ME8593
Design of Machine Elements

Unit – 1

STEADY STRESSES AND VARIABLE STRESSES IN MACHINE MEMBERS

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UNIT I STEADY STRESSES AND VARIABLE STRESSES IN MACHINE MEMBERS

Introduction to the design process - factors influencing machine design, selection of materials based on mechanical properties - Preferred numbers, fits and tolerances – Direct, Bending and torsional stress equations – Impact and shock loading – calculation of principle stresses for various load combinations, eccentric loading – curved beams – crane hook and ‘C’ frame- Factor of safety - theories of failure – Design based on strength and stiffness – stress concentration – Design for variable loading.

UNIT II SHAFTS AND COUPLINGS

Design of solid and hollow shafts based on strength, rigidity and critical speed – Keys, keyways and splines - Rigid and flexible couplings.

UNIT III TEMPORARY AND PERMANENT JOINTS

Threaded fastners - Bolted joints including eccentric loading, Knuckle joints, Cotter joints – Welded joints, riveted joints for structures - theory of bonded joints.

UNIT IV ENERGY STORING ELEMENTS AND ENGINE COMPONENTS

Various types of springs, optimization of helical springs - rubber springs - Flywheels considering stresses in rims and arms for engines and punching machines- Connecting Rods and crank shafts.

UNIT V BEARINGS

Sliding contact and rolling contact bearings - Hydrodynamic journal bearings, Sommerfeld Number, Raimondi and Boyd graphs, -- Selection of Rolling Contact bearings.

OBJECTIVES

- To familiarize the various steps involved in the Design Process
- To understand the principles involved in evaluating the shape and dimensions of a component to satisfy functional and strength requirements.
- To learn to use standard practices and standard data
- To learn to use catalogues and standard machine components

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INTRODUCTION

- Machine Design is the creation of new and better machines and improving the existing ones
- The process of design is a long and time consuming one
- A new or better machine is one which is more economical in the overall cost of production and operation.
- It is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing

Classifications of Machine Design

1. *Adaptive design*. **D**esigner's work is concerned with adaptation of existing designs. The designer makes minor alternation or modification in the existing designs of the product.

2. *Development design*. **T**his type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

3. *New design*. This type of design needs lot of research, technical ability and creative thinking.

(*a*) *Rational design*. This type of design depends upon mathematical formulae of principle of mechanics.

(*b*) *Empirical design*. This type of design depends upon empirical formulae based on the practice and past experience.

(*c*) *Industrial design*. This type of design depends upon the production aspects to manufacture any machine component in the industry.

(*d*) *Optimum design*. It is the best design for the given objective function under the specified constraints. It may be achieved by minimizing the undesirable effects.

(*e*) *System design*. It is the design of any complex mechanical system like a motor car.

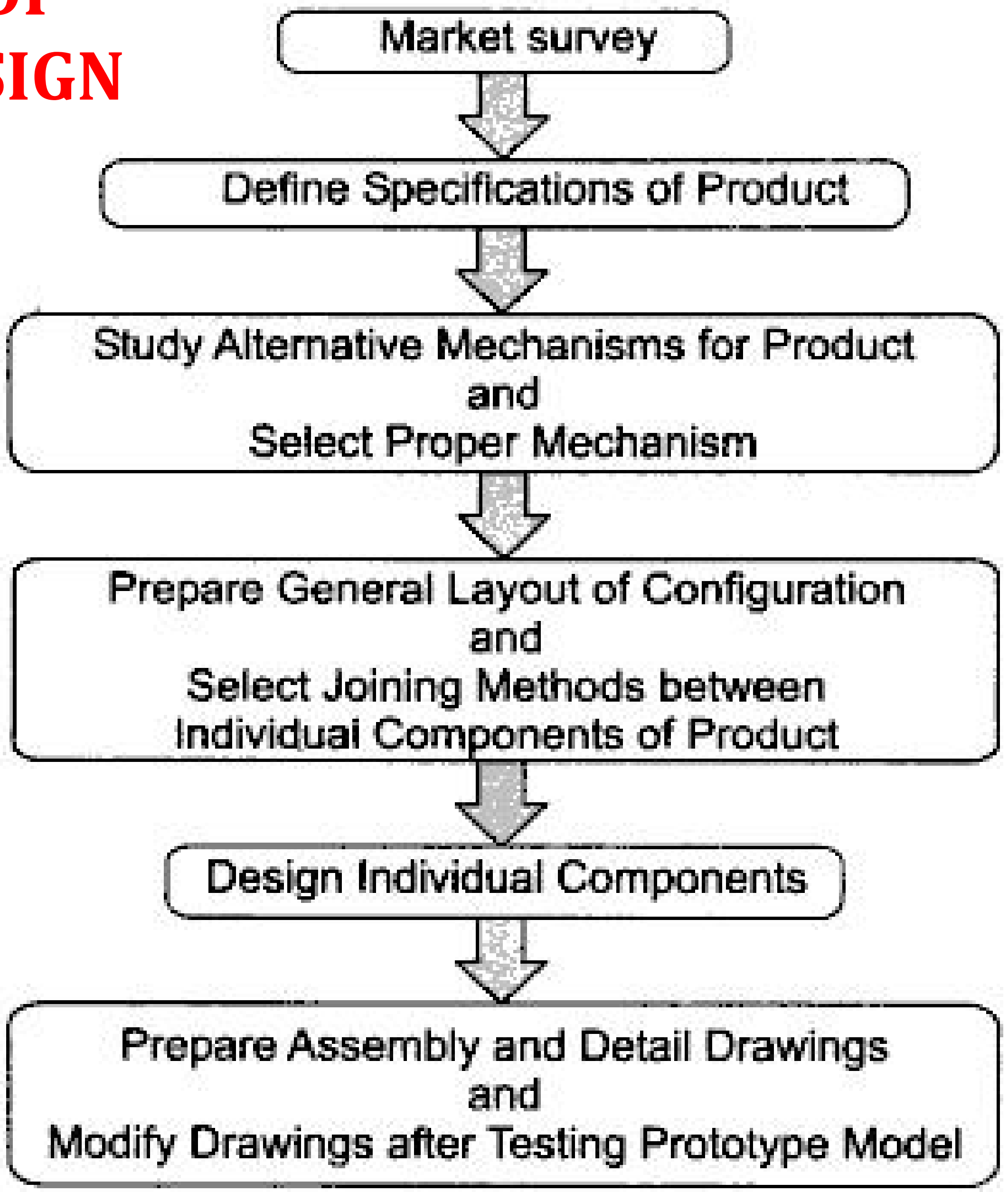
(*f*) *Element design*. It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.

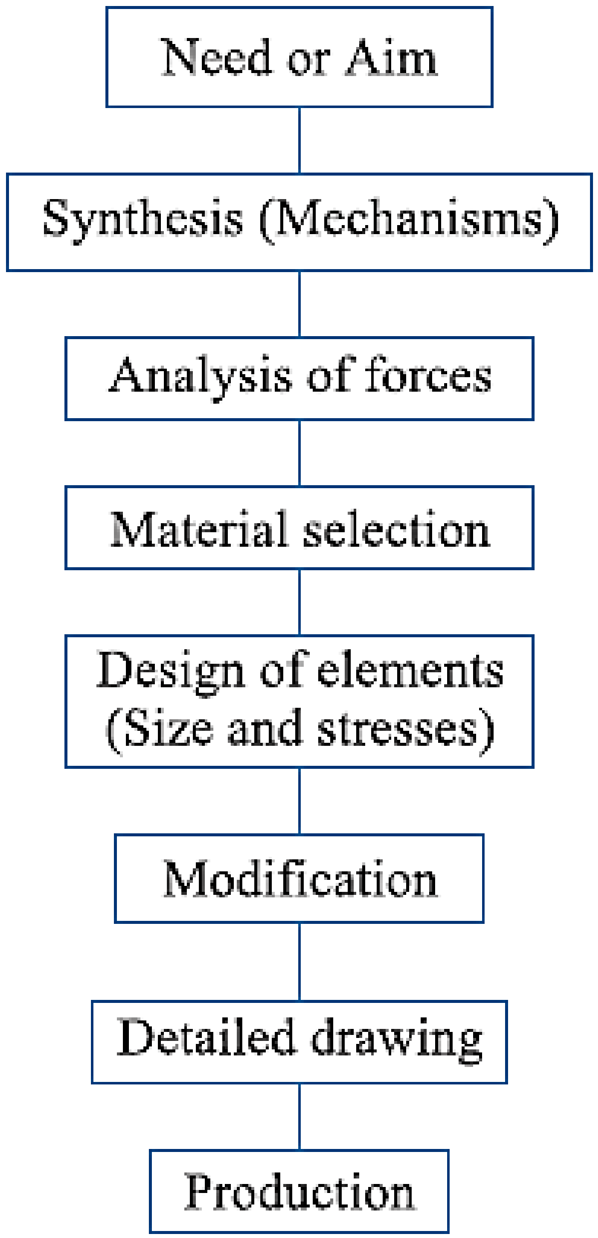
(*g*) *Computer aided design*. This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimization of a design.

General Considerations in Machine Design

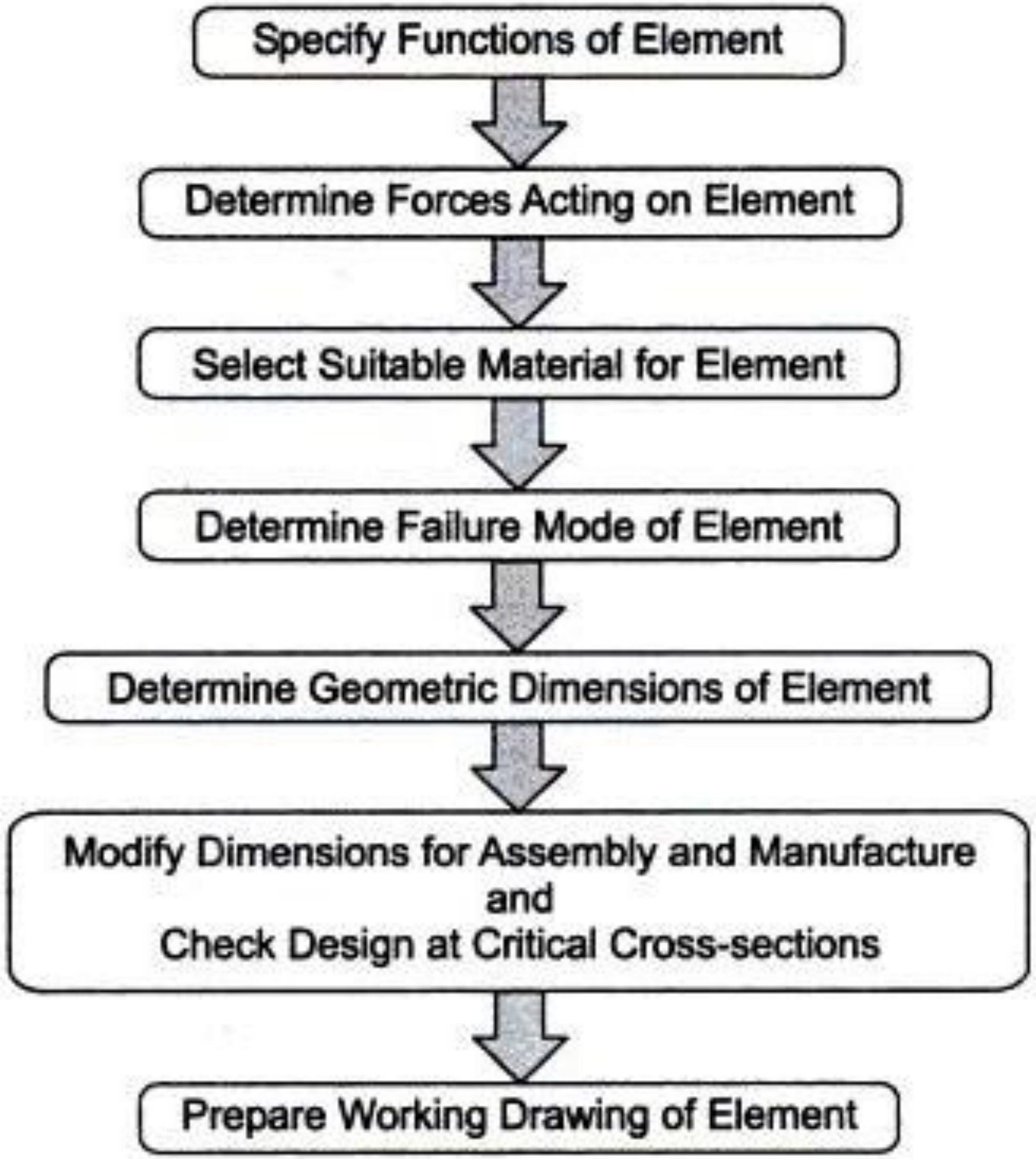
- Type of load and stresses caused by the load.
- Motion of the parts or kinematics of the machine
- Selection of materials
- Form and size of the parts
- Frictional resistance and lubrication
- Convenient and economical features
- Use of standard parts
- Safety of operation
- Workshop facilities
- Number of machines to be manufactured
- Cost of construction
- Assembling

PROCEDURE OF MACHINE DESIGN





DESIGN PROCEDURE OF MACHINE ELEMENTS



- STRENGTH
- RIGIDITY
- WEAR RESISTANCE
- MINIMUM DIMENSION AND WEIGHT
- MANUFACTURABILITY
- SAFETY
- CONFORMANCE TO STANDARDS
- RELIABILITY
- MAINTAINABILITY
- MINIMUM LIFE CYCLE COST
- APPEARANCE
- ERGONOMICS
- OVERALL COST

STANDARDS USED IN ENGINEERING DESIGN

- STANDARDS FOR MATERIALS
- STANDARDS FOR SHAPE AND DIMENSION
- STANDARDS FOR FITS AND TOLERANCE
- STANDARDS FOR TESTING
- STANDARDS FOR ENGINEERING DRAWING

TYPES OF STANDARD

- COMPANY STANDARD
- NATIONAL STANDARD
- INTERNATIONAL STANDARD

SELECTION OF MATERIALS IN DESIGN

- The knowledge of materials and their properties is of great significance for a design engineer.
- The machine elements should be made of such a material which has properties suitable for the conditions of operation.
- The best material is one which serve the desired objective at the minimum cost.

ENGINEERING MATERIALS

a) Metals

b) Non-metals

Metals:

Ferrous – Which contains iron as the major constituent

Ex. Steel, Cast Iron

Non-ferrous – Materials don't contains Iron.

Ex. Copper, Aluminium

Non-Metals:

(i) Ceramic materials – oxides, carbides and nitrides of various metals. Ex. Glass, Brick, Concrete, Cement etc.

(ii) Organic materials – Polymeric materials composed of carbon compounds. Ex: Paper, fuel, rubber, paints, etc.

FACTORS TO BE CONSIDERED FOR THE SELECTION OF MATERIALS

1. Availability
2. Quantity required
3. Size
4. Cost
5. Space Consideration
6. Physical properties
7. Mechanical Properties
8. Manufacturing process

Physical properties

- Density
- Colour
- Electrical conductivity
- Thermal conductivity
- Size
- Shape

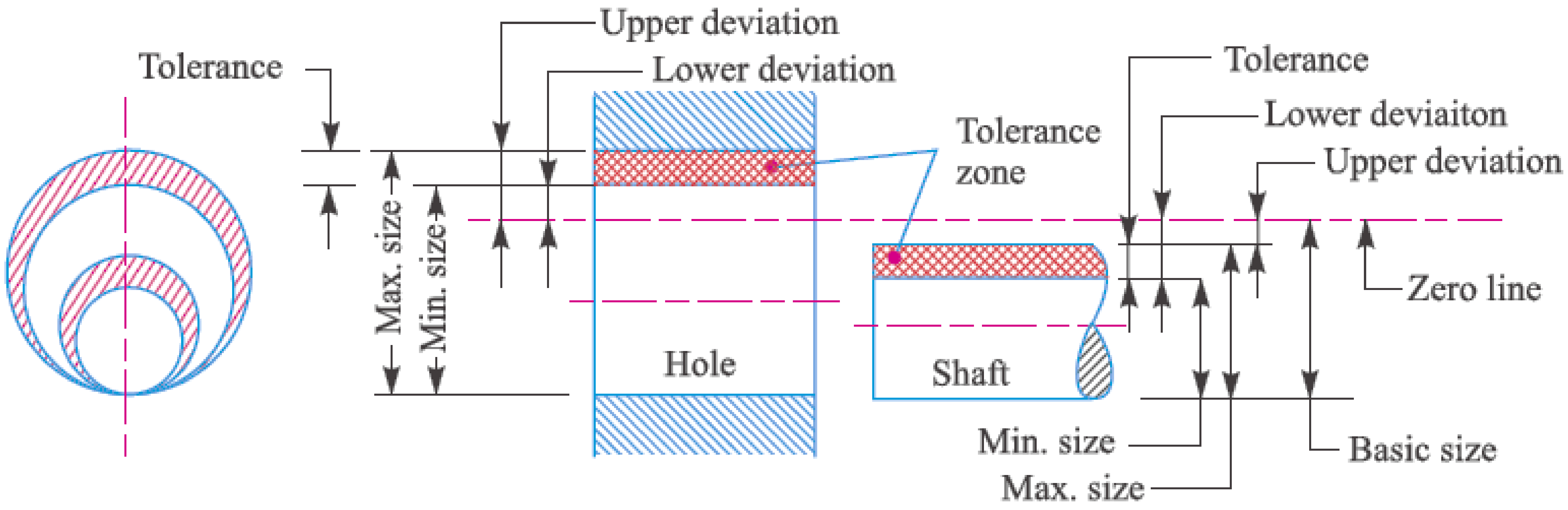
MECHANICAL PROPERTIES

- Strength → ability to resist external forces
- Stiffness → ability to resist deformation under stress
- Elasticity → property to regain its original shape
- Plasticity → property which retains the deformation produced under load
- Ductility → property of a material to be drawn into wire form with using tensile force
- Brittleness → property of breaking a material without any deformation
- Malleability → property of a material to be rolled or hammered into thin sheets
- Toughness → property to resist fracture under impact load
- Machinability → property of a material to be cut
- Resilience → property of a material to absorb energy
- Creep → material undergoes slow and permanent deformation when subjected to constant stress with high temperature
- Fatigue → failure of material due to cyclic loading
- Hardness → resistant to indentation, scratch

Interchangeability

- The term interchangeability is normally employed for the mass production of identical items within the prescribed limits of sizes. A lot of time is required to maintain the sizes of the part within a close degree of accuracy even then there will be small variations. If the variations are within certain limits, all parts of equivalent size will be equally fit for operating in machines and mechanisms. certain variations are recognized and allowed in the sizes of the mating parts to give the required fitting. In order to control the size of finished part, with due allowance for error, for interchangeable parts is called ***limit system***

Important Terms used in Limit System



Indian Standard System of Limits and Fits – IS 919 - 1993

- ***Nominal size*** It is the size of a part specified in the drawing as a matter of convenience.
- ***Basic size*** It is the size of a part to which all limits of variation (*i.e.* tolerances) are applied to arrive at final dimensioning of the mating parts. The nominal or basic size of a part is often the same.
- ***Actual size*** It is the actual measured dimension of the part. The difference between the basic size and the actual size should not exceed a certain limit, otherwise it will interfere with the interchangeability of the mating parts.

- ***Limits of sizes*** There are two extreme permissible sizes for a dimension of the part. The largest permissible size for a dimension of the part is called ***upper*** or ***high*** or ***maximum limit***, whereas the smallest size of the part is known as ***lower*** or ***minimum limit***
- ***Allowance*** It is the difference between the basic dimensions of the mating parts. The allowance may be ***positive*** or ***negative***. When the shaft size is less than the hole size, then the allowance is ***positive*** and when the shaft size is greater than the hole size, then the allowance is ***negative***
- ***Tolerance*** It is the difference between the upper limit and lower limit of a dimension. In other words, it is the maximum permissible variation in a dimension. The tolerance may be ***unilateral*** or ***bilateral***.

- ***Zero line*** It is a straight line corresponding to the basic size. The deviations are measured from this line. The positive and negative deviations are shown above and below the zero line respectively.

FITS

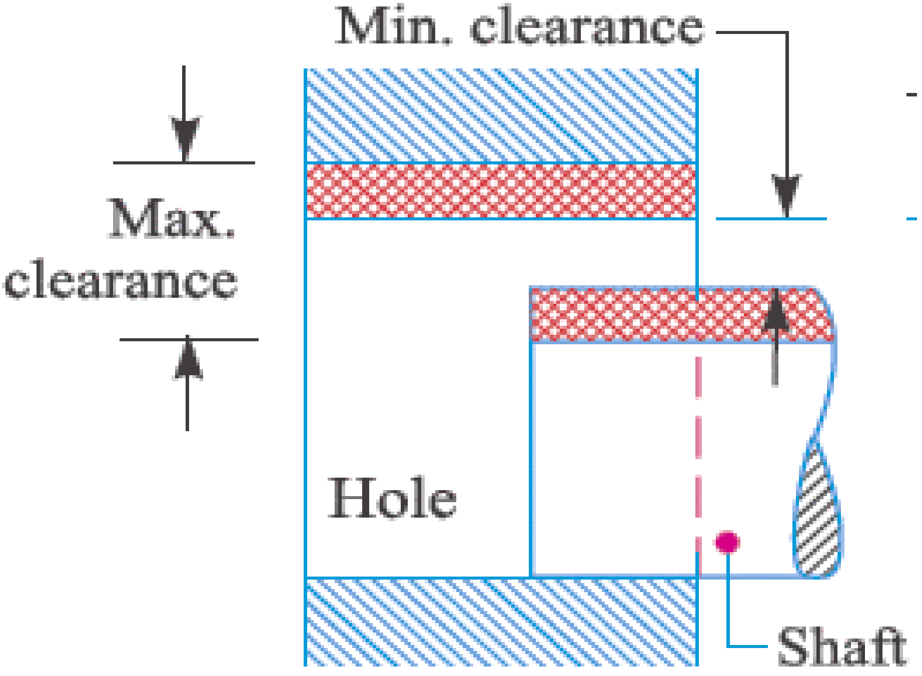
The degree of tightness or looseness between the two mating parts is known as a *fit.* The nature of fit is characterized by the presence and size of clearance and interference.

