{wl^3}{2EI} \qquad y_B = \frac{wl^4}{8EI}$$

Cantilever with UDL from fixed end:

$$?_B = ?_c = \frac{w(l-a)^3}{6EI}$$

$$y_B = \frac{w(l-a)^4}{8EI} + \frac{w(l-a)^3.a}{6EI} \qquad y_c = \frac{w(l-a)^4}{8EI}$$

When a = l/2 ie. UDL acting half of the length

$$y_B = \frac{7wl^3}{384EI}$$

Cantilever with UDL from free end:

$$?_B = \frac{wl^3}{6EI} - \frac{w(l-a)^3}{6EI}$$

$$y_B = \frac{wl^4}{8EI} - \frac{w(l-a)^4}{8EI} + \frac{w(l-a)^3}{6EI} \cdot a$$

Cantilever with UVL:

$?_B = \frac{wl^3}{24EI}$ $\qquad$ $y_B = \frac{wl^4}{30EI}$

A cantilever of length carries a concentrated load 'W' at its free end.

Draw shear force and bending moment.

Solution:

At a section a distance x from free end consider the forces to the left, then F = -W (for all values of x) -ve sign means the shear force to the left of the x-section are in downward direction and therefore negative

Taking moments about the section gives (obviously to the left of the section)

M = -Wx (-ve sign means that the moment on the left hand side of the portion is in the anticlockwise direction and is therefore taken as –ve according to the sign convention)

so that the maximum bending moment occurs at the fixed end i.e. M = -W l

Simplysupported beam -problems

Simply supported beam subjected to a central load (i.e. load acting at the mid-way)

By symmetry the reactions at the two supports would be W/2 and W/2. now consider any section X-X from the left end then, the beam is under the action of following forces.

.So the shear force at any X-section would be = W/2 [Which is constant upto $x < l/2$]

If we consider another section Y-Y which is beyond l/2 then

for all values greater = l/2

SSB with central point load:

$?_B = -wl^3$ $\qquad$ $y_B = wl^4$

$$\frac{}{16EI} \qquad \frac{}{30EI}$$

SSB with eccentric point load:

$$?_B = \frac{-wab}{6EIL}(b+2a) \qquad y_{max} = \frac{-wa}{9v3\ EIL}(b^2 + 2ab)^{3/2}$$

If a >b then

$$y_{max} = \frac{-wb}{9v3\ EIL}(a^2 + 2ab)^{3/2}$$

SSB with UDL:

$$?_B = \frac{wl^3}{24EI} \qquad y_B = \frac{5wl^4}{384EI}$$

Overhanging beams - problems

In the problem given below, the intensity of loading varies from q1 kN/m at one end to the q2 kN/m at the other end.This problem can be treated by considering a U.d.i of intensity q1 kN/m over the entire span and a uniformly varying load of 0 to (q2- q1)kN/m over the entire span and then super impose teh two loadings.

Point of Contraflexure:

Consider the loaded beam a shown below along with the shear force and Bending moment diagrams for It may be observed that this case, the bending moment diagram is completely positive so that the curvature of the beam varies along its length, but it is always concave upwards or sagging.However if we consider a again a loaded beam as shown below along with the S.F and B.M diagrams, then

It may be noticed that for the beam loaded as in this case,

The bending moment diagram is partly positive and partly negative.If we plot the deflected shape of the beam just below the bending moment

This diagram shows that L.H.S of the beam „sags' while the R.H.S of the beam „hogs'

The point C on the beam where the curvature changes from sagging to hogging is a point of contraflexure.

OR

It corresponds to a point where the bending moment changes the sign, hence in order to find the point of contraflexures obviously the B.M would change its sign when it cuts the X-axis therefore to get the points of contraflexure equate the bending moment equation equal to zero.The fibre stress is zero at such sections

Note: there can be more than one point of contraflexure

2.4Stresses in beams

Preamble:

When a beam having an arbitrary cross section is subjected to a transverse loads the beam will bend. In addition to bending the other effects such as twisting and buckling may occur, and to investigate a problem that includes all the combined effects of bending, twisting and buckling could become a complicated one. Thus we are interested to investigate the bending effects alone, in order to do so, we have to put certain constraints on the geometry of the beam and the manner of loading.

Assumptions:

The constraints put on the geometry would form the **assumptions:**

1. Beam is initially **straight** , and has a **constant cross-section.**

2. Beam is made of **homogeneous material** and the beam has a **longitudinal plane of symmetry.**

3. Resultant of the applied loads lies in the plane of symmetry.

4. The geometry of the overall member is such that bending not buckling is the primary cause of failure.

5. Elastic limit is nowhere exceeded and **'E'** is same in tension and compression.

6. Plane cross - sections remains plane before and after bending.

Let us consider a beam initially unstressed as shown in fig 1(a). Now the beam is subjected to a constant bending moment (i.e. „Zero Shearing Force') along its length as would be obtained by applying equal couples at each end. The beam will bend to the radius R as shown in Fig 1(b)

As a result of this bending, the top fibers of the beam will be subjected to tension and the bottom to compression it is reasonable to suppose, therefore, **that some here between the two there are points at which the stress is zero. The locus of all such points is known as neutral axis** . The radius of curvature R is then measured to this axis. For symmetrical sections the N. A. is the axis of symmetry but what ever the section N. A. will always pass through the centre of the area or centroid.

As we are aware of the fact internal reactions developed on any cross-section of a beam may consists of a resultant normal force, a resultant shear force and a resultant couple. In order to ensure that the bending effects alone are investigated, we shall put a constraint on the loading such that the resultant normal and the resultant shear forces are zero on any cross-section perpendicular to the longitudinal axis of the member, That means F = 0 since or M = constant.

Thus, the zero shear force means that the bending moment is constant or the bending is same at every cross-section of the beam. Such a situation may be visualized or envisaged when the beam

or some portion of the beam, as been loaded only by pure couples at its ends. It must be recalled that the couples are assumed to be loaded in the plane of symmetry.

When a member is loaded in such a fashion it is said to be in **pure bending.** The examples of pure bending have been indicated in EX 1and EX 2 as shown below :

When a beam is subjected to pure bending are loaded by the couples at the ends, certain cross-section gets deformed and we shall have to make out the conclusion that,

1. Plane sections originally perpendicular to longitudinal axis of the beam remain plane and perpendicular to the longitudinal axis even after bending , i.e. the cross-section A'E', B'F' (refer Fig 1(a)) do not get warped or curved.

2. In the deformed section, the planes of this cross-section have a common intersection i.e. any time originally parallel to the longitudinal axis of the beam becomes an arc of circle.

We know that when a beam is under bending the fibres at the top will be lengthened while at the bottom will be shortened provided the bending moment M acts at the ends. In between these there are some fibres which remain unchanged in length that is they are not strained, that is they do not carry any stress. The plane containing such fibres is called neutral surface.

The line of intersection between the neutral surface and the transverse exploratory section is called the neutral axisNeutral axis **(N A)** .

Bending Stresses in Beams or Derivation of Elastic Flexural formula :

In order to compute the value of bending stresses developed in a loaded beam, let us consider the two cross-sections of a beam **HE** and **GF** , originally parallel as shown in fig 1(a).when the beam

is to bend it is assumed that these sections remain parallel i.e. **H'E'** and **G'F'** , the final position of the sections, are still straight lines, they then subtend some angle q.

Consider now fiber AB in the material, at adistance y from the N.A, when the beam bends this will stretch to A'B'

Since CD and C'D' are on the neutral axis and it is assumed that the Stress on the neutral axis zero. Therefore, there won't be any strain on the neutral axis

Consider any arbitrary a cross-section of beam, as shown above now the strain on a fibre at a distance „y' from the N.A, is given by the expression

Now the termis the property of the material and is called as a second moment of area of the cross-section and is denoted by a symbol I.

Therefore M/I = sigma/y = E/R

This equation is known as the Bending Theory Equation. The above proof has involved the assumption of pure bending without any shear force being present. Therefore this termed as the pure bending equation. This equation gives distribution of stresses which are normal to cross-section i.e. in x-direction.

Stress variation along the length and in the beam section

Bending Stress and Deflection Equation

In this section, we consider the case of pure bending; i.e., where only bending stresses exist as a result of applied bending moments. To develop the theory, we will take the phenomenological approach to develop what is called the "**Euler-Bernoulli theory of beam bending**." Geometry: Consider a long slender straight beam of length L and cross-sectional area A. We assume the beam is prismatic or nearly so. The length dimension is large compared to the dimensions of the cross-section. While the cross-section may be any shape, we will assume that it is symmetric about the y axis

Loading: For our purposes, we will consider shear forces or distributed loads that are applied in the y direction only (on the surface of the beam) and moments about the z-axis. We have consider examples of such loading in ENGR 211 previously and some examples are shown below:

Kinematic Observations: In order to obtain a "feel" for the kinematics (deformation) of a beam subjected to pure bending loads, it is informative to conduct an experiment. Consider a rectangular lines have been scribed on the beam's surface, which are parallel to the top and bottom surfaces (and thus parallel to a centroidally placed x-axis along the length of the beam). Lines are also scribed around the circumference of the beam so that they are perpendicular to the longitudinals (these circumferential lines form flat planes as shown). The longitudinal and circumferential lines form a square grid on the surface. The beam is now bent by moments at each end as shown in the lower photograph. After loading, we note that the top line has stretched and the bottom line has shortened (implies that there is strain *exx*). If measured carefully, we see that the longitudinal line at the center has not changed length (implies that *exx* = 0 at *y* = 0). The longitudinal lines now appear to form concentric circular lines.

We also note that the vertical lines originally perpendicular to the longitudinal lines remain straight

and perpendicular to the longitudinal lines. If measured carefully, we will see that the vertical lines remain approximately the same length (implies *eyy* = 0). Each of the vertical lines (as well as the planes they form) has rotated and, if extended downward, they will pass through a common point that forms the center of the concentric longitudinal lines (with some radius *?*). The flat planes originally normal to the longitudinal axis remain essentially flat planes and remain normal to the deformed longitudinal lines. The squares on the surface are now quadrilaterals and each appears to have tension (or compression) stress in the longitudinal direction (since the horizontal lines of a square have changed length). However, in pure bending we make the assumption that. If the *x*-axis is along the length of beam and the *y*-axis is normal to the beam, this suggests that we have an axial normal stress *sxx* that is tension above the *x*-axis and compression below the *y*-axis. The remaining normal stresses *syy* and *szz* will generally be negligible for pure bending about the *z*-axis. For pure bending, all shear stresses are assumed to be zero. Consequently, for pure bending, the stress matrix reduces to zero

2.5 Effect of shape of beam section on stress induced

CIRCULAR SECTION :

For a circular x-section, the polar moment of inertia may be computed in the following manner

Consider any circular strip of thickness dr located at a radius 'r'.

Than the area of the circular strip would be dA = 2pr. dr

Thus

Parallel Axis Theorem:

The moment of inertia about any axis is equal to the moment of inertia about a parallel axis through the centroid plus the area times the square of the distance between the axes.

If „ZZ' is any axis in the plane of cross-section and „XX' is a parallel axis through the centroid G, of the cross-section, then

Rectangular Section:

For a rectangular x-section of the beam, the second moment of area may be computed as below :

Consider the rectangular beam cross-section as shown above and an element of area **dA** , thickness **dy** , breadth **B** located at a distance **y** from the neutral axis, which by symmetry passes through the centre of section. The second moment of area **I** as defined earlier would be

Thus, for the rectangular section the second moment of area about the neutral axis i.e., an axis through the centre is given by

Similarly, the second moment of area of the rectangular section about an axis through the lower edge of the section would be found using the same procedure but with integral limits of **0** to **D** .

Therefore

These standards formulas prove very convenient in the determination of INA for build up sections which can be conveniently divided into rectangles. For instance if we just want to find out the Moment of Inertia of an I - section, then we can use the above relation.

Let us consider few examples to determaine the sheer stress distribution in a given X-sections

Rectangular x-section:

Consider a rectangular x-section of dimension b and d

A is the area of the x-section cut off by a line parallel to the neutral axis. is the distance of the centroid of A from the neutral axis

This shows that there is a parabolic distribution of shear stress with y.

The maximum value of shear stress would obviously beat the location y = 0.

Therefore the shear stress distribution is shown as below.

It may be noted that the shear stress is distributed parabolically over a rectangular cross-section, it is maximum at y = 0 and is zero at the extreme ends.

I - section :

Consider an I - section of the dimension shown below.

The shear stress distribution for any arbitrary shape is given as

Let us evaluate the quantity, thequantity for this case comprise the contribution due to flange area and web area

Flange area

Web Area

To get the maximum and minimum values of t substitute in the above relation.

y = 0 at N. A. And y = d/2 at the tip.

The maximum shear stress is at the neutral axis. i.e. for the condition y = 0 at N. A.

Hence, ...(2)

The minimum stress occur at the top of the web, the term bd 2 goes off and shear stress is given by the following expression

............(3)

The distribution of shear stress may be drawn as below, which clearly indicates a parabolic distribution

Note: from the above distribution we can see that the shear stress at the flanges is not zero, but it has some value, this can be analyzed from equation (1). At the flange tip or flange or web interface y = d/2.Obviously than this will have some constant value and than onwards this will have parabolic distribution.

In practice it is usually found that most of shearing stress usually about 95% is carried by the web, and hence the shear stress in the flange is neglible however if we have the concrete analysis i.e. if we analyze the shearing stress in the flange i.e. writing down the expression for shear stress for flange and web separately, we will have this type of variation.

This distribution is known as the "top – hat" distribution. Clearly the web bears the most of the shear stress and bending theory we can say that the flange will bear most of the bending stress.

Shear stress distribution in beams of circular cross-section:

Let us find the shear stress distribution in beams of circular cross-section. In a beam of circular cross-section, the value of Z width depends on y.

Using the expression for the determination of shear stresses for any arbitrary shape or a arbitrary section.

Where òy dA is the area moment of the shaded portion or the first moment of area.

Here in this case „dA' is to be found out using the Pythagoras theorem

The distribution of shear stresses is shown below, which indicates a parabolic distribution

Principal Stresses in Beams

It becomes clear that the bending stress in beam sx is not a principal stress, since at any distance y from the neutral axis; there is a shear stress t (or txy we are assuming a plane stress situation)

In general the state of stress at a distance y from the neutral axis will be as follows.

At some point „P' in the beam, the value of bending stresses is given as

After substituting the appropriate values in the above expression we may get the inclination of the principal planes.

Illustrative examples: Let us study some illustrative examples, pertaining to determination of principal stresses in a beam

1. Find the principal stress at a point A in a uniform rectangular beam 200 mm deep and 100 mm wide, simply supported at each end over a span of 3 m and carrying a uniformly distributed load of 15,000 N/m.

Solution: The reaction can be determined by symmetry

R1 = R2 = 22,500 N

consider any cross-section X-X located at a distance x from the left end.

Hence,

S. F at XX =22,500 – 15,000 x

B.M at XX = 22,500 x – 15,000 x (x/2) = 22,500 x – 15,000 . x2 / 2

Therefore,

S. F at X = 1 m = 7,500 N

B. M at X = 1 m = 15,000 N

Now substituting these values in the principal stress equation,

We get s1 = 11.27 MN/m2

s2 = - 0.025 MN/m2

Bending Of Composite or Flitched Beams

A composite beam is defined as the one which is constructed from a combination of materials. If such a beam is formed by rigidly bolting together two timber joists and a reinforcing steel plate, then it is termed as a flitched beam.

The bending theory is valid when a constant value of Young's modulus applies across a section it cannot be used directly to solve the composite-beam problems where two different materials, and therefore different values of E, exists. The method of solution in such a case is to replace one of the materials by an equivalent section of the other.

Consider, a beam as shown in figure in which a steel plate is held centrally in an appropriate recess/pocket between two blocks of wood .Here it is convenient to replace the steel by an equivalent area of wood, retaining the same bending strength. i.e. the moment at any section must be the same in the equivalent section as in the original section so that the force at any given dy in the equivalent beam must be equal to that at the strip it replaces.

Hence to replace a steel strip by an equivalent wooden strip the thickness must be multiplied by the modular ratio E/E'.

The equivalent section is then one of the same materials throughout and the simple bending theory applies. The stress in the wooden part of the original beam is found directly and that in the steel found from the value at the same point in the equivalent material as follows by utilizing the given relations.

Stress in steel = modular ratio x stress in equivalent wood

The above procedure of course is not limited to the two materials treated above but applies well for any material combination. The wood and steel flitched beam was nearly chosen as a just for the sake of convenience.

Assumption

In order to analyze the behavior of composite beams, we first make the assumption that the materials are bonded rigidly together so that there can be no relative axial movement between them. This means that all the assumptions, which were valid for homogenous beams are valid except the one assumption that is no longer valid is that the Young's Modulus is the same throughout the beam.

The composite beams need not be made up of horizontal layers of materials as in the earlier example. For instance, a beam might have stiffening plates as shown in the figure below.

Again, the equivalent beam of the main beam material can be formed by scaling the breadth of the plate material in proportion to modular ratio. Bearing in mind that the strain at any level is same in both materials, the bending stresses in them are in proportion to the Young's modulus.

Shear stresses in beams

When a beam is subjected to non uniform bending, both bending moments, M, and shear forces, V, act on the cross section. The normal stresses, sx, associated with the bending moments are obtained from the flexure formula. We will now consider the distribution of shear stresses, t, associated with the shear force, V. Let us begin by examining a beam of rectangular cross section. We can reasonably assume that the shear stresses t act parallel to the shear force V. Let us also assume that the distribution of shear stresses is uniform across the width of the beam.

Shear flow

One thing we might ask ourselves now is: Where does maximum shear stress occur? Well, it can be

shown that this always occurs in the center of gravity of the cross-section. So if you want to calculate the maximum shear stress, make a cut through the center of gravity of the cross-section.

CE8395 – STRENGTH OF MATERIALS FOR MECHANICAL ENGINEERS

PREVIOUS YEAR ANNA UNIVERSITY – TWO MARK QUESTION & ANSWERS

UNIT - I - STRESS, STRAIN AND DEFORMATION OF SOLIDS

1. What you mean by thermal stress? (AM-2019, 2015)

When a material is free to expand or contract due to change in temperature, no stress and strain will be developed in the material. But when the material is rigidly fixed at both the ends, the change in length is prevented. Due to change in temperature, stress will be developed in the material. Such stress is known as thermal stress.

2. Define principle stresses and principle plane. (AM 2019)

Principle stress: The magnitude of normal stress, acting on a principal plane is known as principal stresses. **Principle plane:** The planes which have no shear stress are known as principal planes.

3. Define modulus of elasticity. (ND-2016)

The ratio of tensile stress or compressive stress to the corresponding strain is known as modulus of elasticity or young's modulus and is denoted by E.

4. Define Poisons Ratio. (ND-2018)

When a body is stressed within its elastic limit, the ratio of lateral strain to the longitudinal strain is constant for a given material.

5. Give the relationship between Bulk Modulus and Young's Modulus. (MJ 2016)

$E = 3K (1-2\mu)$ V

Where, **E -** Young's Modulus

K - Bulk Modulus **μ-** Poisson's ratio

6. Define Elasticity. (ND- 2015)

Elasticity is the tendency of solid materials to return to their original shape after being deformed. Solid objects will deform when forces are applied on them. If the material is elastic, the object will return to its initial shape and size when these forces are removed.

7. What is principle of super position? (ND 2015)

The resultant deformation of the body is equal to the algebric sum of the deformation of the individual section. Such principle is called as principle of super position.

8. State the relationship between Young's Modulus and Modulus of Rigidity. (ND-2014)

$E = 2G (1+\mu)$ Where, **E -** Young's Modulus **G** – Modulus of rigidity **μ -** Poisson's ratio

9. What is Bulk Modulus of material? (MJ 2014)

The Ratio of direct stress to the corresponding volumetric strain is known as Bulk Modulus.

10. Define Resilience and Proof Resilience. **(MJ-2014)**

The strain energy stored by the body within elastic limit, when loaded externally is called resilience.

The maximum strain energy stored in a body up to elastic limit is known as proof resilience.

11. Define shear strain and Volumetric strain. **(ND-2013)**

The two equal and opposite forces act tangentially on any cross sectional plane of a body tending to slide one part of the body over the other part. The stress induced in that section is called shear stress and the corresponding strain is known as shear strain.

Volumetric strain is defined as the ratio of change in volume to the original volume of the body

12. What is meant by strain energy? **(ND-2013)**

When an elastic material is deformed due to application of external force, internal resistance is developed in the material of the body. Due to deformation, some work is done by the internal resistance developed in the body, which is stored in the form of energy. This energy is known as strain energy. It is expressed in N-m.

13. Define Hooke's law. **(MJ -2013)**

It states that when a material is loaded, within its elastic limit, the stress is directly proportional to the strain.

Stress α strain

E = o / e Unit is N/mm^2

Where, E – Young's Modulus, o –Stress,

e - Strain.

14. Define the term modulus of Resilence. **(MJ-2013)**

It is the Proof resilience of the material per unit volume.

$$\text{Modulus of resilience} = \frac{\text{Proof resilience}}{\text{Volume of the body}}$$

15. Give the relation for change in length of a bar hanging freely under its own weight. (AM - 05)

Change in length, $\partial L = PL/AE$

Where, P – Axial load.

L – Length of the bar.

E – Young's Modulus of the bar.

A – Area of the bar.

.

PRE-YEAR UNIV – PART-B QUESTION

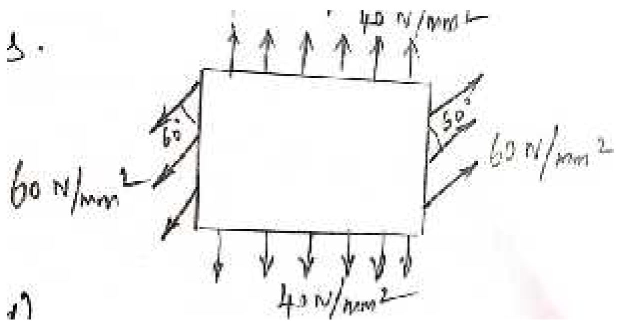
UNIT-I STRESS STRAIN AND DEFORMATION OF SOLIDS

1. (i) A compound tube consist of a steel tube 140mm internal diameter and 160mm external diameter and an outer brass tube 160mm internal diameter and 180mm external diameter. The two tubes are of same length .The compound tube carries an axial compression load of 900KN. Find the stresses and the load carried by each tube and the amount of its shortens. Take E for steel as 2×10^5 N/mm^2 and for a brass $1\text{x}10^5$ N/mm^2.

(ii) Two members are connected to carry a tensile force of 80 KN by a lap joint with two number of 20mm diameter bolt. Fine the shear induced in the bolt.

(ND-2016)

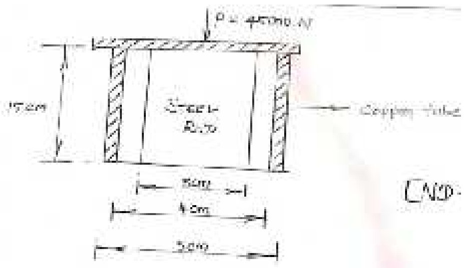
2. **(i)** A point in a strained materials is subjected to the stress as shown in figure . Locate the principle plane and find the principle stresses.



(ii) A steel rod of 20mm diameter passes centrally through a copper tube of 50mm external diameter and 40mm internal diameter. The tube is closed at each end by rigid plates of negligible thickness. The nuts are tightened lightly on the projecting parts of the rod. If the temperature of the assembly is raised by 50˚C, calculate the stress developed in copper and steel. Take E for steel as 2×10^5N/mm^2 and copper as 1×10^5 N/mm^2 and α for steel and copper as 12 x 10^{-6} per ˚C and 18 x 10^{-6} per ˚C. **(ND-2016)**

3. A metallic bar 300mm×100mm×40mm is subjected to a force of 50KN (tensile), 6KN (Tensile) and 4kN (tensile) along X,Y and Z direction respectively. Determine the change in the volume of the block. Take $E=2\times10^5$ N/mm^2 and poisson ratio =0.25. **(ND-2015, 2014)**

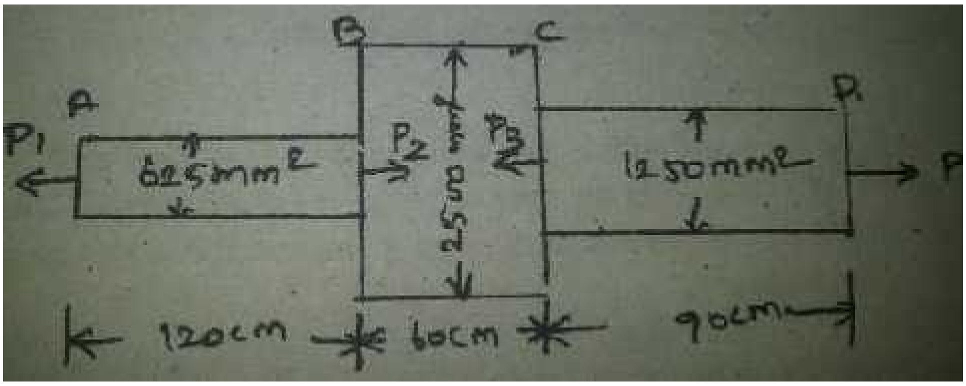
4. A steel rod of 3cm diameter is enclosed centrally in a hollow copper tube of external diameter 5cm and internal diameter of 4cm as shown in fig. The composite bar is then subjected to axial pull of 45000N. (i) The stresses in the rod and tube (ii) Load carried by each bar. Take E for steel $=2.1\times10^5$ N/mm^2 and for copper $=1.1\text{x}10^5$ N/mm^2.



(ND-2015)

5. (i) Derive an expression for change in length of a circular bar with uniformly varying diameter and subjected to an axial tensile load 'P'

(ii) A member ABCD is subjected to point loads P1, P2, P3, P4 as shown in fig. calculate the force P_2, necessary for equilibrium, if P_1 = 45 KN, P_3 = 450 KN and P_4 = 130 KN. Determine the total elongation of the member, assuming the modulus of elasticity to be E= 2.1×10^5 N/mm^2.

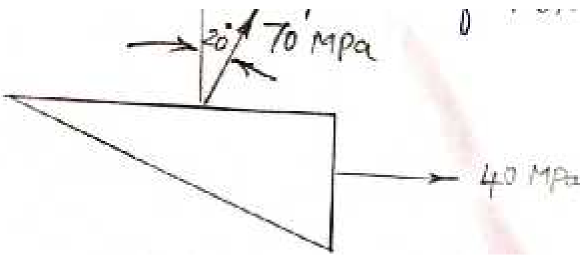


(ND-2014)

6. (i) A bar of 30 mm diameter is subjected to a pull of 60 kN. The measured extension on gauge length of 200 mm is 0.09 mm and the change in diameter is 0.0039 mm. calculate the Poisson's ratio and the values of the three moduli.

(ii) A rectangular block 350mm long, 100mm wide and 80mm thick is subjected to axial load as follows. 50kN tensile in the direction of length. 100KN compression in the direction of thickness and 60KN tensile in the direction of breadth. Determine the change in volume, bulk modulus, modulus of rigidity. Take, E=2×10^5 N/mm^2 and Poission ratio =0.25. **(MJ-2014)**

7. (i) A resultant tensile stress of 70Mpa is action over as shown in figure. Another direct tensile of 40Mpa is acting over plane, which is at right angle to the previous one. Find the resultant stresses in the second plane, the principal planes and stresses and the plan maximum shear intenisity.



(ii) Determine the strain energy due to self weight of a bar of uniform C.S. ‘a’ having length ‘L’ which is hanging vertically down.
(MJ-2014)

8. A reinforced concrete column 500mm×500mm in section is reinforced with 4 steel bars 25mm diameter: one in each corner, the column is carring a load of 1000KN. find the stresses in the concrete and steel bars. Take E steel = 2×10^5 N/mm^2 and E For Concrete $=14\times10^3$ N/mm^2. **(MJ-2013)**

UNIT –II – TRANSVERSE LOADING ON BEAMS AND STRESSES IN BEAM

1. Define point of contra flexure. (AM -2019, ND-2018, AM-2013)

It is a point where the bending moment changes its sign from +ve to –ve or –ve to +ve. At that point bending moment is Zero.

2. What are the assumptions made in the theory of bending? (ND-2018, ND-2015, MJ-2014)

i) The material is perfectly homogeneous and isotropic. It obeys hook's law.

ii) The value of young's modulus is the sane in tension as well as in compression.

iii) The radius of curvature of the beam is very large compared to the cross section dimensions of the beam.

iv) The resultant force on a transverse section of the beam is zero.

3. Define shear force and bending moment at a section? (AM-2015)

Shear force: SF at any cross section is the algebraic sum of all the forces acting either sides of a beam. Bending moment: BM at a cross section is the algebraic sum of the moment of all the forces which are placed either side from that point.

4. Mention the different types of beams. (ND-2015)

i. Cantilever beam,
ii. Simply supported beam, iii. Fixed beam,
iv. Continuous beam and v. Over hanging beam

5. Define shear stress distribution. (MJ - 2013)

` The variation of shear stress along the depth of beam is called shear stress distribution.

6. State the assumptions while deriving the general formula for shear stresses. (MJ- 2011)

i. The material is homogenous, isotropic and elastic.

ii. The modulus of elasticity in tension and compression are same and the shear stress is constant along the beam width.

7. Define bending moment in beam. (ND- 2012)

The bending moment of the beam may be defined as the algebraic sum of the moments of the forces, to the right or left of the section.

8. What is a flitched beam? (ND-2014)

A beam which is constructed by two different materials is called flitched beam or composite beam.

9. What is the value of bending moment corresponding to a point having a zero shear force? (AM-2010)

The value of bending moment is maximum when the shear force changes its sign or zero. In a beam, that point is considered as maximum bending moment.

10. What is meant by Neutral axis of the beam? **(ND- 2012)**

It is an imaginary plane, which divides the section of the beam into the tension and compression zones on the opposite sides of the plane.

11. What is mean by compressive and tensile force? **(ND-2011)**

The forces in the member will be compressive if the member pushes the joint to which it is connected whereas the force in the member will be tensile if the member pulls the joint to which it is connected.

12. What are the benefits of method of sections compared with other methods? **(AM- 2011)**

1. This method is very quick

2. When the forces in few members of the truss are to be determined, then the method of section is mostly used.

13. Define thin cylinder? **(ND-2010)**

If the thickness of the wall of the cylinder vessel is less than 1/15 to 1/20 of its internal diameter, the cylinder vessel is known as thin cylinder.

14. What are types of stress in a thin cylindrical vessel subjected to internal pressure? **(AM-2010)**

These stresses are tensile and are known as

Circumferential stress (or hoop stress)

Longitudinal stress

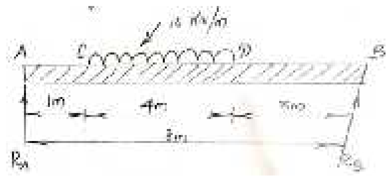
15. What are maximum shear stresses at any point in a cylinder? **(AM- 2010)**

Maximum shear stresses at any point in a cylinder, subjected to internal fluid pressure is given by $(f_1 - f_2) / 2 = pd / 8t$

UNIT-II TRANSVERSE LOADING ON BEAMS AND STRESSES IN BEAM

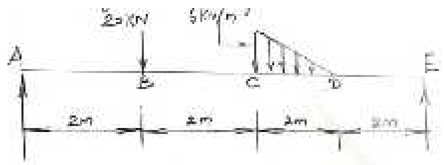
1. (i) A simply supported beam AB of length 5m carries point loads of 8KN, 10KN and 15KN at 1.5m and 2.5m and 4m respectively from left hand support. Draw shear force diagram and bending moment diagram
(ii) A cantilever beam AB of length 2m carries a uniformly distributed load of 12KN/m overentire length. Find the shear stress and bending stress. If the size of the beam is 230mm×300mm. **(ND-2016)**

2. Draw the shear force and B.M diagrams for a simply supported beam of length 8m and carrying a uniformly distributed load of 10KN/m for a distance of 4m as shown in fig. **(ND-2015)**



3. A steel plate of width 120mm and of thickness 20mm is bent into a circular arc of radius 10m. Determine the maximum stress induced and the bending moment which will produce the maximum stress, Take $E=2x10^5$ N/mm^2 **(ND-2015)**

4. Draw SFD and BMD and find the maximum bending moment for the beam given in Fig. **(ND-2014)**



5. Prove that the ratio of depth to width of the strongest beam that can be cut from a circular log of diameter'd' is 1.414. Hence calculate the depth and width of the strongest beam that can be cut out of a cylindrical log of wood whose diameter is 300mm. **(ND-2014)**

6. (i) Draw the shear force and bending moment diagram for a cantilever carrying load whose intensity varies uniformly from zero at the fixed end to 'W' per unit run at the free end.
(ii) Draw the S.F.D and B.M.D For the SSB as shown in fig.