TYPES OF FIT

- Clearance fit
- Transition fit
- Interference fit

Clearance fit

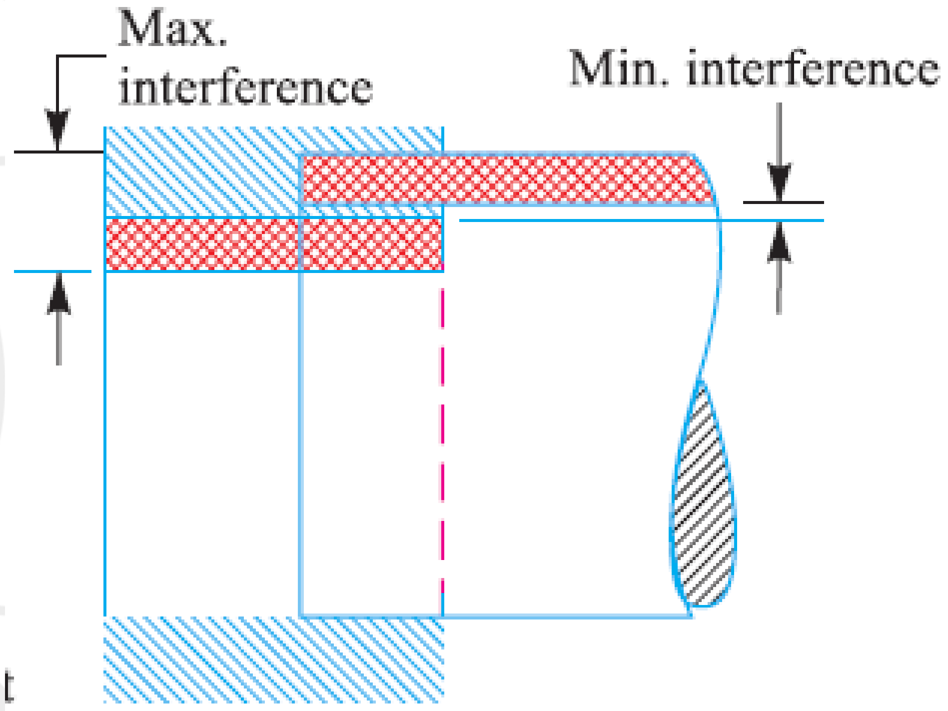
- In this type of fit, the size limits for mating parts are so selected that clearance between them always occur, It may be noted that in a clearance fit, the tolerance zone of the hole is entirely above the tolerance zone of the shaft. In a clearance fit, the difference between the minimum size of the hole and the maximum size of the shaft is known as ***minimum clearance*** whereas the difference between the maximum size of the hole and minimum size of the shaft is called ***maximum clearance*** The clearance fits may be **slide fit, easy sliding fit, running fit, slack running fit and loose running fit.**



(a) Clearance fit.

Interference fit

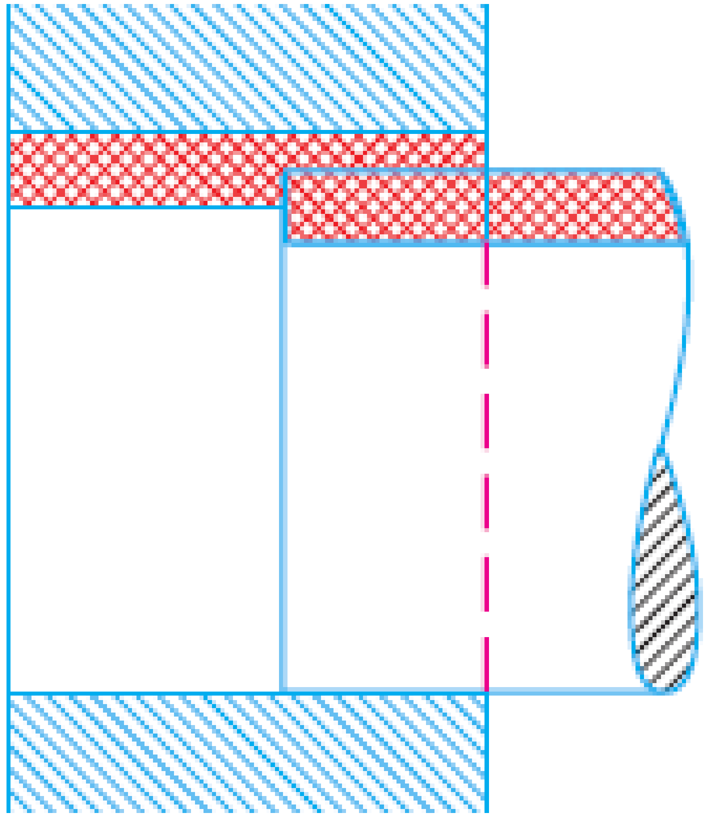
- In this type of fit, the size limits for the mating parts are so selected that interference between them always occur, It may be noted that in an interference fit, the tolerance zone of the hole is entirely below the tolerance zone of the shaft. In an interference fit, the difference between the maximum size of the hole and the minimum size of the shaft is known as ***minimum interference,*** whereas the difference between the minimum size of the hole and the maximum size of the shaft is called ***maximum interference*** **The interference fits may be shrink fit, heavy drive fit and light drive fit**



(*b*) Interference fit.

Transition fit

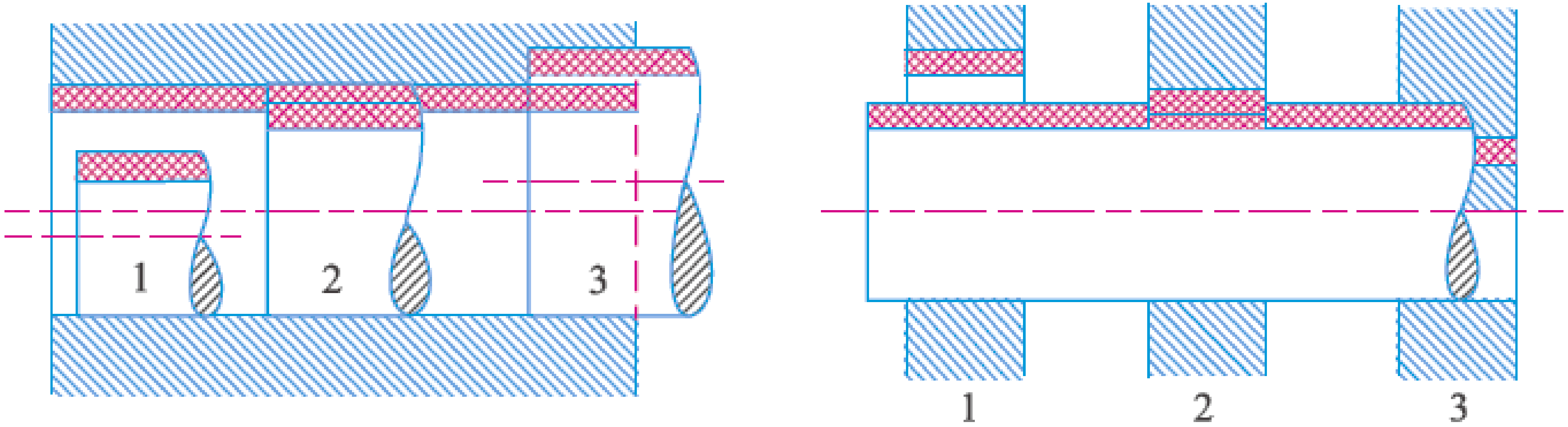
In this type of fit, the size limits for the mating parts are so selected that either a clearance or interference may occur depending upon the actual size of the mating parts, It may be noted that in a transition fit, the tolerance zones of hole and shaft overlap. **The transition fits may be force fit, tight fit and push fit**.



(c) Transition fit.

Basis of Limit System

- ***Hole basis system*** When the hole is kept as a constant member and different fits are obtained by varying the shaft size, then the limit system is said to be on a hole basis.
- ***Shaft basis system*** When the shaft is kept as a constant member and different fits are obtained by varying the hole size, then the limit system is said to be on a shaft basis.



1. Clearance fit. 2. Transition fit. 3. Interference fit.

(*a*) Hole basis system. (*b*) Shaft basis system.

Surface Roughness and its Measurement

- Surfaces produced by different machining operations (*e.g*. turning, milling, shaping, planing, grinding and superfinishing) are of different characteristics. The variation is judged by the degree of smoothness.
- In the assembly of two mating parts, it becomes absolutely necessary to describe the surface finish
- According to Indian standards, following symbols are used to denote the various degrees of surface roughness :

Symbol	Surface roughness (*Ra*) in microns
∇	8 to 25
∇ ∇	1.6 to 8
∇ ∇ ∇	0.025 to 1.6
∇ ∇ ∇ ∇	Less than 0.025

S.No.	Manufacturing process	Surface roughness in microns	S.No.	Manufacturing process	Surface roughness in microns
1.	Lapping	0.012 to 0.016	9	Extrusion	0.16 to 5
2.	Honing	0.025 to 0.40	10.	Boring	0.40 to 6.3
3.	Cylindrical grinding	0.063 to 5	11.	Milling	0.32 to 25
4.	Surface grinding	0.063 to 5	12.	Planing and shaping	1.6 to 25
5.	Broaching	0.40 to 3.2	13.	Drilling	1.6 to 20
6.	Reaming	0.40 to 3.2	14.	Sand casting	5 to 50
7.	Turning	0.32 to 25	15.	Die casting	0.80 to 3.20
8.	Hot rolling	2.5 to 50	16.	Forging	1.60 to 2.5

Factor of Safety

In order to prevent the failure of a machine part, the part is designed assuming such a value to design stress which is very low compared to the yield stress or ultimate stress.

"It is defined, as the ratio of the maximum stress to the working stress/design stress subjected to static loading"

$$\text{Factor of safety} = \frac{\mathit{Maximum\ stress}}{\mathit{Working\ (or)\ Design\ stress}} \rightarrow \quad \text{General}$$

$$\text{Factor of safety} = \frac{\mathit{Yeild\ point\ stress}}{\mathit{Working\ (or)\ Design\ stress}} \rightarrow \quad \text{Ductile Material}$$

$$\text{Factor of safety} = \frac{\mathit{Ultimate\ stress}}{\mathit{Working\ (or)\ Design\ stress}} \rightarrow \quad \text{Brittle Material}$$

Load:

Any external force acting upon a machine member

Types of load:

(i) Dead (or) Steady (or) Static load:

The load which does not change in magnitude and direction.

Ex. Self weight

(ii) Live (or) Varying load:

The load which is continuously changing.

Ex. Vehicle pass over a bridge

(iii) Suddenly applied load (or) shock load:

The load which is applied suddenly

Ex: Blows of a hammer

(iv) Impact load:

The load which is applied with some initial velocity (or) The load which is dropped from certain height.

Ex: forging

Stress:

The internal resistance of force per unit area is called stress.

$$\sigma = \frac{P}{A}$$

Where P = Load or force acting on the body

A = Cross- sectional area of the body

Strain:

The rate of change of deformation (or) It's the ratio of change in dimension to the original dimension.

$$e = \frac{\delta L}{L}$$

Mathematical equitation

$$Stress\ (\sigma) = \frac{P}{A} : P = Load, A = Area$$

$$Strain\ (e) = \frac{\delta l}{l} : l = Length, \delta l = Change\ in\ length$$

$$Shear\,Strain\ (\phi) = \frac{\delta l}{l} : l = Length, \delta l = Change\ in\ length$$

$$Volumetric\ Strain\ (e_v) = \frac{\delta v}{v} : v = Volume, \delta l = Change\ in\ volume$$

% of Elongation

% of Reduction in Area

$$Poission's\ ratio\ (\mu) = \frac{Lateral\ strain}{Logitudinal\ strain}$$

$$Young's\ modulus\ (E) = \frac{\sigma}{e}; \quad \frac{\mathrm{Pl}}{\mathrm{A}\delta l}; \quad \delta l = \frac{Pl}{AE}$$

$$Modulus\ of\ Rigidity(C or\ G) = \frac{\tau}{\phi}$$

$$Bulk\ Modulus\ (K) = \frac{dp}{\frac{-dv}{v}}$$

$$Bending\ Stress = \frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$Torsional\ equation = \frac{\tau}{R} = \frac{C.\theta}{L} = \frac{q}{r} = \frac{T}{J}$$

Relation ship of stress and strain

In Two direction $e_1 = \frac{\sigma_1}{E} - \frac{\sigma_2}{E}.\mu$; $e_2 = \frac{\sigma_2}{E} - \frac{\sigma_1}{E}.\mu$

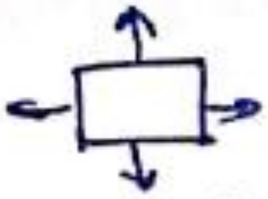
In Three direction

$$e_1 = \frac{\sigma_1}{E} - \frac{\sigma_2}{E}.\mu - \frac{\sigma_3}{E}\mu$$
$$e_2 = \frac{\sigma_2}{E} - \frac{\sigma_1}{E}.\mu - \frac{\sigma_3}{E}\mu$$
$$e_3 = \frac{\sigma_3}{E} - \frac{\sigma_1}{E}.\mu - \frac{\sigma_2}{E}\mu$$

Principal stresses

$$\sigma_n = \sigma \cos^2\theta$$

$$\sigma_T = \frac{\sigma}{2} \sin 2\theta$$



$$\sigma_n = \left(\frac{\sigma_1+\sigma_2}{2}\right) + \left(\frac{\sigma_1}{2} - \frac{\sigma_2}{2}\right) \cdot \cos 2\theta$$

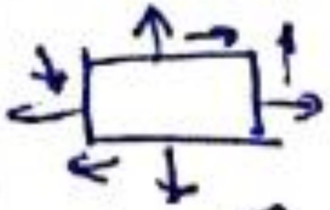
$$\sigma_T = \left(\frac{\sigma_1-\sigma_2}{2}\right) \sin 2\theta$$

$$\sigma_R = \sqrt{\sigma_n^2+\sigma_T^2}$$

$$\tan\phi = \frac{\sigma_T}{\sigma_N}$$

$$\sigma_N = \tau \sin 2\theta$$

$$\sigma_T = -\tau \cos 2\theta$$

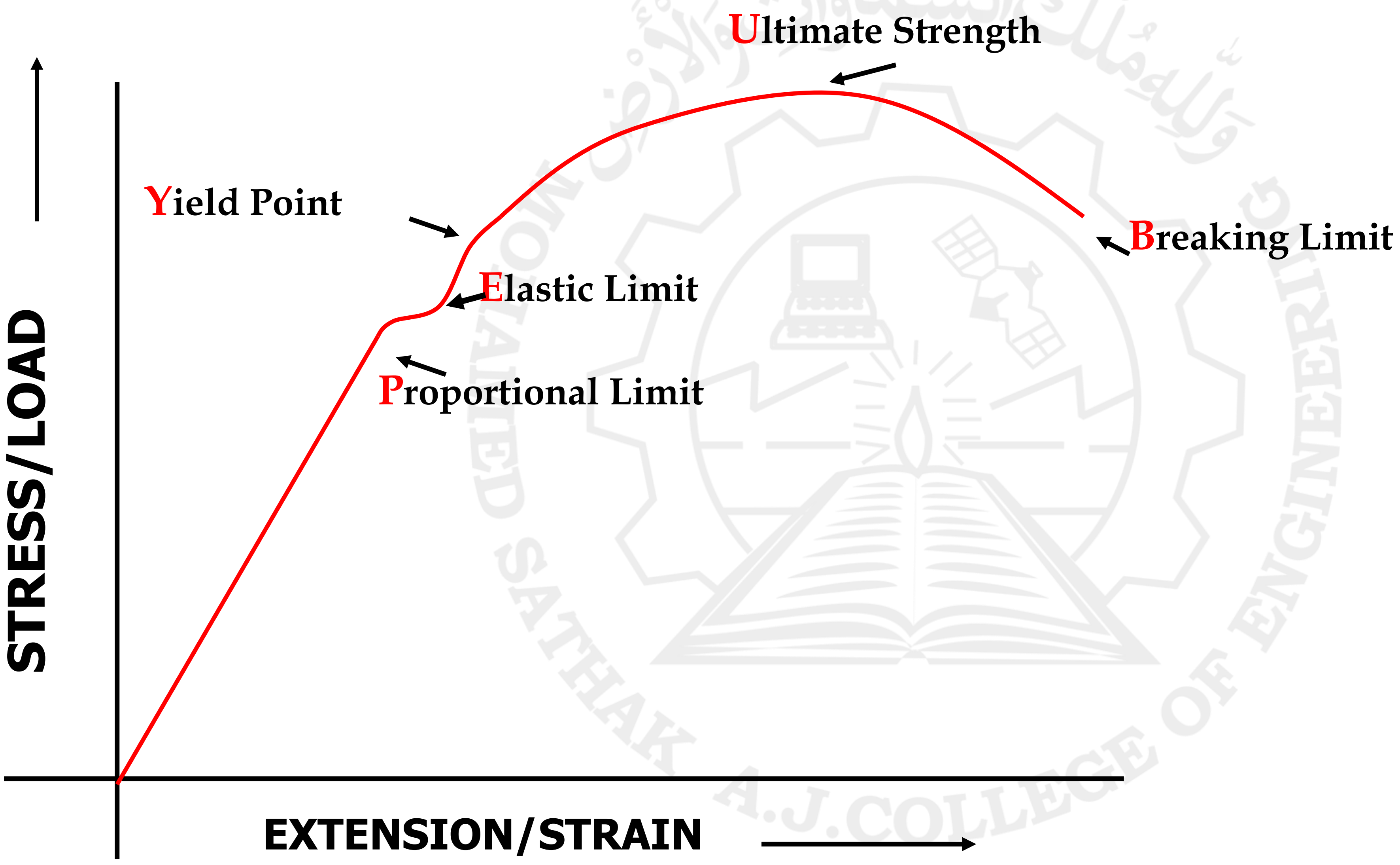


$$\sigma_N = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2} \cos 2\theta + \tau \sin 2\theta$$

$$\sigma_T = \frac{\sigma_1-\sigma_2}{2} \sin 2\theta - \tau \cos 2\theta$$

Major PS $= \dfrac{\sigma_1+\sigma_2}{2} + \sqrt{\left(\dfrac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$

Minor PS $\dfrac{\sigma_1+\sigma_2}{2} - \sqrt{\left(\dfrac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$

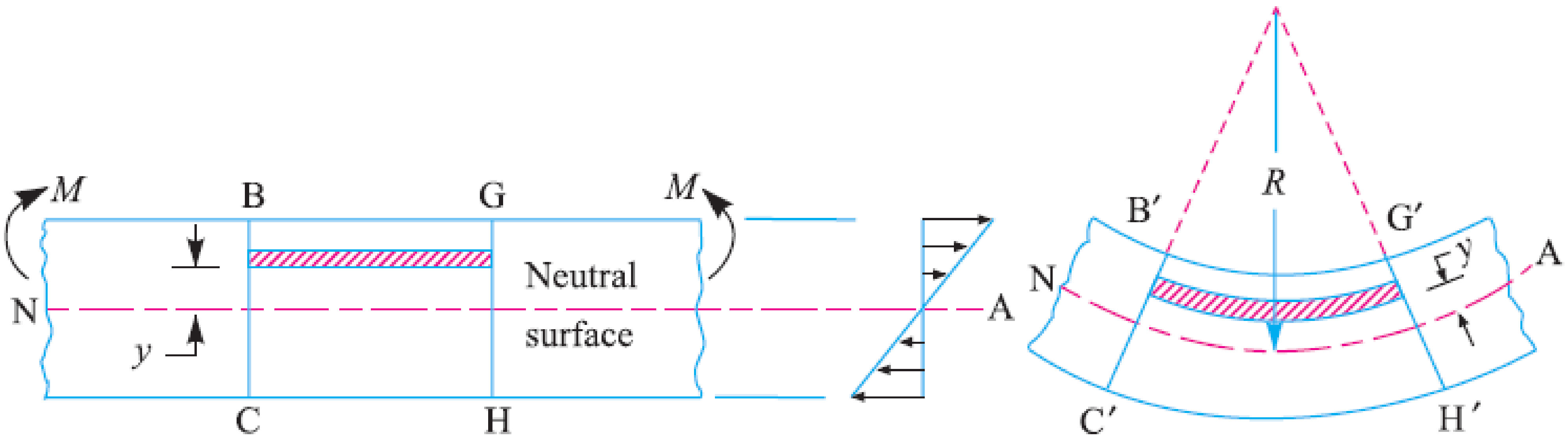


Ultimate Strength
Yield Point
Breaking Limit
Elastic Limit
Proportional Limit
STRESS/LOAD
EXTENSION/STRAIN

Bending Stress in Curved Beams

A curved beam is defined as a beam in which, the neutral axis is unloaded conditions is curved instead of straight

- The straight beams, the neutral axis of the section coincides with its centroidal axis and the stress distribution in the beam is linear. But in case of curved beams, the neutral axis of the cross-section is shifted towards the centre of curvature of the beam causing a non-linear (hyperbolic) distribution of stress. It may be noted that the neutral axis lies between the centroidal axis and the centre of curvature and always occurs within the curved beams. The application of curved beam principle is used in crane hooks, chain links and frames of punches, presses, planers etc.



$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

where

M = Bending moment acting at the given section,

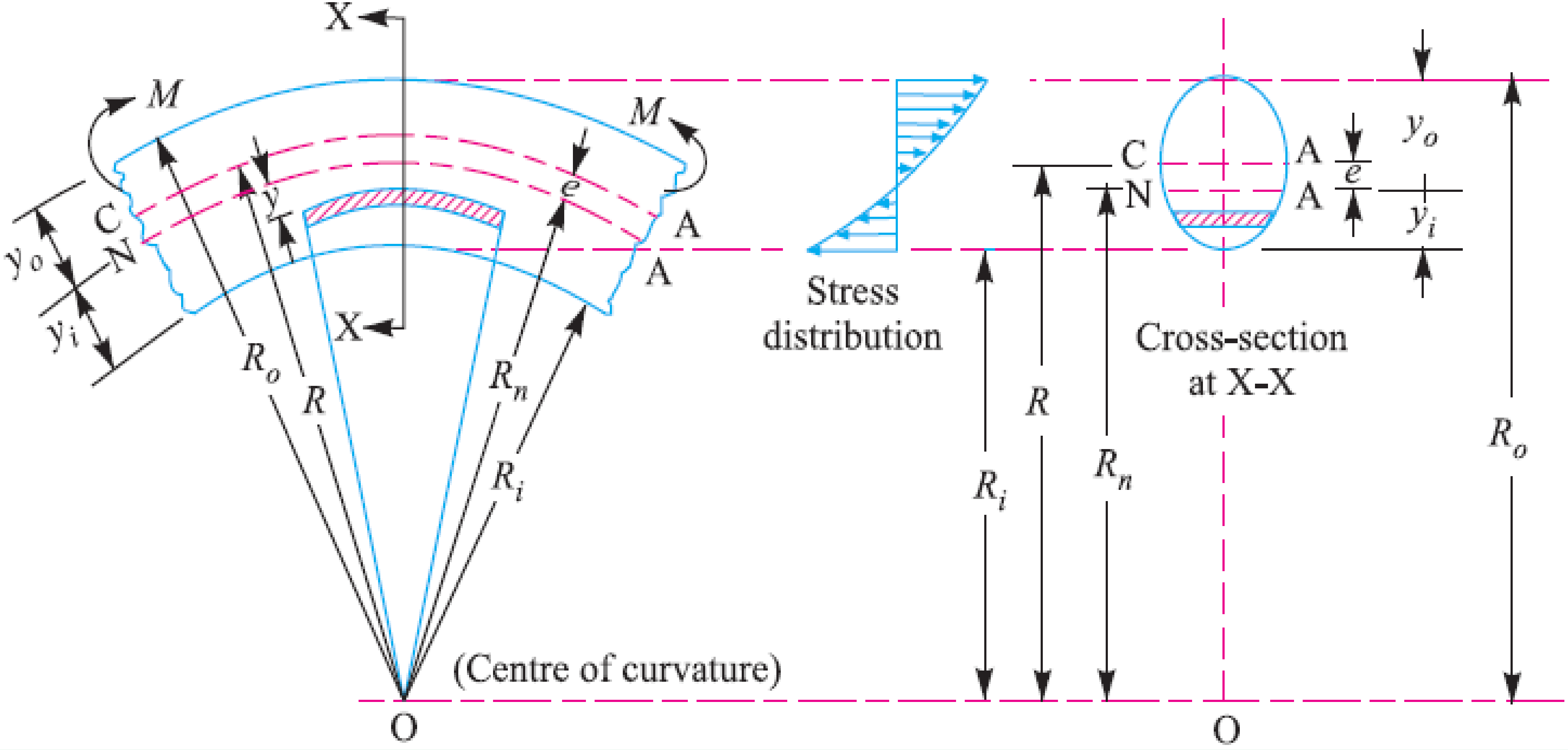
σ = Bending stress,

I = Moment of inertia of the cross-section about the neutral axis,

y = Distance from the neutral axis to the extreme fibre,

E = Young's modulus of the material of the beam, and

R = Radius of curvature of the beam.


X
M
M
C
N
A
A
y_o
y_i
y
e
X
R_o
R
R_n
R_i
(Centre of curvature)
O
Stress
distribution
C
N
A
A
y_o
e
y_i
Cross-section
at X-X
R_i
R
R_n
R_o
O

Consider a curved beam subjected to a bending moment M

- The general expression for the bending stress (σ_b) in a curved beam at any fibre at a distance y from the neutral axis, is given by

$$\sigma_b = \frac{M}{A\,e}\left[\frac{y}{R_n - y}\right]$$

M = Bending moment acting at the given section about the centroidal axis,

A = Area of cross-section,

e = Distance from the centroidal axis to the neutral axis = $R - R_n$,

R = Radius of curvature of the centroidal axis,

R_n = Radius of curvature of the neutral axis, and

y = Distance from the neutral axis to the fibre under consideration. It is positive for the distances towards the centre of curvature and negative for the distances away from the centre of curvature.

- If the section is symmetrical such as a circle, rectangle, I-beam with equal flanges, then the maximum bending stress will always occur at the inside fibre.
- If the section is unsymmetrical, then the maximum bending stress may occur at either the inside fibre or the outside fibre. The maximum bending stress at the inside fibre is given by

$$\sigma_{bi} = \frac{M \cdot y_i}{A \cdot e \cdot R_i}$$

y_i = Distance from the neutral axis to the inside fibre = $R_n - R_i$, and

R_i = Radius of curvature of the inside fibre.

$$\sigma_{bo} = \frac{M \cdot y_o}{A \cdot e \cdot R_o}$$

y_o = Distance from the neutral axis to the outside fibre = $R_o - R_n$, and

R_o = Radius of curvature of the outside fibre.

Theories of Failure

When the component is subjected to several types of loads, combined stresses are induced. The failures of such components are broadly classified into two groups, Elastic & Yielding and Fracture.

Theories of elastic failure provide a relationship between the strength of the machine components subjected to complex state of stresses

The problem of predicting the failure stresses for members subjected to bi-axial or tri-axial stresses is much more complicated, so that a large number of different theories have been formulated

The principal theories of failure for a member subjected to bi-axial stress

- Maximum principal stress theory (Rankine's theory).
- Maximum shear stress theory (Guest's or Tresca's theory).
- Maximum principal strain theory (Saint Venant theory).
- Maximum strain energy theory (Haigh's theory).
- Maximum distortion energy theory (Hencky and Von Mises theory).

MAXIMUM PRINCIPAL STRESS THEOR

(RANKINE'S THEORY)

This Criterion of failure is accredited to the british engineer, WJM RANKINE (1850)

The theory States that the failure of the mechanical Components Subjected to bi axial (or) tri axial stresses occurs when the maximum principal stress reaches the yield (or) ultimate strength of the material.

If σ_1, σ_2 and σ_3 are the three principal stresses at a point on the component. and

$$\sigma_1 > \sigma_2 > \sigma_3$$

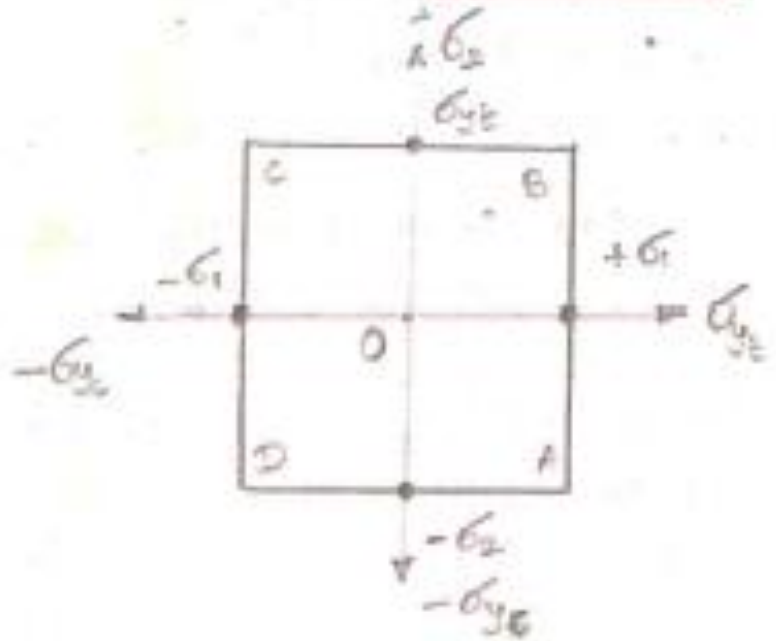
Then according to this theory, the failure occurs whenever

$$\sigma_1 = \sigma_{y_t} \quad \text{or} \quad \sigma_1 = \sigma_{u_{(t)}}$$

The theory considers only the maximum of principal stresses and disregards the influence of the other principal stress

$$\therefore \boxed{\sigma_1 = \frac{\sigma_{y_t}}{F.O.S}} \quad (\text{or}) \quad \boxed{\sigma_1 = \frac{\sigma_{u_t}}{F.O.S}}$$

t = tensile
or
c = compressive.



MAXIMUM SHEAR STRESS THEORY

(CA Coulomb, H Tresca and JJ Guest)

The theory states that the failure of a mechanical component subjected to bi axial, or triaxial stresses occurs when the maximum shear stress at any point in the component becomes equal to the maximum shear stress in the standard specimens of the tension test when yielding starts.

In the tension test, the specimen is subjected to uni axial stress (σ_1) and ($\sigma_2 = 0$)

$$\therefore \tau_{max} = \frac{\sigma_1}{2}$$ when the specimal starts yielding $\sigma_1 = \sigma_{yt}$

$$\therefore \tau_{max} = \frac{\cancel{\sigma_{yt}}}{\cancel{f_{s}}} = \frac{\sigma_{yt}}{2} \qquad \boxed{\tau_{max} = \frac{\sigma_{yt}}{2}}$$

Suppose σ_1, σ_2 and σ_3 are the three principal stresses at a point on the component

$$\therefore \tau_{1-2} = \frac{\sigma_1 - \sigma_2}{2} \qquad \tau_{2-3} = \frac{\sigma_2 - \sigma_3}{2} \qquad \tau_{3-1} = \frac{\sigma_3 - \sigma_1}{2}$$

The largest of these stresses is equated to

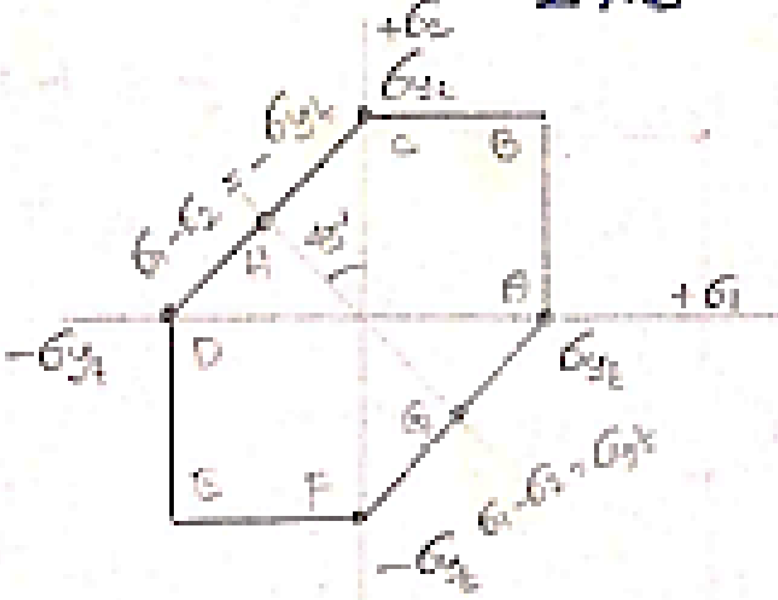
$$\tau_{max} = \frac{\sigma_{yt}}{2}$$

Considering f.o.s. $\dfrac{\sigma_1 - \sigma_2}{2} = \dfrac{\sigma_{yt}}{2\,fos}$

$$\therefore \boxed{\sigma_1 - \sigma_2 = \frac{\sigma_{yt}}{fos}}$$

$$\boxed{\sigma_2 - \sigma_3 = \frac{\sigma_{yt}}{fos}}$$

$$\boxed{\sigma_3 - \sigma_1 = \frac{\sigma_{yt}}{fos}}$$



DISTORTION ENERGY THEORY

(M T Huber, R von Mises & H Hencky)
1904 1913 1925

This theory states that the failure of the mechanical component subjected to biaxial or triaxial stresses occurs when the strain energy of distortion per unit volume at any point in the component becomes equal to the strain energy of distortion per unit volume in the standard specimen of tension test when yielding starts.

A unit cube subjected to the three principal stresses σ_1, σ_2 and σ_3

The total strain energy U of the cube is

$$U = \frac{1}{2}\sigma_1\epsilon_1 + \frac{1}{2}\sigma_2\epsilon_2 + \frac{1}{2}\sigma_3\epsilon_3$$

The Criteria of failure for the distortion energy theory is

$$2\sigma_{y_t}^2 = (\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2$$

$$\sigma_{y_t} = \sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]}$$

Considering factor of safety

$$\boxed{\frac{\sigma_{y_t}}{F.O.S} = \sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right]}} \quad \text{3 dim}$$

For Biaxial stress $\sigma_3 = 0$

$$\boxed{\frac{\sigma_{y_t}}{F.O.S} = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}} \quad \text{2 dim}$$

MAXIMUM PRINCIPAL STRAIN THEORY

(Saint Venant's theory)

This theory states that the failure or yielding occurs at a point in a member when a the maximum principal strain in a biaxial stress system reaches the limiting value of strain (ie. strain at yield point) as determined from a simple tensile test.

$$\epsilon_{max} = \frac{\sigma_1}{E} - \mu \cdot \frac{\sigma_2}{E} \qquad \text{let } \epsilon = \frac{\sigma_{y_t}}{E}$$

$$\therefore \frac{\sigma_{y_t}}{E} = \frac{\sigma_1}{E} - \mu \cdot \frac{\sigma_2}{E} \quad ; \quad \boxed{\sigma_y = \sigma_1 - \mu \cdot \sigma_2} \rightarrow \text{Two Dim}$$

$\sigma_y = \sigma_1 - \mu\sigma_2 - \mu\sigma_3$	; $\sigma_y = \sigma_2 - \mu\sigma_3 - \mu\sigma_1$
$\sigma_y = \sigma_3 - \mu \cdot \sigma_1 - \mu \cdot \sigma_2$	$\rightarrow$ three dim

Whichever is maximum is taken as σ_{y_t}

MAXIMUM STRAIN ENERGY THEORY (Haigh's theory)

This theory states the failure occurs at a point in a member when the strain energy per unit volume in biaxial stress system reaches the limiting strain energy per unit volume as determined from simple tension test.

Strain energy/unit volume in biaxial stress system $\Big\}$ $U_1 = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 - \mu\sigma_1\sigma_2\right]$

limiting strain energy $U_2 = \frac{1}{2E}\left[\frac{\sigma_y}{F_s}\right]^2$

Based on the theory $U_1 = U_2$

$$\frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 - \mu\sigma_1\sigma_2\right] = \frac{1}{2E}\left(\frac{\sigma_y}{F_s}\right)^2$$

$$\boxed{\left(\frac{\sigma_y}{F_s}\right)^2 = \sigma_1^2 + \sigma_2^2 - \mu\sigma_1\sigma_2}$$

ECCENTRIC LOADING

An external load, whose line of action is parallel but does not coincide with the centroidal axis of the machine components is known as an eccentric load. the distance between the centroidal axis of the machine component and the eccentric load is called eccentricity. and is generally denoted by e.

According to the principle of statics the eccentric force P can be replaced by a parallel force P passing through the centroidal axis along with a couple (P×e)

The force P causes a uniformly distributed tensile stress of magnitude P/A

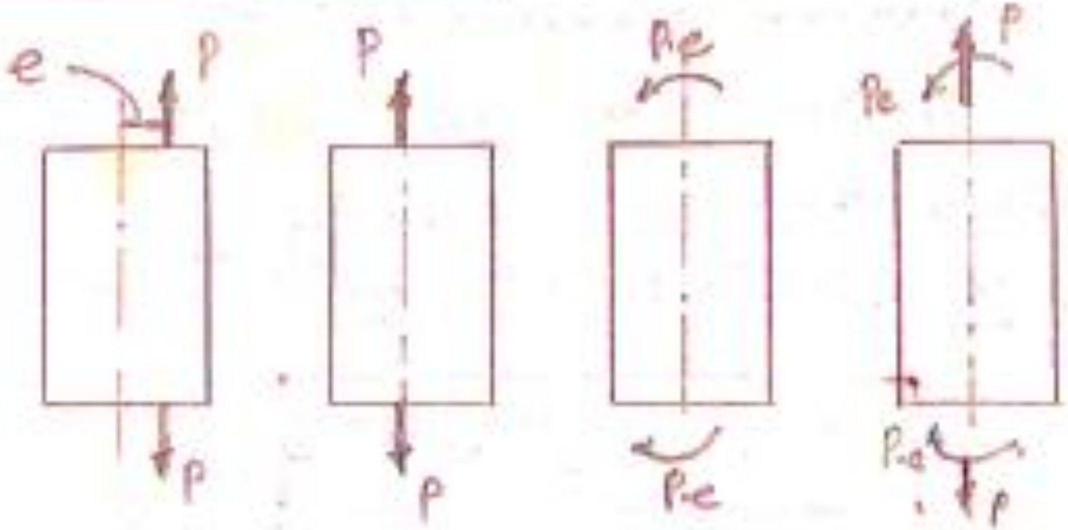
The couple causes bending stress of magnitude $\frac{P.e.y}{I}$

(ie) $\frac{M}{I} = \frac{\sigma_b}{y}$ ~~$= \frac{E}{R}$~~

$\frac{P.e}{I} \times y = \sigma_b$

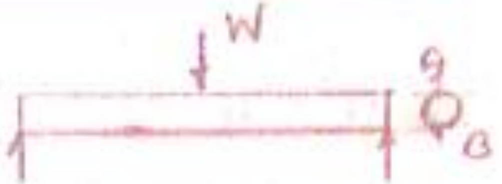
The Resultant stresses at the cross section are obtained by the principle of super imposition of stresses.

$$\therefore \boxed{\sigma = \frac{P}{A} \pm \frac{P.e.y}{I}}$$



COMPLETELY REVERSED OR CYCLIC STRESSES

Consider a rotating beam of circular cross section and carrying a load W. This load induces stresses in the beam which are cyclic in nature.



A little consideration will show that the upper fibers of the beam are under compressive stress and the lower fibre are under tensile stress. After a half revolution the point B occupies the position of point A and the point A occupies the position of point B. Thus the point B is now under compressive stress and the point A under tensile stress. The speed of variation of these stresses depends upon the speed of the beam.

For each Revolution of the beam the stresses are reversed from compressive to tensile. The stresses which vary from one value of compressive to the same value of tensile (or) vise versa are known as Completely reversed (or) Cyclic stresses

" The stresses which vary from a minimum value to a maximum value of the same nature are called fluctuating stresses."

" The stresses which vary from zero to a certain maximum value are called repeated stress"

" The stresses which vary from a minimum value to a maximum value of the opposite nature (i.e) min Tension to max Comp (or) min comp to max Tension) are called alternating stresses."

FATIGUE AND ENDURANCE LIMIT

FATIGUE: When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as fatigue.

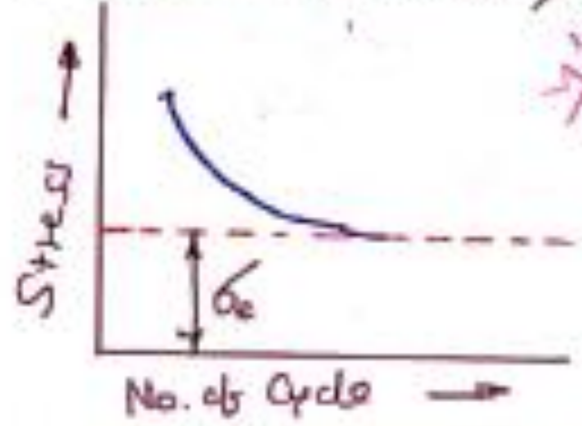
Failure is caused by means of a progressive crack formation which (or) failure may occur even without any prior indication.

ENDURANCE LIMIT:

A speciman is subjected to a completely reversed stress cycle. A record is kept of the number of cycles required to produce failure at a given stress. and the results are plotted in stress - cycle curve. (S-N CURVE)

If the stress is kept below a certain value as shown in dotted line the material will not fail what ever may be the number of cycles. This stress as represented by dotted line is known as Endurance or fatigue limit.