(MJ-2014)

7. A T-section of a SSB has the width of flange =100mm, overall depth =100mm, thickness of flange and stem= 20mm determine the maximum stress in beam when a bending moment of 12KN-m is acting on the section. Also calculate the shear stress at neutral axis and at the junction of web and flange when shear force of 50KN acting on beam. **(MJ-2014)**

8. A simply supported beam of span 6m is carrying a UDL of 2KN/m over the entire span. Calculate the magnitude of shear force and bending moment at every section, 2m from the left support. Also draw shear force and bending moment diagrams. **(MJ-2013)**

Unit - 2

Transverse loading on Beams & Stresses in B

Shear force & Bending Moment Diagrams:-

* A shear force diagram is one which sh the variation of the shear force along the le of the beam

* Bending moment diagram is one which s the variation of the bending moment along the length of the beam.

Types of Beams:-

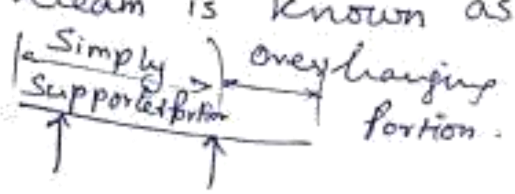
The following are the important types of b

1. Cantilever beam.

2. Simply supported beam

3. Overhanging beam

If the end portion of a beam is extended beyond support, such beam is known as over hanging beam.



4. Fixed beams:-

A beam whose both ends are fixed or built in walls, is known as fixed beam.

5. Continuous beams:-

A beam which is provided more than two supports.

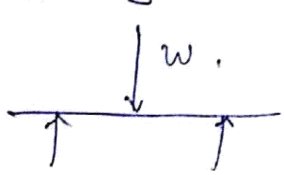
Types of loads:-

A beam is normally horizontal & the loads acting on beams are generally vertical.
The following are the important types of load acting on a beam.

1. Concentrated or Point Load.
2. Uniformly distributed load and
3. Uniformly varying load.

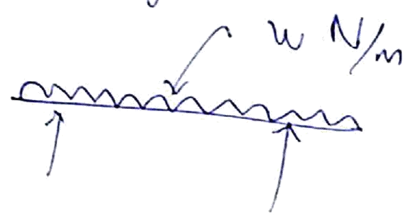
Concentrated or Point load:-

A conc. load is one which is considered to act at a point,



Uniformly distributed load:-

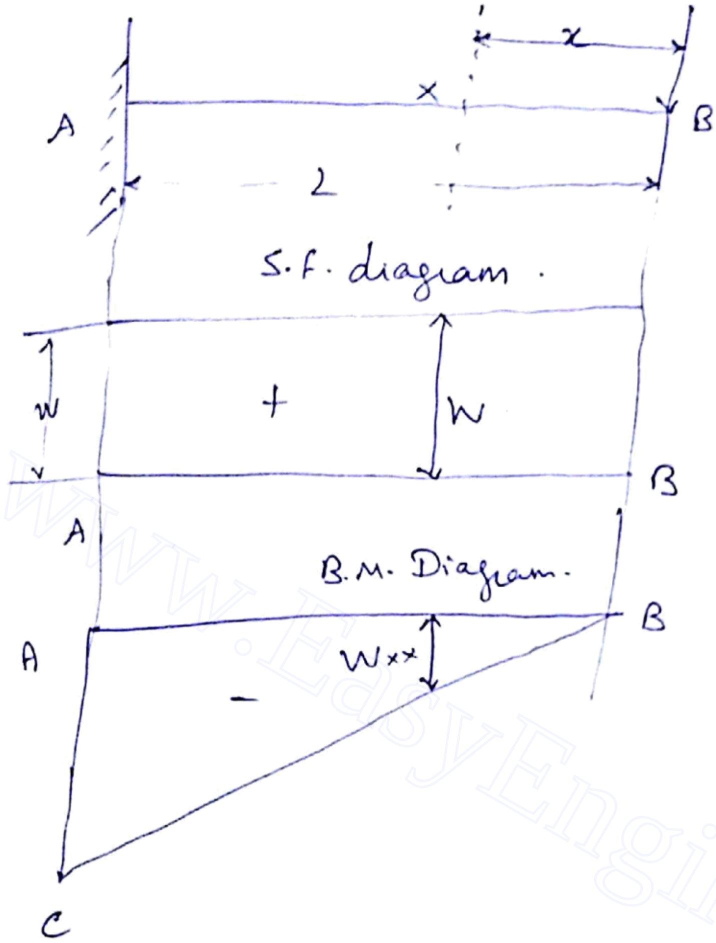
* A uniformly distributed load is one which is spread over a beam in such a manner that rate of loading w is uniform along the length.
* The rate of loading is expressed as w N/m run.
* It's represented as UDL.
* For solving the numerical problems, the total UDL is converted into a pt. load acting at the centre of uniformly distributed load.



Uniformly Varying load:-

* A uniformly varying load is one which is spread over a beam in such a manner that rate of loading varies from pt. to pt along the beam, in which load is zero at one end & increases uniformly to the other end. Such load is known as triangular load.

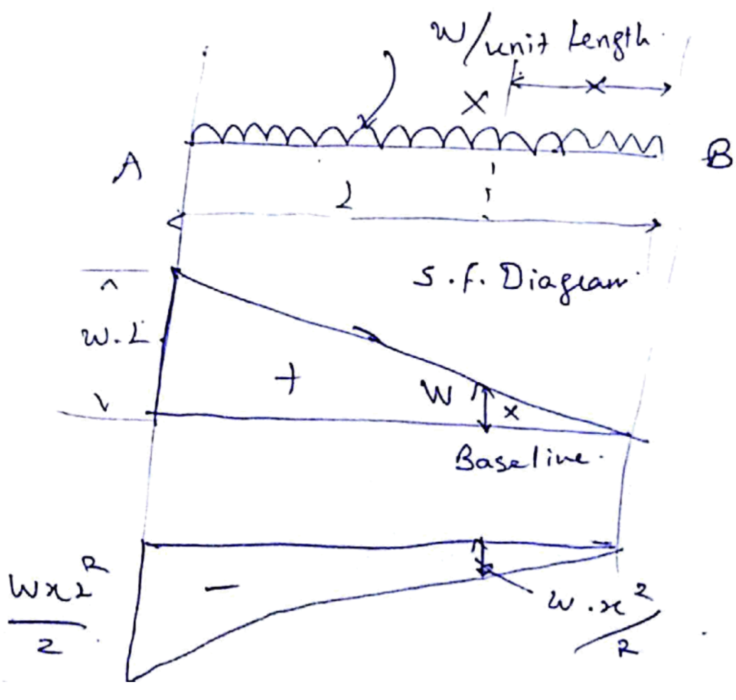
Shear force & Bending Moment diagrams for a cantilever with a point load at the free end :-



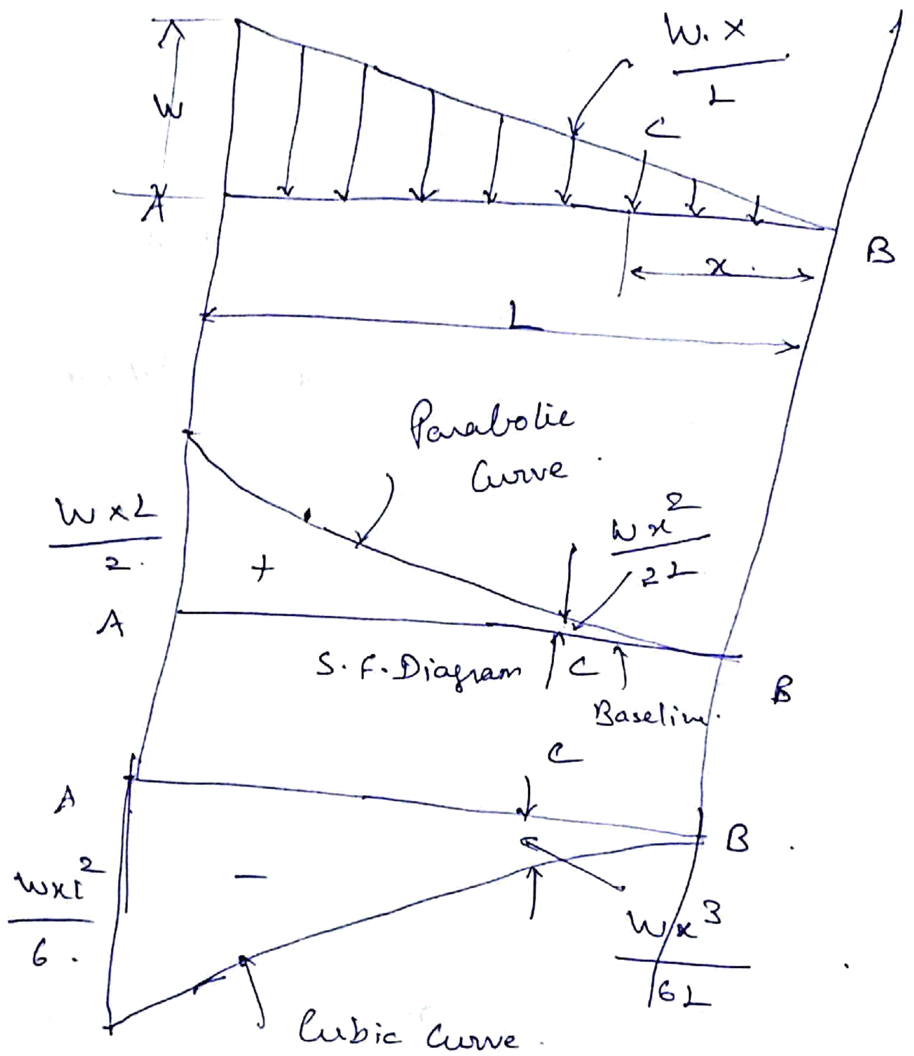
$F_x = +W.$

$M_x = -W x.$

Shear force & Bending Moment Diagrams for a Cantilever with a Uniformly distributed beam:-



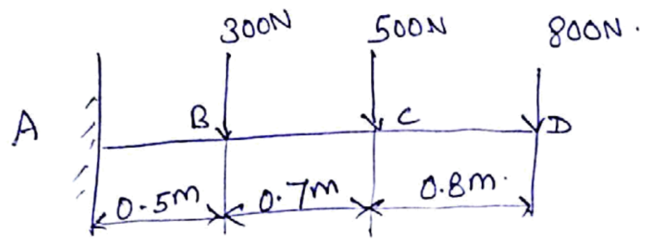
Shear force & Bending Moment diagrams for a Cantilever carrying a gradually varying load.



B.M. diagram.

x

Pb.① A cantilever beam of length 2 m carries the point loads as shown in figure. Draw the shear force & B.M. diagrams for the cantilever beam.



Shear force calulation:-

S.F at D, $F_D = 800\text{ N}$.

S.F at C, $F_C = 800 + 500 = 1300\text{N}$.

S.F. at B, $F_B = 800 + 500 + 300 = 1600\text{N}$.

S.F. at A, $F_A = 1600\text{ N}$.

Bending Moment Diagram:-

The bending moment at D is Zero.

(i) The bending moment at any section b/w C&D at a distance x & D is given by

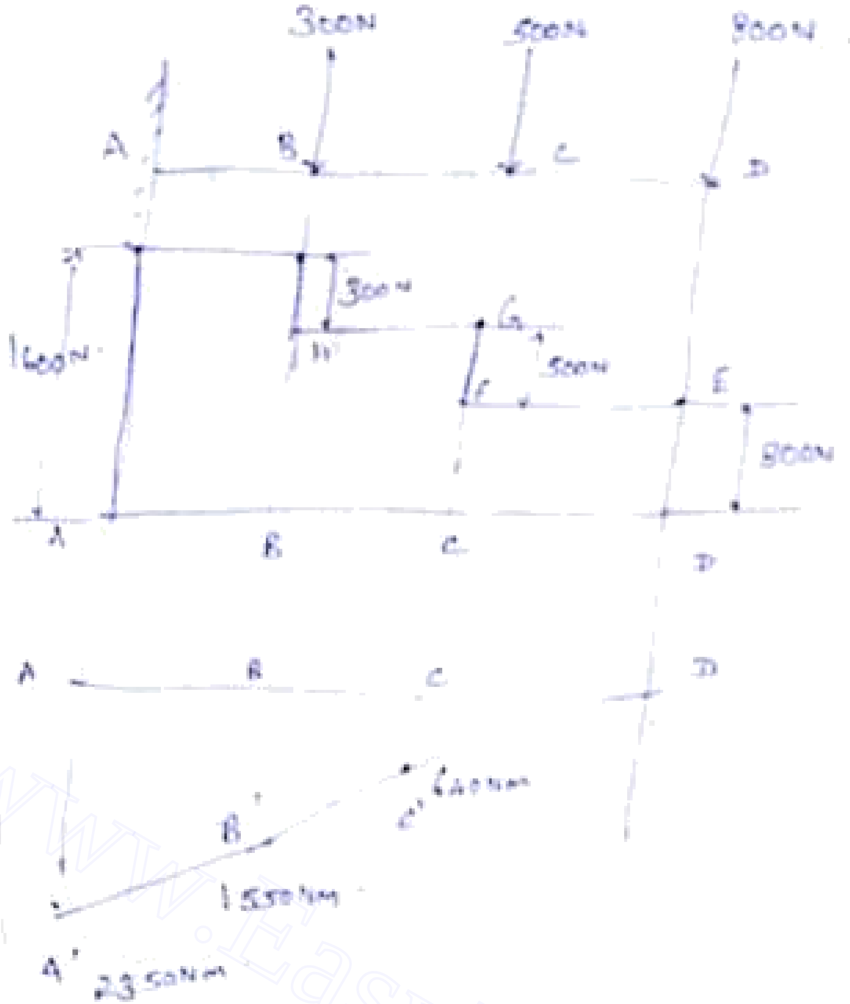
$$M_x = -800 \times x$$

B.M at C, $M_C = -800 \times 0.8 = -640\text{Nm}$

B.M. at B. $M_B = -800 \times 1.5 - 500\,(0.7) = -1550\text{Nm}$.

B.M at A. $M_A = -800 \times 2 - 500\,(1.2) - 300\,(0.5)$

$= -2350\text{Nm}$.



Pb 2 A cantilever of length 2m carries a UDL of 2kN/m length over the whole length & a pt load of 3kN & the free end. Draw the S.F & B.M diagrams for the Cantilever.

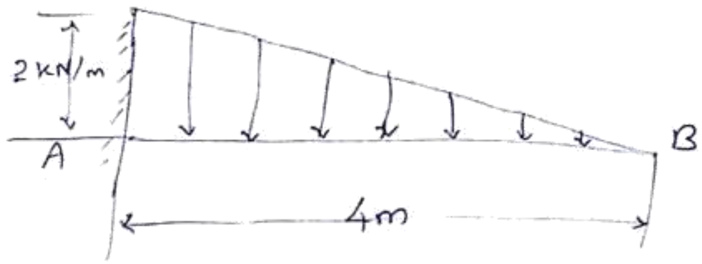
Shear force at B = 3kN.

$$F_x = 3.0 + w \cdot x$$

$$= 3.0 + 2 \times x$$

Pb. 3

A cantilever beam of length 4m carries UVL, zero at the free end to 2kN/m at the fixed end. Draw the S.F & B.M. diagrams.

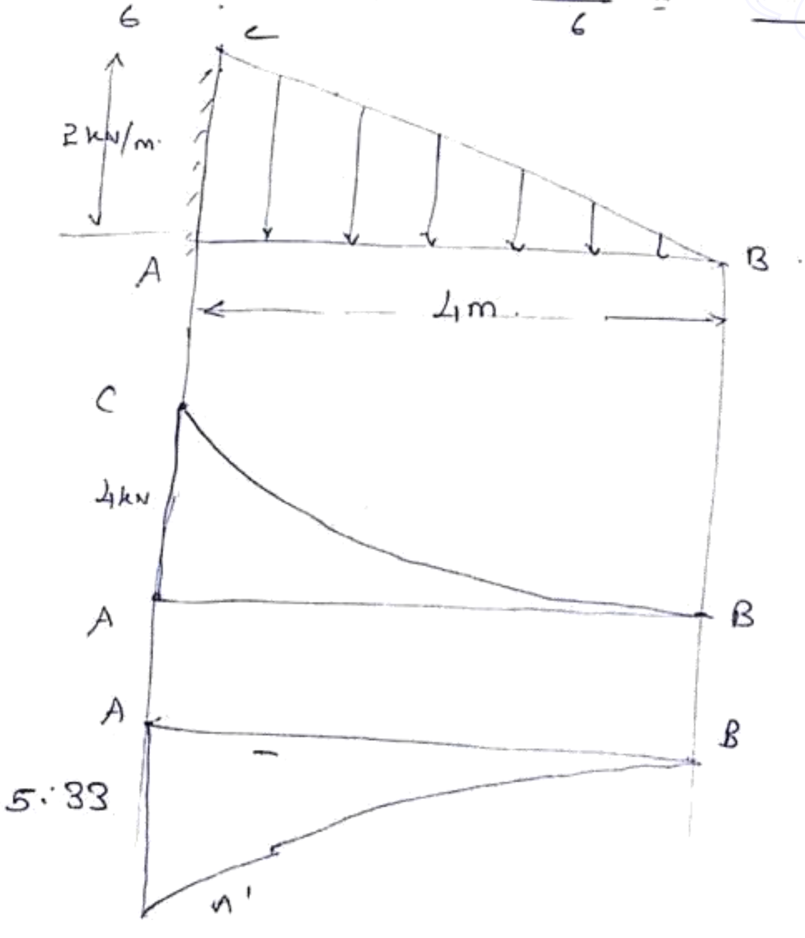


Shear force is zero at B. The shear force at C will be equal to the area of load diagram.

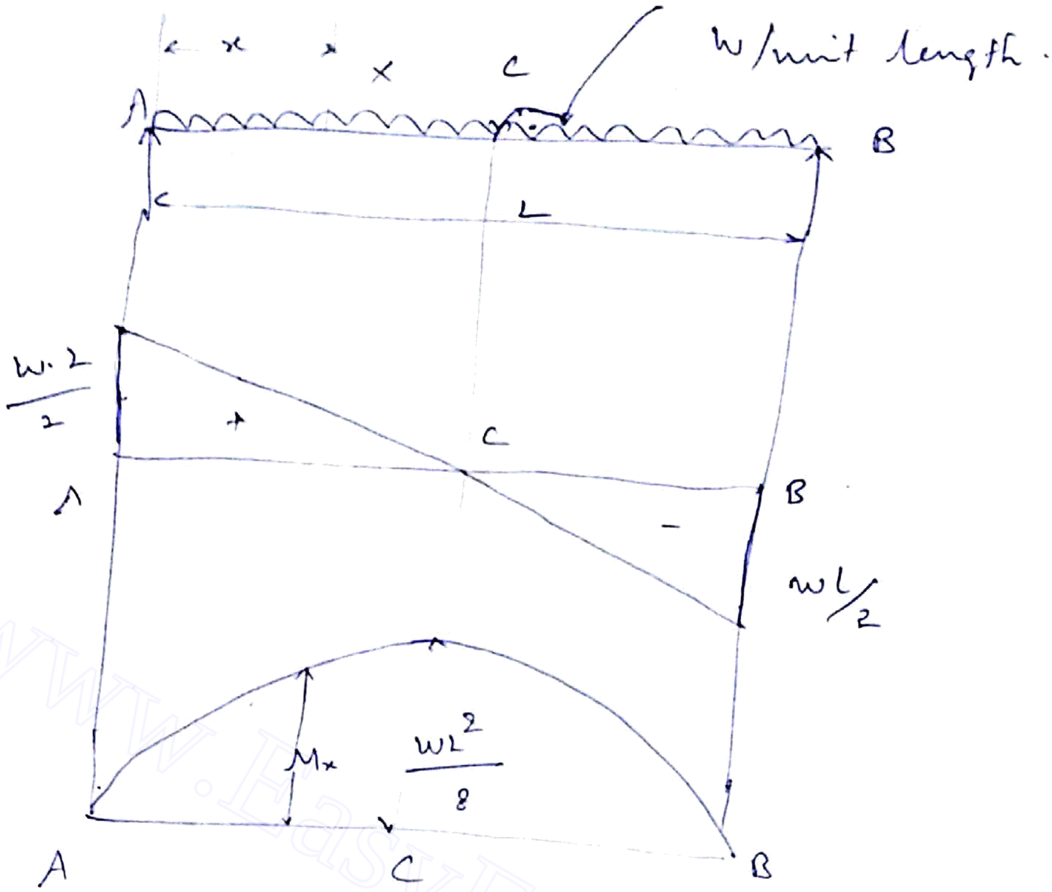
$$\text{Shear force at } C = \frac{4\times 2}{2} = 4\,kN.$$

The B.M. at B is zero, The B.M at A is equal to

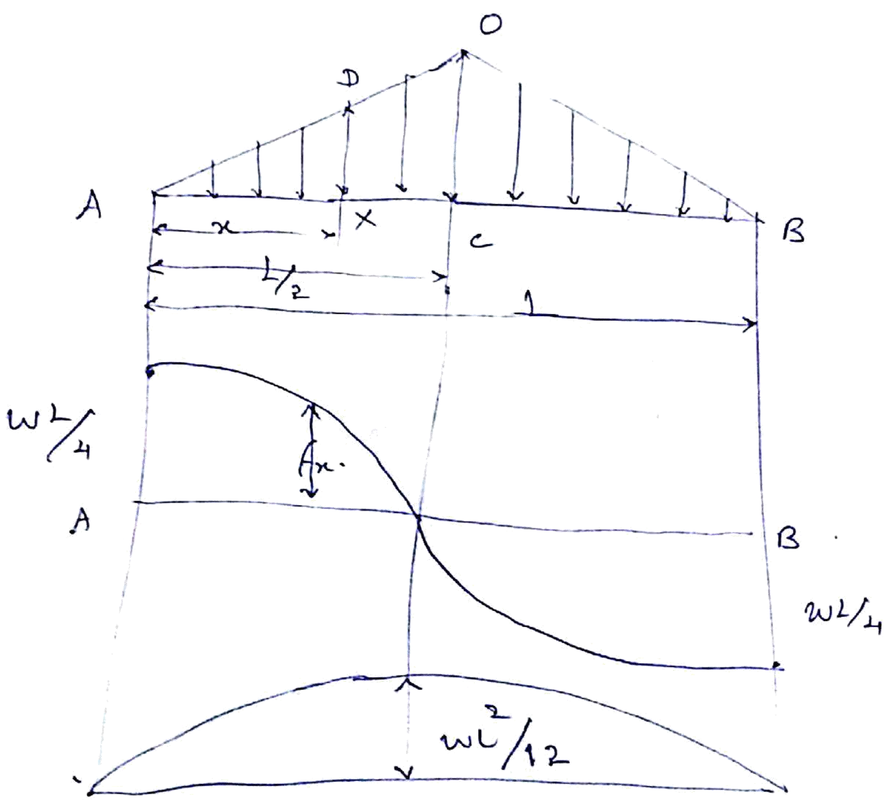
$-\dfrac{wl^2}{6}$: $\quad M_A = -\dfrac{wL^2}{6} = -\dfrac{2\times 4^2}{6} = 5.33\ kNm$



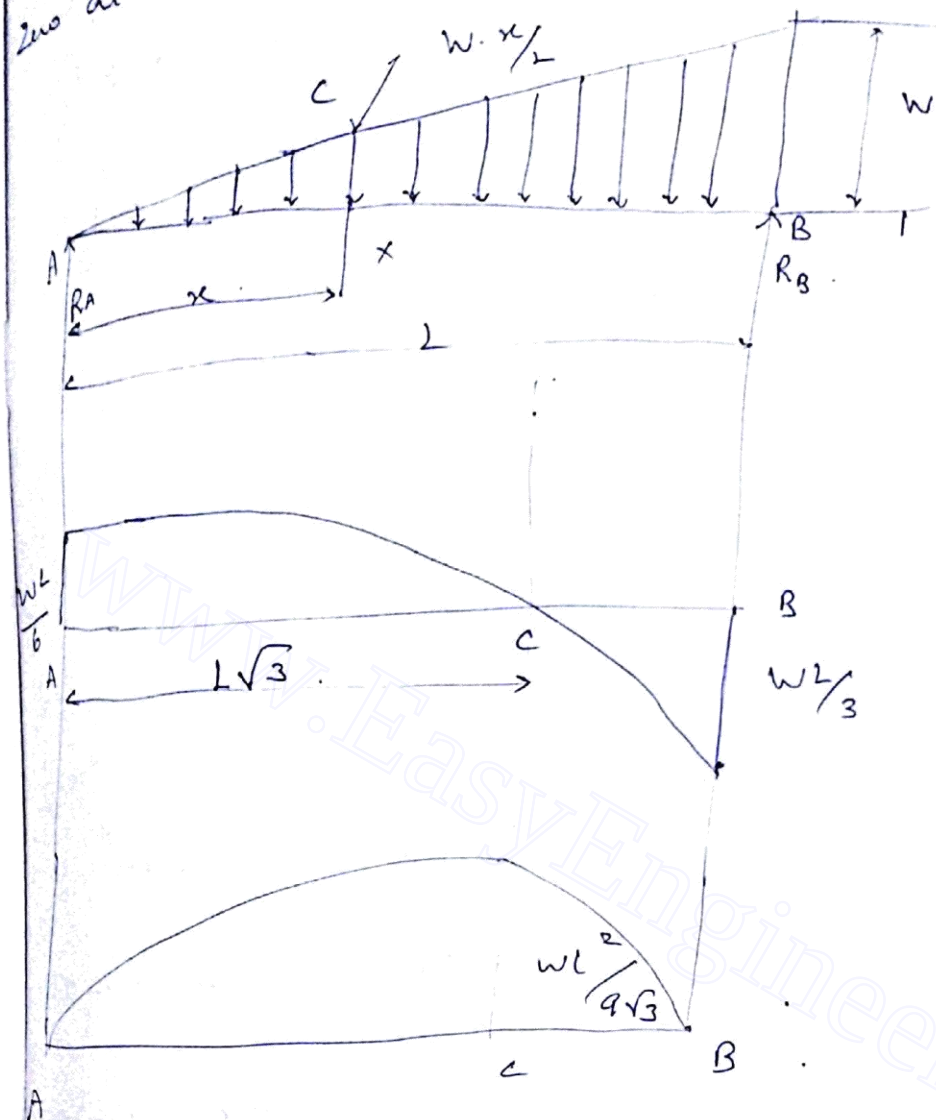
S.F & B.M. Diagrams for SSB carrying UDL :-



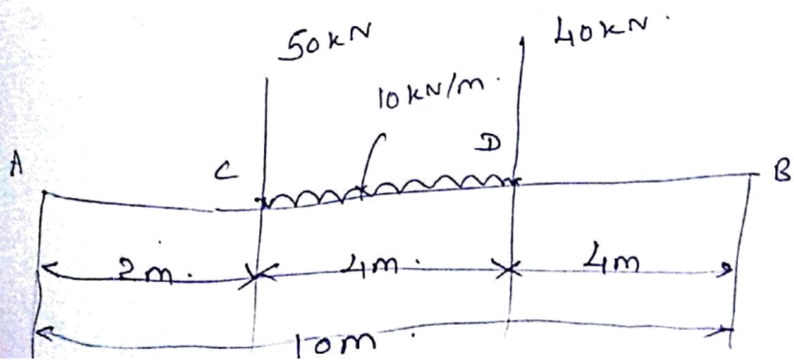
S.F. & B.M. Diagrams for SSB carrying UVL from zero at each end to w/unit length at the centre.



S.F & B.M Diagrams for a SSB carrying UVL from zero at one end to w/unit length at the other end.



Pb.4 A simply supported beam of length 10m, carries the UDL & two pt. loads as shown in figure. Draw the S.F & B.M diagram for the beam. Also calculate the bending moment.



First calculate the reactions R_A & R_B.

Taking moments of all forces about A, we get

$$R_B \times 10 = 50 \times 2 + 10 \times 4 \times \left(2 + \frac{4}{2}\right) + 40(2+4).$$

$$= 500. \quad \boxed{R_B = 50\,kN}.$$

R_A = Total load on beam $- R_B$.

$$= (50 + 10 \times 4 + 40) - 50 \Rightarrow 80\,kN$$

S.F. Diagram

S.F. at A, $F_A = R_A = 80\,kN$

S.F. just on R.H.S of C $= R_A - 50 = 30\,kN$.

S.F. just on L.H.S of D $= R_A - 50 - 10 \times 4 = -10\,kN$

S.F. just on R.H.S. of D $= R_A - 50 - 10 \times 4 - 40 = -50$

S.F. at B $= -50\,kN$

Now shear force at E $= R_A - 50 - 10(x-2)$

$$= 50 - 10x.$$

But shear force at E = 0.

$$50 - 10x = 0; \quad x = 5\,m.$$

B.M Diagram

B.M at C, $M_C = R_A \times 2 = 160$ knm.

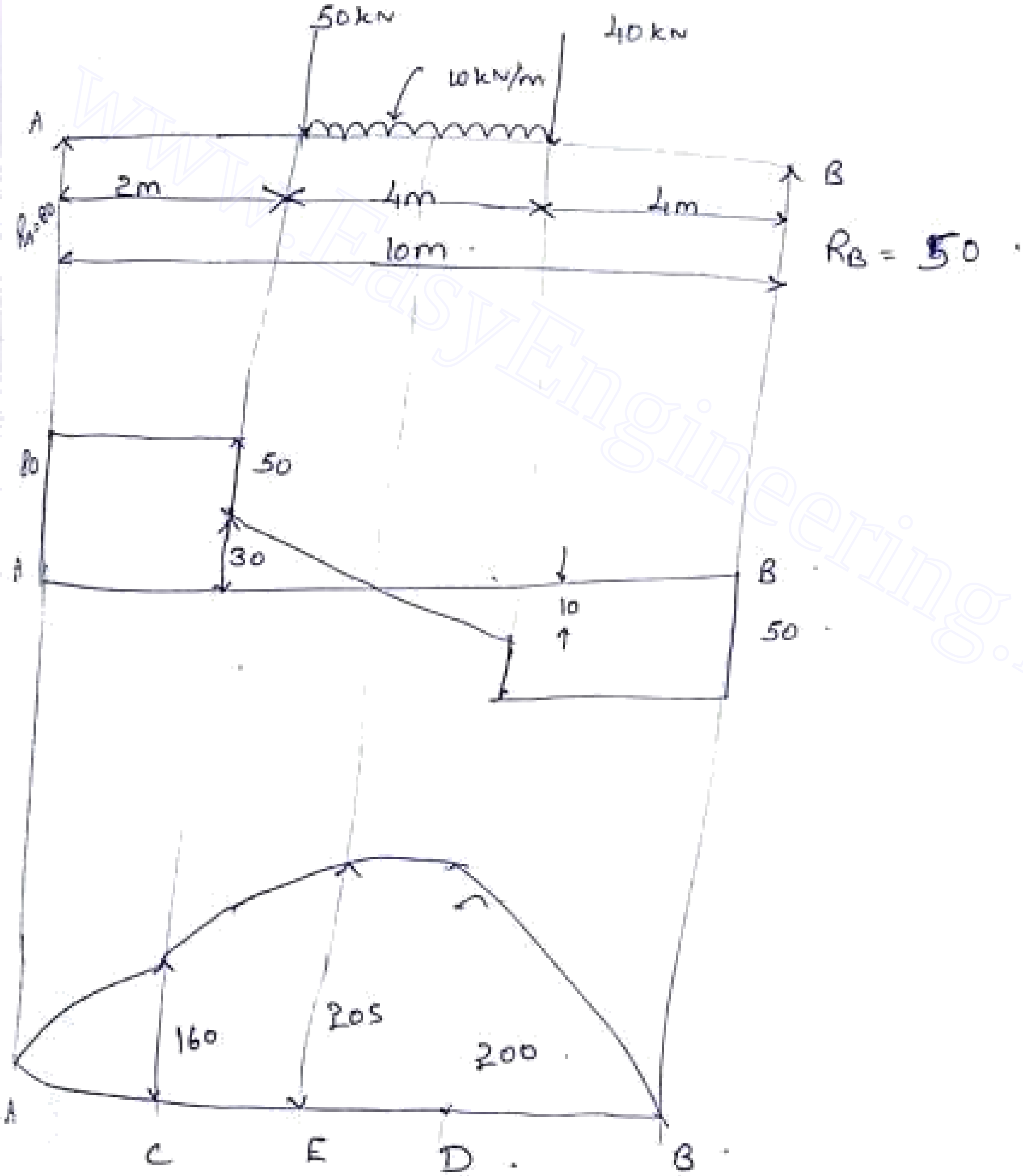
B.M. at D, $M_D = R_A \times 6 - 50 \times 4 - 10 \times 4 \times 4/2$

$= 200$ kNm.