"It is defined as maximum value of the completely reversed bending stress which a polished standard specimen can with stand without failure for infinite number of cycles (usually 10^7 cycles)"



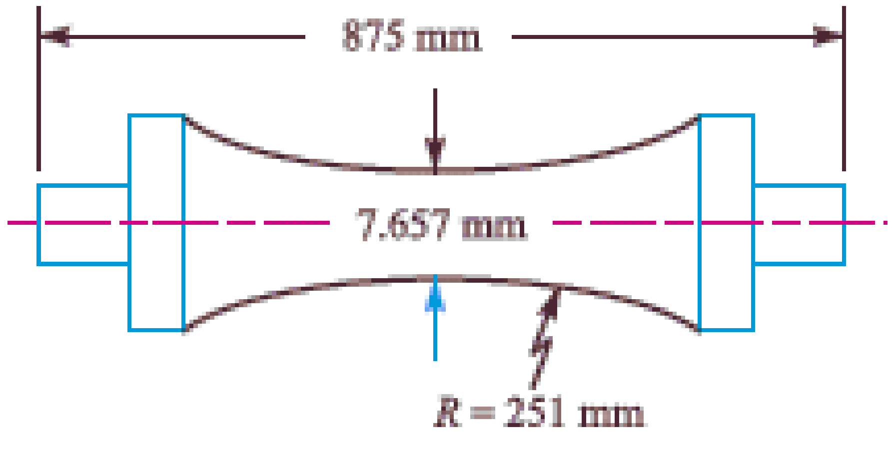
Endurance limit in σ_u

Steel $\sigma_e = 0.5\sigma_u$

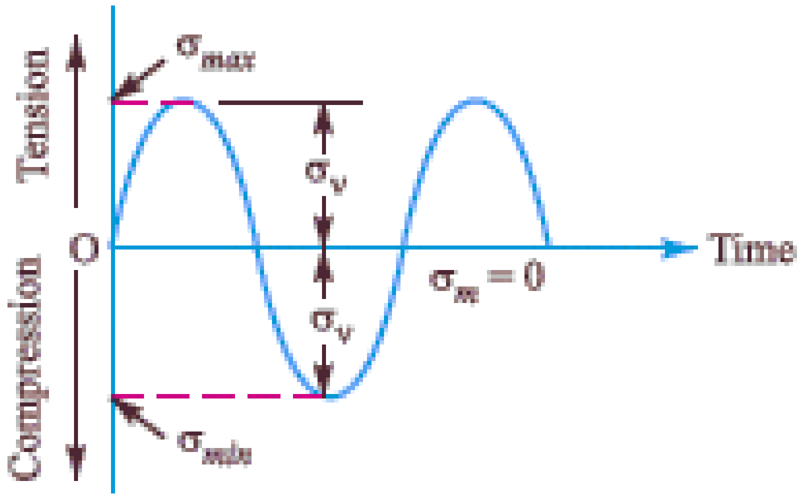
Cast steel $\sigma_e = 0.4\sigma_u$

Cast iron $\sigma_e = 0.35\sigma_u$

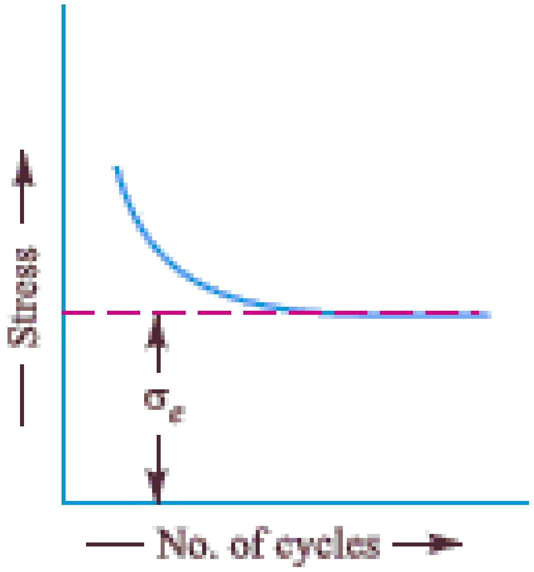
Non ferrous Metal & Alloys $\sigma_e = 0.3\sigma_u$



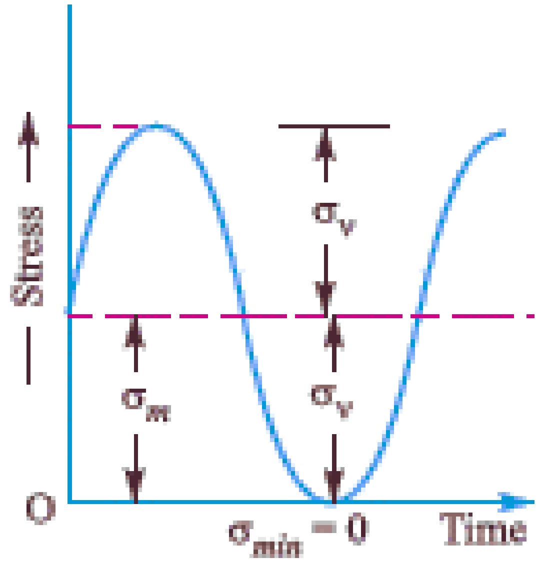
(a) Standard specimen.



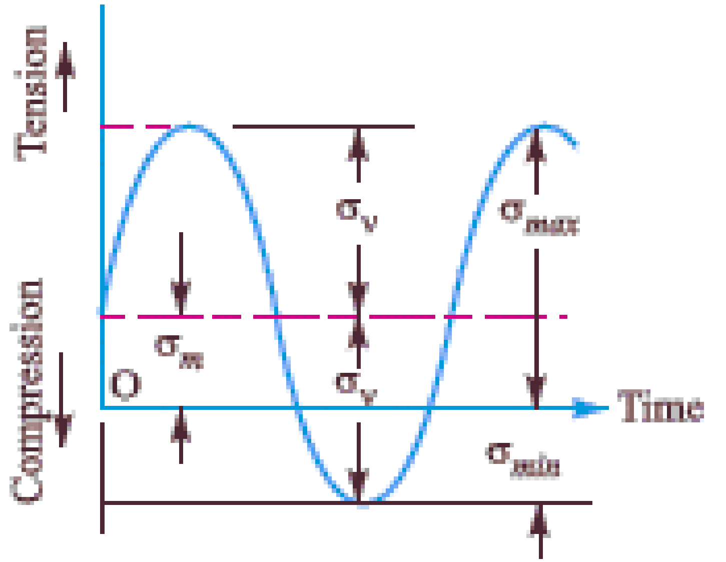
(b) Completely reversed stress.



(c) Endurance or fatigue limit.



(d) Repeated stress.



(e) Fluctuating stress.

1. Mean (or) Average stress $= \frac{\sigma_{max} + \sigma_{min}}{2}$

2. Reversed stress or alternating (or) variable stress $= \frac{\sigma_{max} - \sigma_{min}}{2}$

3. Stress ratio $R = \frac{\sigma_{max}}{\sigma_{min}}$

EFFECT OF LOADING ON ENDURANCE LIMIT – (LOAD FACTOR)

The endurance limit depending upon the type of loading may be modified by load factor.

$K_b \rightarrow$ load correction factor for the reversed (or) rotating or Bending load

$K_a \rightarrow$ load correction factor for axial load

$K_s \Rightarrow$ Torsional or Shear load.

$\sigma_{eb} = \sigma_e K_b$

$\sigma_{ea} = \sigma_e K_a$

$\tau_e = \sigma_e K_s$

Effect of Surface finish on endurance limit – surface finish factor.

K_{sur} = Surface finish factor

$$\sigma_{e_1} = \sigma_e \times K_{sur}$$

Effect of size and endurance limit – Size factor

K_{sz} = Size factor

$$\sigma_{ez} = \sigma_e \times K_{sz}$$

Effect of Miscellaneous factors on endurance limit

$$\sigma_e' = \sigma_{es} \times K_{sur} \cdot K_{sz} \cdot K_r \, K_b \, K_i$$

K_{sur} = Surface finish factor

K_{sz} = Size factor

K_b = load factor

K_r = Reliability factor

K_t = Temperature factor

K_i = impact factor.

NOTCH SENSITIVITY

It may be defined as the degree to which the theoretical effect of stress concentration is actually reached. The stress gradient depends mainly on the radius of the notch, hole, (or) fillet and on the grain size of the material.

Notch sensitivity factor q is used in cyclic loading

$$q = \frac{K_f - 1}{K_t - 1} \qquad K_f = 1 + q(K_t - 1)$$

$$K_{fs} = 1 + q(K_{ts} - 1)$$

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STRESS CONCENTRATION

If a machine component changes the shape of its cross section, the simple stress distribution no longer holds good and the neighbourhood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called stress concentration.

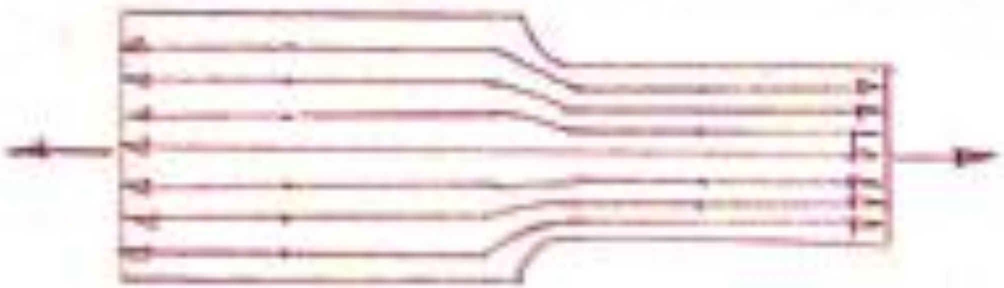
It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness or scratches etc.,

The material near the edges is stressed considerably higher than the average value.

The maximum stress occurs at some point on the fillet and is directed parallel to the boundary at that point.

CAUSES FOR STRESS CONCENTRATION

* Variation in properties of materials
* Load application
* Abrupt changes in section
* Discontinuities in the component
* Machining scratches.



THEORETICAL STRESS CONCENTRATION FACTOR

It is defined as the ratio of the maximum stress in a member (at a notch or a fillet) to the nominal stress at the same section based upon net area.

Mathematically $K_t = \dfrac{\text{Maximum stress}}{\text{Nominal stress}}$

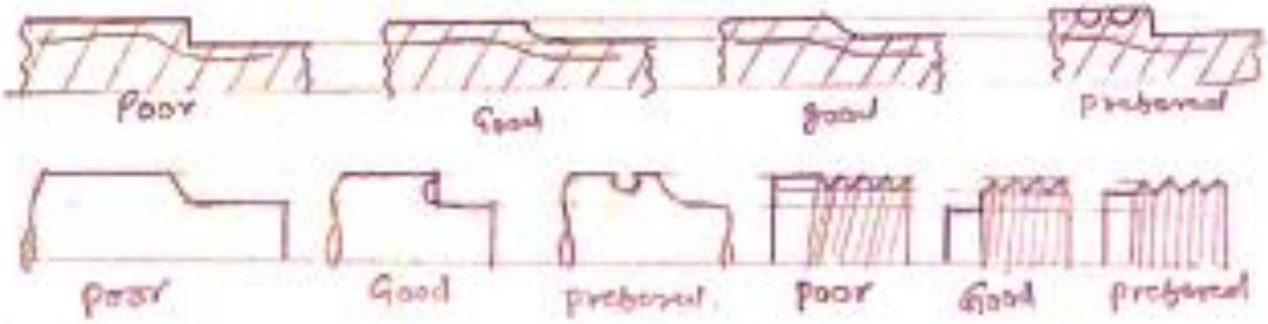
$$\boxed{k_t = \frac{\sigma_{max}}{\sigma}}$$

Stress concentration factor K_t will be referred from PSG Data Book Pg - 7.1 - 7.17 for various types of section.

METHODS OF REDUCING STRESS CONCENTRATION.

"It is not possible to completely eliminate the effect of stress concentration, these are methods to reduce stress concentration"

* Fillet, Radius, undercutting and notch for member in Bending
* Additional notches and holes in Tension member.
* Drilling Additional Holes for shafts
* Reduction of stress Concentration in Thread members.



FACTORS TO BE CONSIDERED WHILE DESIGNING MACHINE PARTS TO AVOID FATIGUE FAILURE

* The variation in the size of the component should be as gradual as possible
* The holes, notches and other stress raisers should be avoided.
* The proper stress de concentrators such as fillets and notches should be provided wherever necessary
* The parts should be protected from corrosive atmosphere
* A smooth finish of outer surface of the component increases the fatigue life
* The material with high fatigue strength should be selected

FATIGUE STRESS CONCENTRATION FACTOR

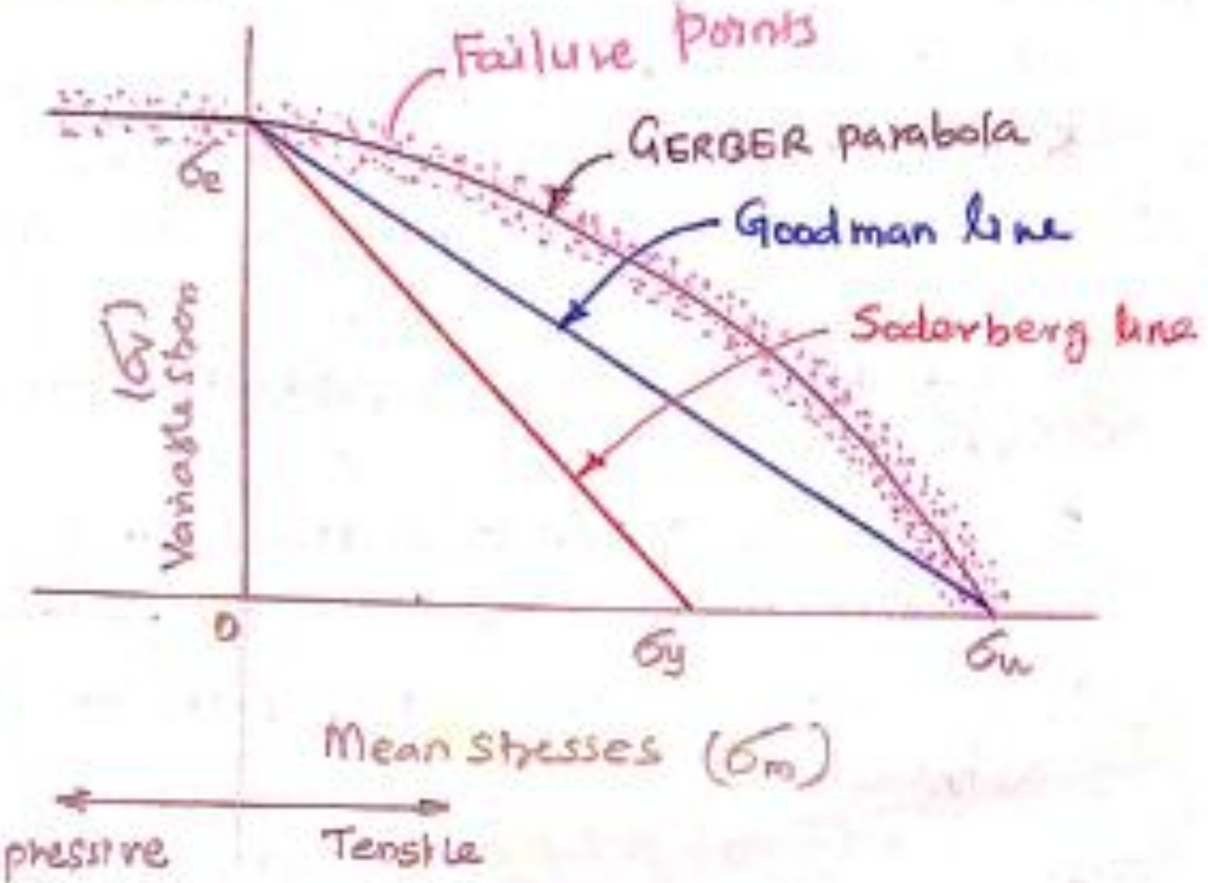
When a member subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoritical stress concentration factor

∴ Fatigue stress Concentration factor (K_f)

$$K_f = \frac{\text{Endurance limit without stress Concentration}}{\text{Endurance limit with stress Concentration}}$$

COMBINED STEADY AND VARIABLE STRESS

The failure points from fatigue tests made with different steels and combination of mean and variable stresses are plotted.



The Combination of stresses are solved by following methods.

1. GERBER method
2. GOODMAN method
3. SODERBERG method

"The test data for ductile material fall closer to Gerber parabola but because of scatter in the test points a straight line relationship is usually prefered in designing machine parts."

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GERBER METHOD FOR COMBINATION OF STRESSES

The relationship between variable stress (σ_v) and Mean stress (σ_m) for axial and bending loading for ductile matl. are plotted in graph.

A parabolic curve between the endurance limit (σ_e) and ultimate tensile strength (σ_u) was proposed by Gerber in ~~1874~~ 1874

pages
PSG DATA : 7.3 – 7.6

According to GERBER,

Variable stress $\sigma_v = \sigma_e \left[\frac{1}{F.S} - \left(\frac{\sigma_m}{\sigma_u}\right)^2 \cdot F.S \right]$

(or) $$\boxed{\frac{1}{F.S} = \left(\frac{\sigma_m}{\sigma_u}\right)^2 F.S + \frac{\sigma_v}{\sigma_e}} \quad -①$$

F.S = Factor of safety

σ_m = mean stress (tensile or compressive)

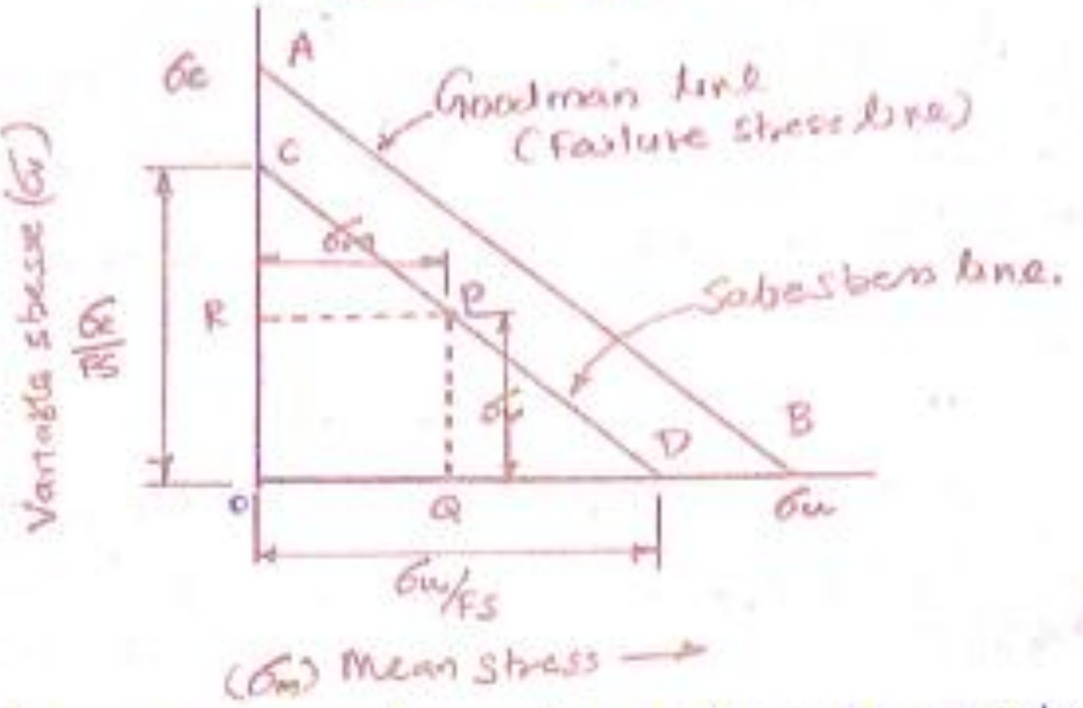
σ_u = Ultimate stress (Tensile or Compressive)

σ_e = Endurance limit for reversal loading

By considering fatigue stress concentration factor (K_f) eq.① will be

$$\boxed{\frac{1}{FS} = \left[\frac{\sigma_m}{\sigma_u}\right]^2 F.S + \frac{\sigma_v \cdot K_f}{\sigma_e}}$$

GOODMAN METHOD FOR COMBINATION OF STRESSES



The straight line connecting the endurance limit (σe) and the ultimate strength (σu) follows the ~~the~~ suggestion of Goodman.

A Goodman line is used when the design is based on ultimate strength and may be used for ductile or brittle materials.

Line AB connecting σ_e and σ_u is called Good'sman failure stress line. If a suitable f.s is applied to endurance limit and ultimate strength a safe stress line CD drawn parallel to AB

From the Δ COD & PQD

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD-OQ}{OD} = 1 - \frac{OQ}{OD}$$

$$\frac{\sigma_v}{\sigma_e/F_S} = 1 - \frac{\sigma_m}{\sigma_u/F_S}$$

$$\boxed{\frac{1}{F.S} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e}}$$

F.S = Factor of safety

σ_m = mean stress

σ_v = Variable stress

σ_u = ultimate stress

σ_e = Endurance stress

By considering K_f

K_f = Fatigue stress concentration factor.

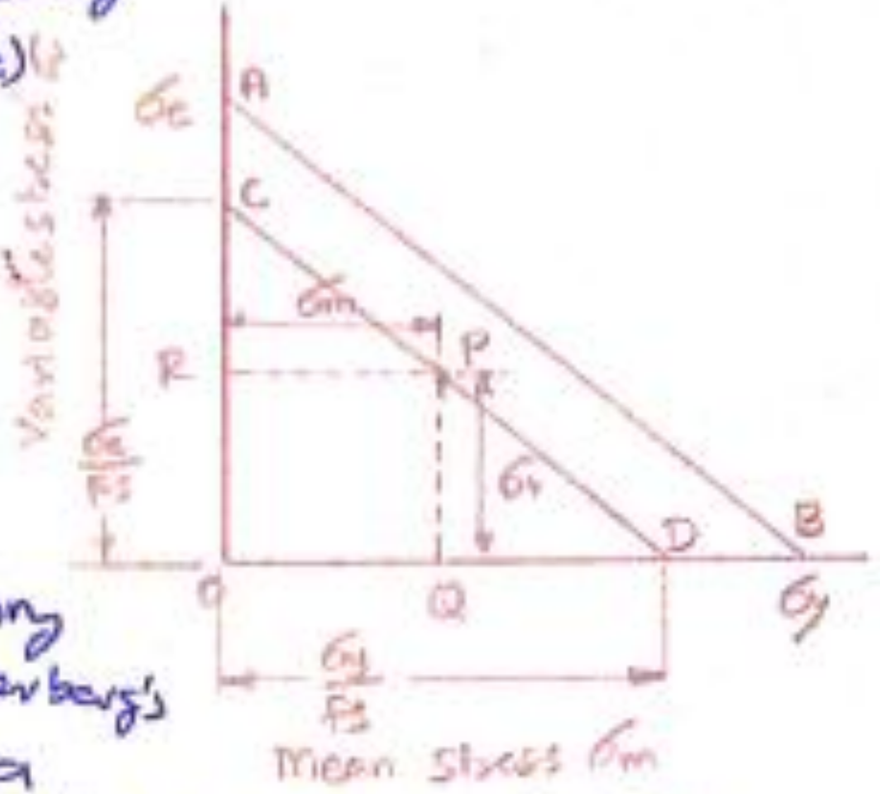
$$\boxed{\frac{1}{F.S} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v}{\sigma_e} \cdot K_f}$$

For shear loading

$$\boxed{\frac{1}{F.S} = \frac{\tau_m}{\tau_u} + \frac{\tau_v}{\tau_e} \cdot K_f}$$

SODERBERG METHOD FOR COMBINATION OF STRESSES

A straight line connecting the endurance limit (σ_e) and the yield strength σ_y follows the suggestion of Soderberg line. This line is used when the design is based on yield strength.



The line AB connecting σ_e and σ_y is called Soderberg's failure stress line. If a suitable factor of safety is applied to the endurance limit and yield strength, a safe stress line CD drawn parallel to the line AB.