At E, $x = 5$m & hence BM at E.

$M_E = R_A \times 5 - 50(5-2) - 10(5-2) \times \left(\frac{5-2}{2}\right)$

$= 205$ kN/m.



S.F & B.M Diagrams for Over hanged beams :-

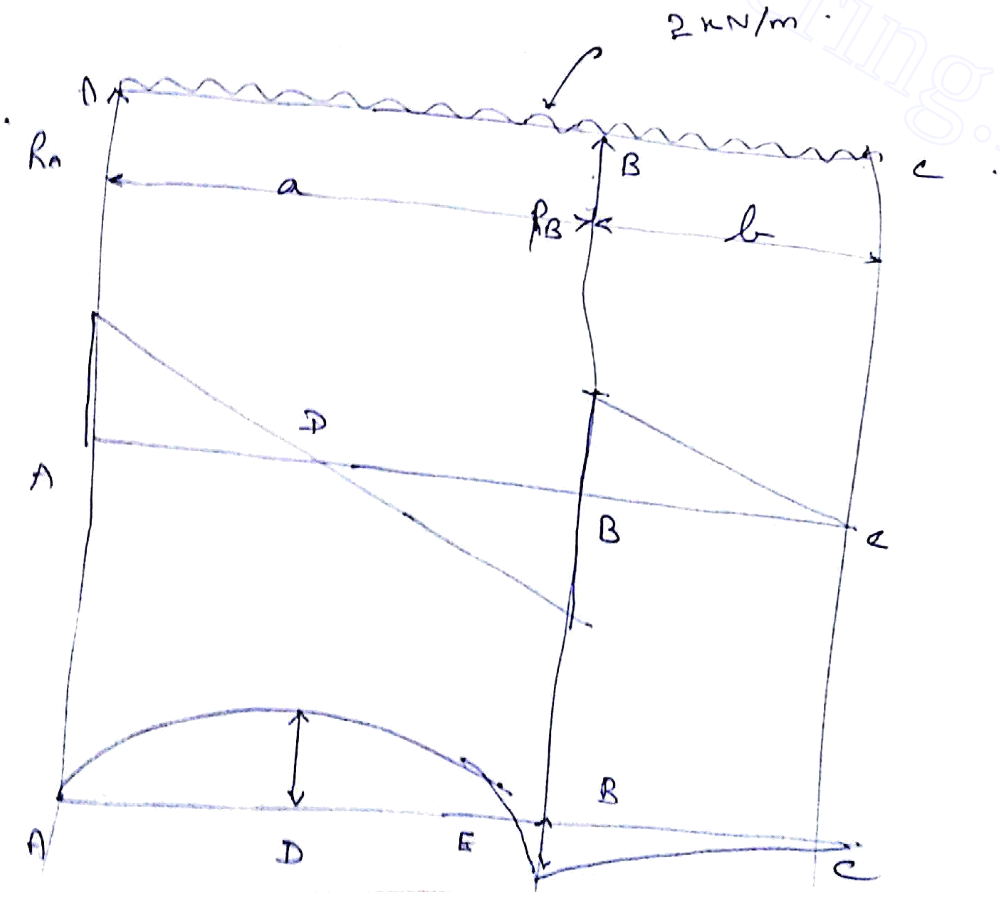
* If the end portion of a beam is extended beyond the support, such beam is known as Overhanging beam.

* B.M is +ve b/w the two supports whereas B.M is -ve for over hanging portion.

* Hence at some pt, the B.M is zero after changing its sign from +ve to -ve.

Point of Contraflexure :-

It's the pt. where B.M is zero after changing its sign from +ve to -ve or vice versa.



Bending stresses in Beams:-

When some external load acts on a beam, the shear force & bending moments are setup at all sections of the beam.

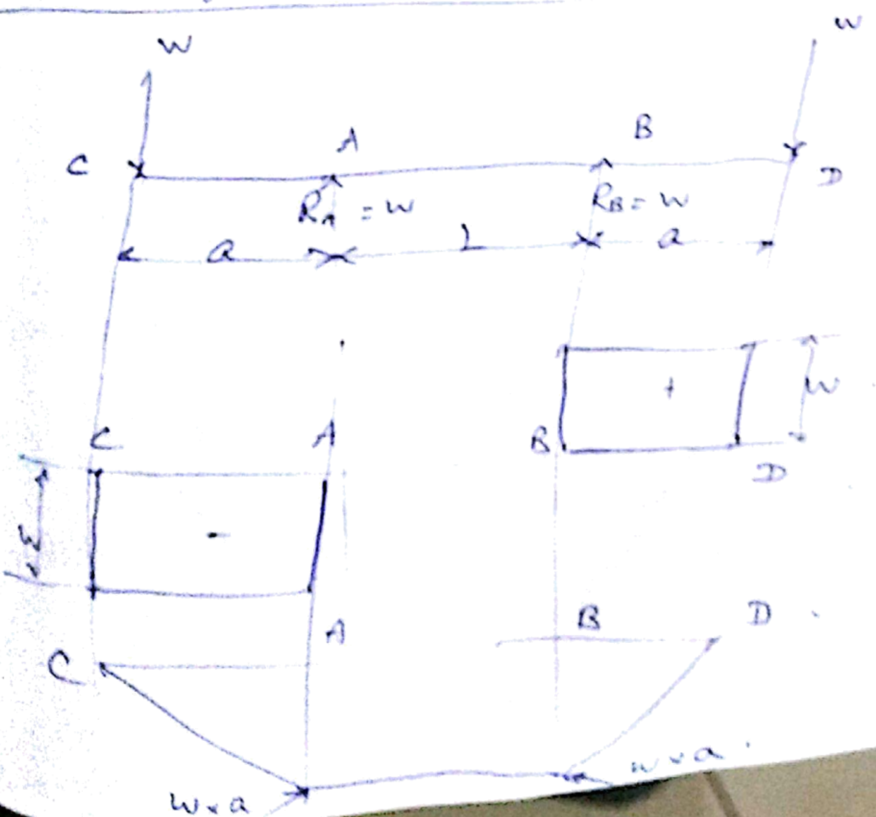
Due to the shear force & bending moment, the beam undergoes certain deformation.

The material of the beam will offer resistance or stresses against these deformations.

These stresses with certain assumptions can be calculated.

The stresses introduced by bending moments are known as bending stresses.

Pure bending (or) Simple bending:-



Theory of simple bending with assumptions ma

1. The material of the beam is homogeno isotropic.

2. The value of Young's modulus of elasticity the same in Tension & compression.

3 The transverse sections which were plane before bending, remain plane after bending also

4. The beam is initially straight & all longi filaments bend into circular area with a common centre of curvature.

5. The radius of curvature is large compar with the dimensions of the cross-section

6. Each layer of the beam is free to expa or contract, independently of the layer, above or below

Section Modulus :-

Section modulus is defined as the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis. It's denoted by symbol Z.

$$Z = \frac{I}{y_{max}}$$

I = M.O.I about neutral axis

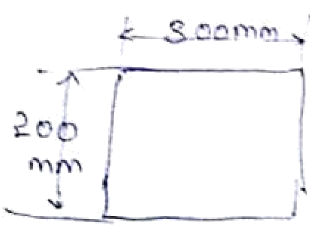
y_{max} = Distance of the outermost layer from the neutral axis

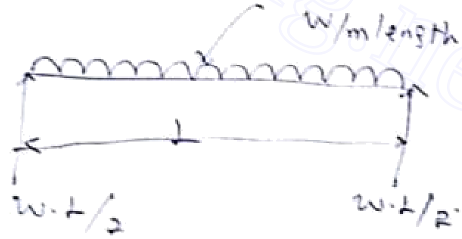
$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

$$M = \sigma_{max} \cdot \frac{I}{y_{max}}$$

$$\boxed{M = \sigma_{max} \cdot Z}$$

A rectangular beam 200mm deep & 300mm wide is SSB over a span of 8m. What UDL/m the beam may carry, if the bending stress is not to exceed $120 N/mm^2$





Given:-

Depth of beam d = 200mm

Width of beam b = 300mm

Length of beam L = 8m

Max. bending stress $\sigma_{max} = 120 N/mm^2$

$$Z = \frac{bd^2}{6} = \frac{300 \times 200^2}{6} = 2{,}000{,}000\,mm^3$$

Max. B.M. for a SSB carrying UDL as shown,

$$M = \frac{wl^2}{8} = w \times 8^2/8 \Rightarrow 8w.\,Nm$$

$$= 8000 \cdot w \cdot Nmm.$$

$$M = \sigma_{max} \cdot Z$$

$$8000w = 120 \times 20{,}00{,}000$$

$$\boxed{w = 30\,kN/m}$$

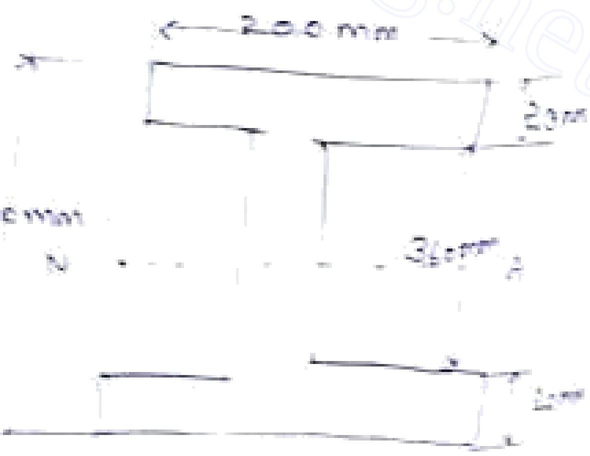
A rolled steel joist of I section has the dimension as shown in fig. This beam of I section carries a UDL of 40kN/m run on a span of 10m. Calculate the Max. stress produced due to bending.

Given:

U.D.L. $w = 40\,kN/m$

$= 40{,}000\,N/m$.

Span $l = 10m$.



Moment of Inertia about the neutral axis

$$= \frac{200 \times 400^3}{12} - \frac{(200-10) \times 360^3}{12}$$

$$= 327946666\,mm^4.$$

B.M is given by.

$$M = \frac{wl^2}{8} = \frac{40.000 \times 10^2}{8} \Rightarrow 5 \times 10^8 \text{ N-mm}$$

Now using the relation,

$$\frac{M}{I} = \frac{\sigma}{y}.$$

$$\sigma = \frac{M}{I} \times y.$$

$$\sigma_{max} = 304.92 \text{ N/mm}^2.$$

x ——————

Shear stresses In beams :-

The following are the important sections over which the shear stress distribution is to be obtained.

1. Rectangular section.

2. Circular section.

3. I. section

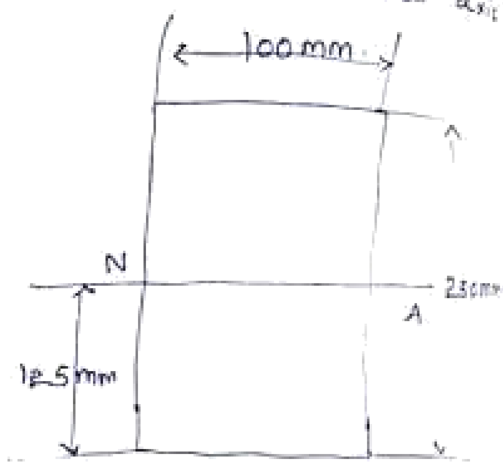
4. T- sections and

5. Miscellaneous sections

A rectangular beam 100mm wide & 250mm deep is subjected to a maximum shear force of 50kN. Determine (i) Average shear stress.

(ii) Maximum shear stress & (iii) Shear stress at a distance of 25mm above the neutral axis

<u>Given</u>



Width, $b = 100mm$

Depth, $d = 250mm$

Maximum shear force, $F = 50kN = 50,000N$.

(i) Average shear stress is given by

$$\tau_{avg} = \frac{F}{Area} = \frac{50,000}{b \times d} = \frac{50,000}{100 \times 250} = 2N/mm^2$$

(ii) Max. shear stress is given by eqn

$$\tau_{max} = 1.5 \times \tau_{avg}$$

$$= 1.5 \times 2 \Rightarrow 3 \, N/mm^2.$$

(iii) The shear stress at a distance y from N.A.

$$\tau = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right).$$

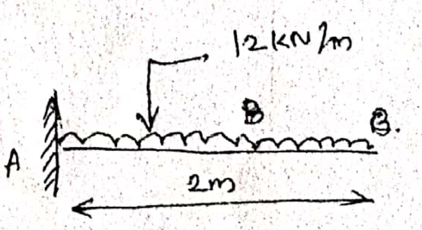
$$= \frac{50,000}{2I}\left(\frac{250^2}{4} - 25^2\right)$$

$$= \frac{50,000 \times 12}{2 \times 100 \times 250^3} \times 15,000 \text{ N/mm}^2$$

$$= 2.88 \text{ N/mm}^2$$

A cantilever beam AB of length 2m carries a uniformly distributed load 12 kN/m over entire length. Calculate the shear stress and bending stress. If the size of beam 230mm x 300mm,

$l = 2m$



UDL, $w = 12$ kN/m

Breadth $b = 230$mm

Depth $d = 300$ mm

To find:

Max. Bending Stress

Shear stress

Soln; Max. Bending Stress.

$M_{max} = 12 \times 2 \times \frac{2}{2} = 24$ kN-m.

$= 24 \times 10^6$ N-mm.

$y = \frac{d}{2} = \frac{300}{2} = 150$ mm.

$I = \frac{bd^3}{12} = \frac{230 \times 300^3}{12}$

$= 51.75 \times 10^7$ mm^4.

By using Relation

$\frac{M}{I} = \frac{\sigma_b}{y}$

$\frac{\times 10^6}{\times 10^6} = \frac{\sigma_b}{150}$

$\sigma_b =$

Max. Shear Stress.

By using relation.

$q_{max} = \frac{3}{2} \times \frac{F}{b.d}$

F = Max. Shear force

$= 12 \times 2 = 24$ kN at A

$q_{max} = \frac{3}{2} \times \frac{24 \times 10^3}{230 \times 300}$

$= 0.522$ N/mm^2.

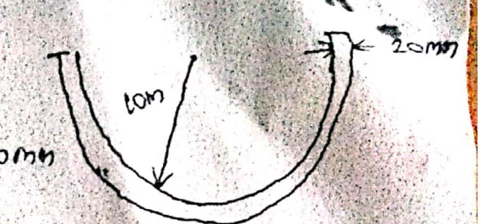
A steel plate of width 120mm and thickness 20mm is bent into a circular arc of radius 10m. Determine the max. stress induced and bending moment which will produce the max. stress, Take $E = 2 \times 10^5$ N/mm^2

Given: width, $b = 120$ mm

thickness, $d = 20$ mm.

Radius of curvature $R = 10m = 10000$ mm

$= \cancel{8000}$



Young's modulus, $E = 2\times10^5$ N/mm^2.

To find $\sigma_{b\,max}$ & M_{max}

Soln, Moment of Inertia, $I = \frac{bt^3}{12}$

$= \frac{120 \times 20^3}{12} = 8\times10^4$ mm^4.

Max. bending stress thickness of plate

$y_{max} = \frac{20}{2} = 10$ mm

$\frac{M}{I} = \frac{E}{R}$

$\sigma_{b\,max} = \frac{E}{R} \times y_{max}$

$= \frac{2\times10^5}{10\times10^3} \times 10 = $ N/mm^2.

$M = \frac{2\times10^5}{10\times10^3} \times 8\times10^4$

$= 1.6\times10^6$ N-mm

$= 1.6$ kN-m.

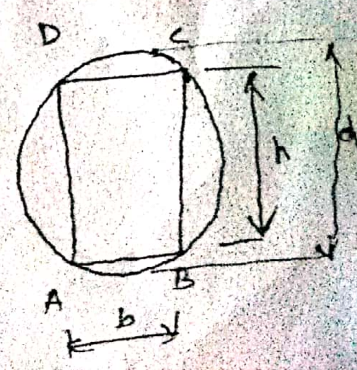
Prove that the ratio of depth to width of strongest beam that can be cut from a circular log of diameter 'd' is 1.414. Hence calculate depth and width of strongest beam that can be cut out of a cylindrical log of wood whose diameter is 300mm.

Dia of log d.

b - width

h - Depth

To find: Prove $\frac{h}{b} = 1.414$.



h, b = ?

$z = \frac{I}{y} = \frac{\left(\frac{bh^3}{12}\right)}{h/2} = \frac{bh^2}{6} \rightarrow (1)$.

$\triangle BCD$, $b^2+h^2 = d^2 \Rightarrow h^2 = d^2 - b^2$. in (1), $z = \frac{1}{6}(bd^2 - b^3)$. $\rightarrow (2)$

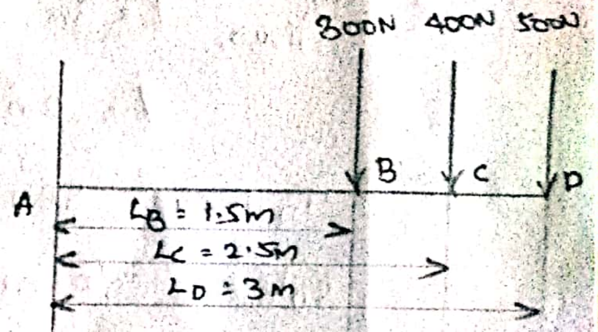
Max value $\frac{dz}{db} = 0$. $\frac{d}{db}\left[\frac{bd^2-b^3}{6}\right] = 0$. $\Rightarrow d^2 = 3b^2 \rightarrow (3)$

Sub in (3), $b^2+h^2 = 3b^2$, $\frac{h}{b} = 1.414$.

$d = 300$ mm, Sub in (3) $d^2 = 3b^2$, $b^2 = \frac{90000}{3}$, $= 173.2$ mm

$h = \sqrt{2} \times b = 244.9$ mm

A Cantilever beam of Length 3m carries the Point loads as shown in figure. Draw the Shear force and bending moment diagrams for cantilever beam.



Given $L_D = 3m$, $W_D = 500N$.

$L_C = 2.5m$, $W_C = 400N$

$L_B = 1.5m$, $W_B = 300N$.

To draw : SFD & BMD.

Soln,

SF is sum of forces on either Left or right side of the Section.

The force acting downward so its Positive

SF at D $= +W_D = +500N$.

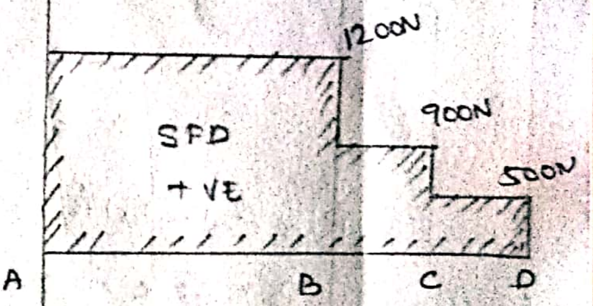
SF btn D and C remains Constant and equal to +500N.

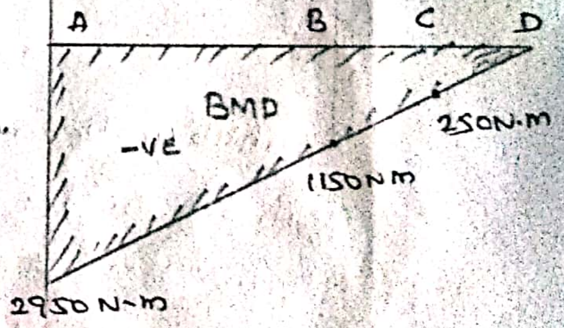
SF at C $= +W_D + W_C = 900N$.

SF btn C & B and equal to 900N

SF at B = +1200N.

SF at A = +1200N.





BM calculation.

BM at Section = force × Distance from Corresponding Section

BM at B $= -W_D(L_D - L_B)$

$= -1150 N\text{-}m$.

BM at D $= -W_D \times$ Distance from D

$= -W_D \times 0 = 0$.

BM at C $= -W_C \times (L_D - L_C)$

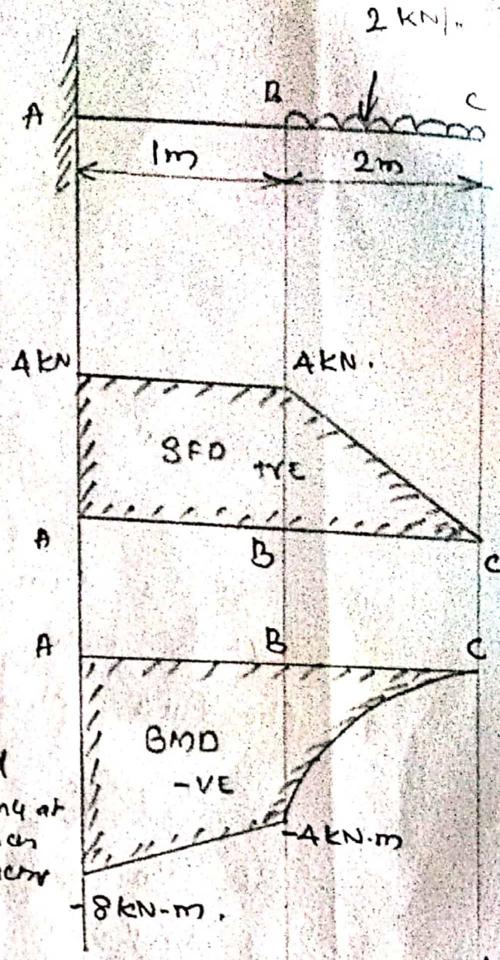
$= -250 N.m$.

BM at A $= -W_D L_D - W_C L_C - W_B L_B$

$= -500 \times 3 - 400 \times 2.5 - 300 \times 1.5$

$= -2950 N.m$.

A cantilever of length 3m carries a UDL of over a length of 2m from the free end. Dr the SF and BM diagrams for the cantilever.



Given, $L = 3m$, $w = 2kN/m$.

$l = 2m$.

To draw, SFD & BMD.

Soln. SF at C = 0.

SF at B = 2kN/m × 2m.

= 4kN.

SF at A = 4kN.

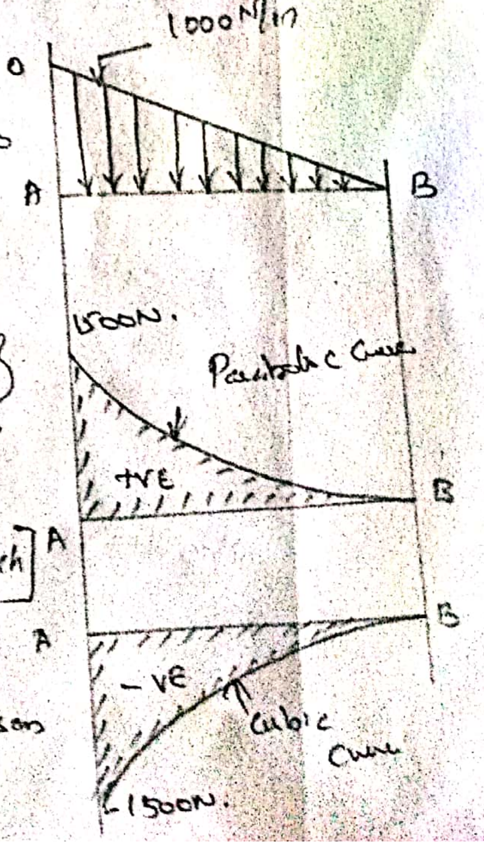
BM calculation:

BM at C = 0. BM at B = Load × Length of load × Distance at which load acts

= −2 × 2 × 2/2

= −4 kNm.

A cantilever beam 3m long carries a gradually varying load zero at the free end to 1000 N/m at the fixed end. Draw SF and BM diagrams for beam.



To draw SFD & BMD.

SF calc. SF at B = 0, SF at A = ½ × 3 × 1000

= 1500N.

BM calculation: BM at B = 0.

BM at A = {Load equ. to point ld} × {Distance of load from A}

$= -\underbrace{\tfrac{1}{2} \times 3 \times 1000}$ × ⅓ × 3.

[Load Equ. to UVL = ½ × L × h]

= −1500N.

[distance is ⅓L from A.

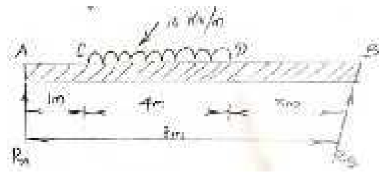
⅔L from B for this case

[#Cognitive Level: K1- Remember, K2 – Understand, K3 – Apply, K4 – Analyze, K5 – Evaluate, K6- Creativity]

1. i. A simply supported beam AB of length 5m carries point loads of 8KN, 10KN and 15KN at 1.5m and 2.5m and 4m respectively from left hand support. Outline shear force diagram and bending moment diagram (CO2) **[K2]**

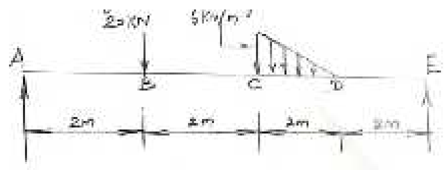
   ii. A cantilever beam AB of length 2m carries a uniformly distributed load of 12KN/m over entire length. Calculate the shear stress and bending stress. If the size of the beam is 230mm×300mm. (CO2) **[K2]** **(ND-2016)**

2. Outline the shear force and B.M diagrams for a simply supported beam of length 8m and carrying a uniformly distributed load of 10KN/m for a distance of 4m as shown in fig. (CO2) **[K2]** **(ND-2015)**



3. A steel plate of width 120mm and of thickness 20mm is bent into a circular arc of radius 10m. Determine the maximum stress induced and the bending moment which will produce the maximum stress, Take $E=2x10^5$ N/mm^2 (CO2) **[K4]** **(ND-2015)**

4. Sketch SFD and BMD and find the maximum bending moment for the beam given in Fig. (CO2) **[K3]** **(ND-2014)**



5. Prove that the ratio of depth to width of the strongest beam that can be cut from a circular log of diameter ‘d’ is 1.414. Hence calculate the depth and width of the strongest beam that can be cut out of a cylindrical log of wood whose diameter is 300mm. (CO2) **[K3]** **(ND-2014)**

6. i. Sketch the shear force and bending moment diagram for a cantilever carrying load whose intensity varies uniformly from zero at the fixed end to ’W’ per unit run at the free end. (CO2) **[K3]**

   ii. Outline the S.F.D and B.M.D For the SSB as shown in fig. (CO2) **[K2]**

7. A T-section of a SSB has the width of flange =100mm, overall depth =100mm, thickness of flange and stem= 20mm determine the maximum stress in beam when a bending moment of 12KN-m is acting on the section. Also calculate the shear stress at neutral axis and at the junction of web and flange when shear force of 50KN acting on beam. (CO2) **[K2]** **(MJ-2014)**

8. A simply supported beam of span 6m is carrying a UDL of 2KN/m over the entire span. Calculate the magnitude of shear force and bending moment at every section, 2m from the left support. Also draw shear force and bending moment diagrams. (CO2) **[K2]** **(MJ-2013)**

UNIT III TORSION

Torsion formulation stresses and deformation in circular and hollows shafts – Stepped shafts– Deflection in shafts fixed at the both ends – Stresses in helical springs – Deflection of helical springs, carriage springs.

OBJECTIVE :

To determine stresses and deformation in circular shafts and helical spring due to torsion.

OUTCOMES :

Students will be able to

Apply basic equation of simple torsion in designing of shafts and helical spring

---

Torsion

In solid mechanics, **torsion** is the twisting of an object due to an applied torque. In sections perpendicular to the torque axis, the resultant shear stress in this section is perpendicular to the radius.

For solid shafts of uniform circular cross-section or hollow circular shafts with constant wall thickness, the torsion relations are:

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\phi}{\ell}$$

where:

- R is the outer radius of the shaft i.e. m, ft.
- t is the maximum shear stress at the outer surface.
- *f* is the angle of twist in radians.
- *T* is the torque (N·m or ft·lbf).
- *l* is the length of the object the torque is being applied to or over.
- *G* is the shear modulus or more commonly the modulus of rigidity and is usually given in gigapascals (GPa), lbf/in^2 (psi), or lbf/ft^2.
- *J* is the torsion constant for the section. It is identical to the polar moment of inertia for a round shaft or concentric tube only. For other shapes J must be determined by other means. For solid shafts the membrane analogy is useful, and for thin walled tubes of arbitrary shape the shear flow approximation is fairly good, if the section is not re-entrant. For thick walled tubes of arbitrary shape there is no simple solution, and finite element analysis (FEA) may be the best method.
- The product *GJ* is called the torsion.

3.1 Beam shear

Beam shear is defined as the internal shear stress of a beam caused by the sheer force applied to the beam.

$$\tau = \frac{VQ}{It},$$

where

V = total shear force at the location in question;

Q = statical moment of area;
t = thickness in the material perpendicular to the shear;
I = Moment of Inertia of the entire cross sectional area.

This formula is also known as the Jourawski formula

Semi-monocoque shear

Shear stresses within a semi-monocoque structure may be calculated by idealizing the cross-section of the structure into a set of stringers (carrying only axial loads) and webs (carrying only shear flows). Dividing the shear flow by the thickness of a given portion of the semi-monocoque structure yields the shear stress. Thus, the maximum shear stress will occur either in the web of maximum shear flow or minimum thickness.

Also constructions in soil can fail due to shear; e.g., the weight of an earth-filled dam or dike may cause the subsoil to collapse, like a small and slide.

Impact shear

The maximum shear stress created in a solid round bar subject to impact is given as the equation:

$$\tau = 2\left(\frac{UG}{V}\right)^{\frac{1}{2}},$$

where

U = change in kinetic energy;
G = shear modulus;
V = volume of rod;

and

$$U = U_{rotating} + U_{applied};$$
$$U_{rotating} = \frac{1}{2}I\omega^2;$$
$$U_{applied} = T\theta_{displaced};$$

3.2 Bars of Solid and hollow circular section

The stiffness, *k*, **of a body** is a measure of the resistance offered by an elastic body to deformation. For an elastic body with a single Degree of Freedom (for example, stretching or compression of a rod), the stiffness is defined as

$$k = \frac{F}{\delta}$$

where

F is the force applied on the body
d is the displacement produced by the force along the same degree of freedom (for instance, the change in length of a stretched spring)

In the International System of Units, stiffness is typically measured in nektons per meter. In English Units, stiffness is typically measured in pound force (lbf) per inch.

Generally speaking, deflections (or motions) of an infinitesimal element (which is viewed as a point) in an elastic body can occur along multiple degrees of freedom (maximum of six DOF at a point). For example, a point on a horizontal beam can undergo both a vertical displacement and a rotation relative to its undeformed axis. When there are M degrees of freedom a M x M matrix must be used to describe the stiffness at the point. The diagonal terms in the matrix are the direct-related stiffnesses (or simply stiffnesses) along the same degree of freedom and the off-diagonal terms are the coupling stiffnesses between two different degrees of freedom (either at the same or different points) or the same degree of freedom at two different points. In industry, the term influence coefficient is sometimes used to refer to the coupling stiffness.