From the triangle COD and PQD

$$\frac{PQ}{CO} = \frac{QD}{OD} = \frac{OD - OQ}{OD} = 1 - \frac{OQ}{OD}$$

$$\frac{\sigma_v}{\sigma_e / F_S} = 1 - \frac{\sigma_m}{\sigma_y / F_S} \quad ; \quad \boxed{\frac{1}{FS} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}}$$

By considering fatigue stress concentration factor K_f

$$\boxed{\frac{1}{FS} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \cdot K_f}{\sigma_e}}$$

For Reversed shear loading

$$\boxed{\frac{1}{FS} = \frac{\tau_m}{\tau_y} + \frac{\tau_v}{\tau_e} K_f}$$

COMBINED VARIABLE NORMAL STRESS AND VARIABLE SHEAR STRESS

When a machine part is subjected to both variables normal stress and a variable shear stress then it is designed by the following theories.

1. Maximum shear stress theory
2. Maximum Normal stress Theory

According to Soderberg's Relation

$$\frac{1}{F_s} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \cdot K_{fs}}{\sigma_e \cdot K_{sur} \times K_{sz}}$$

Multiply by σ_y on Both side

$$\frac{\sigma_y}{F_s} = \sigma_m + \frac{\sigma_v \cdot \sigma_y \cdot K_{fs}}{\sigma_e \cdot K_{sur} \times K_{sz}}$$

$$\frac{\tau_y}{F_s} = \tau_m + \frac{\tau_v \cdot \tau_y \cdot K_{fs}}{\tau_e \cdot K_{sur} \cdot K_{sz}}$$

Right hand side is equivalent shear stress

Equivalent shear stress

$$\boxed{\tau_{es} = \tau_m + \frac{\tau_v \, \tau_y \cdot K_{fs}}{\tau_e \cdot K_{sur} \cdot K_{sz}}}$$

Right hand side is equivalent normal stress due to reverse bending.

$$\therefore \sigma_{neb} = \sigma_m + \frac{\sigma_v \sigma_y k_{fb}}{\sigma_e k_{sur} k_{sz}}$$

IIly equivalent normal stress due to axial loading

$$\sigma_{nea} = \sigma_m + \frac{\sigma_v \sigma_y k_{fa}}{\sigma_e . k_{sur} k_{sz}}$$

$\therefore$ Total equivalent stress

$$\sigma_{ne} = \sigma_{nea} + \sigma_{nb} = \frac{\sigma_y}{FS}$$

Reversed torsional or shear loading

$$\frac{1}{FS} = \frac{\tau_m}{\tau_y} = \frac{\tau_v . k_{fs}}{\tau_e . k_{sur} k_{sz}}$$

multiplied by τ_y on Both side

Maximum shear stress theory is used in designing m/c parts of ductile material

$$\tau_{es(max)} = \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4(\tau_{es})^2} = \frac{\tau_y}{FS}$$

Maximum normal stress theory is used in designing m/c parts of brittle material.

$$\sigma_{ne\,max} = \frac{1}{2}\sigma_{ne} + \frac{1}{2}\sqrt{(\sigma_{ne})^2 + 4(\tau_{es})^2} = \frac{\sigma_y}{FS}$$

Calculate the tolerances, fundamental deviations and limits of sizes for the shaft designated as 40 H8 / f7

Data given:

40 H8/f7	*Dia of hole*	=	*40 mm*
	Dia of shaft	=	*40 mm*

From the Data book Pg. 3.9 and 3.7

$$H8 = \ +^{39}_{00} \qquad\qquad f7 = \ =^{25}_{50}$$

Upper limit of hole = 40.000 + 0.039 = 40.039
Lower limit of hole = 40.000 + 0.000 = 40.000
Tolerance of hole = 40.039-40.000 = 0.039

Upper limit of hole = 40.000 - 0.025 = 39.975
Lower limit of hole = 40.000 - 0.050 = 39.950
Tolerance of hole = 39.975 -39.950 = 0.025

Maximum Allowance = 40.039 - 39.950 = 0.089
Minimum Allowance = 40.000 – 39.975 = 0.025

5) A rectangular block of material is subjected to a tensile stress of 110 N/mm² on one plane and a tensile stress of 47 N/mm² on the plane at right angles to the former. Each of the above stresses is accompanied by a shear stress of 63 N/mm² and that associated with the former tensile stress tends to rotate the block anticlockwise. Find:

(i) Determine the normal and Tangential stress on a oblique plane of 45°

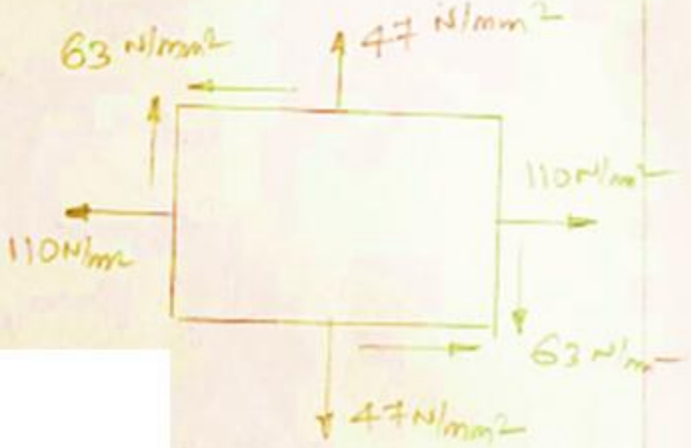
(ii) The direction and magnitude of each of the principal stress and

(iii) Magnitude of the greatest shear stress.

Data given

$\sigma_1 = 110\ N/mm^2$ $\sigma_2 = 47\ N/mm^2$

$\tau = 63\ N/mm^2$ $\theta = 45°$



$$\sigma_N = \frac{\sigma_1+\sigma_2}{2} + \frac{\sigma_1-\sigma_2}{2}\cos 2\theta + \tau \sin 2\theta$$

$$= \frac{110+47}{2} + \frac{110-47}{2}\cos 2(45) + 63 \times \sin(2\times 45)$$

$$78.5 + 0 + 63$$

$$\boxed{\sigma_N = 141.5 \text{ N/mm}^2}$$

$$\sigma_T = \frac{\sigma_1-\sigma_2}{2}\sin 2\theta - \tau\cos 2\theta$$

$$= \frac{110-47}{2} \times \sin 2(45) - 63\cos 2(45)$$

$$\boxed{\sigma_T = 31.5 \text{ N/mm}^2}$$

Major principal stress $= \frac{\sigma_1+\sigma_2}{2} + \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$

$= \frac{110+47}{2} + \sqrt{\left(\frac{110-47}{2}\right)^2 + 63^2}$

$= 78.5 + 70.44$

$\sigma_{max} = 148.94 \ N/mm^2$

Minor principal stress $= \frac{\sigma_1+\sigma_2}{2} - \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$

$= \frac{110+47}{2} - \sqrt{\left(\frac{110-47}{2}\right)^2 + 63^2}$

$= 78.5 - 70.44$

$\sigma_{min} = 8.064 \ N/mm^2$

Direction of principal stresses

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} = \frac{2 \times 63}{110 - 47}$$

$$\tan 2\theta = 2.000 \quad \therefore \theta = 31.71^\circ \text{ (or) } 121.43'$$

Magnitude of the greatest shear stress

$$\sigma_{t\,max} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

$$= \frac{1}{2}\sqrt{(110-47)^2 + 4 \times 63^2}$$

$$\sigma_{r\,max.} = 70.436 \text{ N/mm}^2$$

7) The following results were obtained in a tensile test on a mild steel specimen of original diameter 20 mm and gauge length 40mm

load at limit of proportionality = 80 KN

Extension at 80 KN load = 0.048 mm

load at yield point = 85 KN

Maximum load = 150 KN

When the two parts were fitted together after being broken, the length between gauge length was found to be 55.6 mm and the diameter at the neck was 15.8 mm

Calculate Young's modulus, yield stress, ultimate tensile stress, Percentage elongation and percentage of reduction in area

Data given:

$d = 20\,mm$ $P = 80\,kN$ $dl = 0.048$ @ 80 kN

$l = 40\,mm$ $dl = 15.6\,mm$ @ 150 kN

$P_Y = 85\,kN$ $P_{max} = 150\,kN$ $\Delta d = 15.8\,mm$

$$\sigma = \frac{80\times10^3}{\frac{\pi}{4}\times20^2} = 254.6\,N/mm^2$$

$$\epsilon = \frac{0.048}{40} = 1.2\times10^{-3}$$

$$E = \frac{254.6}{1.2\times10^{-3}}$$

$$\boxed{E = 212166.67\ N/mm^2}$$

Yield stress

$$\sigma_y = \frac{85\times10^3}{\pi/4\times20^2} \quad \boxed{\sigma_Y = 270.50\,N/mm^2}$$

Ultimate Tensile Stress

$$\sigma_u = \frac{150\times10^3}{\frac{\pi}{4}\times20^2} \quad \boxed{\sigma_u = 477.46\ N/mm^2}$$

% of elongation

$$= \frac{55.6-40}{40} = \boxed{39\%}$$

% of Reduction area

$$= \frac{20^2-15.8^2}{20^2} = \boxed{37.59\%}$$

Problem. *A steel shaft 35 mm in diameter and 1.2 m long held rigidly at one end has a hand wheel 500 mm in diameter keyed to the other end. The modulus of rigidity of steel is 80 GPa. 1. What load applied to tangent to the rim of the wheel produce a torsional shear of 60 MPa? 2. How many degrees will the wheel turn when this load is applied?*

Solution. Given : d = 35 mm or r = 17.5 mm ; l = 1.2 m = 1200 mm ; D = 500 mm or R = 250 mm ; C = 80 GPa = 80 kN/mm^2 = 80 × 103 N/mm^2 ; τ = 60 MPa = 60 N/mm2

1. Load applied to the tangent to the rim of the wheel

Let W = Load applied (in newton) to tangent to the rim of the wheel.

We know that torque applied to the hand wheel,

$$T = W.R = W \times 250 = 250\, W \text{ N-mm}$$

and polar moment of inertia of the shaft,

$$J = \frac{\pi}{32} \times d^4 = \frac{\pi}{32}(35)^4 = 147.34 \times 10^3 \text{ mm}^4$$

We know that $\dfrac{T}{J} = \dfrac{\tau}{r}$

$$\therefore \quad \frac{250\, W}{147.34 \times 10^3} = \frac{60}{17.5} \quad \text{or} \quad W = \frac{60 \times 147.34 \times 10^3}{17.5 \times 250} = 2020 \text{ N} \quad \textbf{Ans.}$$

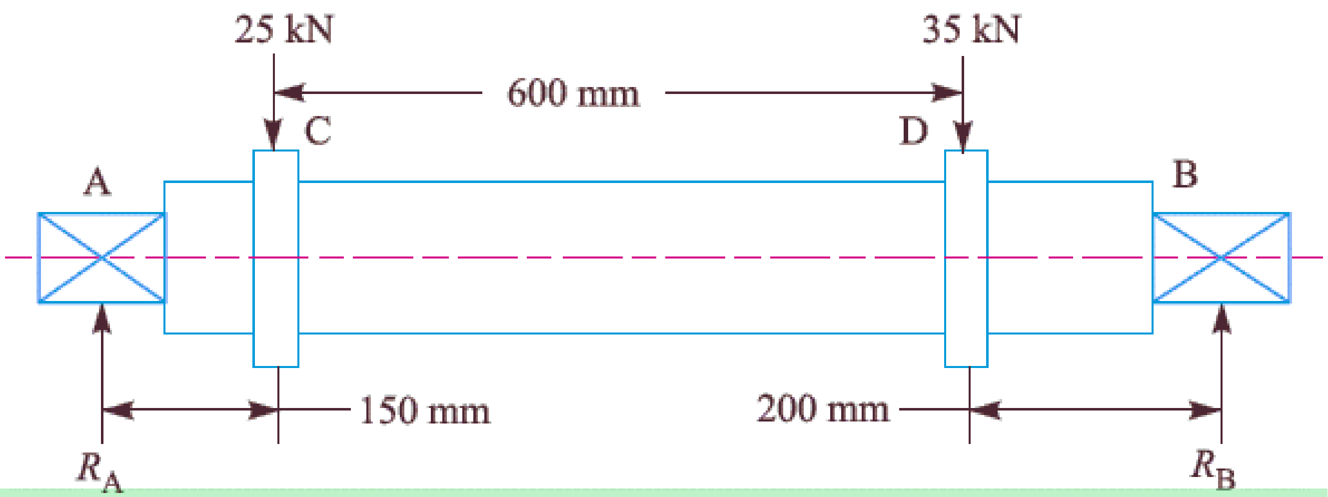
2. *Number of degrees which the wheel will turn when load W = 2020 N is applied*

Let θ = Required number of degrees.

We know that $\frac{T}{J} = \frac{C.\theta}{l}$

$$\therefore \quad \theta = \frac{T.l}{C.J} = \frac{250 \times 2020 \times 1200}{80 \times 10^3 \times 147.34 \times 10^3} = 0.05° \quad \textbf{Ans.}$$

Problem *A pump lever rocking shaft is shown in Fig. 5.5. The pump lever exerts forces of 25 kN and 35 kN concentrated at 150 mm and 200 mm from the left and right hand bearing respectively. Find the diameter of the central portion of the shaft, if the stress is not to exceed 100 MPa.*



Solution. Given : $\sigma_b = 100$ MPa $= 100$ N/mm^2

Let R_A and R_B = Reactions at *A* and *B* respectively.

Taking moments about *A*, we have

$$R_B \times 950 = 35 \times 750 + 25 \times 150 = 30\,000$$

$$\therefore \quad R_B = 30\,000 / 950 = 31.58 \text{ kN} = 31.58 \times 10^3 \text{ N}$$

and

$$R_A = (25 + 35) - 31.58 = 28.42 \text{ kN} = 28.42 \times 10^3 \text{ N}$$

$\therefore$ Bending moment at *C*

$$= R_A \times 150 = 28.42 \times 10^3 \times 150 = 4.263 \times 10^6 \text{ N-mm}$$

and bending moment at $D = R_B \times 200 = 31.58 \times 10^3 \times 200 = 6.316 \times 10^6$ N-mm

We see that the maximum bending moment is at D, therefore maximum bending moment, $M = 6.316 \times 10^6$ N-mm.

Let d = Diameter of the shaft.

$\therefore$ Section modulus,

$$Z = \frac{\pi}{32} \times d^3$$

$$= 0.0982\, d^3$$

We know that bending stress (σ_b),

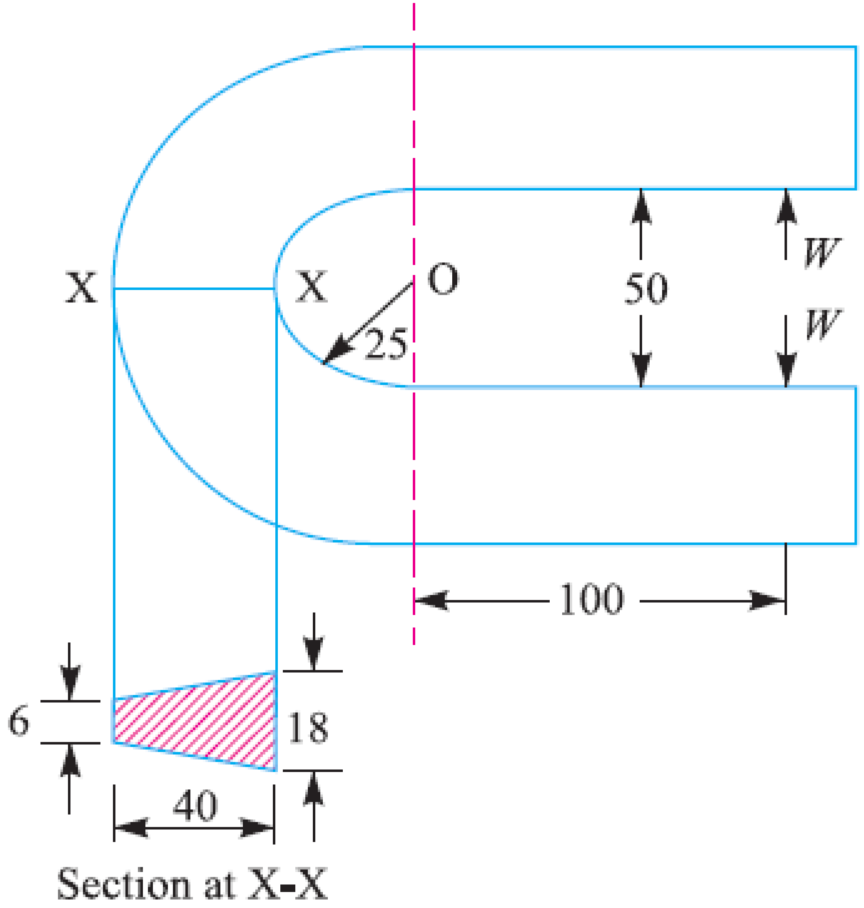
$$100 = \frac{M}{Z}$$

$$= \frac{6.316 \times 10^6}{0.0982\, d^3} = \frac{64.32 \times 10^6}{d^3}$$

$d^3 = 64.32 \times 10^6/100 = 643.2 \times 10^3$ or $d = 86.3$ say 90 mm **Ans.**

Problem *The frame of a punch press is shown in Fig. Find the stresses at the inner and outer surface at section X-X of the frame, if W = 5000 N.*

Solution. Given : W = 5000 N ; b_i = 18 mm ; b_o = 6 mm ; h = 40 mm ; R_i = 25 mm ;
R_o = 25 + 40 = 65 mm



All dimensions in mm.

We know that area of section at X-X,

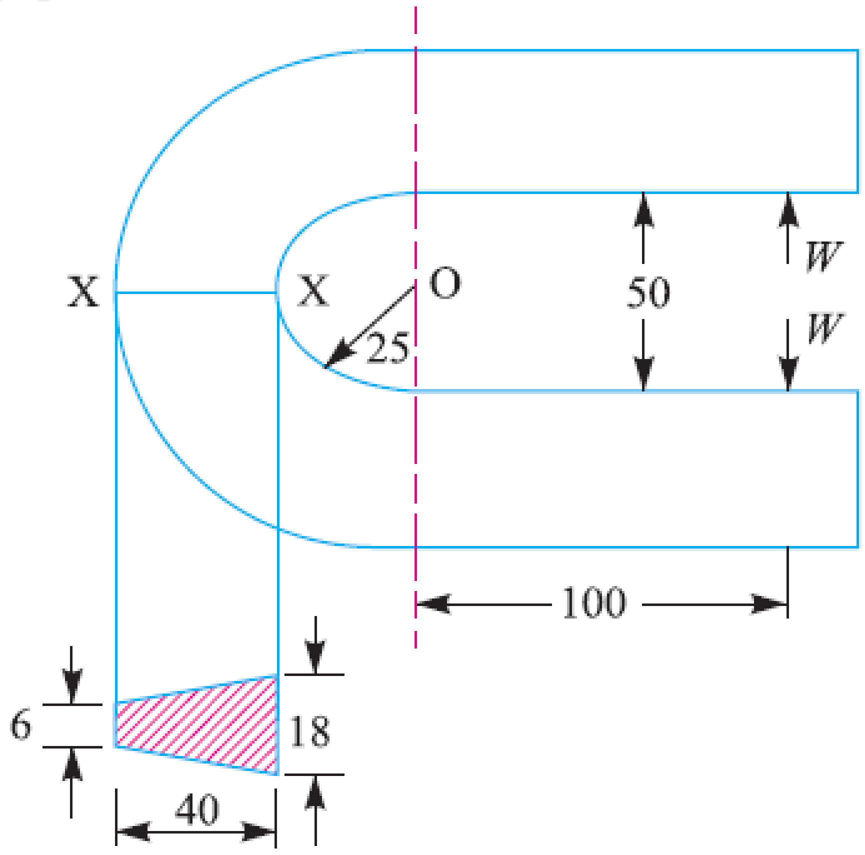
$$A = \frac{1}{2}(18 + 6)\,40 = 480 \text{ mm}^2$$

We know that radius of curvature of the neutral axis,

$$R_n = \frac{\left(\frac{b_i + b_o}{2}\right)h}{\left(\frac{b_i R_o - b_o R_i}{h}\right)\log_e\left(\frac{R_o}{R_i}\right) - (b_i - b_o)}$$

$$= \frac{\left(\frac{18 + 6}{2}\right)\times 40}{\left(\frac{18\times 65 - 6\times 25}{40}\right)\log_e\left(\frac{65}{25}\right) - (18 - 6)}$$

$$= \frac{480}{(25.5\times 0.9555) - 12} = 38.83 \text{ mm}$$



Section at X-X

All dimensions in mm.

and radius of curvature of the centroidal axis,

$$R = R_i + \frac{h\,(b_i + 2b_o)}{3\,(b_i + b_o)} = 25 + \frac{40\,(18 + 2 \times 6)}{3\,(18 + 6)} \text{ mm}$$

$$= 25 + 16.67 = 41.67 \text{ mm}$$

Distance between the centroidal axis and neutral axis,

$$e = R - R_n = 41.67 - 38.83 = 2.84 \text{ mm}$$

and the distance between the load and centroidal axis,

$$x = 100 + R = 100 + 41.67 = 141.67 \text{ mm}$$

∴ Bending moment about the centroidal axis,

$$M = W.x = 5000 \times 141.67 = 708\,350 \text{ N-mm}$$

The section at X-X is subjected to a direct tensile load of $W = 5000$ N and a bending moment of $M = 708\,350$ N-mm. We know that direct tensile stress at section X-X,

$$\sigma_t = \frac{W}{A} = \frac{5000}{480} = 10.42 \text{ N/mm}^2 = 10.42 \text{ MPa}$$

Distance from the neutral axis to the inner surface,

$$y_i = R_n - R_i = 38.83 - 25 = 13.83 \text{ mm}$$

Distance from the neutral axis to the outer surface,

$$y_o = R_o - R_n = 65 - 38.83 = 26.17 \text{ mm}$$

We know that maximum bending stress at the inner surface,

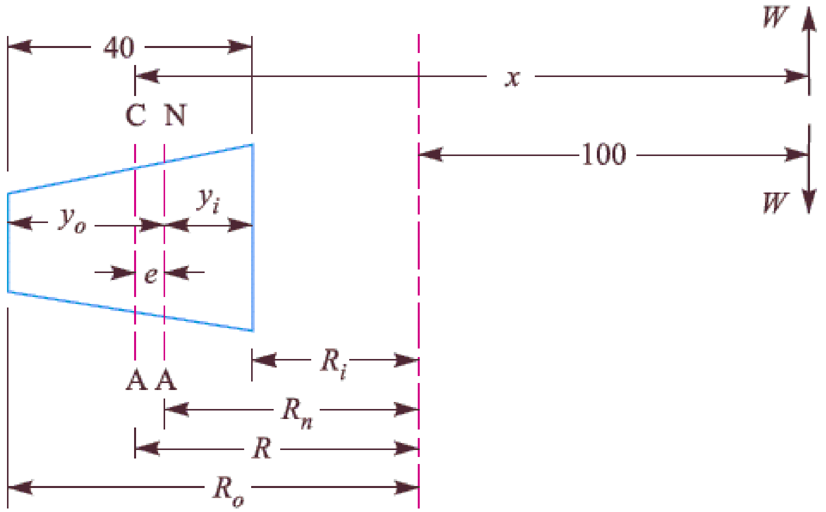
$$\sigma_{bi} = \frac{M \cdot y_i}{A \cdot e \cdot R_i} = \frac{708\,350 \times 13.83}{480 \times 2.84 \times 25} = 287.4 \text{ N/mm}^2$$

$$= 287.4 \text{ MPa (tensile)}$$

and maximum bending stress at the outer surface,

$$\sigma_{b0} = \frac{M \cdot y_o}{A \cdot e \cdot R_o} = \frac{708\,350 \times 26.17}{480 \times 2.84 \times 65} = 209.2 \text{ N/mm}^2$$

$$= 209.2 \text{ MPa (compressive)}$$



$\therefore$ Resultant stress on the inner surface

$$= \sigma_t + \sigma_{bi} = 10.42 + 287.4 = 297.82 \text{ MPa (tensile)} \quad \textbf{Ans.}$$

and resultant stress on the outer surface,

$$= \sigma_t - \sigma_{bo} = 10.42 - 209.2 = -198.78 \text{ MPa}$$

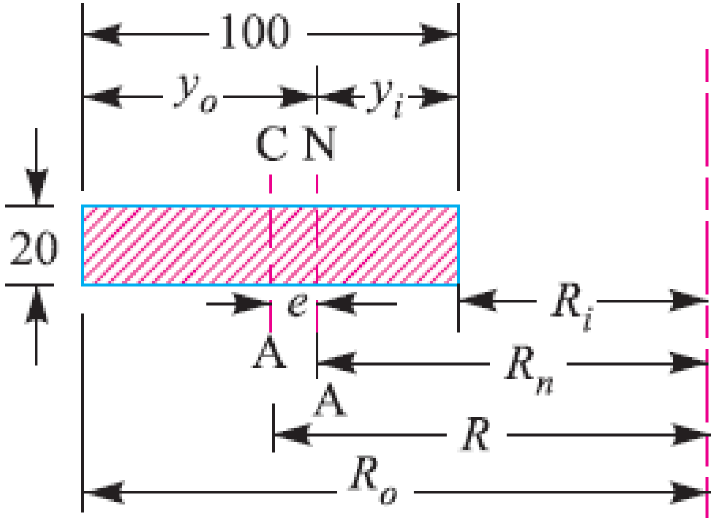
$$= 198.78 \text{ MPa (compressive)} \quad \textbf{Ans.}$$

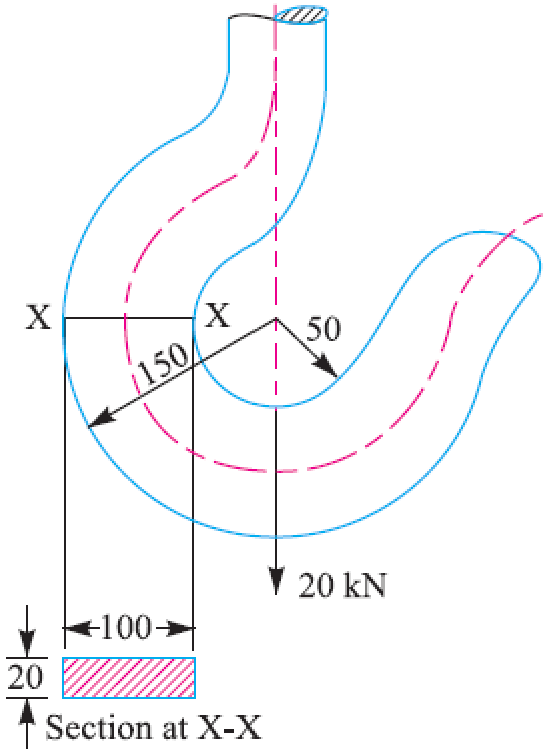
Problem *The crane hook carries a load of 20 kN as shown in Fig The section at X-X is rectangular whose horizontal side is 100 mm. Find the stresses in the inner and outer fibres at the given section.*

Solution. Given : W = 20 kN = 20 × 103 N ; Ri = 50 mm ; Ro = 150 mm ; h = 100 mm ; b = 20 mm

We know that area of section at *X-X*,

$A = b.h = 20 \times 100 = 2000$ mm^2





We know that radius of curvature of the neutral axis,

$$R_n = \frac{h}{\log_e\left(\frac{R_o}{R_i}\right)} = \frac{100}{\log_e\left(\frac{150}{50}\right)} = \frac{100}{1.098} = 91.07 \text{ mm}$$

and radius of curvature of the centroidal axis,

$$R = R_i + \frac{h}{2} = 50 + \frac{100}{2} = 100 \text{ mm}$$

∴ Distance between the centroidal axis and neutral axis,

$$e = R - R_n = 100 - 91.07 = 8.93 \text{ mm}$$

and distance between the load and the centroidal axis,

$$x = R = 100 \text{ mm}$$

∴ Bending moment about the centroidal axis,

$$M = W \times x = 20 \times 10^3 \times 100 = 2 \times 10^6 \text{ N-mm}$$

The section at X-X is subjected to a direct tensile load of $W = 20 \times 10^3$ N and a bending moment of $M = 2 \times 10^6$ N-mm. We know that direct tensile stress at section X-X,

$$\sigma_t = \frac{W}{A} = \frac{20 \times 10^3}{2000} = 10 \text{ N/mm}^2 = 10 \text{ MPa}$$

We know that the distance from the neutral axis to the inside fibre,

$$y_i = R_n - R_i = 91.07 - 50 = 41.07 \text{ mm}$$

and distance from the neutral axis to outside fibre,

$$y_o = R_o - R_n = 150 - 91.07 = 58.93 \text{ mm}$$

∴ Maximum bending stress at the inside fibre,

$$\sigma_{bi} = \frac{M \cdot y_i}{A \cdot e \cdot R_i} = \frac{2 \times 10^6 \times 41.07}{2000 \times 8.93 \times 50} = 92 \text{ N/mm}^2 = 92 \text{ MPa (tensile)}$$

and maximum bending stress at the outside fibre,

$$\sigma_{bo} = \frac{M \cdot y_o}{A \cdot e \cdot R_o} = \frac{2 \times 10^6 \times 58.93}{2000 \times 8.93 \times 150} = 44 \text{ N/mm}^2$$

$$= 44 \text{ MPa (compressive)}$$

∴ Resultant stress at the inside fibre

$$= \sigma_t + \sigma_{bi} = 10 + 92 = 102 \text{ MPa (tensile)}$$ **Ans.**

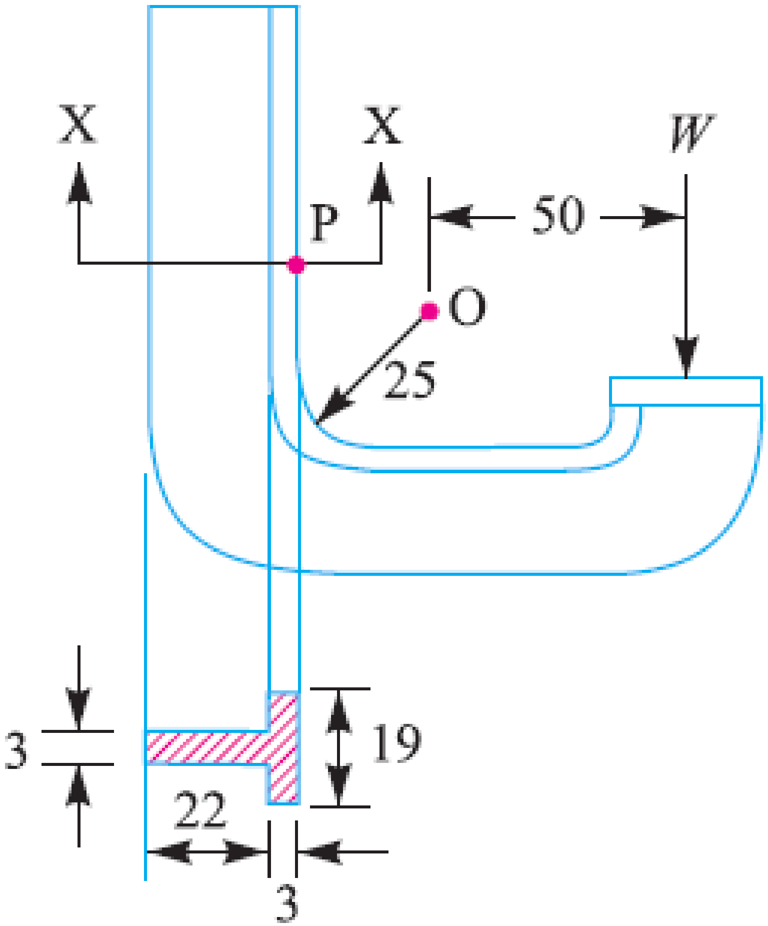
and resultant stress at the outside fibre

$$= \sigma_t - \sigma_{bo} = 10 - 44 = -34 \text{ MPa} = 34 \text{ MPa (compressive)}$$ **Ans.**

Problem *A C-clamp is subjected to a maximum load of W, as shown in Fig. 5.13. If the maximum tensile stress in the clamp is limited to 140 MPa, find the value of load W.*

Solution. Given : $\sigma t(max) = 140$ MPa = 140 N/mm² ;
$Ri = 25$ mm ; $Ro = 25 + 25 = 50$ mm ; $bi = 19$ mm ;
$ti = 3$ mm ; $t = 3$ mm ; $h = 25$ mm

We know that area of section at *X-X*,
$A = 3 \times 22 + 3 \times 19 = 123 \text{ mm}^2$



Section of X-X
All dimensions in mm.

$$R_n = \frac{t_i\ (b_i - t) + t\ .\ h}{(b_i - t)\ \log_e\left(\frac{R_i + t_i}{R_i}\right) + t\ \log_e\left(\frac{R_o}{R_i}\right)}$$

$$= \frac{3\ (19 - 3) + 3 \times 25}{(19 - 3)\ \log_e\left(\frac{25 + 3}{25}\right) + 3\ \log_e\left(\frac{50}{25}\right)}$$

$$= \frac{123}{16 \times 0.113 + 3 \times 0.693} = \frac{123}{3.887} = 31.64\ \text{mm}$$

and radius of curvature of the centroidal axis,

$$R = R_i + \frac{\frac{1}{2} h^2\ .\ t + \frac{1}{2}\ t_i^2\ (b_i - t)}{h\ .\ t + t_i\ (b_i - t)}$$

$$= 25 + \frac{\frac{1}{2} \times 25^2 \times 3 + \frac{1}{2} \times 3^2\ (19 - 3)}{25 \times 3 + 3\ (19 - 3)} = 25 + \frac{937.5 + 72}{75 + 48}$$

$$= 25 + 8.2 = 33.2\ \text{mm}$$

Distance between the centroidal axis and neutral axis,

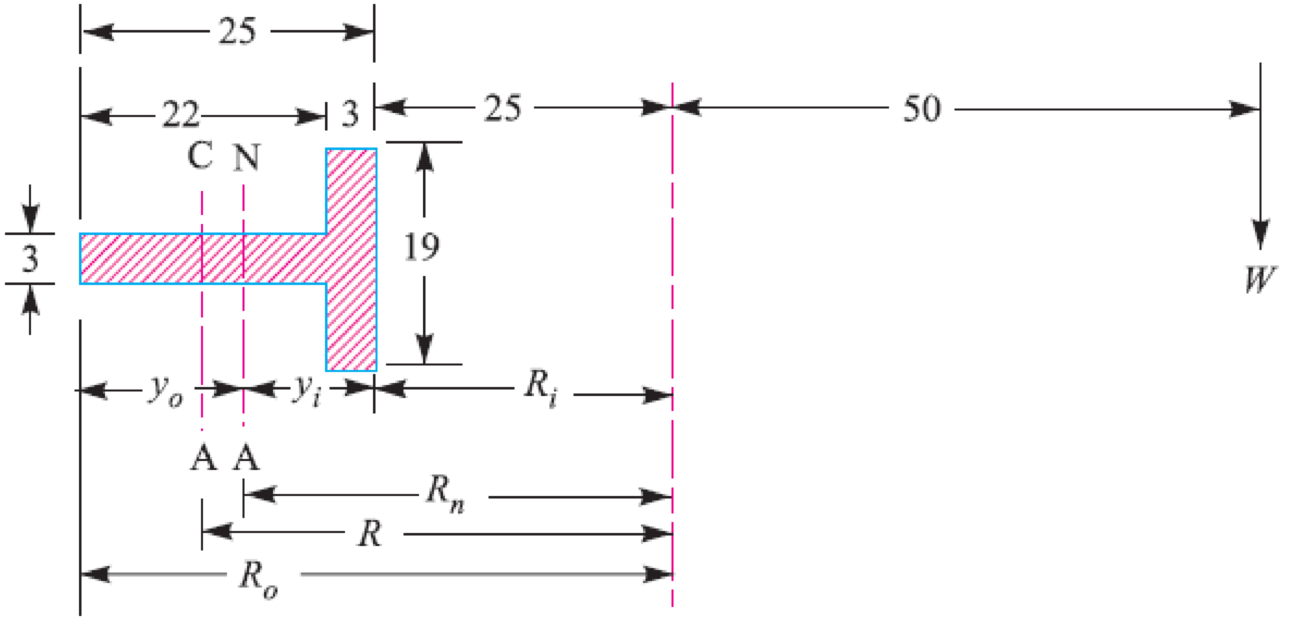
$$e = R - R_n = 33.2 - 31.64 = 1.56 \text{ mm}$$

and distance between the load W and the centroidal axis,

$$x = 50 + R = 50 + 33.2 = 83.2 \text{ mm}$$

$\therefore$ Bending moment about the centroidal axis,

$$M = W.x = W \times 83.2 = 83.2\ W \text{ N-mm}$$



All dimensions in mm.

The section at X-X is subjected to a direct tensile load of W and a bending moment of 83.2 W. The maximum tensile stress will occur at point P (*i.e.* at the inner fibre of the section).

Distance from the neutral axis to the point P,
$yi = Rn - Ri = 31.64 - 25 = 6.64$ mm

Direct tensile stress at section *X-X*,

$$\sigma_t = \frac{W}{A} = \frac{W}{123} = 0.008\ W\ \text{N/mm}^2$$

and maximum bending stress at point *P*,

$$\sigma_{bi} = \frac{M \cdot y_i}{A \cdot e \cdot R_i} = \frac{83.2\ W \times 6.64}{123 \times 1.56 \times 25} = 0.115\ W\ \text{N/mm}^2$$

We know that the maximum tensile stress $\sigma_{t(max)}$,

$$140 = \sigma_t + \sigma_{bi} = 0.008\ W + 0.115\ W = 0.123\ W$$

$$\therefore \quad W = 140/0.123 = 1138\ \text{N}\ \textbf{Ans.}$$

Note : We know that distance from the neutral axis to the outer fibre,

$$y_o = R_o - R_n = 50 - 31.64 = 18.36\ \text{mm}$$

$\therefore$ Maximum bending stress at the outer fibre,

$$\sigma_{bo} = \frac{M \cdot y_o}{A \cdot e \cdot R_o} = \frac{83.2\ W \times 18.36}{123 \times 1.56 \times 50} = 0.16\ W$$

and maximum stress at the outer fibre,

$$= \sigma_t - \sigma_{bo} = 0.008\ W - 0.16\ W = -\ 0.152\ W\ \text{N/mm}^2$$

$$= 0.152\ W\ \text{N/mm}^2\ \text{(compressive)}$$

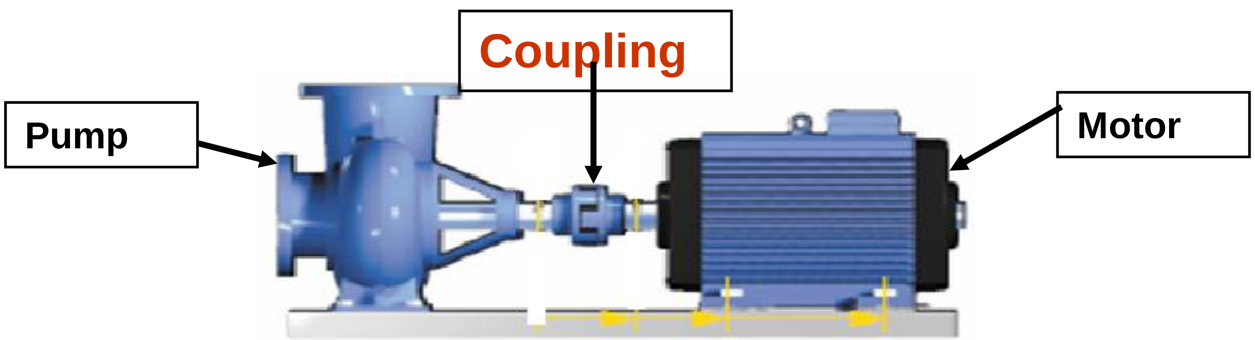
From above we see that stress at the outer fibre is larger in this case than at the inner fibre, but this stress at outer fibre is compressive.

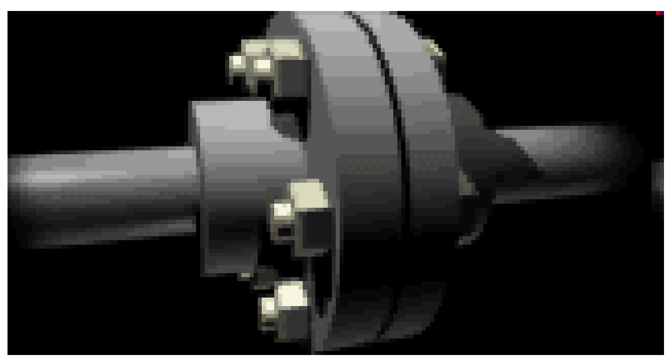
UNIT II
SHAFTS AND COUPLINGS

Design of solid and hollow shafts based on strength, rigidity and critical speed – Keys, keyways and splines - Rigid and flexible couplings.

Couplings

Coupling is a device used to connect two shafts together at their ends for the purpose of transmitting power





Uses of coupling

- To provide connection of shafts of units made separately
- To allow misalignment of the shafts or to introduce mechanical flexibility.
- To reduce the transmission of shock loads
- To introduce protection against overloads.
- To alter the vibration characteristics

Types of Shafts Couplings

1. **Rigid coupling.**

It is used to connect two shafts which are perfectly aligned.

(a) Sleeve or muff coupling.

(b) Clamp or split-muff or compression coupling, and

(c) Flange coupling.

2. **Flexible coupling.**

It is used to connect two shafts having both lateral and angular misalignment.

(a) Bushed pin type coupling,

(b) Universal coupling, and

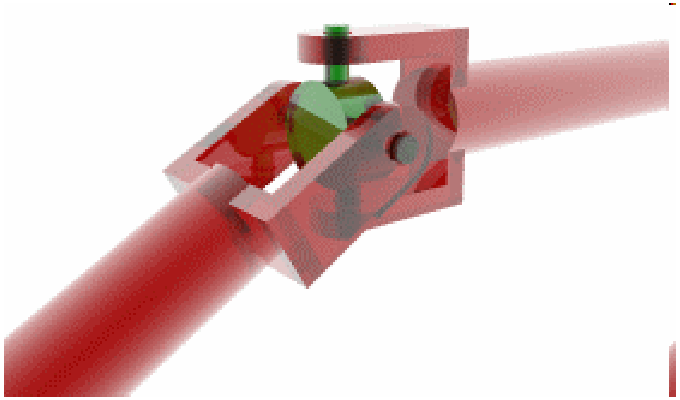
(c) Oldham coupling.

Types of coupling

- Rigid
- Flexible
- Universal

Flexible coupling

Rigid coupling

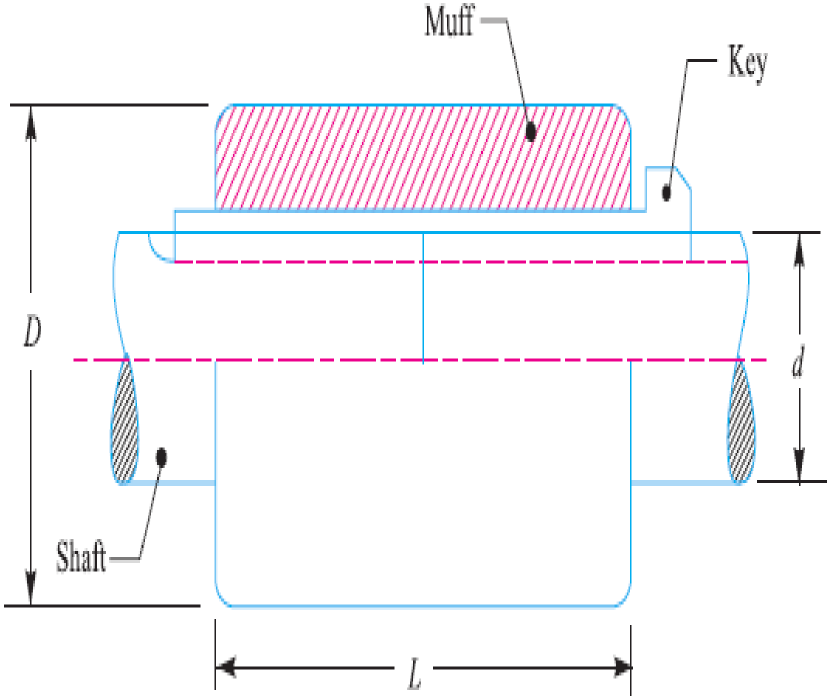


Universal coupling

Requirements of a Good Coupling

1. It should be easy to connect or disconnect.
2. It should transmit the full power from one shaft to the other shaft without losses.
3. It should hold the shafts in perfect alignment.
4. It should reduce the transmission of shock loads from one shaft to another shaft.
5. It should have no projecting parts.