It is noted that for a body with multiple DOF, the equation above generally does not apply since the applied force generates not only the deflection along its own direction (or degree of freedom), but also those along other directions.

For a body with multiple DOF, in order to calculate a particular direct-related stiffness (the diagonal terms), the corresponding DOF is left free while the remaining should be constrained. Under such a condition, the above equation can be used to obtain the direct-related stiffness for the degree of freedom which is unconstrained. The ratios between the reaction forces (or moments) and the produced deflection are the coupling stiffnesses.

The inverse of stiffness is *compliance*, typically measured in units of metres per newton. In rheology it may be defined as the ratio of strain to stress and so take the units of reciprocal stress, *e.g.* 1/Pa.

3.3 Stepped shaft ,Twist and torsion stiffness – Compound shafts – Fixed and simply supported shafts

Shaft: The shafts are the machine elements which are used to transmit power in machines.

Twisting Moment: The twisting moment for any section along the bar / shaft is defined to be the algebraic sum of the moments of the applied couples that lie to one side of the section under consideration. The choice of the side in any case is of course arbitrary.

Shearing Strain: If a generator a ?? b is marked on the surface of the unloaded bar, then after the twisting moment 'T' has been applied this line moves to ab'. The angle ???' measured in radians, between the final and original positions of the generators is defined as the shearing strain at the surface of the bar or shaft. The same definition will hold at any interior point of the bar.

Modulus of Elasticity in shear: The ratio of the shear stress to the shear strain is called the modulus of elasticity in shear OR Modulus of Rigidity and in represented by the symbol

Angle of Twist: If a shaft of length L is subjected to a constant twisting moment T along its length, than the angle ? through which one end of the bar will twist relative to the other is known is the angle of twist.

Despite the differences in the forms of loading, we see that there are number of similarities between bending and torsion, including for example, a linear variation of stresses and strain with position.

In torsion the members are subjected to moments (couples) in planes normal to their axes.

For the purpose of desiging a circular shaft to withstand a given torque, we must develop an equation giving the relation between twisting moment, maximum shear stress produced, and a quantity representing the size and shape of the cross-sectional area of the shaft.

Not all torsion problems, involve rotating machinery, however, for example some types of vehicle suspension system employ torsional springs. Indeed, even coil springs are really curved members in torsion as shown in figure.

Many torque carrying engineering members are cylindrical in shape. Examples are drive shafts, bolts and screw drivers.

Simple Torsion Theory or Development of Torsion Formula : Here we are basically interested to derive an equation between the relevant parameters

Relationship in Torsion:

1 st Term: It refers to applied loading ad a property of section, which in the instance is the polar second moment of area.

2 nd Term: This refers to stress, and the stress increases as the distance from the axis increases.

3 rd Term: it refers to the deformation and contains the terms modulus of rigidity &

combined term (??? l) which is equivalent to strain for the purpose of designing a circular shaft to with stand a given torque we must develop an equation giving the relation between Twisting moments max m shear stain produced and a quantity representing the size and shape of the cross ??sectional area of the shaft.

Refer to the figure shown above where a uniform circular shaft is subjected to a torque it can be shown that every section of the shaft is subjected to a state of pure shear, the moment of resistance developed by the shear stresses being every here equal to the magnitude, and opposite in sense, to the applied torque. For the purpose of deriving a simple theory to describe the behavior of shafts subjected to torque it is necessary make the following base assumptions.

Assumption:

(i) The materiel is homogenous i.e of uniform elastic properties exists throughout the material.

(ii) The material is elastic, follows Hook's law, with shear stress proportional to shear strain.

(iii) The stress does not exceed the elastic limit.

(iv) The circular section remains circular

(v) Cross section remain plane.

(vi) Cross section rotate as if rigid i.e. every diameter rotates through the same angle.

Consider now the solid circular shaft of radius R subjected to a torque T at one end, the other end being fixed Under the action of this torque a radial line at the free end of the shaft twists through an angle , point A moves to B, and AB subtends an angle ' at the fixed end. This is then the angle of distortion of the shaft i.e the shear strain.

Since angle in radius = arc / Radius

arc AB = R?

= L ? [since L and ? also constitute the arc AB]

Thus, ? = R? / L (1)

From the definition of Modulus of rigidity or Modulus of elasticity in shear

Stresses: Let us consider a small strip of radius r and thickness dr which is subjected to shear stress??'.

The force set up on each element

= stress x area

= ?' x 2? r dr (approximately)

This force will produce a moment or torque about the center axis of the shaft.

= ?' . 2 ? r dr . r

= 2 ???' . r2. dr

The total torque T on the section, will be the sum of all the contributions.

Since ?' is a function of r, because it varies with radius so writing down??' in terms of r from the equation (1).

Where

T = applied external Torque, which is constant over Length L;

J = Polar moment of Inertia

[D = Outside diameter ; d = inside diameter]

G = Modules of rigidity (or Modulus of elasticity in shear)

? = It is the angle of twist in radians on a length L.

Tensional Stiffness: The tensional stiffness k is defined as the torque per radius twist

i.e, k = T /???= GJ / L

Problem 1

A stepped solid circular shaft is built in at its ends and subjected to an externally applied torque. T0 at the shoulder as shown in the figure. Determine the angle of rotation ?0 of the shoulder section where T0 is applied ?

Solution: This is a statically indeterminate system because the shaft is built in at both ends. All that we can find from the statics is that the sum of two reactive torque TA and TB at the built ?? in ends of the shafts must be equal to the applied torque T0

Thus TA+ TB = T0 ------ (1)

[from static principles]

Where TA ,TB are the reactive torque at the built in ends A and B. wheeras T0 is the applied torque

From consideration of consistent deformation, we see that the angle of twist in each portion of the shaft must be same.

i.e ?a = ? b = ? 0

using the relation for angle of twist

N.B: Assuming modulus of rigidity G to be same for the two portions

So the defines the ratio of TA and TB

So by solving (1) & (2) we get

Non Uniform Torsion: The pure torsion refers to a torsion of a prismatic bar subjected to torques acting only at the ends. While the non uniform torsion differs from pure torsion in a sense that the bar / shaft need not to be prismatic and the applied torques may vary along the length.

Here the shaft is made up of two different segments of different diameters and having torques applied at several cross sections. Each region of the bar between the applied loads between changes in cross section is in pure torsion, hence the formula's derived earlier may be applied. Then form the internal torque, maximum shear stress and angle of rotation for each region can be calculated from the relation

The total angle to twist of one end of the bar with respect to the other is obtained by summation using the formula

If either the torque or the cross section changes continuously along the axis of the bar, then the ? (summation can be replaced by an integral sign (?). i.e We will have to consider a differential element.

After considering the differential element, we can write

Substituting the expressions for Tx and Jx at a distance x from the end of the bar, and then integrating between the limits 0 to L, find the value of angle of twist may be determined.

Application to close-coiled helical springs

Closed Coiled helical springs subjected to axial loads:

Definition: A spring may be defined as an elastic member whose primary function is to deflect or distort under the action of applied load; it recovers its original shape when load is released.

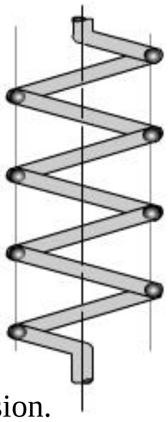
or

Springs are energy absorbing units whose function is to store energy and to restore it slowly or rapidly depending on the particular application.

Important types of springs are:

There are various types of springs such as

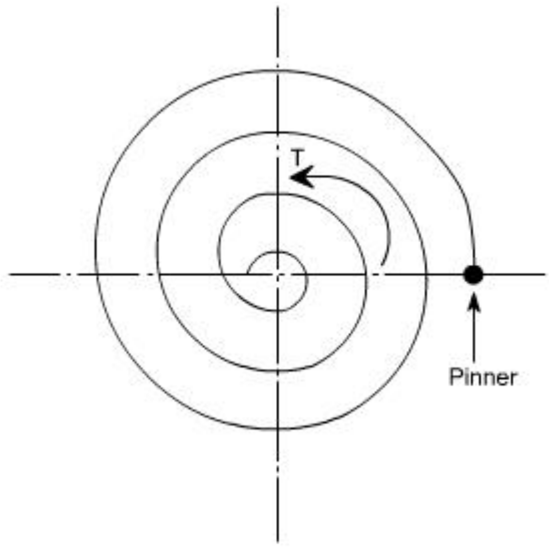
(i) helical spring: They are made of wire coiled into a helical form, the load being applied along the axis of the helix. In these type of springs the major stresses is torsional shear stress due to twisting. They are



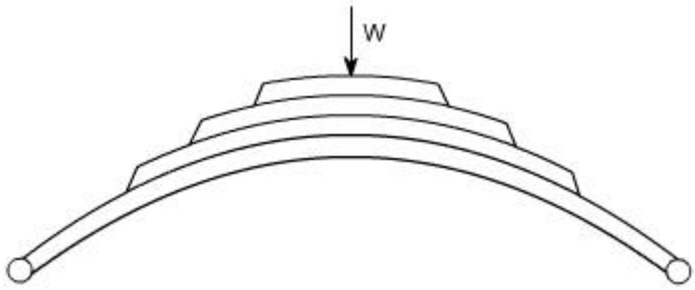
both used in tension and compression.

(ii) Spiral springs: They are made of flat strip of metal wound in the form of spiral and loaded in torsion.

In this the major stresses are tensile and compression due to bending.



(iv) Leaf springs: They are composed of flat bars of varying lengths clamped together so as to obtain greater efficiency . Leaf springs may be full elliptic, semi elliptic or cantilever types, In these type of springs the major stresses which come into picture are tensile & compressive.



These type of springs are used in the automobile suspension system.

Uses of springs :

(a) To apply forces and to control motions as in brakes and clutches.

(b) To measure forces as in spring balance.

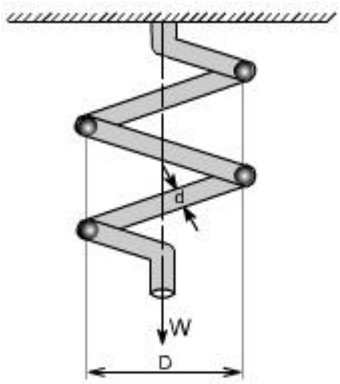
(c) To store energy as in clock springs.

(d) To reduce the effect of shock or impact loading as in carriage springs.

(e) To change the vibrating characteristics of a member as inflexible mounting of motors.

Derivation of the Formula :

In order to derive a necessary formula which governs the behaviour of springs, consider a closed coiled spring subjected to an axial load W.



Let

W = axial load

D = mean coil diameter

d = diameter of spring wire

n = number of active coils

C = spring index = D / d For circular wires

l = length of spring wire

G = modulus of rigidity

x = deflection of spring

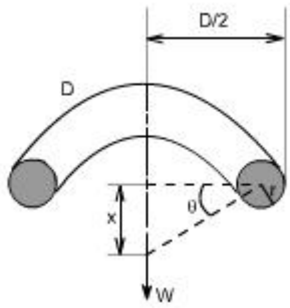
q = Angle of twist

when the spring is being subjected to an axial load to the wire of the spring gets be twisted like a shaft.

If q is the total angle of twist along the wire and x is the deflection of spring under the action of load W along the axis of the coil, so that

x = D / 2 . □

again l = □ D n [consider ,one half turn of a close coiled helical spring]



Maximum shear stress in spring section including Wahl Factor

Wahl's factor

Assumptions: (1) The Bending & shear effects may be neglected

(2) For the purpose of derivation of formula, the helix angle is considered to be so small that it may be neglected.

Any one coil of a such a spring will be assumed to lie in a plane which is nearly □^{r} to the axis of the spring. This requires that adjoining coils be close together. With this limitation, a section taken perpendicular to the axis the spring rod becomes nearly vertical. Hence to maintain equilibrium of a segment of the spring, only a shearing force V = F and Torque T = F. r are required at any X – section. In the analysis of springs it is customary to assume that the shearing stresses caused by the direct shear force is uniformly distributed and is negligible

so applying the torsion formula.

Using the torsion formula i.e

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G.\theta}{l}$$

and substitituting $J = \frac{\pi d^4}{32}; T = w.\frac{d}{2}$

$$\theta = \frac{2.x}{D}; l = \pi D.x$$

3.4 SPRING DEFLECTION

$$\frac{w.d/2}{\frac{\pi d^4}{32}} = \frac{G.2x/D}{\pi D.n}$$

Thus,

$$x = \frac{8w.D^3.n}{G.d^4}$$

Spring striffness: The stiffness is defined as the load per unit deflection therefore

$$k = \frac{w}{x} = \frac{w}{\frac{8w.D^3.n}{G.d^4}}$$

Therefore

$$k = \frac{G.d^4}{8.D^3.n}$$

Shear stress

$$\frac{w.d/2}{\frac{\pi d^4}{32}} = \frac{\tau_{max^m}}{d/2}$$

$$\text{or } \tau_{max^m} = \frac{8wD}{\pi d^3}$$

3.5 WAHL'S FACTOR :

In order to take into account the effect of direct shear and change in coil curvature a stress factor is defined, which is known as Wahl's factor

K = Wahl' s factor and is defined as $K = \frac{4c-1}{4c-4} + \frac{0.615}{c}$

Where C = spring index

= D/d

if we take into account the Wahl's factor than the formula for the shear stress becomes $\tau_{max^m} = \frac{16.T.k}{\pi d^3}$

Strain Energy : The strain energy is defined as the energy which is stored within a material when the work has been done on the material.

In the case of a spring the strain energy would be due to bending and the strain energy due to bending is given by the expansion

$$U = \frac{T^2 L}{2EI}$$
$$L = \pi D n$$
$$I = \frac{\pi d^4}{64}$$
so after substitution we get
$$U = \frac{32T^2 Dn}{E.d^4}$$

Deflection of helical coil springs under axial loads

Deflection of springs

Example: A close coiled helical spring is to carry a load of 5000N with a deflection of 50 mm and a maximum shearing stress of 400 N/mm^2 .if the number of active turns or active coils is 8.Estimate the following:

(i) wire diameter

(ii) mean coil diameter

(iii) weight of the spring.

Assume G = 83,000 N/mm^2 ; □ = 7700 kg/m^3

solution :

(i) for wire diametre if W is the axial load, then

$$\frac{\frac{w.d/2}{\frac{\pi d^4}{32}}}{} = \frac{\tau_{max^m}}{d/2}$$
$$D = \frac{400}{d/2} \cdot \frac{\pi d^4}{32} \cdot \frac{2}{W}$$
$$D = \frac{400.\pi d^3.2}{5000.16}$$
$$D = 0.0314\, d^3$$

Futher, deflection is given as

$$x = \frac{8wD^3.n}{G.d^4}$$
on substituting the relevant parameters we get
$$50 = \frac{8.5000.(0.0314\, d^3)^3.8}{83,000.d^4}$$
$$d = 13.32\text{mm}$$

Therefore,

$D = .0314 \times (13.317)^3$mm

$=74.15$mm

$D = 74.15$ mm

Weight

mass or weight = volume . density

= area . length of the spring . density of spring material

$= \frac{\pi d^2}{4} . \pi D n . \rho$

On substituting the relevant parameters we get

Weight = 1.996 kg

= 2.0 kg

Design of helical coil springs

Helical spring design

Springs in Series: If two springs of different stiffness are joined endon and carry a common load W, they are said to be connected in series and the combined stiffness and deflection are given by the following equation.

$$\frac{W}{k} = x_1 + x_2 = \frac{W}{k_1} + \frac{W}{k_2}$$

or

$$\boxed{\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}}$$

Springs in parallel: If the two spring are joined in such a way that they have a common deflection ‘x' ; then they are said to be connected in parallel.In this care the load carried is shared between the two springs and total load $W = W_1 + W_2$

$$x = \frac{W}{k} = \frac{W_1}{k_1} = \frac{W_2}{k_2}$$

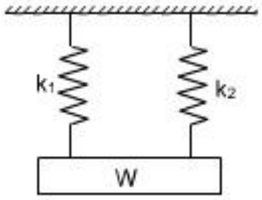
Thus $W_1 = \frac{Wk_1}{k}$

$$W_2 = \frac{Wk_2}{k}$$

Futher

$$W = W_1 + W_2$$

thus $\boxed{k = k_1 + k_2}$



stresses in helical coil springs under torsion loads

Stresses under torsion

Shear Stress in the Shaft

When a shaft is subjected to a torque or twisting, a shearing stress is produced in the shaft. The shear stress varies from zero in the axis to a maximum at the outside surface of the shaft.

The shear stress in a solid circular shaft in a given position can be expressed as:

$s = T\, r / I_p$ *(1)*

where

s = shear stress (MPa, psi)

T = twisting moment (Nmm, in lb)

r = distance from center to stressed surface in the given position (mm, in)

I_p *= "polar moment of inertia" of cross section (mm^4, in^4)*

The "polar moment of inertia" is a measure of an object's ability to resist torsion.

Circular Shaft and Maximum Moment

Maximum moment in a circular shaft can be expressed as:

$T_{max} = s_{max}\, I_p / R$ *(2)*

where

T_{max} = maximum twisting moment (Nmm, in lb)

s_{max} = maximum shear stress (MPa, psi)

R = radius of shaft (mm, in)

Combining (2) and (3) for a **solid shaft**

$T_{max} = (p/16)\ s_{max}\ D^3$ *(2b)*

Combining (2) and (3b) for a **hollow shaft**

$T_{max} = (p/16)\ s_{max}\ (D^4 - d^4) / D$ *(2c)*

Circular Shaft and Polar Moment of Inertia

Polar moment of inertia of a circular solid shaft can be expressed as

$I_p = p\ R^4/2 = p\ D^4/32$ *(3)*

where

D = shaft outside diameter (mm, in)

Polar moment of inertia of a circular hollow shaft can be expressed as

$I_p = p\ (D^4 - d^4) /32$ *(3b)*

where

d = shaft inside diameter (mm, in)

Diameter of a Solid Shaft

Diameter of a solid shaft can calculated by the formula

$D = 1.72\ (T_{max}/s_{max})^{1/3}$ *(4)*

Torsional Deflection of Shaft

The angular deflection of a torsion shaft can be expressed as

$? = L\ T / I_p\ G$ *(5)*

where

? = angular shaft deflection (radians)

L = length of shaft (mm, in)

G = modulus of rigidity (Mpa, psi)

The angular deflection of a torsion solid shaft can be expressed as

$? = 32\ L\ T / (G\ p\ D^4)$ (5a)

The angular deflection of a torsion hollow shaft can be expressed as

$? = 32\ L\ T / (G\ p\ (D^4 - d^4))$ (5b)

The angle in degrees can be achieved by multiplying the angle *?* in radians with *180/p*

*S*olid shaft (*p* replaced)

$?_{degrees}$ ˜ $584\ L\ T / (G\ D^4)$ (6a)

Hollow shaft (p replaced)

$?_{degrees}$ ˜ $584\ L\ T / (G\ (D^4 - d^4)$ (6b)

PART A

1. Define torsion **(May 2014)**

A shaft is said to be in torsion, when equal and opposite torques are applied at the two ends of the shaft. The torque is equal to the product of the force applied (tangentially to the ends of a shaft) and radius of the shaft.

2. What are the assumptions made in the theory of torsion? **(May 2010)**

The material of the shaft is uniform throughout.

The twist along the shaft is uniform.

Normal cross sections of the shaft, which were plane and circular before

Twist; remain plane and circular after twist.

All diameters of the normal cross section which were straight before twist, remain straight with their magnitude unchanged, after twist.

3. Write the expression for power transmitted by a shaft in Watts

$P=2\Pi NT/60$

Where

N--- Speed of the shaft in rpm

T—Mean torque transmitted in Nm

P---- Power

4. The torque transmitted by a hollow shaft is given by

$T = \Pi/16 \times \tau \, (D^4-d^4)/D$

Where τ -maximum shear stress induced at the outer surface.

D- External diameter

d-internal diameter

5. Define polar modulus.

Polar modulus is defined as the ratio of the polar moment of inertia to the radius of the shaft. It is also called torsional section modulus and is denoted by Zp.

6. Define torsional rigidity **(May 2012)**

Let a twisting moment T produce a twist of radian in a length l then

$$T/J = C\theta/L$$

Where C—modulus of rigidity of the material.

7. Why hollow circular shafts are preferred when compared to solid circular shafts?

Comparison by strength;

The torque transmitted by the hollow shaft is greater than the solid shaft, therebyhollow shaft is stronger than the solid shaft.

Comparison by weight:

For the same material, length and given torque, weight of a hollow shaft will be less. So hollow shafts are economical when compared to solid shafts, when torque is acting.

8. What is mean by spring? Name the two important types of springs.

Spring is a device which is used to absorb energy by taking very large change in its form without permanent deformation and then release the same when its required.

TYPES

Torsion spring

Bending spring

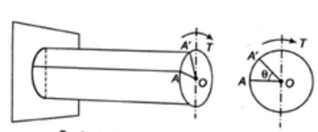
9. Distinguish between close and open helical coil springs.

If the angle of the helix of the coil is so small that the bending effects can be neglected, then the spring is called a closed –coiled spring. Close –coiled spring is a torsion spring The pitch between two adjacent turns is small. If the slope of the helix of the coil is quite appreciable then both the bending as well as torsional shear stresses are introduced in the spring, then the spring is called open coiled spring.

10. Define stiffness of a spring? In what unit it is measured?

Stiffness of a spring is defined as load per unit deflection. It is denoted by K and unit is N/mm.

11. Draw shear stress distribution of a circular section due to torque.



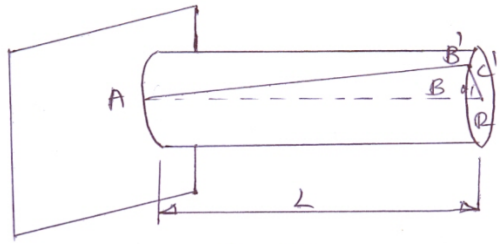
Torsion in shaft and combined stresses

UNIT-III

TORSION

PART B

1. Derive Torsion equation.



Consider a shaft of length L and circular cross section of radius R subjected to torque T as shown in figure.

A line AB on the surface of the shaft, which is straight in absence of the torque, becomes distorted.

The point B moves to the B' as shown

$\theta = \angle BOB'$ is known as the angle of twist.

$\phi = \angle BAB'$ is the shear strain.

For the extreme fibres (which are at distance R from axis of the shaft) like AB,

$$BB' = L\phi = R\theta$$

$$\phi = \frac{R\theta}{L} \rightarrow (1)$$

for a fibre at any distance r from axis of shaft (r < R).

$$CC' = L\phi' = r\theta$$

where ϕ' is the Shear strain on fibre at C.

$$\phi = \frac{r\theta}{L}$$

$\frac{\text{Shear stress}}{\text{Shear strain}}$ = Modulus of rigidity

$$\frac{\tau'}{\phi'} = G$$

where,

τ' = shear stress on the fibre at c.

$\tau' = G\phi'$

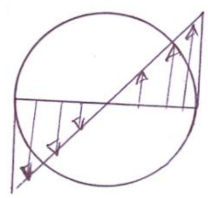
$$\tau' = \frac{Gr\theta}{L} \rightarrow (2).$$

As G, θ and L are constants,

$\tau' \propto r$.

for $r = 0$, $\tau' = 0$ and for $r = R$, $\tau' = \tau$, the maximum shear stress.

The variation of shear stress with radial distance from the axis of the shaft is shown in figure (2).



At $r = R$,

$$\tau = \frac{GR\theta}{L} \rightarrow (3)$$

from equations (1) & (2),

$$\tau'/r = \tau/R = \frac{G\theta}{L} \rightarrow (4)$$

Consider an elementary area dA at a section of the shaft as shown in figure (3).

Let df = Force an area element dA.

Then, the torque on area element is

$$dT = df.r$$

$$T = \int_0^R df.r.$$

$$dF = \tau' dA = \left(\frac{G\theta r}{L}\right) dA.$$

From equation ②,

$$T = \int_0^R G\theta/L \, r^2 \, dA.$$

$$= G\theta/L \int_0^R r^2 \, dA$$

But, $\int_0^R r^2 \, dA = J$, the polar moment of Inertia.

$$T = \frac{G\theta}{L} J$$

$$T/J = \frac{G\theta}{L} \rightarrow ⑤.$$

From eqn ④ & ⑤,

$$T/J = \tau/R = \frac{G\theta}{L}$$ is known as Torsion equation.

2. A hollow shaft with diameter ratio 3/5 is required to transmit 450 kW at 120 rpm. The Shearing stress in the shaft must not be exceed 60 N/mm^2 and twist in a length of 2.5 m is not to exceedd 1°. Calculate the minimum external diameter of the shaft $C = 80 \text{ kN/mm}^2$.

Given:

$d/D = 3/5$

$d = 0.6D$

$P = 450 \text{ kW}$

$N = 120 \text{ rpm}$

$\tau = 60 \text{ N/mm}^2$

$L = 2.5 \text{ m} = 2500 \text{ mm}$

Twist in the shaft $= 1^\circ = \frac{1 \times \pi}{180^\circ} = 0.0174$ radius

$$C = 80\ kN/mm^2 = 80 \times 10^3\ N/mm^2.$$

To.

External diameter of the hollow Shaft

Solution:

$$P = 2\pi NT/60$$

$$450 = \frac{2\pi \times 120 \times T}{60.}$$

$$T = 35.80\ KN-m.$$

CASE I:

Shear stress considering.

$$T = \pi/16 \times I \times \left[\frac{D^4 - d^4}{D}\right]$$

$$35.80 \times 10^6 = \pi/16 \times 60 \times \left[\frac{D^4 - (0.6D)^4}{D}\right]$$

$$35.80 \times 10^6 = \pi/16 \times 60 \times \frac{D^4}{D}\left[1 - (0.6)^4\right]$$

$$35.80 \times 10^6 = 11.78 \times D^3 \times (0.870)$$

$$D^3 = 3493160.041$$

$$D = 151.73mm$$

CASE II:

$$\frac{T}{J} = \frac{C\theta}{l}$$

$$J = \frac{\pi}{32}\left[D^4 - d^4\right]$$

$$= \frac{\pi}{32}\left[D^4 - (0.6D)^4\right]$$

$$\frac{35.80 \times 10^6}{\pi/32\left[D^4 - (0.6D)^4\right]} = \frac{80 \times 10^3 \times 0.0174}{2500}$$

$$\frac{35.80 \times 10^6}{\pi/32 \times D^4\left[1 - (0.6)^4\right]} = 0.5568$$

$$\frac{35.80 \times 10^6}{\pi/32 \, D^4 (0.870)} = 0.5568$$

$$\frac{35.80 \times 10^6}{\pi/32 \times 0.870 \times 0.5568} = D^4$$

$$7.527 \times 10^8 = D^4$$

$$D = 165.6 \text{mm}.$$

3. A close coiled helical spring is to have a stiffness of 1.5 N/mm of compression. under a maximum load of 60 N. The maximum shearing stress produced in the wire of the spring is 125 N/mm² Solid length of the spring is 50 mm. find the diameter of coil, diameter of wire and number of coils $C = 4.5 \times 10^4$ N/mm².

Given:

$K = 1.5$ N/mm

$W = 60$ N

$\tau_{max} = 125$ N/mm²

$G = C = 4.5 \times 10^4$ N/mm²

Solid length (nd) = 50 mm

To:

Number of turns

Diameter of the wire.

Solution:

$$n = 50/d.$$

$$K = \frac{Gd^4}{64R^3 n}.$$

$$1.5 = \frac{4.5 \times 10^4 d^4}{64 R^3 n.}$$

$$1.5 = \frac{4.5 \times 10^4 d^4}{64 R^3 (50/d)}$$

$$\frac{d^5}{R^3} = 0.1067 \rightarrow (1)$$

$$\tau_{max} = \frac{16 W R}{\pi d^3}$$

$$125 = \frac{16 \times 60 R}{\pi d^3}$$

$$R = \frac{d^3}{2.445} \rightarrow (2).$$

Substitute in eqn (1).

$$\frac{d^5}{\left(\frac{d^3}{2.445}\right)^3} = 0.1067$$

$$\frac{d^5 \times 2.445^3}{d^9} = 0.1067$$

$$d^4 = \frac{2.445^3}{0.1067}.$$

$$d = 3.42 \text{ mm}$$

Radius of coil (R) $= \frac{3.42^3}{2.445}$

$R = 16.36$ mm

Diameter of Coil (D) $= 2R$

$= 2 \times 16.36$

$D = 32.72$ mm

$$n = \frac{50}{d}$$

$$n = 50/3.42$$

$$n = 14.62$$

$$n = 15.$$

4. A steel shaft is required to transmit 75 kW power at 100 rpm and the maximum twisting moment is 30% greater than the mean. Find the diameter of the steel shaft if the maximum stress is 70 N/mm². Also determine the angle of twist in a length of 3m of the shaft. Assume the modulus of rigidity for steel as 90 kN/mm².

Solution:

$$P = \frac{2\pi N T_{mean}}{60,000} \text{ kW}$$

$$75 = \frac{2\pi \times 100\, T_{mean}}{60,000}$$

$$T_{mean} = 7.162 \times 10^3 \text{ Nm}$$

$$T_{max} = T_{mean} + \frac{30}{100} T_{mean} = 1.3\, T_{mean}$$

$$T_{max} = 1.3 \times 7.162 \times 10^3$$

$$T_{max} = 9.3106 \times 10^3 \text{ Nm}$$

$$\tau_{max} = \frac{TR}{J} = \frac{T_{max}(D/2)}{(\pi/32\, D^4)}$$

$$T_{max} = \pi/16\, D^3\, \tau_{max}$$

$$\tau_{max} = 70 \text{ N/mm}^2.$$

8

$T_{max} = \pi/16 \, D^3 \times \tau$

$9.3106 \times 10^6 = \pi/16 \, D^3 \times 70$

$D = 87.82 \text{ mm}$

$\theta = TL/GJ$

$J = \pi/32 \, D^4 = \pi/32 \times 87.82^4$

$J = 5.84 \times 10^6 \text{ mm}^4$

$L = 3\text{m} = 3 \times 10^3 \text{mm}$

$G = 90 \text{kN/mm}^2$

$= 90 \times 10^3 \text{N/mm}^2$

$T = 9.3106 \times 10^6 \text{ Nmm}$

$$\theta = \frac{9.3106 \times 10^6 \times 3 \times 10^3}{90 \times 10^3 \times 5.84 \times 10^6}$$

$\theta = 0.053145 \text{ rad}$

$= 0.053143 \times 180/\pi \text{ degree}$

$\theta = 3.045^\circ.$

5. A soled cercular shaft transmits 75KW power at 200 rpm. Calculate the shaft diameter, if the twist in the shaft is not to exceed 1° in 2m length of shaft, and shear stress is limited to 50 N/mm². Take C = 1×10⁵ N/mm².

Given data

$P = 75 kW = 75\times10^3 W$

$N = 200 rpm$

$\theta = 1^\circ$

$= \pi/180 \times 1 = 0.01745$ rad.

$L = 2m = 2\times10^3 mm$

$\tau = 50 N/mm^2$

$P = \frac{2\pi N T}{60} \Rightarrow 75\times10^3 = \frac{2\pi\times200\times T}{60}$

$T = \frac{75\times10^3\times60}{2\pi\times200} = 3580.98 Nm$

To find

d.

Solution

(i) Diameter of the shaft when maximum shear stresses is limited to 50 N/mm².