Sleeve or Muff-coupling



Sleeve or Muff-coupling

- It is the simplest type of rigid coupling, made of cast iron.
- It consists of a hollow cylinder whose inner diameter is the same as that of the shaft.
- It is fitted over the ends of the two shafts by means of a gib head key.
- The power is transmitted from one shaft to the other shaft by means of a key and a sleeve.
- It is, therefore, necessary that all the elements must be strong enough to transmit the torque.

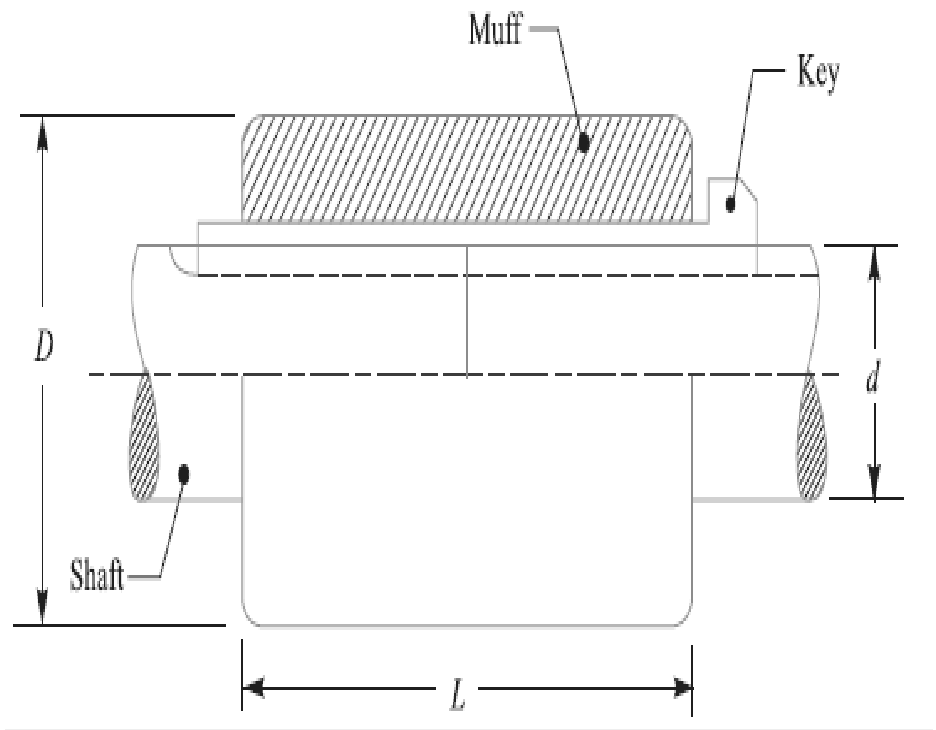
The usual proportions of a cast iron sleeve coupling are as follows :(from page no 5.16)

Outer diameter of the sleeve,

$D = 2d + 13$ mm

and length of the sleeve,

$L = 3.5\ d$



where d is the diameter of the shaft.

Sleeve or Muff-coupling

1. *Design for sleeve*

The sleeve is designed by considering it as a hollow shaft.

- We know that torque transmitted by a hollow section,

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \times D^3 \, (1 - k^4) \qquad \ldots (\because k = d/D)$$

2. Design for key

The length of the coupling key is at least equal to the length of the sleeve (*i.e. 3.5 d)*.

l =3.5 d / 2 =$L/2$

Sleeve or Muff-coupling

Check for shearing and crushing stresses:

$$T = l \times w \times \tau \times \frac{d}{2} \qquad \text{... (Considering shearing of the key)}$$

$$= l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \text{... (Considering crushing of the key)}$$

PBM:1 Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.

Given data:

$P = 40\ kW = 40 \times 103\ W;$

$N = 350\ r.p.m.;$

$\tau s = 40\ MPa = 40\ N/mm2;$

$\sigma cs = 80\ MPa = 80\ \mathrm{N/mm2};$

$\tau c = 15\ MPa = 15\ N/mm2$

solution

1. *Design for shaft*

Let d = Diameter of the shaft.

We know that the torque transmitted by the shaft, key and muff,

$$T = \frac{P \times 60}{2\pi N} = \frac{40 \times 10^3 \times 60}{2\pi \times 350} = 1100 \text{ N-m}$$

$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted (T),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore \quad d^3 = 1100 \times 10^3/7.86 = 140 \times 10^3$ or $d = 52$ say 55 mm **Ans.**

2. *Design for sleeve*

We know that outer diameter of the muff,

$$D = 2d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say } 125 \text{ mm}$$ **Ans.**

and length of the muff,

$$L = 3.5\, d = 3.5 \times 55 = 192.5 \text{ say } 195 \text{ mm}$$ **Ans.**

Let us now check the induced shear stress in the muff.

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \times D^3 (1 - k^4) \qquad \ldots (\because k = d/D)$$

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125} \right]$$

$$= 370 \times 10^3 \, \tau_c$$

$$\therefore \qquad \tau_c = 1100 \times 10^3 / 370 \times 10^3 = 2.97 \text{ N/mm}^2$$

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm^2, therefore the design of muff is safe.

3. Design for key

- From data book page no 5.16 take the value of width and height w=16 h=10

We know that length of key in each shaft,

$$l = L / 2 = 195 / 2 = 97.5 \text{ mm } \textbf{Ans.}$$

Let us now check the induced shear and crushing stresses in the key. First of all, let us consider shearing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 16 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3\ \tau_s$$

$$\therefore \qquad \tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$$

Now considering crushing of the key. We know that torque transmitted (T),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{10}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3\ \sigma_{cs}$$

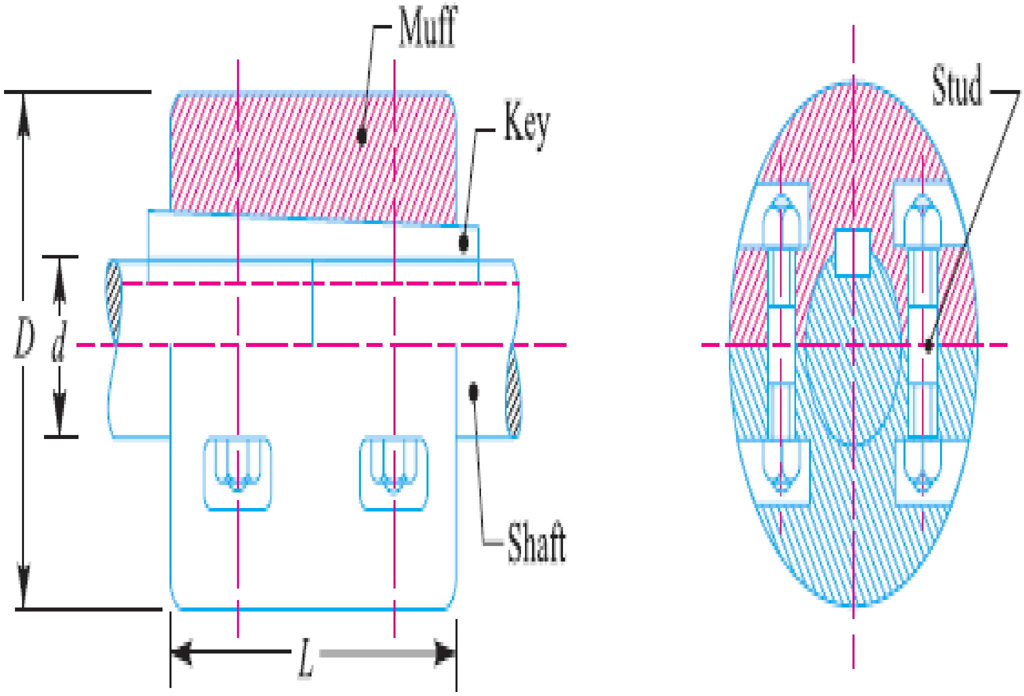
$$\therefore \qquad \sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2$$

Since the induced shear and crushing stresses are less than the permissible stresses, therefore the design of key is safe.

Clamp or Compression Coupling

- It is also known as split muff coupling.
- In this case, the muff or sleeve is made into two halves
- and are bolted together
- The halves of the muff are made of cast iron.
- The shaft ends are made to a butt each other and a single key is fitted directly in the keyways of both the shafts.
- One-half of the muff is fixed from below and the other half is placed from above.
- Both the halves are held together by means of mild steel studs or bolts and nuts.

split muff coupling



1. *Design of muff and key*

The muff and key are designed in the similar way as discussed in muff coupling (Art. 13.14).

2. *Design of clamping bolts*

Let T = Torque transmited by the shaft,

d = Diameter of shaft,

d_b = Root or effective diameter of bolt,

n = Number of bolts,

σ_t = Permissible tensile stress for bolt material,

μ = Coefficient of friction between the muff and shaft, and

L = Length of muff.

We know that the force exerted by each bolt

$$= \frac{\pi}{4} (d_b)^2 \sigma_t$$

$\therefore$ Force exerted by the bolts on each side of the shaft

$$= \frac{\pi}{4} (d_b)^2 \sigma_t \times \frac{n}{2}$$

Let p be the pressure on the shaft and the muff surface due to the force, then for uniform pressure distribution over the surface,

$$p = \frac{\text{Force}}{\text{Projected area}} = \frac{\frac{\pi}{4} (d_b)^2 \sigma_t \times \frac{n}{2}}{\frac{1}{2} L \times d}$$

$\therefore$ Frictional force between each shaft and muff,

$$F = \mu \times \text{pressure} \times \text{area} = \mu \times p \times \frac{1}{2} \times \pi d \times L$$

PBM:1 Design a clamp coupling to transmit 30 kW at 100 r.p.m. The allowable shear stress for the shaft and key is 40 MPa and the number of bolts connecting the two halves are six. The permissible tensile stress for the bolts is 70 MPa. The coefficient of friction between the muff and the shaft surface may be taken as 0.3.

Given data :

$P = 30\ kW = 30 \times 103\ W$;

$N = 100$ r.p.m. ;

$\tau = 40\ MPa = 40\ N/mm2$;

$n = 6$;

$\sigma t = 70\ MPa = 70\ N/mm2$; $\mu = 0.3$

Solution:

1. *Design for shaft*

Let d = Diameter of shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{30 \times 10^3 \times 60}{2\,\pi \times 100} = 2865 \text{ N-m} = 2865 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted by the shaft (T),

$$2865 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 2865 \times 10^3 / 7.86 = 365 \times 10^3$ or $d = 71.4$ say 75 mm **Ans.**

2. *Design for muff*

We know that diameter of muff,

$$D = 2d + 13 \text{ mm} = 2 \times 75 + 13 = 163 \text{ say } 165 \text{ mm}$$ **Ans.**

and total length of the muff,

$$L = 3.5\, d = 3.5 \times 75 = 262.5 \text{ mm}$$ **Ans.**

3. ***Design for key***

The width and thickness of the key for a shaft diameter of 75 mm (from Table 13.1) are as follows :

Width of key, w = 22 mm **Ans.**

Thickness of key, t = 14 mm **Ans.**

and length of key = Total length of muff = 262.5 mm **Ans.**

4. ***Design for bolts***

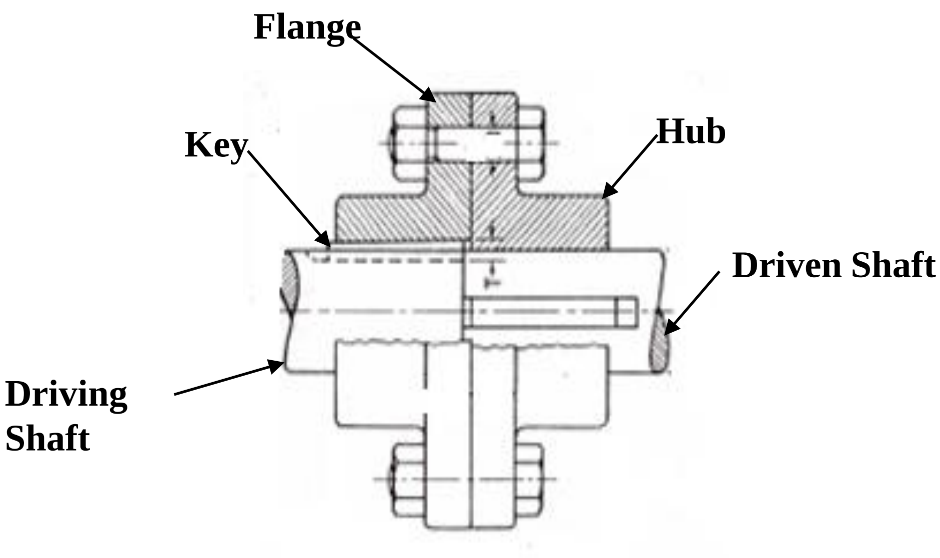
Let d_b = Root or core diameter of bolt.

We know that the torque transmitted (T),

$$2865 \times 10^3 = \frac{\pi^2}{16} \times \mu (d_b)^2 \sigma_t \times n \times d = \frac{\pi^2}{16} \times 0.3\,(d_b)^2\ 70 \times 6 \times 75 = 5830(d_b)^2$$

$$\therefore \quad (d_b)^2 = 2865 \times 10^3 / 5830 = 492 \quad \text{or} \quad d_b = 22.2 \text{ mm}$$

Flange Coupling



Flanged Coupling

❖A flange coupling usually applies to a coupling having two separate cast iron flanges.

❖Each flange is mounted on the shaft end and keyed to it.

❖One of the flange has a projected portion and the other flange has a corresponding recess.

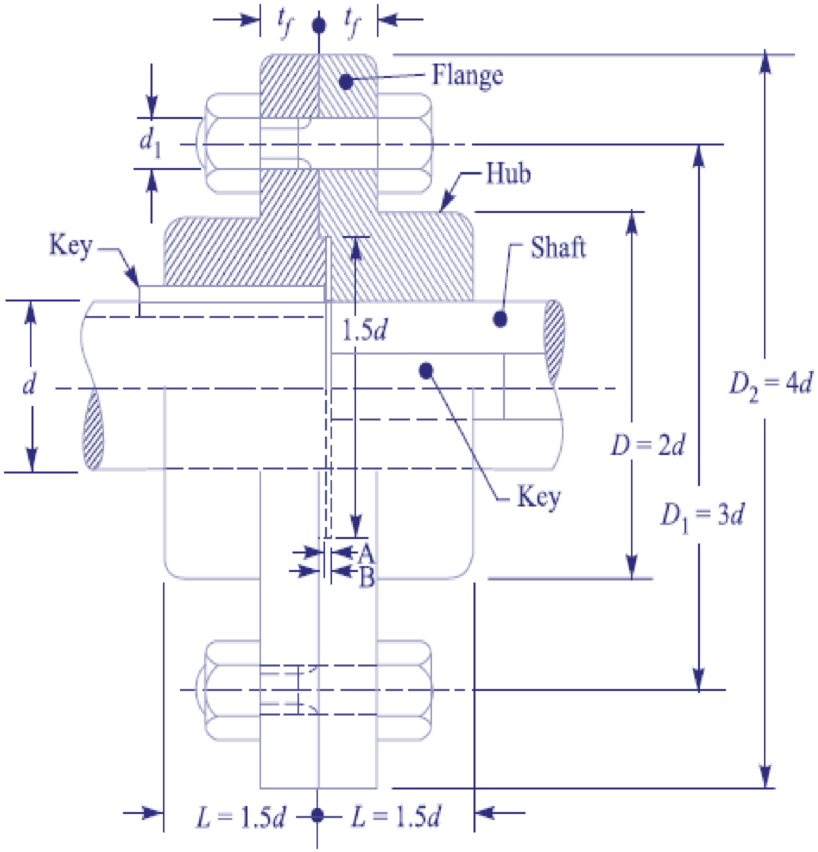
Types of flange couplings :

1. Unprotected type :
   - In an unprotected type flange coupling, each shaft is keyed to the boss of a flange with a counter sunk key and the flanges are coupled together by means of bolts.
   - Generally, three four or six bolts are used.
2. Protected type flange coupling.

   In a protected type flange coupling, the protruding bolts and nuts are protected by flanges on the two halves of the coupling, in order to avoid danger to the workman
3. Marine type flange coupling.

   In a marine type flange coupling, the flanges are forged integral with the shafts with The flanges are held together by means of tapered headless bolts, numbering from four to twelve depending upon the diameter of shaft.

Design of Flange Coupling



Let

d = Diameter of shaft or inner diameter of hub,

D = Outer diameter of hub,

d_1 = Nominal or outside diameter of bolt,

D_1 = Diameter of bolt circle,

n = Number of bolts,

t_f = Thickness of flange,

τ_s, τ_b and τ_k = Allowable shear stress for shaft, bolt and key material respectively

τ_c = Allowable shear stress for the flange material *i.e.* cast iron,

σ_{cb}, and σ_{ck} = Allowable crushing stress for bolt and key material respectively.

If d is the diameter of the shaft or inner diameter of the hub, then

Outside diameter of hub,

$$D = 2\,d$$

Length of hub, $L = 1.5\,d$

Pitch circle diameter of bolts,

$$D_1 = 3d$$

Outside diameter of flange,

$$D_2 = D_1 + (D_1 - D) = 2\,D_1 - D = 4\,d$$

Thickness of flange, $t_f = 0.5\,d$

Number of bolts = 3, for d upto 40 mm

= 4, for d upto 100 mm

= 6, for d upto 180 mm

Design procedure for Flange Coupling

1. *Design for hub*

The hub is designed by considering it as a hollow shaft, transmitting the same torque (T) as that of a solid shaft.

$$\therefore \qquad T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right)$$

The outer diameter of hub is usually taken as twice the diameter of shaft. Therefore from the above relation, the induced shearing stress in the hub may be checked.

The length of hub (L) is taken as 1.5 d.

2. *Design for key*

The key is designed with usual proportions and then checked for shearing and crushing stresses.

3. *Design for flange*

The flange at the junction of the hub is under shear while transmitting the torque. Therefore, the troque transmitted,

T = Circumference of hub × Thickness of flange × Shear stress of flange × Radius of hub

$$= \pi D \times t_f \times \tau_c \times \frac{D}{2} = \frac{\pi D^2}{2} \times \tau_c \times t_f$$

The thickness of flange is usually taken as half the diameter of shaft. Therefore from the above relation, the induced shearing stress in the flange may be checked.

4. *Design for bolts*

The bolts are subjected to shear stress due to the torque transmitted. The number of bolts (n) depends upon the diameter of shaft and the pitch circle diameter of bolts (D_1) is taken as 3 d. We know that

$$\text{Load on each bolt} = \frac{\pi}{4}(d_1)^2\ \tau_b$$

∴ Total load on all the bolts

$$= \frac{\pi}{4}(d_1)^2\ \tau_b \times n$$

and torque transmitted,
$$T = \frac{\pi}{4}(d_1)^2\ \tau_b \times n \times \frac{D_1}{2}$$

From this equation, the diameter of bolt (d_1) may be obtained. Now the diameter of bolt may be checked in crushing.

We know that area resisting crushing of all the bolts

$$= n \times d_1 \times t_f$$

and crushing strength of all the bolts

$$= (n \times d_1 \times t_f)\ \sigma_{cb}$$

∴ Torque,
$$T = (n \times d_1 \times t_f \times \sigma_{cb})\ \frac{D_1}{2}$$

From this equation, the induced crushing stress in the bolts may be checked.

PBM: 1 Design a cast iron protective type flange coupling to transmit 15 kW at 900 r.p.m. from an electric motor to a compressor. The service factor may be assumed as 1.35. The following permissible stresses may be used :

Shear stress for shaft, bolt and key material = 40 MPa

Crushing stress for bolt and key = 80 MPa

Shear stress for cast iron = 8 MPa

Draw a neat sketch of the coupling.

Given data:

$P = 15\ kW = 15 \times 103\ W$;

$N = 900$ r.p.m. ;

Service factor = 1.35 ;

$\tau s = \tau b = \tau k = 40\ MPa = 40\ N/mm2$;

$\sigma cb = \sigma ck = 80\ MPa = 80\ N/mm2$;

$\tau c = 8\ MPa = 8\ N/mm2$

1. *Design for hub*

First of all, let us find the diameter of the shaft (d). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 900} = 159.13 \text{ N-m}$$

Since the service factor is 1.35, therefore the maximum torque transmitted by the shaft,

$$T_{max} = 1.35 \times 159.13 = 215 \text{ N-m} = 215 \times 10^3 \text{ N-mm}$$

We know that the torque transmitted by the shaft (T),

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 215 \times 10^3 / 7.86 = 27.4 \times 10^3$ or $d = 30.1$ say 35 mm **Ans.**

We know that outer diameter of the hub,

$$D = 2d = 2 \times 35 = 70 \text{ mm} \quad \textbf{Ans.}$$

and length of hub, $L = 1.5\, d = 1.5 \times 35 = 52.5$ mm **Ans.**

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted (T_{max}).

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(70)^4 - (35)^4}{70}\right] = 63\,147\, \tau_c$$

$\therefore$ $\tau_c = 215 \times 10^3/63\,147 = 3.4 \text{ N/mm}^2 = 3.4 \text{ MPa}$

Since the induced shear stress for the hub material (*i.e.* cast iron) is less than the permissible value of 8 MPa, therefore the design of hub is safe.