$T = \pi/16 \, \tau D^3$

$3580980 = \pi/16 \times 50 \times D^3$

$D = 71.3 mm.$

(ii) Diameter of the Shaft when the twist in the shaft is not to exceed 1°

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$\frac{3580980}{\pi/32\, D^4} = \frac{10^5 \times 0.01745}{2000} \qquad J = \pi/32\, D^4$$

$$D = \left[\frac{32 \times 2000 \times 3580980}{\pi \times 10^5 \times 0.01745}\right]^{1/4} = 80.4\,mm.$$

PART C

6. Two Solid Shafts AB + BC of aluminium and Steel respectively are rigidly fastened together at B and attached to two rigid supports at A and C. Shaft AB is 7.5 cm in diameter and 2m in length. Shaft BC is 5.5 cm in diameter and 1m length. A torque of 20000 N cm is applied at the junction B. Compute the maximum shearing stresses in each material. What is the angle of twist at the junction. Take the modulus of rigidity of the materials $C_{Al} = 0.3 \times 10^5\, N/mm^2$ + $C_{St} = 0.9 \times 10^5\, N/mm^2$.

Given data

Allumenium	Steel
$L_1 = 2m = 2000\,mm$	$L_2 = 1m = 1000\,mm$
$d_1 = 7.5\,cm = 75\,mm$	$d_2 = 5.5\,cm = 55\,mm$
$C_1 = 0.3 \times 10^5\, N/mm^2$	$C_2 = 0.9 \times 10^5\, N/mm^2$

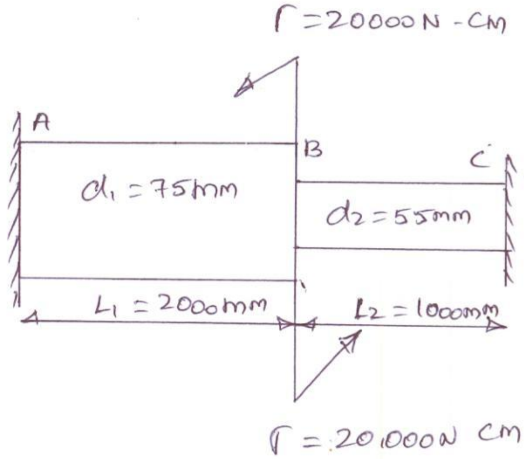
$T = 20{,}000$ N cm

$= 200000$ N mm

To find

Maximum Shear Stress

Solution



The torque is applied at junction B, hence angle of twist in shaft AB + BC will be same $(\theta_1 = \theta_2 = \theta)$

$$T_1 + T_2 = 20{,}000 \text{ N mm}$$

$$\frac{T}{J} = \frac{C\theta}{L}$$

Shaft AB

$$\frac{T_1}{J_1} = \frac{C_1 \theta_1}{L_1} \; ; \; \theta_1 = \frac{T_1 \times L_1}{J_1 \times C_1} \qquad J_1 = \frac{\pi}{32} d_1^4 = \pi/32 \times 75^4$$

$$\theta_1 = \frac{T_1 \times 20000}{\frac{\pi}{32} \times 75^4 \times 0.3 \times 10^5} = \frac{T_1 \times 2000 \times 32}{\pi \times 75^4 \times 0.3 \times 10^5}$$

Shaft BC

$$\theta_2 = \frac{T_2 \times L_2}{J_2 \times C_2} \qquad J = \pi/32 \times 55^4$$

$$= \frac{T_2 \times 1000}{\pi/32 \times 55^2 \times 0.9 \times 10^5}$$

$$= \frac{T_2 \times 1000 \times 32}{\pi \times 55^4 \times 0.9 \times 10^5}$$

$$\theta_1 = \theta_2$$

$$\frac{T_1 \times 2000 \times 32}{\pi \times 75^4 \times 0.3 \times 10^5} = \frac{T_2 \times 1000 \times 32}{\pi \times 55^4 \times 0.9 \times 10^5}$$

$$\frac{2T_1}{75^4 \times 0.3} = \frac{T_2}{55^4 \times 0.9}$$

$$T_1 = \frac{75^4 \times 0.3}{55^4 \times 0.9 \times 2} T_2$$

$$T_1 = 0.576\, T_2$$

$$T_1 + T_2 = 20{,}000$$

$$0.576\, T_2 + T_2 = 200000$$

$$1.576\, T_2 = 20{,}0000$$

$$T_2 = \frac{200000}{1.576} = 126900 \text{ N mm}$$

$$T_1 + T_2 = 200000$$

$$T_1 = 20000 - T_2 = 200000 - 126900$$

$$= 73100 \text{ N mm.}$$

$$\frac{T}{J} = \frac{\tau}{R}$$

$$\frac{T_1}{J_1} = \frac{\tau_1}{R_1}, \quad \tau_1 = \frac{T_1 \times R_1}{J_1} = \frac{73100 \times 37.5}{\pi/32 \times 75^4}$$

$$\tau_1 = 0.882 \text{ N/mm}^2.$$

13

$$\frac{T_2}{J_2} = \frac{\tau_2}{R_2}$$

$$\tau_2 = \frac{T_2 \times R_2}{J_2} = \frac{126900 \times 27.5}{\pi/32 \times 55^4}$$

$$= \frac{126900 \times 27.5 \times 32}{\pi \times 55^4} = 3.884\ N/mm^2.$$

RESULT

$\tau_1 = 0.882\ N/mm^2$

$\tau_2 = 3.884\ N/mm^2.$

A laminated spring 1m long is made up of plates each 5cm wide and 1cm thick. If the bending stress in the plate is limited to $100\ N/mm^2$, how many plates would be required to enable the spring to carry a central point load of 2kN. If $E = 2.1 \times 10^5\ N/mm^2$ what is the deflection under the load.

Given data

$l = 1m = 1000mm$

$b = 5cm = 50mm$

$t = 1cm = 10mm$

$\sigma = 100\ N/mm^2$

$W = 2kN = 2000N$

$E = 2.1 \times 10^5\ N/mm^2$

To find

n - no of plates

δ - Deflection

$$\sigma = \frac{3wt}{2nbt^2}$$

$$100 = \frac{3\times2000\times1000}{2\times n\times50\times60^2}$$

$$n = \frac{3\times2000\times1000}{100\times2\times50\times100} = 6.$$

Deflection

$$\delta = \frac{\sigma\times l^3}{4E\times t} = \frac{100\times1000^2}{4\times2.1\times10^5\times60} = 11.9\ mm.$$

Result

$n = 6$

$\delta = 11.9\ mm.$

8. An open coil helical spring made of 5mm diameter wire has 16 coils 100 mm inner diameter with helix angle of 16°. Calculate the deflection maximum direct + shear stresses induced due to an axial load of 300N. Take G = 90GPa + E = 200Gpa.

Given data

$d = 5\ mm$

$n = 16$

$D_i = 100\ mm$

$D = D_i + d = 100+5 = 105\ mm$

$R = 52.5\ mm.$

$\alpha = 16^\circ$

$W = 300N$

$E = 200GPa = 200 \times 10^3 N/mm^2$

$G = 90GPa = 90 \times 10^3 N/mm^2$.

To find

Deflection

Maximum direct + Shear Stresses.

Solution

$$\delta = \frac{64WR^3 n \sec\alpha}{d^4}\left[\frac{\cos^2\alpha}{C} + \frac{2\sin^2\alpha}{E}\right]$$

$$= \frac{64 \times 300 \times 52.5^3 \times 16 \times \sec 16^\circ}{5^4}$$

$$= 815.36\ mm.$$

Bending Stress $\sigma_b = \dfrac{32WR\sin\alpha}{\pi d^3}$

$$= \frac{32 \times 300 \times 52.5 \times \sin 16}{\pi \times 5^3}$$

$$= 353.76\ N/mm^2$$

Shear Stress $\tau = \dfrac{16WR\cos\alpha}{\pi d^3}$

$$= \frac{16 \times 300 \times 52.5 \times \cos 16}{\pi \times 5^3}$$

$$\tau = 616.85\ N/mm^2.$$

Maximum Shear Stress

$$\tau_{max} = \frac{16WR}{\pi d^3}$$

$$= \frac{16 \times 300 \times 52.5}{\pi \times 5^3}$$

$$= 641.71 \, N/mm^2.$$

maximum principle stress

$$\sigma_{b1} = \frac{16WR}{\pi d^3} (\sin\alpha + 1)$$

$$= \frac{16 \times 300 \times 52.5}{\pi \times 5^3} (\sin 16^\circ + 1)$$

$$\sigma_{b1} = 818.59 \, N/mm^2.$$

Result

$$\delta = 815.87 mm$$

$$\tau_{max} = 641.71 \, N/mm^2$$

$$\sigma_{b1} = 818.59 \, N/mm^2.$$

UNIT III TORSION			
Torsion formulation stresses and deformation in circular and hollows shafts – Stepped shafts– Deflection in shafts fixed at the both ends – Stresses in helical springs – Deflection of helical springs, carriage springs.			
PART-A(2 Marks)			
Q.No	**Questions**	**BT Level**	**Competence**
1	Define torsional rigidity of the solid circular shaft.	(BT1)	Remember
2	Differentiate between closed coil helical spring and open coil helical spring.	(BT2)	Understand
3	List out of the applications of helical springs.	(BT1)	Remember
4	When the hollow circular shafts are more suitable than solid circular shafts?	(BT1)	Remember
5	Describe the term polar modulus.	(BT1)	Remember
6	Define torsion.	(BT1)	Remember
7	Measure the torque which a shaft of 50 mm diameter can transmit safely, if the allowable shear stress is 75 N/mm^2.	(BT5)	Evaluate
8	Quote the expressions for polar modulus of solid and hollow circular shaft.	(BT1)	Remember
9	Express the stiffness of a close coiled helical spring mathematically.	(BT2)	Understand
10	Summarize the assumptions made in torsional equation.	(BT2)	Understand
11	Give the expression for the angle of twist for a hollow circular shaft with external diameter D, internal diameter, length l and rigidity modulus G.	(BT2)	Understand
12	Calculate the minimum diameter of shaft required to transmit a torque of 29820 Nm if the maximum shear stress is not to exceed 45 N/mm^2.	(BT3)	Application
13	Classify springs with example.	(BT3)	Application
14	Show the difference in stiffness of two springs when they are connected in series and in parallel.	(BT3)	Application
15	Explain the term spring index.	(BT4)	Analyze
16	Point out any two applications of leaf spring.	(BT4)	Analyze
17	Compare helical spring and carriage spring.	(BT4)	Analyze
18	Evaluate the axial deformation, when a load of 50N is acting in the spring of stiffness 10N/mm.	(BT5)	Evaluate
19	Combine the expressions for deflection and shear stress of close coiled spring.	(BT6)	Create
20	Formulate the mathematical expression for deflection of an open coiled helical spring.	(BT6)	Create

	PART-B(13 Marks)			
Q.No	**Questions**	**Marks**	**BT Level**	**Competence**
1	The internal and external diameter of a hollow shaft is in the ratio of 2:3. The hollow shaft is to transmit a 400kW power at 120rpm. The maximum expected torque is 15% greater than the mean value. If the shear stress not to exceed 50MPa, find section of the shaft which would satisfy the shear stress and twist condition. Take G= 0.85×10^5MPa.	(13)	(BT4)	Analyze
2	(a) What are the assumptions made in the torque equations?	(5)	(BT1)	Remember
	(b) Derive the expression for power transmitted by a shaft.	(8)	(BT4)	Analyze
3	(a) A steel shaft is to require to transmit 75kW power at 100 rpm and the maximum twisting moment is 13% greater than the mean. Find the diameter of the steel shaft if the maximum stress is 70N/mm^2. Also determine the angle of twist in a length of 3m of the shaft. Assume the modules of rigidity for steel as 90kN/mm^2.	(7)	(BT3)	Application

	(b) Obtain a relation for the torque and power, a solid shaft can transmit.	(6)	(BT5)	Evaluate
4	(a) Find the diameter of the solid shaft to transmit 90kW at 160 rpm such that the shear stress is limited to $60 N/mm^2$. The maximum torque is likely to exceed the mean torque by 20%. Also find the permissible length of the shaft, if the twist is not to exceed 1° over the entire length. Take rigidity modulus as $0.8\times10^5 N/mm^2$.	(7)	(BT5)	Evaluate
	(b) What do you mean by the strength of the shaft? Compare the strength of solid and hollow circular shafts.	(6)	(BT2)	Understand
5	(a) Determine the dimensions of a hollow circular shaft with a diameter ratio of 3:4 which is to transmit 60kW at 200 rpm. The maximum shear stress in the shaft is limited to 70 GPa and the angle of twist to 3.8° in a length of 4m. For the shaft material, the modulus of rigidity is 80 GPa.	(7)	(BT5)	Evaluate
	(b) Derive the expression for the shear stress produced in a circular solid shaft subjected to torsion.	(6)	(BT4)	Analyze
6	(a) Calculate the power that can be transmitted at 300 rpm by a hollow steel shaft of 75mm external diameter and 50mm internal diameter when the permissible shear stress for the steel is $70 N/mm^2$ and the maximum torque is 1.3 times the mean. Compare the strength of this	(7)	(BT5)	Evaluate
	hollow shaft with that of a solid shaft. The same material, weight and length of both the shafts are same.			
	(b) Derive the expression for angle of twist of two shafts when they are connected in series.	(6)	(BT4)	Analyze
7	A steel shaft ABCD having a total length of 2400 mm is contributed by three different sections as follows. The portion AB is hollow having outside and inside diameters 80 mm and 50 mm respectively, BC is solid and 80 mm diameter. CD is also solid and 70 mm diameter. If the angle of twist is same for each section, determine the length of each portion and the total angle of twist. Maximum permissible shear stress is 50 Mpa and shear modulus 0.82 x 105 MPa	(13)	(BT4)	Analyze
8	(a) The stiffness of the closed coil helical spring at mean diameter 20 cm is made of 3 cm diameter rod and has 16 turns. A weight of 3 KN is dropped on this spring. Find the height by which the weight should be dropped before striking the spring so that the spring may be compressed by 18 cm. Take C= $8\times10^4 N/mm^2$.	(7)	(BT4)	Analyze
	(b) Derive the expression for stiffness of two closed coil helical springs when connected in series.	(6)	(BT3)	Application
9	(a) It is required to design a closed coiled helical spring which shall deflect 1mm under an axial load of 100 N at a shear stress of 90 Mpa. The spring is to be made of round wire having shear modulus of 0.8 x 105 MPa. The mean diameter of the coil is 10 times that of the coil wire. Find the diameter and length of the wire.	(8)	(BT4)	Analyze
	(b) Deduce the expression for strain energy stored in a closed coil helical spring when subjected to axial loading.	(5)	(BT1)	Remember
10	(a) A helical spring of circular cross-section wire 18 mm in diameter is loaded by a force of 500 N. The mean coil diameter of the spring is 125mm. The modulus of rigidity is 80 kN/mm2. Determine the maximum shear stress in the material of the spring. What number of coils must the spring have for its deflection to be 6 mm?	(8)	(BT5)	Evaluate

10	(a) A helical spring of circular cross-section wire 18 mm in diameter is loaded by a force of 500 N. The mean coil diameter of the spring is 125mm. The modulus of rigidity is 80 kN/mm2. Determine the maximum shear stress in the material of the spring. What number of coils must the spring have for its deflection to be 6 mm?	(8)	(BT5)	Evaluate
	(b) Derive the expression for stiffness of two closed coil helical springs when connected in parallel.	(5)	(BT3)	Application
11	A close coiled helical spring is to have a stiffness of 1.5 N/mm of compression under a maximum load of 60 N. the maximum shearing stress produced in the wire of the spring is 125N/mm^2 .The solid length of the spring is 50mm. Find the diameter of coil, diameter of wire and number of coils .C = 4.5 xl04N/mm^2.	(13)	(BT3)	Application
12	A closely coiled helical spring of round steel wire 10 mm in diameter having 10 complete turns with a mean diameter of 12 cm is subjected to an axial load of 250 N. Determine a) The deflection of the spring b) Maximum shear stress in the wire c) Stiffness of the spring and d) Frequency of vibration. Take C = 0.8 x 105 N/mm^2	(13)	(BT4)	Analyze
13	A leaf spring of semi elliptical type has 10 plates, each 60mm wide and 5 mm thick. The longest plate is 700 mm long. Find the greatest central load on the spring so that the bending stress shall not exceed 150 N/mm^2 and the central deflection shall not exceed 10 mm. take E=2×10^5 N/mm^2.	(13)	(BT4)	Analyze
14	A leaf spring is made of 12 steel plates of 50mm wide and 5mm thick. It carries a load of 4 kN at the centre. If the bending stress is limited to 140 N/mm^2, determine the following: i) Length of the spring and ii) Deflection at the centre of the spring. Take E = 2 x 10^5 N/mm^2.	(13)	(BT3)	Application

	PART-C(15 Marks)			
Q.No	**Questions**	**Marks**	**BT Level**	**Competence**
1	A hollow shaft with diameter ratio 3/5 is required to transmit 450kW at 120rpm. The shearing stress in the shaft must not exceed 60N/mm^2 and the twist in a length of 2.5m is not to exceed 1°. Calculate the maximum external diameter of the shaft. C= 80 kN/mm^2.	(15)	(BT5)	Evaluate
2	A close coiled helical spring is required to absorb 2250 joules of energy. Determine the diameter of the wire, the mean coil diameter of the wire, the mean coil diameter of the spring and the number of coils necessary if i) the maximum stress is not to exceed 400MPa, ii) the maximum compression of the spring is limited to 250mm and iii) the mean diameter of the spring is eight times the wire diameter. For the spring material, rigidity modulus is 70GPa.	(15)	(BT4)	Analyze
3	A solid shaft is to transmit 300kW at 100 rpm if the shear stress is not to exceed 80N/mm2. Find diameter of the shaft. If this shaft was to be replaced by hollow shaft of same material and length with an internal diameter of 0.6 times the external diameter. What percentage saving in weight is possible?	(15)	(BT6)	Create
4	A close coiled helical spring has stiffness of 10N/mm. Its length when fully compressed with adjacent coils touching each other is 400mm. The modulus of rigidity of the material of the spring is 80GPa. i) Determine the wire diameter and mean coil diameter if their ratio is 1/10. ii) If the gap between any two adjacent coils is 2mm, what maximum load can be applied before the spring becomes solid. iii) What is the corresponding maximum shear stress in the spring?	(5) (5) (5)	(BT5)	Evaluate

UNIT IV DEFLECTION OF BEAMS

Double Integration method – Macaulay's method – Area moment method for computation of slopes and deflections in beams - Conjugate beam and strain energy – Maxwell's reciprocal theorems.

OBJECTIVE :

To compute slopes and deflections in determinate beams by various methods.

OUTCOMES :

Students will be able to

Calculate the slope and deflection in beams using different methods

DEFLECTION OF BEAMS

Elastic curve of neutral axis

Assuming that the I-beam is symmetric, the neutral axis will be situated at the midsection of the beam. The neutral axis is defined as the point in a beam where there is neither tension nor compression forces. So if the beam is loaded uniformly from above, any point above the neutral axis will be in compression, whereas any point below it will be in tension .

However, if the beam is NOT symmetric, then you will have to use the following methodology to calculate the position of the neutral axis.

1. Calculate the total cross-sectional area of the beam (we shall call this A). Let x denote the position of the neutral axis from the topmost edge of the top flange of the beam .

2. Divide the I-beam into rectangles and find the area of these rectangles (we shall denote these areas as A1, A2, and A3 for the top flange, web and bottom flange respectively). Additionally, find the distance from the edge of the top flange to the midsection of these 3 rectangles (these distances will be denoted as x1, x2 and x3)

3. Now, to find the position of the neutral axis, the following general formula must be used:

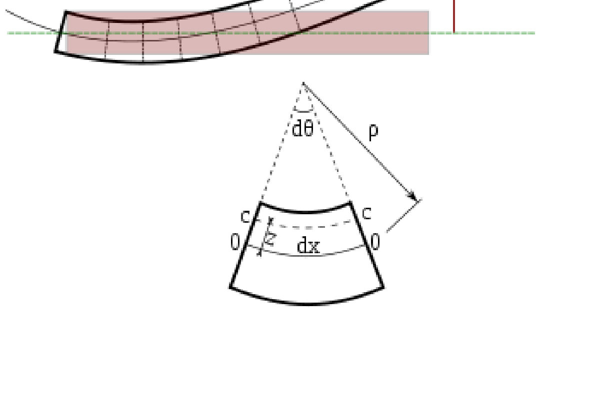
A*x = A1*x1 + A2*x2 + A3*x3

We know all the variables in the above formula, except for x (the position of the neutral axis from the top edge of the top flange). So it is just a case of rearranging the formula to find x.

4.1 Evaluation of beam deflection and slope

Beam deflection

Static beam equation



Bending of an Euler-Bernoulli beam. Each cross-section of the beam is at 90 degrees to the neutral axis.

The Euler-Bernoulli equation describes the relationship between the beam's deflection and the applied load

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(EI\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\right) = q.$$

The curve $w(x)$ describes the deflection w of the beam at some position x (recall that the beam is modeled as a one-dimensional object). q is a distributed load, in other words a force per unit length (analogous to pressure being a force per area); it may be a function of x, w, or other variables.

Note that E is the elastic modulus and that I is the second moment of area. I must be calculated with respect to the centroidal axis perpendicular to the applied loading. For an Euler-Bernoulli beam not under any axial loading this axis is called the neutral axis.

Often, $w = w(x)$, $q = q(x)$, and EI is a constant, so that:

$$EI\frac{\mathrm{d}^4 w}{\mathrm{d}x^4} = q(x).$$

This equation, describing the deflection of a uniform, static beam, is used widely in engineering practice. Tabulated expressions for the deflection $w$for common beam configurations can be found in engineering handbooks. For more complicated situations the deflection can be determined by solving the Euler-Bernoulli equation using techniques such as the "slope deflection method", "moment distribution method", "moment area method, "conjugate beam method", "the principle of virtual work", "direct integration", "Castigliano's method", "Macaulay's method" or the "direct stiffness method".

Successive derivatives of w have important meanings:

- w is the deflection.
- $\frac{\mathrm{d}w}{\mathrm{d}x} = \varphi$ is the slope of the beam.
- $-EI\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} = M$ is the bending moment in the beam.
- $-\frac{\mathrm{d}}{\mathrm{d}x}\left(EI\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\right) = Q$ is the shear force in the beam.

The stresses in a beam can be calculated from the above expressions after the deflection due to a given load has been determined.

A number of different sign conventions can be found in the literature on the bending of beams and care should be taken to maintain consistency.[6] In this article, the sign convention has been chosen so the coordinate system is right handed. Forces acting in the positive *x* and *z* directions are assumed positive. The sign of the bending moment is chosen so that a positive value leads to a tensile stress at the bottom cords. The sign of the shear force has been chosen such that it matches the sign of the bending moment.

Double integration method

The double integration method is a powerful tool in solving deflection and slope of a beam at any point because we will be able to get the equation of the elastic curve.

Thus, EI / M = 1 / y''

.

Macaulay Method

The starting point for Maucaulay's method is the relation between bending moment and curvature from Euler-Bernoulli beam theory

$$\pm EI\frac{d^2w}{dx^2} = M$$

This equation[7] is simpler than the fourth-order beam equation and can be integrated twice to find *w* if the value of *M* as a function of *x* is known. For general loadings, *M* can be expressed in the form

$$M = M_1(x) + P_1\langle x - a_1\rangle + P_2\langle x - a_2\rangle + P_3\langle x - a_3\rangle + \dots$$

where the quantities $P_i\langle x - a_i\rangle$ represent the bending moments due to point loads and the quantity $\langle x - a_i\rangle$ is a Macaulay bracket defined as

$$\langle x - a_i\rangle = \begin{cases} 0 & \text{if } x < a_i \\ x - a_i & \text{if } x > a_i \end{cases}$$

Ordinarily, when integrating *P*(*x* - *a*) we get

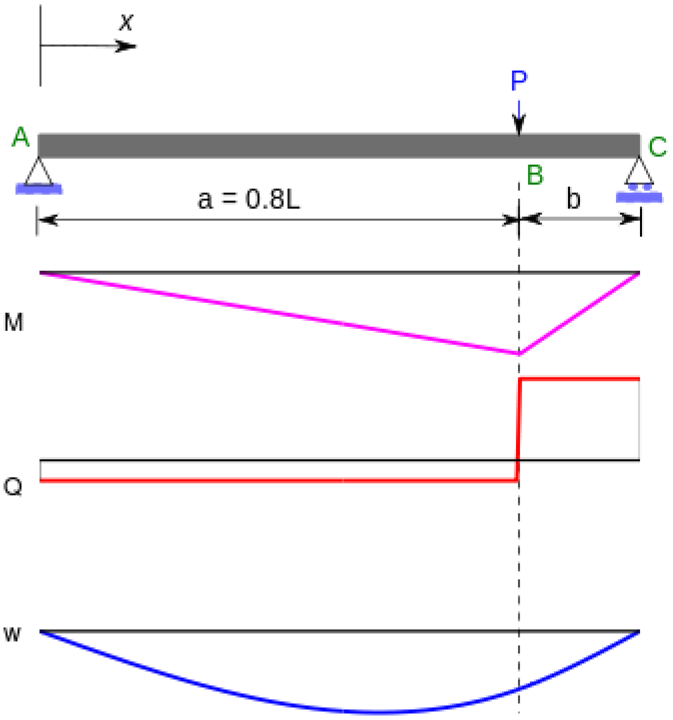
$$\int P(x - a)\ dx = P\left[\frac{x^2}{2} - ax\right] + C$$

However, when integrating expressions containing Macaulay brackets, we have

$$\int P\langle x-a\rangle \, dx = P\frac{\langle x-a\rangle^2}{2} + C_m$$

with the difference between the two expressions being contained in the constant C_m. Using these integration rules makes the calculation of the deflection of Euler-Bernoulli beams simple in situations where there are multiple point loads and point moments. The Macaulay method predates more sophisticated concepts such as Dirac delta functions and step functions but achieves the same outcomes for beam problems.

Example: Simply supported beam with point load



Simply supported beam with a single eccentric concentrated load.

An illustration of the Macaulay method considers a simply supported beam with a single eccentric concentrated load as shown in the adjacent figure. The first step is to find *M*. The reactions at the supports A and C are determined from the balance of forces and moments as

$$R_A + R_C = P, \quad LR_C = Pa$$

Therefore $R_A = Pb / L$ and the bending moment at a point D between A and B ($0 < x < a$) is given by

$M = R_A x = Pbx / L$

Using the moment-curvature relation and the Euler-Bernoulli expression for the bending moment, we have

$$EI\frac{d^2w}{dx^2} = \frac{Pbx}{L}$$

Integrating the above equation we get, for $0 < x < a$,

$$EI\frac{dw}{dx} = \frac{Pbx^2}{2L} + C_1 \qquad \text{(i)}$$
$$EIw = \frac{Pbx^3}{6L} + C_1x + C_2 \qquad \text{(ii)}$$

At $x = a_-$

$$EI\frac{dw}{dx}(a_-) = \frac{Pba^2}{2L} + C_1 \qquad \text{(iii)}$$
$$EIw(a_-) = \frac{Pba^3}{6L} + C_1a + C_2 \qquad \text{(iv)}$$

For a point D in the region BC ($a < x < L$), the bending moment is

$M = R_A x - P(x - a) = Pbx / L - P(x - a)$

In Macaulay's approach we use the Macaulay bracket form of the above expression to represent the fact that a point load has been applied at location B, i.e.,

$$M = \frac{Pbx}{L} - P\langle x - a\rangle$$

Therefore the Euler-Bernoulli beam equation for this region has the form

$$EI\frac{d^2w}{dx^2} = \frac{Pbx}{L} - P\langle x - a\rangle$$

Integrating the above equation, we get for $a < x < L$

$$EI\frac{dw}{dx} = \frac{Pbx^2}{2L} - P\frac{\langle x-a\rangle^2}{2} + D_1 \qquad \text{(v)}$$

$$EIw = \frac{Pbx^3}{6L} - P\frac{\langle x-a\rangle^3}{6} + D_1x + D_2 \qquad \text{(vi)}$$

At $x = a_+$

$$EI\frac{dw}{dx}(a_+) = \frac{Pba^2}{2L} + D_1 \qquad \text{(vii)}$$

$$EIw(a_+) = \frac{Pba^3}{6L} + D_1a + D_2 \qquad \text{(viii)}$$

Comparing equations (iii) & (vii) and (iv) & (viii) we notice that due to continuity at point B, $C_1 = D_1$ and $C_2 = D_2$. The above observation implies that for the two regions considered, though the equation for bending moment and hence for the curvature are different, the constants of integration got during successive integration of the equation for curvature for the two regions are the same.

The above argument holds true for any number/type of discontinuities in the equations for curvature, provided that in each case the equation retains the term for the subsequent region in the form $\langle x-a\rangle^n, \langle x-b\rangle^n, \langle x-c\rangle^n$ etc. It should be remembered that for any x, giving the quantities within the brackets, as in the above case, -ve should be neglected, and the calculations should be made considering only the quantities which give +ve sign for the terms within the brackets.

Reverting back to the problem, we have

$$EI\frac{d^2w}{dx^2} = \frac{Pbx}{L} - P\langle x-a\rangle$$

It is obvious that the first term only is to be considered for $x < a$ and both the terms for $x > a$ and the solution is

$$EI\frac{dw}{dx} = \left[\frac{Pbx^2}{2L} + C_1\right] - \frac{P\langle x-a\rangle^2}{2}$$

$$EIw = \left[\frac{Pbx^3}{6L} + C_1x + C_2\right] - \frac{P\langle x-a\rangle^3}{6}$$

Note that the constants are placed immediately after the first term to indicate that they go with the first term when $x < a$ and with both the terms when $x > a$. The Macaulay brackets help as a reminder that the quantity on the right is zero when considering points with $x < a$.

4.2 Moment area method

Theorems of Area-Moment Method

Theorem I

The change in slope between the tangents drawn to the elastic curve at any two points A and B is equal to the product of 1/EI multiplied by the area of the moment diagram between these two points.

Theorem II

The deviation of any point B relative to the tangent drawn to the elastic curve at any other point A, in a direction perpendicular to the original position of the beam, is equal to the product of 1/EI multiplied by the moment of an area about B of that part of the moment diagram between points A and B.

Rules of Sign

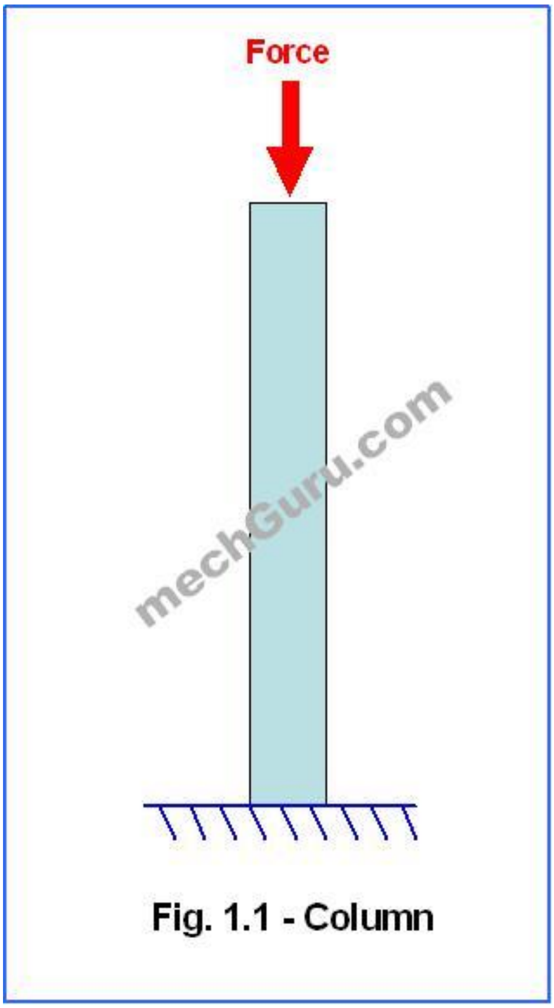
1. The deviation at any point is positive if the point lies above the tangent, negative if the point is below the tangent.
2. Measured from left tangent, if ? is counterclockwise, the change of slope is positive, negative if ? is clockwise.

Columns – End conditions

Columns -end conditions

What is a Column or Strut?

Any machine member, subjected to the axial compressive loading is called a strut and the vertical strut is called column



The columns are generally categorized in two types: **short** columns and **long** columns. The one with length less than eight times the diameter (or approximate diameter) is called short column and the one with length more than thirty times the diameter (or approximate diameter) is called long column.

Ideally, the columns should fail by crushing or compressive stress and it normally happens for the short columns, however, the long columns, most of the times, failure occurs by buckling.

Euler's Buckling Formula

To get the correct results, this formula should only be applied for the long columns. The buckling load calculated by the Euler formula is given by:

Fbe = (C*?^{2}*E*I)/

Equivalent length of a column

Strength Of Columns

A stick of timber, a bar of iron, etc., when used to sustain end loads which act lengthwise of the pieces, are called columns, posts, or struts if they are so long that they would bend before breaking. When they are so short that they would not bend before breaking, they are called short blocks, and their compressive strengths are computed by means of equation 1. The strengths of columns cannot, however, be so simply determined, and we now proceed to explain the method of computing them.

77. End Conditions. The strength of a column depends in part on the way in which its ends bear, or are joined to other parts of a structure, that is, on its " end conditions." There are practically but three kinds of end conditions, namely:

1. "Hinge" or "pin" ends,

2. " Flat" or " square " ends, and

3. "Fixed" ends.

(1) When a column is fastened to its support at one end by means of a pin about which the column could rotate if the other end were free, it is said to be "hinged" or "pinned" at the former end. Bridge posts or columns are often hinged at the ends.

(2) A column either end of which is flat and perpendicular to its axis and bears on other parts of the structure at that surface, is said to be "flat" or " square" at that end.

(3) Columns are sometimes riveted near their ends directly to other parts of the structure and do not bear directly on their ends; such are called " fixed ended." A column which bears on its flat ends is often fastened near the ends to other parts of the structure, and such an end is also said to be " fixed." The fixing of an end of a column stiffens and therefore strengthens it more or less, but the strength of a column with fixed ends is computed as though its ends were flat. Accordingly we have, so far as strength is concerned, the following classes of columns:

78. Classes of Columns. (1) Both ends hinged or pinned; (2) one end hinged and one flat; (3) both ends flat.

Other things being the same, columns of these three classes are unequal in strength. Columns of the first class are the weakest, and those of the third class are the strongest.

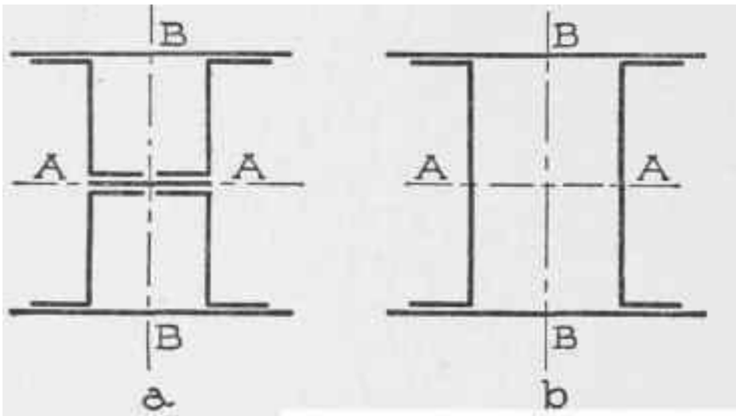


Fig. 46.

70. Cross=sections of Columns. Wooden columns are usually solid, square, rectangular, or round in section; but sometimes they are "built up" hollow. Cast-iron columns are practically always made hollow, and rectangular or round in section. Steel columns are made of single rolled shapes - angles, zees, channels, etc.; but the larger ones are usually "built up" of several shapes. Fig. 46, a, for example, represents a cross-section of a "Z-bar" column; and Fig. 46, b, that of a "channel" column.

80. Radius of Gyration. There is a quantity appearing in almost all formulas for the strength of columns, which is called "radius of gyration." It depends on the form and extent of the cross-section of the column, and may be defined as follows:

The radius of gyration of any plane figure (as the section of a column) with respect to any line, is such a length that the square of this length multiplied by the area of the figure equals the moment of inertia of the figure with respect to the given line.

Thus, if A denotes the area of a figure; I, its moment of inertia with respect to some line; and r, the radius: of gyration with respect to that line; then

$$r^2\mathrm{A} = \mathrm{I}; \text{ or } r = \sqrt{\mathrm{I} \div \mathrm{A}}.$$

(9)

In the column formulas, the radius of gyration always refers to an axis through the center of gravity of the cross-section, and usually to that axis with respect to which the radius of gyration (and moment of inertia) is least. (For an exception, see example 3. Art. 83.) Hence the radius of gyration in this connection is often called for brevity the "least radius of gyration," or simply the "least radius."

Examples. 1. Show that the value of the radius of gyration given for the square in Table A, page 54, is correct.

The moment of inertia of the square with respect to the axis is 1/12 a4- Since A = a2, then, by formula 9 above,

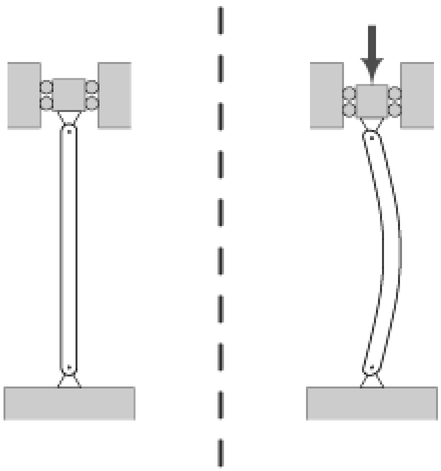
$$r = \sqrt{\frac{1}{12}a^4 \div a^2} = \sqrt{\frac{1}{12}a^2} = a\sqrt{\frac{1}{12}}.$$

2. Prove that the value of the radius of gyration given for the hollow square in Table A, page 54, is correct.

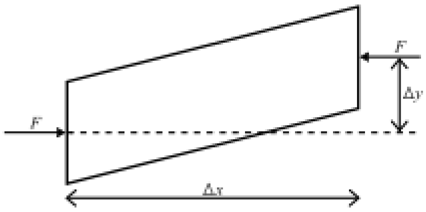
The value of the moment of inertia of the square with respect to the axis is 1/12 (a4 - a1 4). Since A = a2 - a12,

$$r = \sqrt{\frac{\frac{1}{12}(a^4 - a_1^4)}{a^2 - a_1^2}} = \sqrt{\frac{1}{12}(a^2 + a_1^2)}.$$

4.3 Euler equation



A column under a concentric axial load exhibiting the characteristic deformation of buckling



The eccentricity of the axial force results in a bending moment acting on the beam element.

The ratio of the effective length of a column to the least radius of gyration of its cross section is called the *slenderness ratio* (sometimes expressed with the Greek letter lambda, ?). This ratio affords a means of classifying columns. Slenderness ratio is important for design considerations. All the following are approximate values used for convenience.

- A short steel column is one whose slenderness ratio does not exceed 50; an intermediate length steel column has a slenderness ratio ranging from about 50 to 200, and are dominated by the strength limit of the material, while a long steel column may be assumed to have a slenderness ratio greater than 200.
- A short concrete column is one having a ratio of unsupported length to least dimension of the cross section not greater than 10. If the ratio is greater than 10, it is a long column (sometimes referred to as a slender column).
- Timber columns may be classified as short columns if the ratio of the length to least dimension of the cross section is equal to or less than 10. The dividing line between intermediate and long timber columns cannot be readily evaluated. One way of defining the lower limit of long timber columns would be to set it as the smallest value of the ratio of length to least cross sectional area that would just exceed a certain constant K of the material. Since K depends on the modulus of elasticity and the allowable compressive stress parallel to the grain, it can be seen that this arbitrary limit would vary with the species of the timber. The value of K is given in most structural handbooks.

If the load on a column is applied through the center of gravity of its cross section, it is called an axial load. A load at any other point in the cross section is known as an eccentric load. A short column under the action of an axial load will fail by direct compression before it buckles, but a long column loaded in the same manner will fail by buckling (bending), the buckling effect being so large that the effect of the direct load may be neglected. The intermediate-length column will fail by a combination of direct compressive stress and bending.

In 1757, mathematician Leonhard Euler derived a formula that gives the maximum axial load that a long, slender, ideal column can carry without buckling. An ideal column is one that is perfectly straight, homogeneous, and free from initial stress. The maximum load, sometimes called the critical load, causes the column to be in a state of unstable equilibrium; that is, the introduction of the slightest lateral force will cause the column to fail by buckling. The formula derived by Euler for columns with no consideration for lateral forces is given below. However, if lateral forces are taken into consideration the value of critical load remains approximately the same.

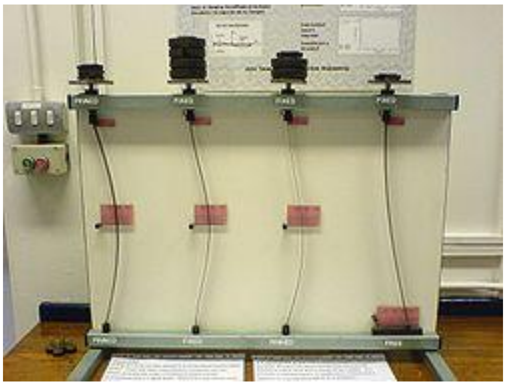
$$F = \frac{\pi^2 EI}{(KL)^2}$$

where

F = maximum or critical force (vertical load on column),
E = modulus of elasticity,
I = area moment of inertia,
L = unsupported length of column,
K = column effective length factor, whose value depends on the conditions of end support of the column, as follows.
For both ends pinned (hinged, free to rotate), *K* = 1.0.
For both ends fixed, *K* = 0.50.
For one end fixed and the other end pinned, *K* = 0.699....
For one end fixed and the other end free to move laterally, *K* = 2.0.
KL is the effective length of the column.

Examination of this formula reveals the following interesting facts with regard to the load-bearing ability of slender columns.

1. Elasticity and not compressive strength of the materials of the column determines the critical load.
2. The critical load is directly proportional to the second moment of area of the cross section.
3. The boundary conditions have a considerable effect on the critical load of slender columns. The boundary conditions determine the mode of bending and the distance between inflection points on the deflected column. The closer together the inflection points are, the higher the resulting capacity of the column.



A demonstration model illustrating the different "Euler" buckling modes. The model shows how the boundary conditions affect the critical load of a slender column. Notice that each of the columns are identical, apart from the boundary conditions.

The strength of a column may therefore be increased by distributing the material so as to increase the moment of inertia. This can be done without increasing the weight of the column by distributing the material as far from the principal axis of the cross section as possible, while keeping the material thick enough to prevent local buckling. This bears out the well-known fact that a tubular section is much more efficient than a solid section for column service.

Another bit of information that may be gleaned from this equation is the effect of length on critical load. For a given size column, doubling the unsupported length quarters the allowable load. The restraint offered by the end connections of a column also affects the critical load. If the connections are perfectly rigid, the critical load will be four times that for a similar column where there is no resistance to rotation (hinged at the ends).

Since the moment of inertia of a surface is its area multiplied by the square of a length called the radius of gyration, the above formula may be rearranged as follows. Using the Euler formula for hinged ends, and substituting $A \cdot r^2$ for I, the following formula results.

$$\sigma = \frac{F}{A} = \frac{\pi^2 E}{(\ell/r)^2}$$

where *F* / *A* is the allowable stress of the column, and *l* / *r* is the slenderness ratio.

Since structural columns are commonly of intermediate length, and it is impossible to obtain an ideal column, the Euler formula on its own has little practical application for ordinary design. Issues that cause deviation from the pure Euler strut behaviour include imperfections in geometry in combination with plasticity/non-linear stress strain behaviour of the column's material. Consequently, a number of empirical column formulae have been developed to agree with test data, all of which embody the slenderness ratio. For design, appropriate safety factors are introduced into these formulae. One such formular is the Perry Robertson formula which estimates of the critical buckling load based on an initial (small) curvature. The Rankine Gordon fomular is also based on eperimental results and surgests that a strut will buckle at a load Fmax given by:

$$\frac{1}{Fmax} = \frac{1}{Fe} + \frac{1}{Fc}$$

where Fe is the euler maximum load and Fc is the maximum compresive load. This formular typically produces a conservative estimate of Fmax.

Self-buckling

A free-standing, vertical column, with density ?, Young's modulus *E*, and radius *r*, will buckle under its own weight if its height exceeds a certain critical height:[1][2][3]

$$h_{crit} = \left(\frac{9B^2}{4} \frac{EI}{\rho g \pi r^2} \right)^{1/3}$$

where *g* is the acceleration due to gravity, *I* is the second moment of area of the beam cross section, and *B* is the first zero of the Bessel function of the first kind of order -1/3, which is equal to 1.86635...

Slenderness ratio

Euler's Theory : The struts which fail by buckling can be analyzed by Euler's theory. In the following sections, different cases of the struts have been analyzed.

Case A: Strut with pinned ends:

Consider an axially loaded strut, shown below, and is subjected to an axial load „P' this load „P' produces a deflection „y' at a distance „x' from one end. Assume that the ends are either pin jointed or rounded so that there is no moment at either end.

Assumption:

The strut is assumed to be initially straight, the end load being applied axially through centroid.

In this equation „M' is not a function „x'. Therefore this equation can not be integrated directly as has been done in the case of deflection of beams by integration method.

Though this equation is in „y' but we can't say at this stage where the deflection would be maximum or minimum.

So the above differential equation can be arranged in the following form

Let us define a operator

D = d/dx

(D2 + n2) y =0 where n2 = P/EI

This is a second order differential equation which has a solution of the form consisting of complimentary function and particular integral but for the time being we are interested in the complementary solution only[in this P.I = 0; since the R.H.S of Diff. equation = 0]

Thus y = A cos (nx) + B sin (nx)

Where A and B are some constants.

Therefore

In order to evaluate the constants A and B let us apply the boundary conditions,

(i) at x = 0; y = 0

(ii) at x = L ; y = 0

Applying the first boundary condition yields A = 0.

Applying the second boundary condition gives

From the above relationship the least value of P which will cause the strut to buckle, and it is called the " **Euler Crippling Load** " Pe from which w obtain.

The interpretation of the above analysis is that for all the values of the load P, other than those which make sin nL = 0; the strut will remain perfectly straight since

y = B sin nL = 0

For the particular value of

Then we say that the strut is in a state of neutral equilibrium, and theoretically any deflection which it suffers will be maintained. This is subjected to the limitation that „L' remains sensibly constant and in practice slight increase in load at the critical value will cause the deflection to increase appreciably until the material fails by yielding.

Further it should be noted that the deflection is not proportional to load, and this applies to all strut problems; like wise it will be found that the maximum stress is not proportional to load.

The solution chosen of nL = p is just one particular solution; the solutions nL= 2p, 3p, 5p etc are equally valid mathematically and they do, infact, produce values of „Pe' which are equally valid for modes of buckling of strut different from that of a simple bow. Theoretically therefore, there are an infinite number of values of Pe , each corresponding with a different mode of buckling.

The value selected above is so called the fundamental mode value and is the lowest critical load producing the single bow buckling condition.

The solution nL = 2p produces buckling in two half – waves, 3p in three half-waves etc.

If load is applied sufficiently quickly to the strut, then it is possible to pass through the fundamental mode and to achieve at least one of the other modes which are theoretically possible. In practical loading situations, however, this is rarely achieved since the high stress associated with the first critical condition generally ensures immediate collapse.

Unit - IV

BEAM DEFLECTION

PART A

1. **Write the maximum value of deflection for a cantilever beam of length L, constant EI and carrying concentrated load W at the end.**

   Maximum deflection at the end of a cantilever due to the load =WL3/3El

2. **What are the different methods used for finding deflection and slope of beams?**

   Double integration method

   Mecaulay‘s method

   Strain energy method

   Moment area method

   Unit load method

3. **State the two theorems in moment area method. (May 2014)**

   Mohr‘s Theorem-I: the angle between tangents at any two points A and B onThe bend beam is equal to total area of the corresponding position of the bending moment diagram divided by EI.

   Mohr‘s Theorem-II: The deviation of B from the tangent at A is equal to the statically moment of the B.M.D. area between A and B with respect to B divided by EI.

4. **What is meant by elastic curve?**

   The deflected shape of a beam under load is called elastic curve of the beam, Within elastic limit.

5. **When Macaulay‘s method is preferred?**

   This method is preferred for determining the deflections of a beam subjected to several concentrated loads or a discontinuous load.

6. **What are the boundary conditions for a cantilever beam?**

   The boundary conditions for a cantilever beam are:

   (i) Deflection at the fixed end is zero.

   (ii)Slope is zero at the fixed end.

7. **What is meant by Double-Integration method? (May 2013)**

Double-integration method is a method of finding deflection and slope of a Bent beam. In this method the differential equation of curvature of bent beam, $EI\ d^2y/dx^2=M$

M is integrated once to get slope and twice to get deflection. Here the constants of integration C1 and C2 are evaluated from known boundary condition.

8. What is Modulus of resilience? (May/Jun 2013)

It is the proof resilience of the material per unit volume.

Modulus of resilience= proof resilience / Volume of the body

9. What are the limitations of double integration method? (Dec 2014)

1. Double integration method can be used only for beams with uniform cross section
2. It is useful only in cases where there is no change in loading.

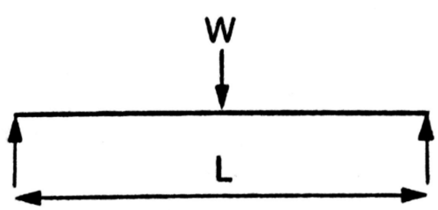
10. Define strain energy. (Dec 2014)

Strain energy is the energy absorbed or stored by a member when work is done on it to deform it.

11. State Maxwell's reciprocal theorem.

The work done by the first system of loads due to displacements caused by a second system of loads equal the work done by the second system of loads due to displacemrnts caused by the first system of loads.

12. Write down the equation for the maximum deflection of a cantilever beam carring a central point load W.



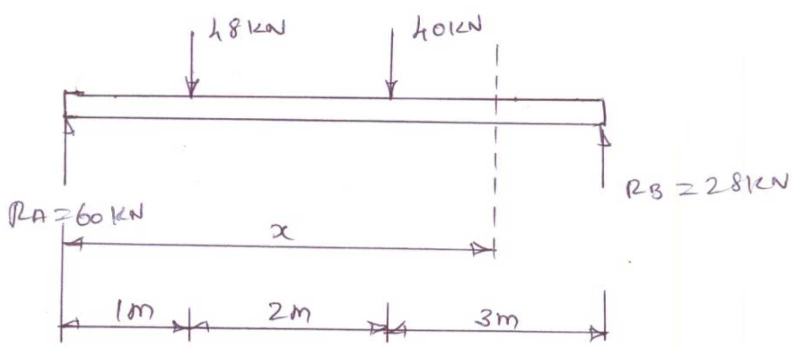
$$\text{DEFLECTION } y_C = \frac{W\ L^3}{48\ E\ I}$$

UNIT-IV

DEFLECTION OF BEAMS

PART B

1. A beam of length 6m is simply supported at the ends and carries two point loads of 48 KN and 40 KN at a distance of 1 m and 3 m respectively from the left support. Compute the slope and deflection under each loads. Assume $EI = 17000 \text{ KN-m}^2$.



Solution:

The free body diagram is shown.

$$\Sigma M_A = 0$$

$$-48(1) - 40(3) + R_B(6) = 0$$

$$R_B = 28 \text{ KN}.$$

$$\Sigma f_y = 0$$

$$R_A - 48 - 40 + R_B = 0$$

$$R_A = 60 \text{ KN}.$$

Take section portion CD at distance x from A as shown in fig.

$$EI \frac{d^2y}{dx^2} = M$$

$$EI\frac{d^2y}{dx^2} = 60x \;\Big|\; -48(x-1) \;\Big|\; -40(x-3 \;\Big|\; KNm.$$

$$EI\frac{dy}{dx} = 30x^2 + C_1 \;\Big|\; -24(x-1)^2 \;\Big|\; -20(x-3)^2 \;\Big|\; KNm^2$$

$$EIy = 10x^3 + C_1x + C_2 \;\Big|\; -8\,(x-1)^3 \;\Big|\; \frac{-20(x-3)^3}{3} \;\Big|\; KNm^2$$

At $x=0, y=0,$

$$0 = c^2$$

At $x=6m, y=0,$

$$0 = 10\times6^3 + C_1\times6 - 8\,(6-1)^3 - \frac{20(6-3)^3}{3}$$

$$C_1 = -163.33$$

At $c, x = 1m$

$$EI\left(\frac{dy}{dx}\right)_C = 30\times1^2 - 163.33$$

$$EI\left(\frac{dy}{dx}\right)_C = -133.33\ KNm^2$$

$$EI = 17000\ KNm^2.$$

$$17000\left(\frac{dy}{dx}\right)_C = -133.33$$

$$\left(\frac{dy}{dx}\right)_C = -7.843\times10^{-3}\ radians.$$

$$\left(\frac{dy}{dx}\right)_C = -0.45^\circ \Rightarrow \left(\frac{dy}{dx}\right)_C = 0.45^\circ$$

At $C, x = 3m$

$$17000\left(\frac{dy}{dx}\right)_D = 30\times3^2 - 163.33 - 24\,(3-1)^2$$

$$\left(\frac{dy}{dx}\right)_D = -6.276 \times 10^{-4} \text{ rad}$$

$$\left(\frac{dy}{dx}\right)_D = -0.036^\circ.$$

$$\left(\frac{dy}{dx}\right)_D = 0.036^\circ$$

At C,

$$17000\, y_c = 10 \times 1^3 - 163.33 \times 1$$

$$y_c = -9.02 \times 10^{-3} \text{ m}$$

$$y_c = 9.02 \text{ mm} \downarrow$$

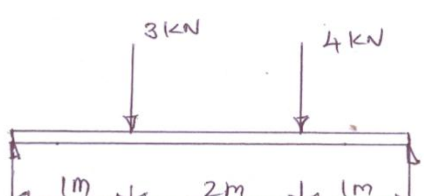
At D,

$$17000\, y_D = 10 \times 3 - 163.33 \times 3 - 8$$

$$D = -0.0167 \text{ m}$$

$$y_D = 16.7 \text{ mm}.$$

2. Using Conjugate beam method, determine the (i) slope at each end and under each load.
(ii) Deflection under each load. For the given beam shown in figure. Take $E = 2 \times 10^5 \text{ N/mm}^2$ and $I = 10^8 \text{ mm}^4$.



Solution:

$$\Sigma M_A = 0$$

$$-3(1) - 4(3) + R_D(4) = 0$$

$$R_D = 3.75 \text{ KN}$$

$\Sigma fy = 0$

$RA - 3 - 4 + RD = 0$

$RA = 3.25\ kN$

$EI = 2 \times 10^5 \times 10^8$

$= 2 \times 10^{13}\ N mm^2$

$= 2 \times 10^{4}\ kN/m^2$.

The beam, SFD, $\frac{M}{EI}$ diagram and Conjugate beam are shown in fig ①.

At A; $\frac{M}{EI} = 0$.

At B, $\frac{M}{EI} = \frac{0 + 3.25 + 1}{2 \times 10^4} = 1.625 \times 10^{-4}\ m^{-1}$

At C $\frac{M}{EI} = \frac{3.25 + 0.25 \times 2}{2 \times 10^4} = 1.875 \times 10^{-4}\ m^{-1}$

At D, $\frac{M}{EI} = 0$.

for Conjugate Beam,

$\Sigma MA = 0$

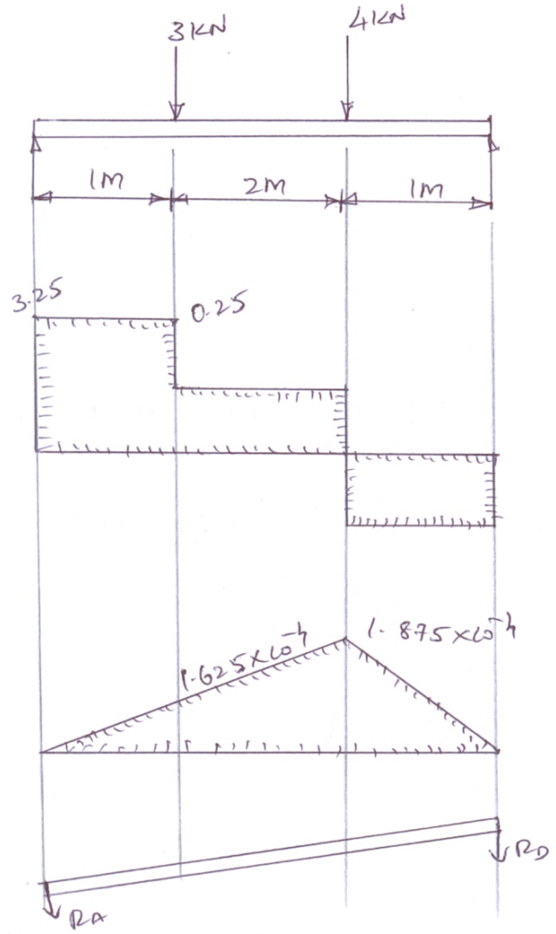
$$-R_D \times 4 + \left(\tfrac{1}{2} \times 1 \times 1.625 \times 10^{-4}\right) \times \left(\tfrac{2}{3}\right) + \left(1.625 \times 10^{-4} \times 2\right) \times 2$$
$$+ \left(\tfrac{1}{2} \times 2 \times 0.25 \times 10^{-4}\right) \times \left(1 + \tfrac{4}{3}\right) + \left(\tfrac{1}{2} \times 1 \times 1.875 \times 10^{-4}\right) \times \left(3 + \tfrac{1}{3}\right) = 0.$$

$R_D = 2.6875 \times 10^{-4} \downarrow$

$\Sigma fy = 0$

$$-R_A - R_D + \tfrac{1}{2} \times 1 \times 1.625 \times 10^{-4} + 1.625 \times 10^{-4} \times 2 + \tfrac{1}{2} \times 2 \times 0.25 \times 10^{-4} \times \tfrac{1}{2} \times 1 \times 1.875 \times 10^{-4} = 0$$

$R_A = 2.5625 \times 10^{-4}$



$\frac{M}{EI}$ diagram

Slope at A = Shear force at A on Conjugate beam,

$= -2.5625 \times 10^{-4}$ rad

$= -0.0147$

$A = 0.0147°$

Slope at D = shear force at D on Conjugate beam

$= +2.6875 \times 10^{-4}$ rad

$= 0.0154°$

$D = 0.0154°$

Slope at B = Shear force at B on Conjugate beam

$= -2.5625 \times 10^{-4} \times 1/2 \times 1 \times 1.625 \times 10^{-4}$

$= -1.75 \times 10^{-4}$ rad

$= -0.01°$

$B = 0.01° \circlearrowright$

Slope at C = Shear force at C on Conjugate beam

$= 2.6875 \times 10^{-4} - 1/2 \times 1 \times 1.875$

$= 1.75 \times 10^{-4}$ rad

$= 0.01°$.

$C = 0.01° \circlearrowleft$

Deflection at B = Bending moment at B on Conjugate beam

$y_B = -2.5625 \times 10^{-4} \times 1 + 1/2 \times 1 \times 1.625 \times 10^{-4} \times 1/3$

$= -2.2914 \times 10^{-4}$ m

$= -0.229$ mm.

$y_B = 0.229$ mm $\downarrow$

Deflection at C = Bending moment at C on Conjugate beam

$y_C = -2.6875 \times 10^{-4} \times 1 + (1/2 \times 1 \times 1.875 \times 10^{-4}) \times 1/3$

$= -2.375 \times 10^{-4}$ m

$y_C = 0.2375$ mm $\downarrow$.

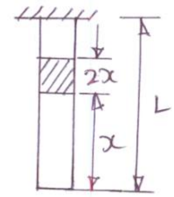
3. Determine the strain energy due to self weight of a bar of uniform cross section 'a' having length 'l' which is hanging vertically down.

Solution:

Consider an element at a distance 'x' from the lower end of the bar as shown in fig.

Let 'dx' be the thickness of the element.

The section 'x-x' will be acted upon by the weight of the bar of length 'x'.



Let W_x = Weight of the bar of length 'x'.

= Volume of the bar of length $x \times$ (weight of unit volume)

$= [A \cdot x] \rho$

$W_x = \rho A x.$

As a result of this weight, the portion 'dx' will experience a small elongation 'ds' then

$$\text{Strain in portion, } dx = \frac{\text{Elongation in } dx}{\text{Length of } x}$$

$$dx = \frac{ds}{dx}$$

$$\text{Stress in portion } dx = \frac{\text{Weight acting on section } x-x}{\text{Area of section}}$$

$$= \rho A x / A$$

$$= \rho x.$$

$$\text{Young's modulus, } E = \frac{\text{Stress}}{\text{Strain}} = \frac{\rho x}{(df/dx)}$$

$$E = \frac{\rho x \, dx}{df}$$

$$df = \frac{\rho x \, dx}{E}$$

Now the strain energy stored in portion 'dx' is given by

$$dv = \text{Average weight} \times \text{Elongation}$$

$$= \left[\tfrac{1}{2} \times W_x\right] \times d_5 = \left[\tfrac{1}{2} \times PAx\right] \times \frac{Px\,dx}{E}$$

$$= \tfrac{1}{2} \times P^2 A x \frac{dx}{E}.$$

Total strain energy stored within the bar due to its self weight 'w' is obtained by Integrating the above equation from 0 to L.

$$u = \int_0^L dv = \int_0^L \tfrac{1}{2} \times P^2 A x^2 \, dx / E$$

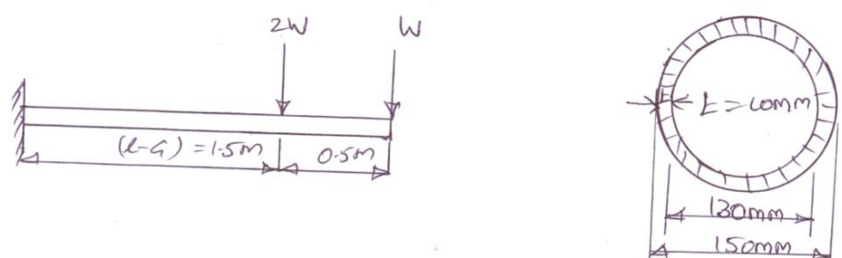
$$= \tfrac{1}{2} \frac{P^2 A}{E} \int_0^L x^2 dx = \tfrac{1}{2} \frac{P^2 A}{E} \left(x^3/3\right)_0^l$$

$$= \tfrac{1}{2} \frac{P^2 A}{E} \frac{L^3}{3} = \frac{AP^2 L^3}{6E}$$

$$U = \frac{AP^2 L^3}{6E}.$$

Double Integration Method.

4. A 2 m long cantilever made up of steel tube section 150 mm external diameter and 10 mm thick is loaded as shown in Fig.



Take $E = 200$ GPa. Calculate

(i) The value of w, So that the max bending stress is 150 MPa.

(ii) The maximum deflection for the loading.

Solution:

Case(i).

$$M_A = 2W \times 1500 + W \times 2000$$

$$= 5000\,W$$

$$\frac{E}{R} = \frac{M}{I} = \frac{\sigma b}{y}.$$

$$I = \pi/64\ (D^4 - D^4)$$

$$= \pi/64\ (150^4 - 130^4)$$

$$I = 10.86 \times 10^6\ mm^4$$

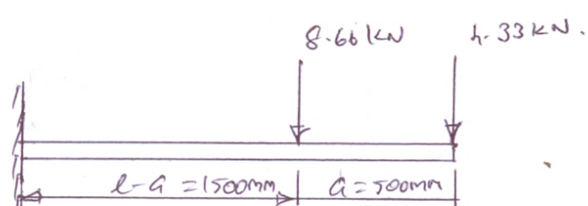
$$\frac{M}{I} = \sigma b/y$$

$$Y = P/d = 150/2 = 75\ mm.$$

Therefore,

$$\frac{5000\,W}{10.83 \times 10^6} = 150/75$$

Case(ii).



Double Integration Method.

(i) Deflection at free end due to the load 4.33 kN alone.

$$= \frac{W_1 L^3}{3EI}$$

(ii) Deflection at free end due to load 8.66 kN alone.

$$= \frac{WL(L-a)^3}{3EI} + \frac{W_L (L-a)^2}{2EI} \cdot a$$

$$y_B = \frac{W_1 L^3}{3EI} + \frac{WL(L-a)^3}{3EI} + \frac{W_2 (L-a)^2}{2EI} \cdot a$$

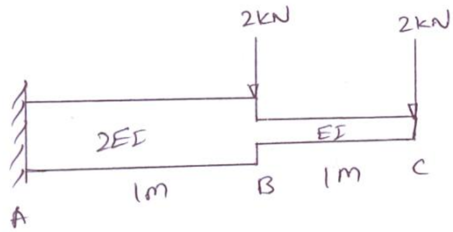
$$= \frac{4.33 \times 10^3 \times 2000^3}{3 \times 200 \times 10^3 \times 10.86 \times 10^6} + \frac{8.33 \times 10^3 \times 1500^3}{3 \times 200 \times 10^3 \times 10.86 \times 10^6}$$

$$+ \frac{8.33 \times 10^3 \times 1500^2}{2 \times 200 \times 10^3 \times 10.86 \times 10^6} \times 500$$

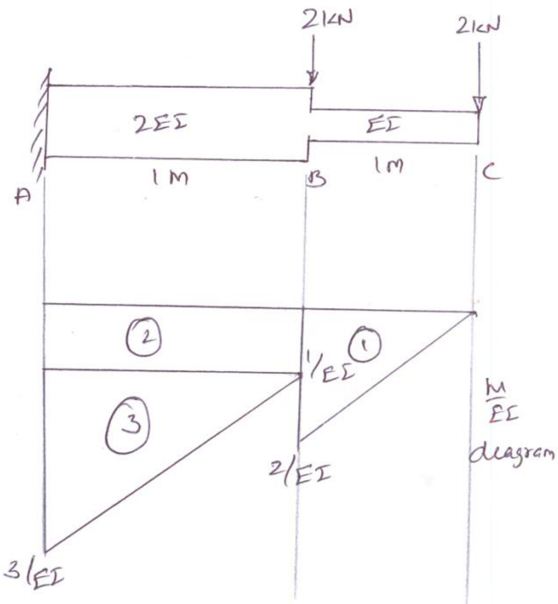
$$= 5.316 + 4.314 + 2.157$$

$$Y_B = 11.78 \text{ mm}.$$

5. For the cantilever beam shown in fig. find the deflection and slope at the free end. $EI = 10000 \text{ kN}/m^2$.



$A_1 = \frac{1}{2} \times 1 \times \frac{2}{EI} = \frac{1}{EI}$



$\bar{x}_1 = \frac{2}{3} \times 1 = \frac{2}{3} m$

$A_2 = \frac{1}{EI} \times 1 = \frac{1}{EI}$

$\bar{x}_2 = 1.5 m$

$A_3 = \frac{1}{2} \times 1 \times \frac{2}{EI} = \frac{1}{EI}$

$\bar{x}_3 = 1 + \frac{2}{3} \times 1 = \frac{5}{3} m$

Slope $C = A_1 + A_2 + A_3$

$= \frac{1}{EI} + \frac{1}{EI} + \frac{1}{EI}$

$= \frac{3}{EI}$

Deflection at 'C'

$= A_1 \bar{x}_1 + A_2 \bar{x}_2 + A_3 \bar{x}_3$

$= \frac{1}{EI} \times \frac{2}{3} + \frac{1}{EI} \times 1.5 + \frac{1}{EI} \times \frac{5}{3}$

$= \frac{11.50}{3EI}$

RESULT

Slope $= \frac{3}{10{,}000} = 3 \times 10^{-4}$ rad

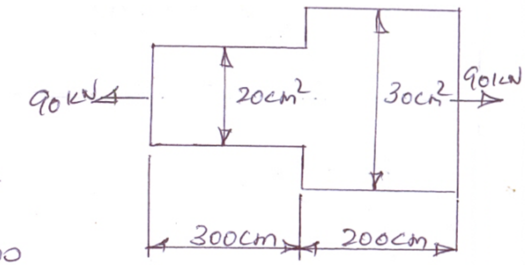
Deflection $= \frac{11.5}{30{,}000} = 3.83 \times 10^{-4}$ m.

PART C

6. A tension bar is made of two parts. the length of first part is 300 cm and area is 20 cm^2 while the second part of length 200 cm and area 30 cm^2. An axial load of 90 kN is gradualy applied. find the total strain energy produced in the bar and compare this value with

that obtain in a uniform bar of same length and having same volume under same load. Take $E = 2\times10^5\ N/mm^2$.

Given data



$L_1 = 300cm = 3000mm$

$A_1 = 20cm^2 = 2000mm^2$

$V_1 = A_1 \times L_1 = 2000\times3000$

$= 6\times10^6 mm^3$.

$L_2 = 200cm = 2000mm$

$A_2 = 30cm^2 = 3000\ mm^2$

$V_2 = A_2 \times L_2 = 3000\times2000$

$= 6\times10^6 mm^3$.

$P = 90KN = 90\times10^3 N$

$E = 2\times10^5 N/mm^2$.

To find

Total strain energy produced in the bar

To compare strain energy produced in this bar and uniform bar

Solution

$$\sigma_1 = \frac{P}{A} = \frac{90\times10^3}{2000} = 45 N/mm^2$$

$$\text{Strain energy } U_1 = \frac{\sigma_1^2}{2E}\times V_1 = \frac{45^2}{2\times2\times10^5}\times6\times10^6$$

$$= 30375 N\text{-}mm = 30.3 Nm.$$

$$U_1 = 30.3 N\text{-}m.$$

Part-II $\sigma_2 = \frac{P}{A_2} = \frac{90000}{3000}$

$= 30\ N/mm^2$

$U_2 = \frac{\sigma_2^2}{2E} \times V_2 = \frac{30^2}{2\times2\times10^5} \times 6\times10^6$

$= 13.5 Nm.$

Strain energy Stored in a Uniform bar

$V = V_1 + V_2$

$= 6\times10^6 + 6\times10^6 = 12\times10^6 mm^2$

$L = L_1 + L_2$

$= 3000 + 2000 = 5000\ mm.$

$V = A \times L$

$12000000 = A \times 5000$

$A = 2400\ mm^2$

Stress in Uniform bar $\sigma = \frac{P}{A} = \frac{90000}{2400}$

$= 37.5\ N/mm^2$

Strain energy Stored in Uniform bar $U = \frac{\sigma^2}{2E} \times V$

$= \frac{37.5^2}{2\times2\times10^5} \times 12000000$

$= 42{,}187 N\text{-}mm$

$U = 42.187 N\text{-}m$

$\frac{\text{Strain energy in given bar}}{\text{Strain energy in Uniform bar}} = \frac{43.8}{42.187} = 1.03$

Result

$U = 43.8 N\text{-}M$

Ratio of Strain energy $= 1.03$

UNIT V - THIN CYLINDERS, SPHERES AND THICK CYLINDERS

Stresses in thin cylindrical shell due to internal pressure circumferential and longitudinal stresses and deformation in thin and thick cylinders – spherical shells subjected to internal pressure –Deformation in spherical shells – Lame's theorem.

OBJECTIVE:

To study the stresses and deformations induced in thin and thick shells.

OUTCOMES:

Students will be able to

Analyze and design thin and thick shells for the applied internal and external pressures

5.1 Triaxial Stress, Biaxial Stress, and Uniaxial Stress

Triaxial stress refers to a condition where only normal stresses act on an element and all shear stresses (t_{xy}, t_{xz}, and t_{yz}) are zero. An example of a triaxial stress state is hydrostatic pressure acting on a small element submerged in a liquid.

A two-dimensional state of stress in which only two normal stresses are present is called *biaxial stress*. Likewise, a one-dimensional state of stress in which normal stresses act along one direction only is called a *uniaxial stress* state.

Pure Shear

Pure shear refers to a stress state in which an element is subjected to plane shearing stresses only, as shown in **Figure 3**. Pure shear occurs in elements of a circular shaft under a torsion load.

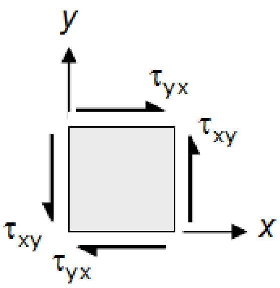


Figure 3. Element in pure shear

Thin cylindrical and spherical shells

Thin-walled assumption

For the thin-walled assumption to be valid the vessel must have a wall thickness of no more than about one-tenth (often cited as one twentieth) of its radius. This allows for treating the wall as a surface, and subsequently using the Young–Laplace equation for estimating the hoop stress created by an internal pressure on a thin wall cylindrical pressure vessel:

$$\sigma_\theta = \frac{Pr}{t} \quad \text{(for a cylinder)}$$

$$\sigma_\theta = \frac{Pr}{2t} \quad \text{(for a sphere)}$$

where

- P is the internal pressure
- t is the wall thickness
- r is the inside radius of the cylinder.
- σ_θ is the hoop stress.

The hoop stress equation for thin shells is also approximately valid for spherical vessels, including plant cells and bacteria in which the internalturgor pressure may reach several atmospheres.

Inch-pound-second system (IPS) units for *P* are pounds-force per square inch (psi). Units for *t*, and *d* are inches (in). SI units for *P* are pascals (Pa), while *t* and *d*=2*r* are in meters (m).

When the vessel has closed ends the internal pressure acts on them to develop a force along the axis of the cylinder. This is known as the axial stress and is usually less than the hoop stress.

$$\sigma_z = \frac{F}{A} = \frac{Pd^2}{(d+2t)^2 - d^2}$$

Though this may be approximated to

$$\sigma_z = \frac{Pr}{2t}$$

Also in this situation a radial stress σ_r is developed and may be estimated in thin walled cylinders as:

$$\sigma_r = \frac{-P}{2}$$

5.2 Deformation in thin cylindrical and spherical shells

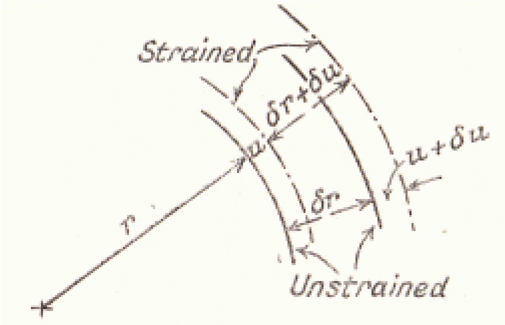
Thick cylinders and shells

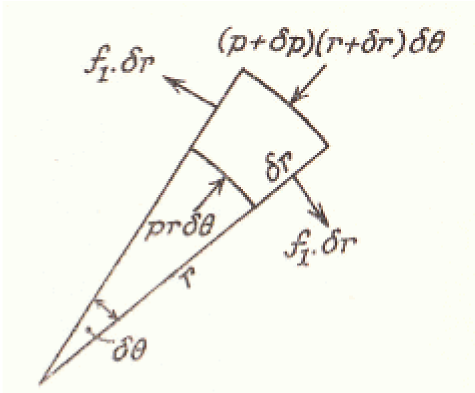
Thick Walled Cylinders

Under the action of radial Pressures at the surfaces, the three Principal Stresses will be . These Stresses may be expected to vary over any cross-section and equations will be found which give their variation with the radius r.

It is assumed that the longitudinal Strain e is constant. This implies that the cross-section remains plain after straining and that this will be true for sections remote from any end fixing.

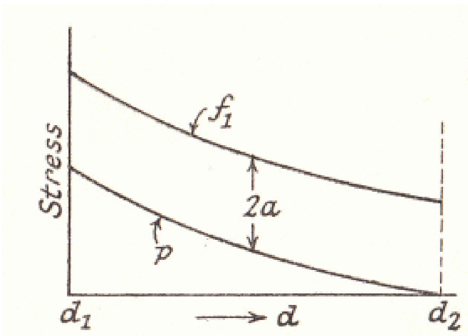
Let u be the radial shift at a radius r. i.e. After Straining the radius r becomes (r + u). and it should be noted that u is small compared to r.





Internal Pressure Only

Pressure Vessels are found in all sorts of engineering applications. If it assumed that the Internal Pressure is at a diameter of and that the external pressure is zero (Atmospheric) at a diameter then using equation (22)



The Error In The "thin Cylinder" Formula

If the thickness of the cylinder walls is t then and this can be substituted into equation

5.4 Principal planes and stresses

Principal stresses and planes

Principal Directions, Principal Stress

The normal stresses ($\sigma_{x'}$ and $\sigma_{y'}$) and the shear stress ($\tau_{x'y'}$) vary smoothly with respect to the rotation angle accordance with the coordinate transformation equations. There exist a couple of particular angles where the stresses take on special values.

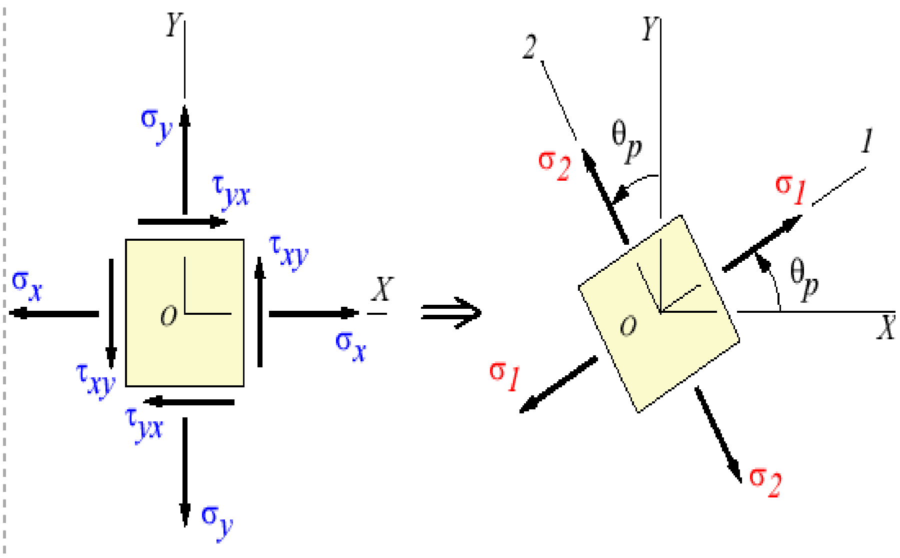
First, there exists an angle θ_p where the shear stress $\tau_{x'y'}$ becomes zero. That angle is found by setting $\tau_{x'y'}$ to zero in the above shear transformation equation and solving for θ (set equal to θ_p). The result is,

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

The angle θ_p defines the *principal directions* where the only stresses are normal stresses. These stresses are called *principal stresses* and are found from the original stresses (expressed in the *x,y,z* directions) via,

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

The transformation to the principal directions can be illustrated as:



Stresses in given coordinate system

Principal stresses

Maximum Shear Stress Direction

Another important angle, θ_s, is where the maximum shear stress occurs. This is found by finding the maximum of the shear stress transformation equation, and solving for θ. The result is,

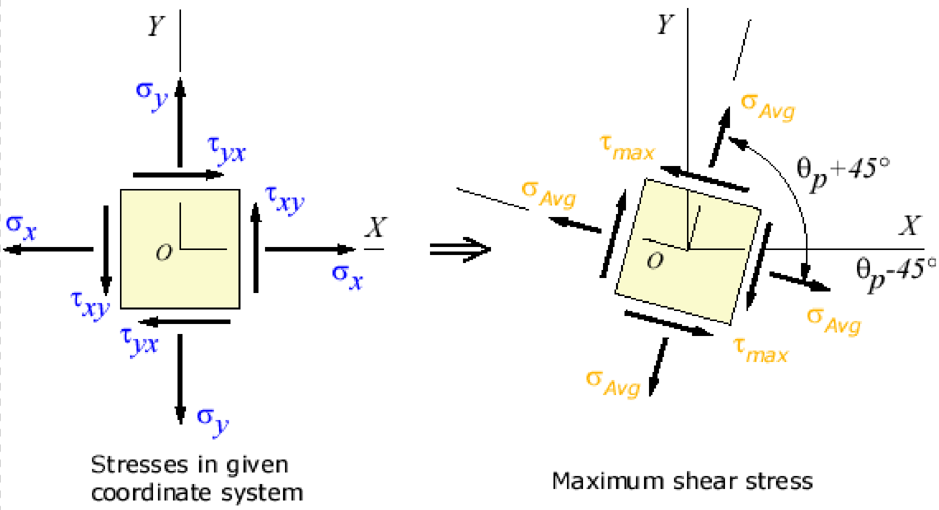
$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\Rightarrow \theta_s = \theta_p \pm 45^\circ$$

The maximum shear stress is equal to one-half the difference between the two principal stresses,

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_1 - \sigma_2}{2}$$

The transformation to the maximum shear stress direction can be illustrated as:



5.5 Mohr's circle for biaxial stresses

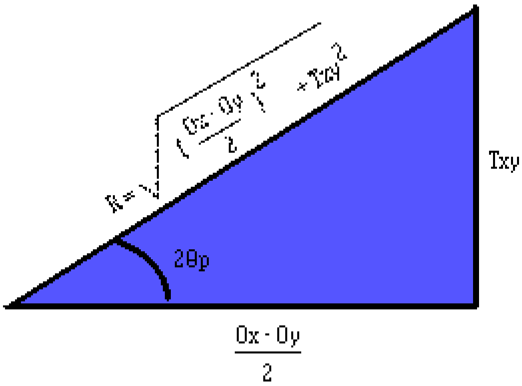
To find the maximum and minimum normal stresses throughout the entire range of angles, one can easily take the first derivative of (3) with respect to theta, set it to zero, and solve for the angle. This will give what is called the **principal plane** on which the **principal stresses** act. If this all sounds overly complicated... you're right! Why not just use the tried and true terminology "maximizing and minimizing the function" instead of inventing these two new terms with unrelated and unclear meaning? Well.... that's civil engineers for you.

$$^{dOx1}/_{d(theta)} = -(Ox - Oy)\sin(2theta) + 2Txy\cos(2theta) = 0$$

$$\tan 2theta_p = (2txy)/(Ox - Oy) \qquad (5)$$

Where $theta_p$ defines the orientation of the principal planes, and its two values, differing by 180^o, are called the **principal angles**.

Now is where we begin to get into the unnecessary jargon. All the excess baggage some engineer created to make it so that utilizing these relationships would not require higher math. This (and many other examples of engineer idiotcy) most likely stems from the fact that most engineers slept through their higher level math classes, and suffer from acute mathematical insecurities (and probably rightly so.) It's these abstract constructions which attempt to simplify the work, yet ultimately make it more difficult for those of us more mathematically inclined, that really piss me off. If you represent equation (5) geometrically with a 90^o triangle, (left), we can obtain general formulas for the principal stresses. First, we note that the hypotenuse of the triangle is,



$$R = SQR\{ [(Ox - Oy)/2]^2 + Txy^2\} \qquad (6)$$

The quantity R is defined as a positive number, and, like the other two sides of the triangle, has the completely meaningless units of "stress". From the triangle we obtain two additional relations:

$$\cos(2theta_p) = (Ox - Oy)/(2R) \qquad \sin(2theta_p) = Txy/R \qquad (7, 8)$$

Which is all very well and good, because it actually leads to the USEFUL equation for the general formula for the principal stresses:

$$O_{1,2} = (Ox + Oy)/2 \ {}^{+}/\text{-}\ R \qquad (9)$$

But such usefulness is short lived as we approach MOHR'S CIRCLE..... Actually, Mohr's circle isn't all that bad in many cases. It supplies its practitioners a clever and easy way to compute otherwise hairy moments of inertia, allows strain analyses to be handled quickly. However, in this case, its application seems to me a bit of a stretch, and what you wind up with is this hopelessly complicated graphical representation that seems so much more difficult than the original equations (3) and (4) that it's hardly worth the effort to learn at all. HOWEVER.... because certain bastich elements in the civil engineering department here at the U of A are requiring their students (many of whom, myself included, will NEVER use these relationships again after the class has ended) to use this technique in spite of the fact that we know of a perfectly valid and correct alternative.

The equations of Mohr's circle can be derived from the transformation equations (3) and (4). By simply rearranging the first equation, we find that the two expressions comprise the equation of a circle in parametric form.

$$Ox_1 - (Ox + Oy)/2 = [(Ox - Oy)/2]\cos(2theta) + Txy\ \sin(2theta) \qquad (10)$$
$$Tx_1y_1 = - \{(Ox - Oy)/2\}\sin(2theta) + Txy\ \cos(2theta) \qquad (11)$$

To eliminate the 2theta parameter, we square each relationship and add the two equations together. This ultimately leads to (after simplification),

$$(Ox_1 - \{Ox + Oy\}/2)^2 + Tx_1y_1{}^2 = \{(Ox - Oy)/2\}^2 + Txy^2 \qquad (12)$$

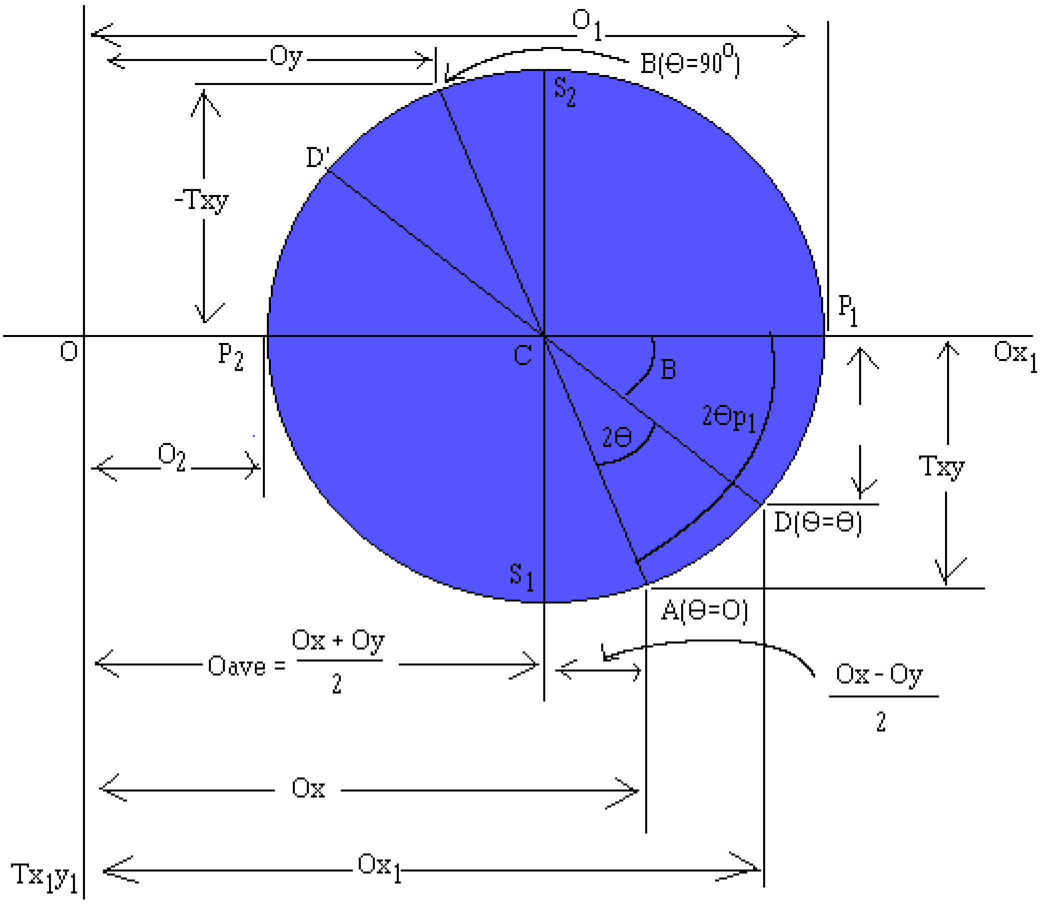
However, by resubstitution of equation (6) and by recognizing that the average stress value between the X and Y axis, Oave, is,

$$Oave = (Ox + Oy)/2 \qquad (12.a)$$

equation (12) can be simplified into the semi friendly equation of a circle in standard algebraic form,

$$(Ox_1 - Oave)^2 + Tx_1y_1{}^2 = R^2 \qquad (13)$$

However, don't let this nice looking equation for a circle fool you. Hidden in this simple equation are some of the most hairy, complicated, and down-right nasty relationships I think I have ever encountered. This makes my studies in the Frobenious theorem for solving differential equations with non-constant singular coefficients seem tame.



With Ox, Oy, and Txy known, the procedure for constructing Mohr's circle is as follows:

1. Draw a set of coordinate axis with Ox_1 and Tx_1y_1 (with T positive downwards. From now on, for simplicity, O and T will represent their respective axis.)
2. The center of the circle, by equation (13) is located at T=0 and O=Oave. Oave is nothing more than (12.a), so the center of the circle is located at:

   $$C = (Ox + Oy)/2$$
3. Locate point A, representing the stress conditions on the X face of the normal oriented element (Figure 1, extreme top left, non-rotated section). Plot coordinates O = Ox, T = Txy. Here, it is important to note that at point A, the inclination angle, theta, is zero.
4. Locate point B, representing the stress conditions on the Y face of the normal oriented element (Figure 1, again, extreme top left, non-rotated section). Again, plot coordinates O = Oy, T = Txy. Note that this point, B, will be diametrically opposite from point A. Also note, that the angle of inclination at B, theta, will be 90^o, as it could also be achieved on the X face by rotating it by 90^o.
5. Draw a line from point A to point B through the center C. This line is a diameter of the circle.
6. Using point C as the center, draw Mohr's circle through points A and B. The circle will have a radius of R, which is the same R as in equation (6).

Now that you have Mohr's circle drawn, you can use it to analyze the problem. (Remember, that this method is every bit as valid as simply using equations (3) and (4) above, except it requires less mathematical skill, and many more memorized relationships.)

$O_{1,2}$, representing the maximum and minimum normal stresses and their respective angles away from point A (where theta = 0^o) can be found by simply looking at the O values when T = 0. In the drawing above, O_1 represents the maximum, and O_2 the minimum. Furthermore, $T_{max/min}$, representing the maximum and minimum shear stresses and their respected angles can be found by locating the T values when O = Oave. At this point, T is simply equal to the radius, R, or equation (6).

In addition to these helpful points, all other possible points for the shear and normal stresses can be found on this circle. In order to find another value of Ox, Oy for a given rotation, one must simply start at the A and B points (A representing the Ox value and B, the Oy value), and rotate in a positive theta direction (by the orientation shown above, this is in a counterclockwise direction, in keeping with the right hand rule) for 2theta (from equations (3) and (4) above). The resulting points, D and D', will yield the Ox, Txy, and Oy, Txy (respectively) for that rotation.

As I have likely mentioned before (likely because, I can't really recall) to me this seems all very abstract and difficult to use. However, the aforementioned bastiches will be requiring this on my upcoming test, so I felt a need to more fully understand it. Granted, I still don't understand it as fully as I would hope, but it ought to be enough to get me through this one, insignificant little test.

P.S.: I apologize for my editorializing and opinionated presentation of this topic. I rarely do this when I analyze problems I don't understand (even when I do not like the method, such as the Lewis Dot structure). This time, however, I have some very strong feelings about my predicament. Also, in all fairness, if you were given the problem where O1 = O2 and Tmax = 0, i.e. the Mohr's circle was simply a little dot with R = 0, using the Mohr's circle method would arrive you at any and all answers much quicker than using equations (3) and (4). However, I don't think this extreme simplification of one special case warrants the abstraction being a required bit of knowledge for civil engineers.

UNIT IV DEFLECTION OF BEAMS			
Double Integration method – Macaulay's method – Area moment method for computation of slopes and deflections in beams - Conjugate beam and strain energy – Maxwell's reciprocal theorems.			
PART-A(2Marks)			
Q.No	**Questions**	**BT Level**	**Competence**
1	List the important methods used to find slope and deflection.	(BT1)	Remember
2	Where does the maximum deflection occur in cantilever beam?	(BT1)	Remember
3	Where does the maximum deflection occur for simply supported beam loaded symmetrically about mid-point and having same cross- section through their length?	(BT1)	Remember
4	Tensile load = 30 kN; length = 1m; width = 25 mm; thickness = 20 mm. calculate the stored stain energy. Take E = 200 GPa.	(BT3)	Application
5	Classify the types of loading on a body.	(BT3)	Application
6	Define modulus of resilience.	(BT1)	Remember
7	Define proof resilience.	(BT1)	Remember
8	Discuss the advantages of macaulay's method.	(BT2)	Understand
9	Give the disadvantage of double integration method.	(BT2)	Understand
10	Explain the conjugate beam method.	(BT4)	Analyze
11	Define strain energy.	(BT1)	Remember
12	Express the units of slope and deflection?	(BT2)	Understand
13	Express the value of slope at the free end of a cantilever beam of constant EI.	(BT2)	Understand
14	Formulate the expression for the stress induced in a body when impact load is applied.	(BT6)	Create
15	Calculate the maximum deflection of a simply supported beam carrying a point load of 100 KN at mid span. Span = 6 m, E= 20000 kN/m^2.	(BT3)	Application
16	Modify the cantilever beam with a point load at free end into conjugate beam.	(BT6)	Create
17	Compare the moment area method with conjugate beam method for finding the deflection of a simply supported beam with UDL over the entire span.	(BT5)	Evaluate
18	Explain Mohr's first theorem.	(BT4)	Analyze
19	Analyze the strain energy method.	(BT4)	Analyze
20	A cantilever beam of spring 2 m is carrying a point load of 20 kN at its free end. Measure the slope at the free end. Assume EI = 12 x 103 $kN\text{-}m^2$.	(BT5)	Evaluate

	PART-B(13 Marks)			
Q.No	**Questions**	**Marks**	**BT Level**	**Competence**
1	A beam AB of length 8 m is simply supported at its ends and carries two point loads of 50 kN and 40 kN at a distance of 2 m and 5 m respectively from left support A. Determine, deflection under each load, maximum deflection and the position at which maximum deflection occurs. Take E = 2 x 10^5 N/mm^2 and I = 8.5 X10^6 mm^4.	(13)	(BT4)	Analyze
2	Explain the Macaulay's method for finding the slope and deflection of beams with example.	(13)	(BT4)	Analyze
3	(a) A beam is simply supported at its ends over a span of 10 m and carries two concentrated loads of 100 kN and 60 kN at a distance of 2 m and 5 m respectively from the left support. Calculate (i) slope at the left support (ii) slope and deflection under the 100 kN load. Assume EI = 36 x 104 kN-m^2.	(7)	(BT5)	Evaluate
	(b) Explain the moment area method for finding the deflection and slope of beams with example.	(6)	(BT4)	Analyze
4	(a) Explain the conjugate beam method for finding the deflection of beams with example.	(6)	(BT4)	Analyze
	(b) A horizontal beam is freely supported at its ends 8m apart and carries a UDL of 15 kN/m over the entire span. Find the maximum deflection. Take E = 2 x 10^5 N/mm^2 and I = 2 x 10^9 mm^4.	(7)	(BT4)	Analyze
5	Explain double integration method for finding deflection of beams with example.	(13)	(BT4)	Analyze
6	Simply supported beam of length 8m is loaded as shown in fig. Calculate the slope and deflection at each point using Moment area method. Take E = 2.1 x 10^5 N/mm^2 and I = 78 x 10^6 mm^4.	(13)	(BT5)	Evaluate
7	(a) A simply supported beam of uniform flexural rigidity EI and span l, carries two symmetrically placed loads P at one–third of the span from each end. Find the slope at the supports and the deflection at mid–span. Use moment area theorems.	(7)	(BT5)	Evaluate
	(b) Derive the expression for strain energy stored in a body when load is applied with impact.	(6)	(BT2)	Understand
8	A circular bar of 60mm diameter and 7m long subjected to gradually applied load of 80 kN. Calculate i) Stretch in the rod, ii) Stress in the rod and iii) Strain energy absorbed by the rod. E = 2×10^5 N/mm^2.	(13)	(BT3)	Application
9	A unknown weight falls through a height of 1cm on a collar rigidly attached to lower end of a vertical bar 5000mm long and 600mm^2 in section. If the maximum extension of the rod is to be 0.002m, what is the corresponding stress and magnitude of the unknown weight? Take E = 2×10^5 N/mm^2.	(13)	(BT4)	Analyze
10	(a) A cantilever AB, 2m long, is carrying a load of 20 kN at free end and 30 kN at a distance 1m from the free end. Find the slope and deflection at free end. Take E = 200 GPa and I = 150 x 10^6 mm^4.	(7)	(BT5)	Evaluate
	(b) Derive the expression for strain energy stored in a body when load is applied suddenly.	(6)	(BT2)	Understand

11	A simply supported beam AB of span 4m, carrying a load of 100 kN at the mid span C has cross sectional moment of inertia 24 x 10^6 mm^4 over the left half of the span and 48 x 10^6 mm^4 over the right half. Find the slope at two supports and the deflection under the load. Take E = 200 GPa.	(13)	(BT3)	Appli cation
12	(a) A simply supported beam of length 4m is loaded with a point load of 10kN and 20kN at a distance of 1m and 2m respectively from the left support. The beam is 200mm wide and 400mm deep. Find the slopes at the supports, deflections under the loads and location and magnitude of the maximum deflections using Macualay's method.. Take E=2×10^4 N/mm^2.	(8)	(BT5)	Evalu ate
	(b) State and explain Maxwell's reciprocal theorem.	(5)	(BT1)	Reme mber
13	(a) A simply supported beam subjected to uniformly distributed load of w kN/m for the entire span. Calculate the maximum deflection by double integration method.	(7)	(BT5)	Evalu ate
	(b) Derive the expression for strain energy stored in a body due to shear stress.	(6)	(BT2)	Unde rstand
14	A simply supported beam AB of span 5m carries a point load of 40 kN at its centre. The value of moment of inertia for the left half is 2 x 10^8 mm^4 and for the right half portion is 4 x 10^8 mm^4. Find the slopes at the two supports and deflection under the load. Take E = 200GN/m^2.	(13)	(BT3)	Appli cation

PART-C(15 Marks)

Q.No	Questions	Marks	BT Level	Competence
1	(a) A 3 m long cantilever of uniform rectangular cross–section 150 mm wide and 300 mm deep is loaded with a point load of 3 kN at the free end and a UDL of 2 kN/m over the entire length. Find the maximum deflection. E = 210 kN/mm2. Use Macaulay's method.	(9)	(BT3)	Applicatio n
	(b) Describe the terms Resilience, Proof resilience and modulus of resilience.	(6)	(BT1)	Remembe r
2	(a) A simply supported beam of span 6 m is subjected to a udl of 2 kN/m over the entire span and a point load of 3 kN at 4 m from the left support. Find the deflection under the point load in terms of EI. Use strain energy method.	(10)	(BT4)	Analyze
	(b) Explain any one theorem in the moment area method.	(5)	(BT1)	Remembe r
3	A tension bar is made of two parts. The length of first part is 3000mm and area is 2000mm^2, while the second part is of length 2000mm and area 3000mm^2. An axial load of 90kN is gradually applied. Find the total strain energy produced in the bar and compare this value with that obtained in a uniform bar of same length and having same volume under same load. Take E = 2 x 10^5 N/mm^2.	(15)	(BT5)	Evaluate
4	A simply supported beam of length 5m carries a point load of 5kN at a distance of 3m from the left end. If E = 2 x 10^5 N/mm^2 and I = 10^8 mm^4, determine the slope at the left support and deflection under the point load using conjugate beam method.	(15)	(BT6)	Create

UNIT - V

THIN CYLINDER, SPHERES AND THICK CYLINDER

1. Distinguish between thin walled cylinder and thick walled cylinder?

In thin walled cylinder, thickness of the wall of the cylindrical vessel is less than1/15 to 1/20 of its internal diameter. Stress distribution is uniform over thethickness of the wall. If the ratio of thickness to its internal diameter is more than 1/20, then cylindrical shell is known as thick cylinders. The stress distribution is not uniform over the thickness of the wall.

2. What are the two type of stress developed in thin cylinder subjected to internal pressure. (Dec 2011,May 2012)

1. Hoop stress
2. Longitudinal stress

3. Define hoop and longitudinal stress (May 2013,Dec 2014)

Hoop stress:

The stress acting along the circumference of the cylinder is called circumference or hoop stress

Longitudinal stress:

The stress acting along the length of the cylinder is known as longitudinal stress

4. For what purpose are the cylindrical and spherical shells used?

The cylindrical and spherical shells are used generally as containers for storage of liquids and gases under pressure.

5. What are assumptions made in the analysis of thin cylinders?

Radial stress is negligible.

Hoop stress is constant along the thickness of theshell.

Material obeys Hooke‘s law.

Material is homogeneous and isotropic.

6. Write the change in diameter and change in length of a thin cylindrical shell due to internal pressure, P.

Change in diameter $\delta d = PD^2 / 2tE(1-1/2m)$

Change in length δl= PDl /2tE(1/2-1/m)
Where P=internal pressure of fluid D= diameter of the cylindrical shell
t = thickness of the cylindrical shell L= length of cylindrical
1/m = Poisson ratio

7. What are the assumptions in lames theorem?

i) The material is homogeneous and isotropic

ii) The material is stressed within elastic limit

8. How many stresses are developed in thick cylinders? Name them.(May/Jun 2012)

Three types of stresses are developed in thick cylinders.
i)Radial stress
ii)Hoop stress
iii) Longitudinal stress

9. Write lames equation to find out stress in thick cylinder(Dec 2014)

Radial stress $\sigma_r = b/r^2 - a$

Hoop stress $\sigma_c = b/r^2 + a$

10. In a thick cylinder will the radial stress vary over the thickness of wall?

Yes, in the thick cylinder radial stress is maximum at inner and minimum at the outer radius

11. Define radial pressure in thin cylinder.

The radial stress for a thick-walled cylinder is equal and opposite to the gauge pressure on the inside surface, and zero on the outside surface. The circumferential stress and longitudinal stresses are usually much larger for pressure vessels, and so for thin-walled instances, radial stress is usually neglected.

12. How does a thin cylinder fail due to internal fluid pressure?

Failure of materials under combined tensile and shear stresses is not simple to predict.
Maximum Principal Stress Theory
Component fails when one of the principal stresses exceeds the value that causes failure in simple tension
Maximum Shear Stress Theory
Component fails when maximum shear stress exceeds the shear stress that causes failure in simple tension
Maximum Strain Energy Theory
Component fails when strain energy per unit volume exceeds the value that causes failure in simple tension

UNIT-V

Thin Cylinders, spheres and Thick Cylinders

PART B

1. A Cylinder shell 800 mm in diameter, 3 m long is having 10 mm metal thickness. If the shell is subjected to an internal pressure of 2.5 N/mm².

(i) The change in diameter.

(ii) The change in length

(iii) The change in volume.

Assume the modulus of elasticity and poisson's ratio of the material of the shell as 200 kN/mm² and 0.25 respectively.

Solution:

(i) Change in diameter is given by

$$\partial d = \frac{Pd^2}{4tE}(L-V)$$

$$\partial d = \frac{2.5 \times 800^2}{4 \times 10 \times 200 \times 10^3}(2-0.25)$$

$$\partial d = 0.35 \text{ mm.}$$

(ii) Change in length is given by

$$\partial L = \frac{PdL}{4tE}(1-2V)$$

$$\partial L = \frac{2.5 \times 800 \times 3 \times 10^3}{4 \times 10 \times 200 \times 10^3}(1-2 \times 0.25)$$

$\delta L = 0.375$ mm.

(iii) Change in volume is given by

$$\delta v = \frac{Pd}{4tE}(5-4\nu)v$$

$$V = \pi/4\, d^2 L$$

$$\delta v = \frac{2.5 \times 800}{4 \times 10 \times 200 \times 10^3}(5-4 \times 0.25) \times \pi/4 \times 800^2 \times 3 \times 10^3$$

$$\delta v = 1.508 \times 10^6 \, mm^3.$$

2. A Spherical shell of 2m diameter is made up of 10 mm thick plates. Calculate the change in diameter and volume of the shell, when its subjected to an internal pressure of 1.6 MPa. Take E = 200 GPa and $1/3m = 0.3$.

Solution:

$$\delta d = \frac{Pd^2}{4tE}(1-\nu)$$

$P = 1.6$ MPa

$d = 2$m $= 2 \times 10^3$ mm

$t = 10$ mm

$E = 200$ GPA $= 200 \times 10^3$ Mpa

$\nu = 1/3 = 0.3$

$$\delta d = \frac{1.6 \times (2 \times 10^3)^2 (1-0.3)}{4 \times 10 \times 200 \times 10^3}$$

$$\delta d = 0.56 \text{ mm}$$

$$\delta V = \frac{\pi P d^4}{8tE}(1-\gamma)$$

$$= \frac{\pi \times 1.6 \times (2\times10^3)^4 (1-0.3)}{8 \times 10 \times 200 \times 10^3}$$

$$\delta V = 3.52 \times 10^6 \ mm^4.$$

Thick Cylinder.

3. Find the thickness of metal necessary for a thick cylindrical shell of internal diameter 160 mm to withstand an internal pressure of $8 \ N/mm^2$. The maximum hoop stress in the section is not exceed $35 \ N/mm^2$.

Solution:

$x = 80 \ mm$

$P_x = 8 \ N/mm^2$

$\sigma_x = 35 \ N/mm^2$

By Lame's equation,

$$P_x = \frac{B}{x^2} - A$$

$$8 = \frac{B}{80^2} - A \rightarrow (1)$$

$$\sigma_x = \frac{B}{x^2} + A$$

$$35 = \frac{B}{80^2} + A \rightarrow (2)$$

From equations (1) and (2)

$A = 13.5$

$B = 137600$

$$P_x = \frac{137600}{x^2} - 13.5$$

At the outer surface , $P_x = 0$, $x = 80 + t$

$$0 = \frac{137600}{(80+t)^2} - 13.5$$

$$(80+t)^2 = \frac{137600}{13.5}$$

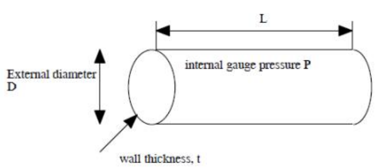
$$t = 20.96\ mm$$

RESULT:

$\sigma_x = 35\ N/mm^2$

$t = 20.96\ mm.$

4. Derivation - Deformation in Thin Cylinder.



When a thin cylindrical shell is subjected to an internal pressure, there will be an increase in diameter as well as length of the shell there by subjected to lateral and linear strains,

Consider a thin cylinder of radius (r) and Thickness (t).

Let,

L = Length of the cylindrical shell

r = Radius of the cylindrical shell.

t = Thickness of the cylindrical shell

P = Intensity of pressure inside shell

E = Young's Modulus of material shell.

$\mu = 1/m$ = Poisson's ratio.

d = Diameter of cylindrical shell.

(i) Increase in diameter.

Circumferential strain in shell

$$e_c = \frac{\text{Increase in Diameter}}{\text{Original diameter}}$$

$$e_c = \frac{\delta d}{d} = \frac{\sigma_c}{E} = \frac{\sigma_L}{\mu E}$$

where,

σ_c = Circumferential or hoop stress

σ_L = Longtitudinal stress.

$$e_c = \frac{1}{E}\left[\sigma_c - \sigma_L/m\right]$$

$$e_c = 1/E\left[\frac{Pr}{t} - Pr/2tm\right]$$

$$e_c = 1/E \; \frac{Pr}{t}\left[1 - 1/2m\right]$$

$$= Pr/Et\left[1 - \mu/2\right] \qquad \left[\because 1/m = \mu \text{ poisson's ratio}\right]$$

Circumferential strain

$$e_c = \frac{Pd}{2Et}\left[1 - \mu/2\right].$$

Now increase in diameter $\delta d = \frac{Pd^2}{2ET}\left[1 - \mu/2\right]$

$$= \frac{Pd^2}{4ET}\left[2 - \mu\right]$$

ii) Increase in length (δL) :

We know that the longitudinal Strain is given by,

$$e_l = \frac{\text{Increase in length}}{\text{Original length}} = \frac{\delta L}{L} = \frac{\sigma_L}{E} - \frac{\sigma_c}{mE}$$

$$e_l = 1/E \left[\frac{Pr}{2t} - \frac{Pr}{tm} \right] = \frac{Pr}{Et} \left[1/2 - \mu \right]$$

Longitudinal strain (e_l) $= \frac{Pr}{2Et} [1 - 2\mu]$

$$e_l = \frac{Pd}{4Et} [1 - 2\mu]$$

Now increase in length (δL) $= e_l \cdot L$

Increase in length (δL) $= \frac{PdL}{4Et} [1 - 2\mu]$

(iii) Increase in the volume (δv):

We know that volumetric strain

$$e_v = \frac{\text{Change in volume} (\delta v)}{\text{Initial volume} (v)}$$

$$= \frac{V_2 - V_1}{V_1}$$

$$e_v = \frac{\pi/4 \, (d + \delta d)^2 (L + \delta L) - \pi d^2/4 \, L}{\pi d^2/4 \, L.}$$

$$e_v = \frac{\left[d^2 + 2.d.\delta d + (\delta d)^2\right]\left[1+\delta L\right] - \delta^2 L\right]}{d^2 l.}$$

$$e_v = \frac{\left[\delta^2 L + d^2.\delta L + 2.d.\delta d.L + 2.d.\delta d.L + L.\delta d^2 + \delta L.\delta d^2 - d^2 L\right]}{d^2 L.}$$

Neglecting, higher powers of δd, δd and other small quantities, we have

$$e_v = \frac{d_2.\delta L + 2.d.L.\delta d}{d^2 L}$$

$$e_v = \frac{2.\delta d}{d} + \delta L/L = 2e_c + e_l$$

$$e_v = 2.\left[\frac{Pd}{2Et}\left(1 - \mu/2\right)\right] + \frac{Pd}{4Et}\left[1 - 2\mu\right]$$

$$e_v = \frac{Pd}{Et}\left[1 - \mu/2 + 1/4\ (1 - 2\mu)\right]$$

$$= \frac{Pd}{Et}\left[5 - 4\mu/4\right]$$

Change or increase in volume

$$\delta V = ev.v$$

Increase or change in

$$\text{volume } (\delta v) = \frac{Pd.v}{4Et}\left[5 - 4\mu\right]$$

5. Determine the maximum and minimum hoop stress across the section of a pipe of 400 mm internal diameter and 100 mm thick when the pipe contains a fluid at a pressure of $8 N/mm^2$. Also sketch the radial pressure distribution and hoop stress distribution across the section.

Given data

Internal dia d_1 = 400mm

$r_1 = 200mm$

$t = 100mm$

$d_2 = 400 + 2 \times 100 = 600mm$

$r_2 = 300 mm$

Fluid pressure $P_0 = 8N/mm^2$

To find

Maximum + minimum hoop stress

Solution

B.C (I) $x = r_1 = 200mm$; $P_x = 8N/mm^2$

(II) $x = r_2 = 300mm$ $P_x = 0$.

$$8 = \frac{b}{200^2} - a = \frac{b}{40000} - a$$

$$0 = \frac{b}{300^2} - a = \frac{b}{90000} - a$$

$$\therefore 8 = \frac{b}{40000} - \frac{b}{90000} = \frac{9b - 4b}{360000}$$

$$= \frac{5b}{360000}$$

$$b = \frac{360000 \times 8}{5} = 576000$$

$$0 = \frac{576000}{90000} - a \; ; \; a = \frac{576000}{90000} = 6.4$$

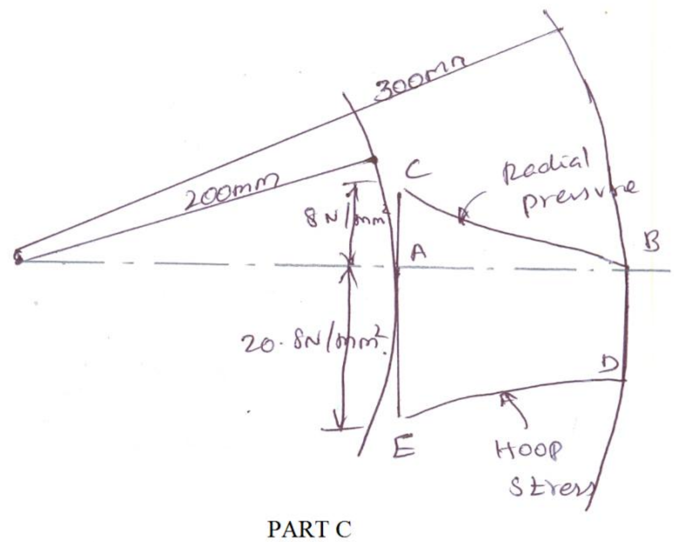
$$\sigma_x = \frac{b}{x^2} + a = \frac{576000}{x^2} + 6.4$$

At $x = 200mm$ $\quad \sigma_{200} = \frac{576000}{200^2} + 6.4 = 14.4 + 6.4$

$$= 20.8 \; N/mm^2$$

$x = 300mm$ $\quad \sigma_{300} = \frac{576000}{300^2} + 6.4$

$$= 6.4 + 6.4 = 12.8 \; N/mm^2$$

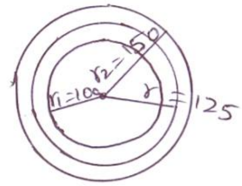


PART C

6. A compound cylinder is made by shrinking a cylinder of external diameter 300 mm and internal diameter of 250mm over another cylinder of external diameter 250mm and internal diameter 200mm. The radial pressure at the junction after shrinking

is $8 N/mm^2$. find the final stresses set up across the section, when the compound cylinder is subjected to an internal fluid pressure of $84.5 N/mm^2$.

Given data



External diameter $= 300mm$

$r_2 = 150 mm$

Internal diameter $= 250mm$

$r^* = 125mm$

for Inner Cylinder

Internal diameter $= 200mm$

$r_1 = 100mm$

Radial pressure $p^* = 8N/mm^2$.

fluid pressure in the compound cylinder

$P = 84.5 N/mm^2$.

Solution

(i) stresses due to shrinking in the Outer + inner cylinders before the fluid pressure is admitted.

for Outer cylinder

$$P_x = \frac{b_1}{x^2} - a_1 \quad + \quad \sigma_x = \frac{b_1}{x^2} + a_1$$

$x = 150\text{ mm}\ ;\ P_x = 0$

$$0 = \frac{b_1}{150^2} - a_1 = \frac{b_1}{22500} - a_1$$

$x = r^* = 125\text{mm},\quad P_x = p^* = 8\text{N/mm}^2$

$$8 = \frac{b}{125^2} - a_1 = \frac{b_1}{15625} - a_1$$

$$\therefore\ 8 = -\frac{b_1}{22500} + \frac{b_1}{15625} = \frac{(-15625 + 22500)\,b}{22500 \times 15625}$$

$$b_1 = \frac{8 \times 22500 \times 15625}{(-15625 + 22500)} = 409090.9$$

$$0 = \frac{409090.9}{22500} - a_1 \ \text{(or)}\ a_1 = \frac{409090.9}{22500}$$

$$= 18.18$$

$$\sigma_x = \frac{409090.9}{x^2} + 18.18$$

$$\sigma_{150} = \frac{409090.9}{150^2} + 18.18 = 36.36\text{ N/mm}^2$$

$$\sigma_{125} = \frac{409090.9}{125^2} + 18.18 = 44.36\text{ N/mm}^2.$$

for inner cylinder

$$P_x = \frac{b_2}{x^2} - a_2 \qquad \sigma_x = \frac{b_2}{x^2} + a_2$$

$x = r_1 = 100\text{mm}\ ;\quad P_x = 0.$

$$0 = \frac{b_2}{100^2} - a_2 = \frac{b_2}{10000} - a_2$$

$x = r^* , = 125\ mm \quad P_x = p^* = 8\ N/mm^2$

$$8 = \frac{b_2}{125^2} - a_2 = \frac{b_2}{15625} - a_2$$

$$8 = \frac{b_2}{15625} - \frac{b_2}{10000} = \frac{-5625 b_2}{15625 \times 10000}$$

$$b_2 = \frac{8 \times 15625 \times 10000}{5625} = -222222.2$$

$$0 = \frac{-222222.2}{x^2} - a_2$$

$$a_2 = -22.22$$

$$\sigma_x = \frac{-222222.2}{x^2} - 22.22$$

$$\sigma_{125} = \frac{-222222.2}{125^2} - 22.22 = -36.22\ N/mm^2$$

$$\sigma_{100} = \frac{-222222.2}{100^2} - 22.22 = -44.44\ N/mm^2$$

(ii) Stresses due to fluid pressure alone

$$P_x = \frac{B}{x^2} - A \qquad \sigma_x = \frac{B}{x^2} + A$$

$x = 100\ mm \quad P_x = P = 84.5\ N/mm^2$

$x = 150\,mm \quad P_x = 0$

$$0 = \frac{B}{150^2} - A = \frac{B}{22500} - A$$

$$84.5 = \frac{B}{10000} - \frac{B}{22500}$$

$$B = \frac{84.5 \times 10000 \times 22500}{12500} = 1521000$$

$$0 = \frac{1521000}{22500} - A \; ; \quad A = \frac{1521000}{22500}$$

$$= 67.6$$

$$\sigma_x = \frac{1521000}{x^2} + 67.6$$

$$\sigma_{100} = \frac{1521000}{100^2} + 67.6 = 219.7\,N/mm^2$$

$$\sigma_{125} = \frac{1521000}{125^2} + 67.6 = 164.94\,N/mm^2$$

$$\sigma_{150} = \frac{1521000}{150^2} + 67.6 = 135.2\,N/mm^2$$

Inner cylinder

$f_{100} = \sigma_{100}$ Shrinkage $+ \sigma_{100}$ internal fluid pressure

$$= -44.44 + 219.7 = 175.26\,N/mm^2$$

$$f_{125} = -36.22 + 164.94 = 128.72\,N/mm^2$$

Outer cylinder.

$f_{125} = \sigma_{125}$ shrinkege $+ \sigma_{125}$ internal flued pressure

$= 44.36 + 164.94 = 209.3\ N/mm^2.$

$f_{150} = \sigma_{150}$ shrinkege $+ \sigma_{150}$ internal flued pressure

$= 36.36 + 135.2 = 171.56\ N/mm^2.$

UNIT V THIN CYLINDERS, SPHERES AND THICK CYLINDERS

Stresses in thin cylindrical shell due to internal pressure circumferential and longitudinal stresses and deformation in thin and thick cylinders – spherical shells subjected to internal pressure –Deformation in spherical shells – Lame's theorem.

PART-A(2 Marks)

Q.No	Questions	BT Level	Competence
1	A cylindrical pipe of diameter 1.5 m and thickness 1.5 cm is subjected to an internal fluid pressure of 1.2 N/mm^2. Calculate the longitudinal stress developed in the pipe.	(BT3)	Application
2	Estimate the thickness of the pipe due to an internal pressure of 10 N/mm^2 if the permissible stress is 120 N/mm^2. The diameter of pipe is 750 mm.	(BT2)	Understand
3	Define circumferential stress.	(BT1)	Remember
4	A spherical shell of 1 m diameter is subjected to an internal pressure 0.5N/mm^2. Discover the thickness of the shell, if the allowable stress in the material of the shell is 75 N/mm^2.	(BT3)	Application
5	Describe longitudinal stress.	(BT1)	Remember
6	Deduce the expression for longitudinal stress in a thin cylinder subjected to a uniform internal fluid pressure.	(BT5)	Evaluate
7	A cylinder of diameter 1.3m and thickness 12mm is subjected to an internal pressure of 1 N/mm^2. Identify the type of cylinder.	(BT1)	Remember
8	Where the hoop stresses and longitudinal stresses are acting in a thin cylindrical shell?	(BT1)	Remember
9	Name the various methods of reducing the hoop stresses.	(BT1)	Remember
10	Formulate the mathematical expressions of Lame's theorem.	(BT6)	Create
11	Formulate an expression for the longitudinal stress in a thin cylinder subjected to an uniform internal fluid pressure.	(BT6)	Create
12	When is the longitudinal stress in a thin cylinder is zero?	(BT1)	Remember
13	Discuss about wire wounded thin cylinder.	(BT2)	Understand
14	Compare the cylindrical shell and spherical shell.	(BT4)	Analyze
15	Differentiate the thick cylinder from thin cylinder.	(BT4)	Analyze
16	List out the formulae for finding change in diameter, change in length and change in volume of a thin cylindrical shell subjected to internal fluid pressure p?	(BT1)	Remember
17	Distinguish between Circumferential stress and longitudinal stress?	(BT2)	Understand
18	Assess the thickness of the pipe due to an internal pressure of 10 N/mm^2 if the permissible stress is 120 N/mm2. The diameter of pipe is 750 mm.	(BT5)	Evaluate
19	Explain briefly about thick compound cylinder.	(BT4)	Analyze
20	Give the expression for hoop stress for thin spherical shells.	(BT2)	Understand

PART-B(13 Marks)

Q.No	Questions	Marks	BT Level	Competence
1	Derive the expressions for change in dimensions of a thin cylinder due to internal pressure.	(13)	(BT2)	Understand
2	A cylindrical thin drum 80cm in diameter and 3m long has a shell thickness of 1cm. If the drum is subjected to an internal pressure of 2.5 N/mm2, determine: 1. Change in diameter, 2. Change in length and 3.	(13)	(BT4)	Analyze

	Change in volume. Take E = 2 x 105 N/mm2 and Poisson's ratio = 0.25.			
3	A thin cylindrical shell 3 m long has 1m internal diameter and 15 mm metal thickness. Calculate the circumferential and longitudinal stresses induced and also the change in the dimensions of the shell, if it is subjected to an internal pressure of 1.5 N/mm^2 Take E = $2x10^5$ N/mm^2 and poison's ratio =0.3. Also calculate change in volume.	(13)	(BT5)	Evaluate
4	(a) A closed cylindrical vessel made of steel plates 4 mm thick with plane ends, carries fluid under pressure of 3 N/mm^2 The diameter of the cylinder is 25cm and length is75 cm. Calculate the longitudinal and hoop stresses in the cylinder wall and determine the change in diameter, length and Volume of the cylinder. Take E=$2.1x10^5$ N/mm^2 and 1/m = 0.286.	(7)	(BT4)	Analyze
	(b) Explain briefly about thin spherical shell and derive the expression for hoop stress in thin spherical shell.	(6)	(BT5)	Evaluate
5	(a) A cylindrical shell 3 m long, 1 m internal diameter and 10 mm thick is subjected to an internal pressure of 1.5 N/mm^2. Calculate the changes in length, diameter and volume of the cylinder. E = 200 kN/mm^2, Poisson's ratio = 0.3.	(7)	(BT3)	Applicat ion
	(b) Derive the expressions for change in dimensions of spherical shell due to internal pressure.	(6)	(BT2)	Underst and
6	(a) A steel cylindrical shell 3 m long which is closed at its ends, had an internal diameter of 1.5 m and a wall thickness of 20 mm. Calculate the circumferential and longitudinal stress induced.	(7)	(BT3)	Applicat ion
	(b) For the given cylindrical shell determine the change in dimensions of the shell if it is subjected to an internal pressure of 1.0N/mm^2. Assume the modulus of elasticity and Poisson's ratio for steel as 200kN/mm^2 and 0.3 respectively.	(6)	(BT4)	Analyze
7	A thin cylindrical shell 3m long, 1.2m diameter is subjected to an internal pressure of 1.67N/mm^2. If the thickness of the shell is 13mm, E = 2 x 10^5 N/mm^2 and 1/m = 0.28 (a) Find the circumferential and longitudinal stresses. (b) Find the maximum shear stress and change in dimensions of the shaft.	(7) (6)	(BT3)	Applicat ion
8	(a) A cylindrical shell 3 m long which is closed at the ends has an internal diameter 1m and wall thickness of 15 mm. Calculate the change in dimensions and change in volume if the internal pressure is 1.5 N/mm^2,E = 2 x 10^5 N/mm^2, μ= 0.3.	(7)	(BT4)	Analyze
	(b) List out the assumptions made on Lame's theory.	(6)	(BT3)	Applicat ion
9	A cylindrical vessel 2m long and 500mm in diameter with 10mm thick plates is subjected to an internal pressure of 3 MPa. Calculate the change in volume of the vessel. Take E = 200 GPa and Poisson's ratio = 0.3 for the vessel material.	(13)	(BT3)	Applicat ion
10	A spherical shell of 2m diameter is made up of 10mm thick plates. Calculate the change in diameter and volume of the shell, when it is subjected to an internal pressure of 1.6 MPa. Take E = 200 GPa and 1/m = 0.3.	(13)	(BT3)	Applicat ion
11	(a) Determine the maximum hoop stress across the section of a pipe of external diameter 600mm and internal diameter 440mm. when the pipe is subjected to an internal fluid pressure of 50N/mm^2.	(7)	(BT5)	Evaluate

	(b) Explain the concept of thick cylinder and deduce the expressions for various stresses induced in thick cylinders.	(6)	(BT3)	Application
12	(a) A steel cylinder of 300mm external diameter is to be shrunk to another steel cylinder of 150mm internal diameter. After shrinking the diameter at the junction is 250mm and radial pressure at the common junction is 40 N/mm^2. Find the original difference in radii at the junction. Take E = 2 x 10^5 N/mm^2.	(7)	(BT4)	Analyze
	(b) A spherical shell of 1.5m internal diameter and 12mm shell thickness is subjected to pressure of 2 N/mm^2. Determine the stress induced in the material of the shell.	(6)	(BT3)	Application
13	(a) Find the thickness of metal necessary for a cylindrical shell of internal diameter 150mm to withstand an internal pressure of $25N/mm^2$. The maximum hoop stress in the section is not to exceed 125 N/mm^2.	(7)	(BT4)	Analyze
	(b) Describe the stresses in compound thick cylinder.	(6)	(BT1)	Remember
14	(a) Determine the maximum and minimum hoop stress across the section of a pipe of 400mm internal diameter and 100mm thick, when the pipe contains a fluid at a pressure of 8 N/mm^2.	(7)	(BT3)	Application
	(b) Sketch the radial pressure distribution and hoop stress distribution across the section of the given pipe.	(6)	(BT5)	Evaluate

	PART-C(15 Marks)			
Q.No	**Questions**	**Marks**	**BT Level**	**Competence**
1	(a) A thin cylinder 1.5 m internal diameter and 5 m long is subjected to an internal pressure of 2 N/mm2. If the maximum stress is limited to 160 N/mm2 find the thickness of the cylinder. E= 200 kN/mm2 and Poisson's ratio = 0.3. Also find the changes in diameter, length and volume of the cylinder.	(8)	(BT3)	Application
	(b) Explain and derive the hoop stress and longitudinal stress in thin cylinders.	(7)	(BT4)	Analyze
2	(a) A cylinder has an internal diameter of 230 mm, wall thickness 5 mm and is 1 m long. It is found to change in internal volume by 12 × 10-6 m3 when filled with a liquid at a pressure 'p'. Taking E = 200 GPa and 1/m = 0.25, determine the stresses in the cylinder, the changes in its length and internal diameter.	(8)	(BT5)	Evaluate
	(b) A spherical shell of internal diameter 1.2m and of thickness 12mm is subjected to an internal pressure of 4 N/mm2. Determine the increase in diameter and increase in volume. Take E = 2 x 105 N/mm2 and μ = 0.33	(7)	(BT4)	Analyze
3	A boiler is subjected to an internal steam pressure of 2 N/mm2. The thickness of boiler plate is 2.6cm and permissible tensile stress is 120 N/mm2. Find the maximum diameter, when efficiency of longitudinal joint is 90% and that of circumferential joint is 40%.	(15)	(BT5)	Evaluate

4	(a) A thin spherical shell 1m in diameter with its wall of 1.2cm thickness is filled with a fluid at atmospheric pressure. What intensity of pressure will be developed in it if 175cm3 more of fluid is pumped into it? Also calculate the circumferential stress at the pressure and the increase in diameter. Take E = 200GN/mm2 and 1/m = 0.3.	(7)	(BT6)	Create
	(b) A thin seamless spherical shell of diameter 1.5m and thickness 8mm. It is filled with a liquid so that the internal pressure is 1.5N/mm2. Determine the increase in diameter and capacity of the shell. Take E = 2 x 105 N/mm2 and μ = 0.3.	(8)	(BT6)	Create