2. *Design for key*

Since the crushing stress for the key material is twice its shear stress (*i.e.* $\sigma_{ck} = 2\tau_k$), therefore a square key may be used. From Table 13.1, we find that for a shaft of 35 mm diameter,

Width of key, $w = 12$ mm **Ans.**

and thickness of key, $t = w = 12$ mm **Ans.**

The length of key (l) is taken equal to the length of hub.

$\therefore$ $l = L = 52.5$ mm **Ans.**

Let us now check the induced stresses in the key by considering it in shearing and crushing.

Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11\ 025\ \tau_k$$

$$\therefore \quad \tau_k = 215 \times 10^3 / 11\ 025 = 19.5 \text{ N/mm}^2 = 19.5 \text{ MPa}$$

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 52.5 \times \frac{12}{2} \times \sigma_{ck} \times \frac{35}{2} = 5512.5\ \sigma_{ck}$$

$$\therefore \quad \sigma_{ck} = 215 \times 10^3 / 5512.5 = 39 \text{ N/mm}^2 = 39 \text{ MPa}$$

Since the induced shear and crushing stresses in the key are less than the permissible stresses, therefore the design for key is safe.

3. *Design for flange*

The thickness of flange (t_f) is taken as 0.5 d.

$\therefore$ $t_f = 0.5\ d = 0.5 \times 35 = 17.5$ mm **Ans.**

Let us now check the induced shearing stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (70)^2}{2} \times \tau_c \times 17.5 = 134\ 713\ \tau_c$$

$\therefore \qquad \tau_c = 215 \times 10^3 / 134\ 713 = 1.6\ \text{N/mm}^2 = 1.6\ \text{MPa}$

Since the induced shear stress in the flange is less than 8 MPa, therefore the design of flange is safe.

4. *Design for bolts*

Let $\qquad d_1$ = Nominal diameter of bolts.

Since the diameter of the shaft is 35 mm, therefore let us take the number of bolts,

$$n = 3$$

and pitch circle diameter of bolts,

$$D_1 = 3d = 3 \times 35 = 105\ \text{mm}$$

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted (T_{max}),

$$215 \times 10^3 = \frac{\pi}{4} (d_1)^2\ \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4} (d_1)^2\ 40 \times 3 \times \frac{105}{2} = 4950\ (d_1)^2$$

$\therefore \qquad (d_1)^2 = 215 \times 10^3 / 4950 = 43.43$ or $d_1 = 6.6$ mm

Assuming coarse threads, the nearest standard size of bolt is M 8. **Ans.**

Other proportions of the flange are taken as follows :

Outer diameter of the flange,

$D_2 = 4\ d = 4 \times 35 = 140$ mm **Ans.**

Thickness of the protective circumferential flange,

$t_p = 0.25\ d = 0.25 \times 35 = 8.75$ say 10 mm **Ans.**

Pbm:2 Design and draw a cast iron flange coupling for a mild steel shaft transmitting 90 kW at 250 r.p.m. The allowable shear stress in the shaft is 40 MPa and the angle of twist is not to exceed 1° in a length of 20 diameters. The allowable shear stress in the coupling bolts is 30 MPa.

Given data :

$P = 90$ kW $= 90 \times 10^3$ W ;

$N = 250$ r.p.m. ;

$\tau_s = 40$ MPa $= 40$ N/mm^2 ;

$\theta = 1° = \pi / 180 = 0.0175$ rad ;

$\tau_b = 30$ MPa $= 30$ N/mm^2

Solution:

shaft,

$$T = \frac{P \times 60}{2\,\pi\,N} = \frac{90 \times 10^3 \times 60}{2\,\pi \times 250} = 3440 \text{ N-m} = 3440 \times 10^3 \text{ N-mm}$$

Considering strength of the shaft, we know that

$$\frac{T}{J} = \frac{\tau_s}{d/2}$$

$$\frac{3440 \times 10^3}{\frac{\pi}{32} \times d^4} = \frac{40}{d/2} \quad \text{or} \quad \frac{35 \times 10^6}{d^4} = \frac{80}{d} \qquad \ldots \left(\because J = \frac{\pi}{32} \times d^4\right)$$

$$\therefore \quad d^3 = 35 \times 10^6 / 80 = 0.438 \times 10^6 \text{ or } d = 76 \text{ mm}$$

Considering rigidity of the shaft, we know that

$$\frac{T}{J} = \frac{C \times \theta}{l}$$

$$\frac{3440 \times 10^3}{\frac{\pi}{32} \times d^4} = \frac{84 \times 10^3 \times 0.0175}{20\,d} \quad \text{or} \quad \frac{35 \times 10^6}{d^4} = \frac{73.5}{d} \qquad \ldots (\text{Taking } C = 84 \text{ kN/mm}^2)$$

$$\therefore \quad d^3 = 35 \times 10^6 / 73.5 = 0.476 \times 10^6 \text{ or } d = 78 \text{ mm}$$

Taking the larger of the two values, we have

d = 78 say 80 mm **Ans.**

Let us now design the cast iron flange coupling of the protective type as discussed below :

1. *Design for hub*

We know that the outer diameter of hub,

$$D = 2d = 2 \times 80 = 160 \text{ mm } \textbf{Ans.}$$

and length of hub, $L = 1.5\,d = 1.5 \times 80 = 120$ mm **Ans.**

Let us now check the induced shear stress in the hub by considering it as a hollow shaft. The shear stress for the hub material (which is cast iron) is usually 14 MPa. We know that the torque transmitted (T),

$$3440 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(160)^4 - (80)^4}{160}\right] = 754 \times 10^3\ \tau_c$$

$\therefore$ $\tau_c = 3440 \times 10^3 / 754 \times 10^3 = 4.56 \text{ N/mm}^2 = 4.56$ MPa

Since the induced shear stress for the hub material is less than 14 MPa, therefore the design for hub is safe.

2. *Design for key*

From Table 13.1, we find that the proportions of key for a 80 mm diameter shaft are :

Width of key, $w = 25$ mm **Ans.**

and thickness of key, $t = 14$ mm **Ans.**

The length of key (l) is taken equal to the length of hub (L).

$\therefore$ $l = L = 120$ mm **Ans.**

Assuming that the shaft and key are of the same material. Let us now check the induced shear stress in key. We know that the torque transmitted (T),

$$3440 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 120 \times 25 \times \tau_k \times \frac{80}{2} = 120 \times 10^3\ \tau_k$$

$$\tau_k = 3440 \times 10^3 / 120 \times 10^3 = 28.7 \text{ N/mm}^2 = 28.7 \text{ MPa}$$

Since the induced shear stress in the key is less than 40 MPa, therefore the design for key is safe.

3. *Design for flange*

The thickness of the flange (t_f) is taken as 0.5 d.

$\therefore$ $t_f = 0.5\,d = 0.5 \times 80 = 40$ mm **Ans.**

Let us now check the induced shear stress in the cast iron flange by considering the flange at the junction of the hub under shear. We know that the torque transmitted (T),

$$3440 \times 10^3 = \frac{\pi\,D^2}{2} \times t_f \times \tau_c = \frac{\pi\,(160)^2}{2} \times 40 \times \tau_c = 1608 \times 10^3\,\tau_c$$

$\therefore$ $\tau_c = 3440 \times 10^3/1608 \times 10^3 = 2.14$ N/mm^2 = 2.14 MPa

Since the induced shear stress in the flange is less than 14 MPa, therefore the design for flange is safe.

4. *Design for bolts*

Let d_1 = Nominal diameter of bolts.

Since the diameter of the shaft is 80 mm, therefore let us take number of bolts,

$$n = 4$$

and pitch circle diameter of bolts,

$$D_1 = 3\,d = 3 \times 80 = 240 \text{ mm}$$

The bolts are subjected to shear stress due to the torque transmitted. We know that torque transmitted (T),

$$3440 \times 10^3 = \frac{\pi}{4}\,(d_1)^2\,n \times \tau_b \times \frac{D_1}{2} = \frac{\pi}{4}\,(d_1)^2 \times 4 \times 30 \times \frac{240}{2} = 11\,311\,(d_1)^2$$

$\therefore$ $(d_1)^2 = 3440 \times 10^3/11\,311 = 304$ or $d_1 = 17.4$ mm

Assuming coarse threads, the standard nominal diameter of bolt is 18 mm. **Ans.**

The other proportions are taken as follows :

Outer diameter of the flange,

$$D_2 = 4\,d = 4 \times 80 = 320 \text{ mm } \textbf{Ans.}$$

Thickness of protective circumferential flange,

$$t_p = 0.25\,d = 0.25 \times 80 = 20 \text{ mm } \textbf{Ans.}$$

Example 13.7. *Design and draw a protective type of cast iron flange coupling for a steel shaft transmitting 15 kW at 200 r.p.m. and having an allowable shear stress of 40 MPa. The working stress in the bolts should not exceed 30 MPa. Assume that the same material is used for shaft and key and that the crushing stress is twice the value of its shear stress. The maximum torque is 25% greater than the full load torque. The shear stress for cast iron is 14 MPa.*

Solution. Given : $P = 15$ kW $= 15 \times 10^3$ W ; $N = 200$ r.p.m. ; $\tau_s = 40$ MPa $= 40$ N/mm^2 ; $\tau_b = 30$ MPa $= 30$ N/mm^2 ; $\sigma_{ck} = 2\tau_k$; $T_{max} = 1.25\ T_{mean}$; $\tau_c = 14$ MPa $= 14$ N/mm^2

The protective type of cast iron flange coupling is designed as discussed below :

1. *Design for hub*

First of all, let us find the diameter of shaft (d). We know that the full load or mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 200} = 716 \text{ N-m} = 716 \times 10^3 \text{ N-mm}$$

and maximum torque transmitted,

$$T_{max} = 1.25\ T_{mean} = 1.25 \times 716 \times 10^3 = 895 \times 10^3 \text{ N-mm}$$

We also know that maximum torque transmitted (T_{max}),

$$895 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\ d^3$$

$\therefore \qquad d^3 = 895 \times 10^3 / 7.86 = 113\ 868 \quad$ or $d = 48.4$ say 50 mm **Ans.**

We know that the outer diameter of the hub,

$$D = 2\,d = 2 \times 50 = 100 \text{ mm } \textbf{Ans.}$$

and length of the hub, $\quad L = 1.5\,d = 1.5 \times 50 = 75$ mm **Ans.**

Let us now check the induced shear stress for the hub material which is cast iron, by considering it as a hollow shaft. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \left(\frac{(100)^4 - (50)^2}{100} \right) = 184\ 100\ \tau_c$$

$$\therefore \qquad \tau_c = 895 \times 10^3 / 184\ 100 = 4.86 \text{ N/mm}^2 = 4.86 \text{ MPa}$$

Since the induced shear stress in the hub is less than the permissible value of 14 MPa, therefore the design for hub is safe.

2. *Design for key*

Since the crushing stress for the key material is twice its shear stress, therefore a square key may be used.

From Table 13.1, we find that for a 50 mm diameter shaft,

Width of key, $w = 16$ mm **Ans.**

and thickness of key, $t = w = 16$ mm **Ans.**

The length of key (l) is taken equal to the length of hub.

$\therefore$ $l = L = 75$ mm **Ans.**

Let us now check the induced stresses in the key by considering it in shearing and crushing. Considering the key in shearing. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 75 \times 16 \times \tau_k \times \frac{50}{2} = 30 \times 10^3\ \tau_k$$

$$\therefore \quad \tau_k = 895 \times 10^3 / 30 \times 10^3 = 29.8\ \text{N/mm}^2 = 29.8\ \text{MPa}$$

Considering the key in crushing. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 75 \times \frac{16}{2} \times \sigma_{ck} \times \frac{50}{2} = 15 \times 10^3\ \sigma_{ck}$$

$$\therefore \quad \sigma_{ck} = 895 \times 10^3 / 15 \times 10^3 = 59.6\ \text{N/mm}^2 = 59.6\ \text{MPa}$$

Since the induced shear and crushing stresses in key are less than the permissible stresses, therefore the design for key is safe.

3. *Design for flange*

The thickness of the flange (t_f) is taken as 0.5 *d*.

$$\therefore \qquad t_f = 0.5 \times 50 = 25 \text{ mm } \textbf{Ans.}$$

Let us now check the induced shear stress in the flange, by considering the flange at the junction of the hub in shear. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (100)^2}{2} \times \tau_c \times 25 = 392\ 750\ \tau_c$$

$$\therefore \qquad \tau_c = 895 \times 10^3 / 392\ 750 = 2.5 \text{ N/mm}^2 = 2.5 \text{ MPa}$$

Since the induced shear stress in the flange is less than the permissible value of 14 MPa, therefore the design for flange is safe.

4. *Design for bolts*

Let d_1 = Nominal diameter of bolts.

Since the diameter of shaft is 50 mm, therefore let us take the number of bolts,

$$n = 4$$

and pitch circle diameter of bolts,

$$D_1 = 3\,d = 3 \times 50 = 150 \text{ mm}$$

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted (T_{max}),

$$895 \times 10^3 = \frac{\pi}{4}(d_1)^2\,\tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2\,30 \times 4 \times \frac{150}{4} = 7070\,(d_1)^2$$

$$\therefore \quad (d_1)^2 = 895 \times 10^3 / 7070 = 126.6 \quad \text{or} \quad d_1 = 11.25 \text{ mm}$$

Assuming coarse threads, the nearest standard diameter of the bolt is 12 mm (M 12). **Ans.**

Other proportions of the flange are taken as follows :

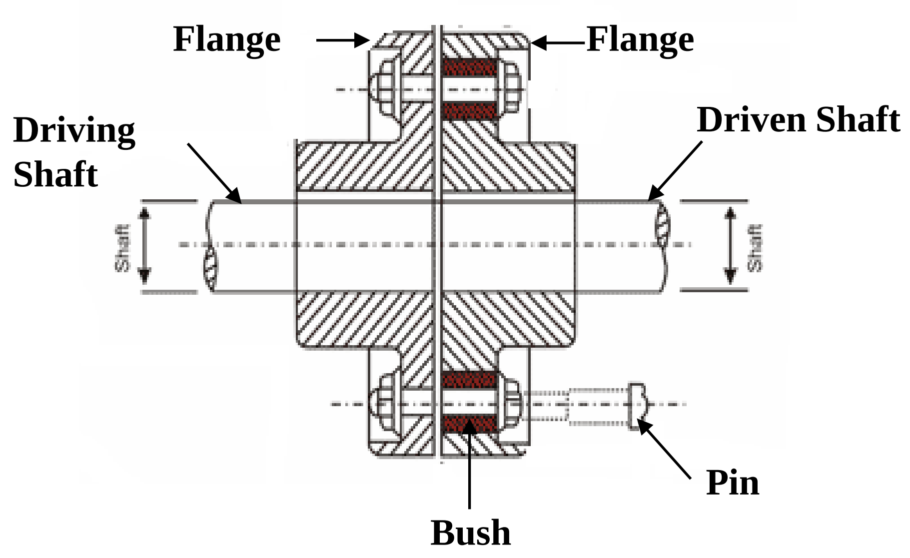
Outer diameter of the flange,

$$D_2 = 4\,d = 4 \times 50 = 200 \text{ mm} \quad \textbf{Ans.}$$

Thickness of the protective circumferential flange,

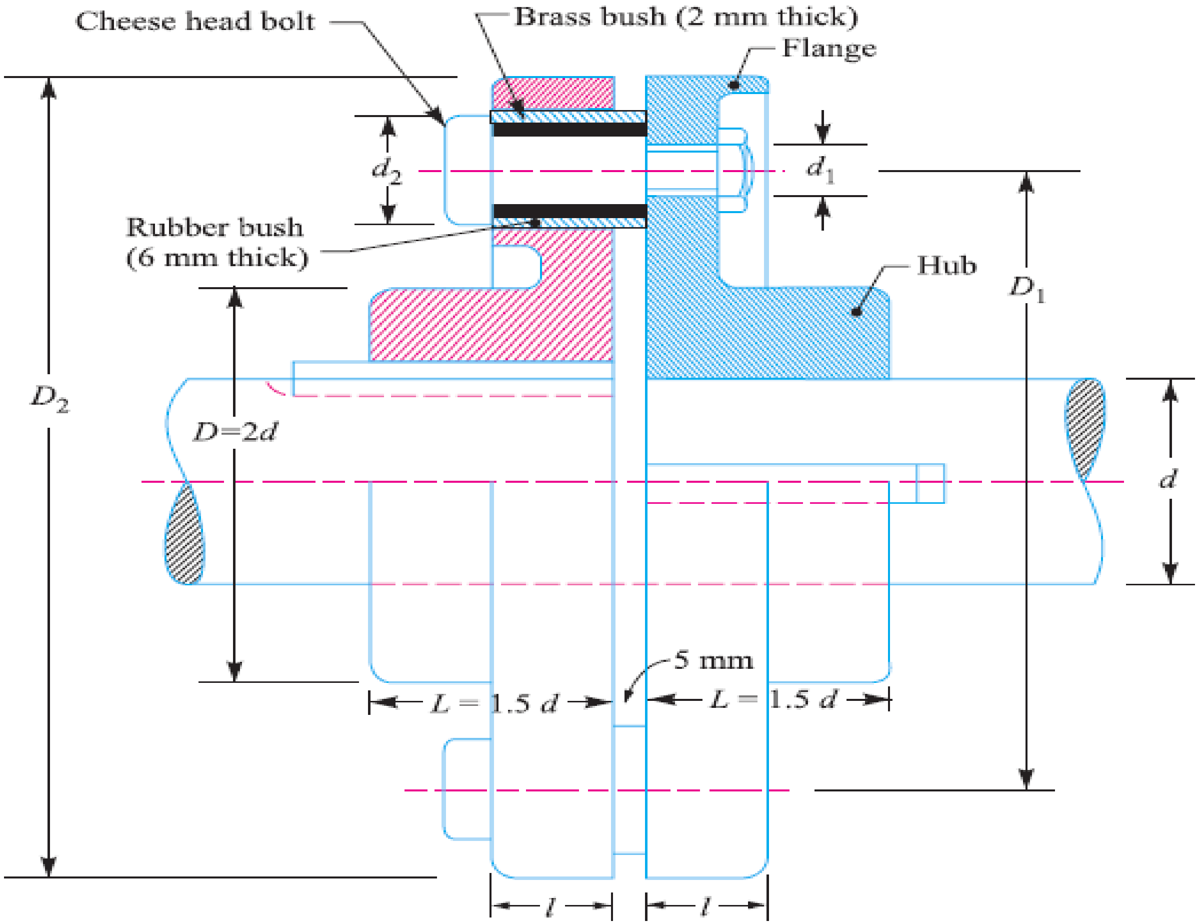
$$t_p = 0.25\,d = 0.25 \times 50 = 12.5 \text{ mm} \quad \textbf{Ans.}$$

Flexible Coupling



- A flexible coupling permits with in certain limits, relative rotation and variation in the alignment of shafts
- Pins (Bolts) covered by rubber washer or bush is used connect flanges with nuts
- The rubber washers or bushes act as a shock absorbers and insulators.

Bushed-pin Flexible Coupling



Bushed-pin flexible coupling

- It is a modification of the rigid type of flange coupling. The coupling bolts are known as pins. The rubber or leather bushes are used over the pins.
- The two halves of the coupling are dissimilar in construction. A clearance of 5 mm is left between the face of the two halves of the coupling.
- There is no rigid connection between them and the drive takes place through the medium of the compressible rubber or leather bushes.
- The main modification is to reduce the bearing pressure on the rubber or leather bushes and it should not exceed 0.5 N/mm2.

Let l = Length of bush in the flange,

d_2 = Diameter of bush,

p_b = Bearing pressure on the bush or pin,

n = Number of pins, and

D_1 = Diameter of pitch circle of the pins.

We know that bearing load acting on each pin,

$$W = p_b \times d_2 \times l$$

∴ Total bearing load on the bush or pins

$$= W \times n = p_b \times d_2 \times l \times n$$

torque transmitted by the coupling,

$$T = W \times n\left(\frac{D_1}{2}\right) = p_b \times d_2 \times l \times n\left(\frac{D_1}{2}\right)$$

Direct shear stress due to pure torsion in the coupling halves,

$$\tau = \frac{W}{\frac{\pi}{4}(d_1)^2}$$

Since the pin and the rubber or leather bush is not rigidly held in the left hand flange, therefore the tangential load (W) at the enlarged portion will exert a bending action on the pin as shown in Fig. 13.16. The bush portion of the pin acts as a cantilever beam of length l. Assuming a uniform distribution of the load W along the bush, the maximum bending moment on the pin,

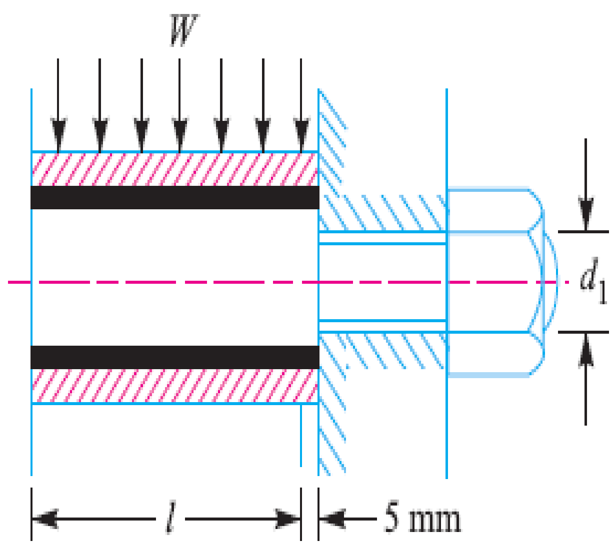


Fig. 13.16

$$M = W\left(\frac{l}{2} + 5 \text{ mm}\right)$$

We know that bending stress,

$$\sigma = \frac{M}{Z} = \frac{W\left(\frac{l}{2} + 5 \text{ mm}\right)}{\frac{\pi}{32}(d_1)^3}$$

Maximum principal stress

$$= \frac{1}{2}\left[\sigma + \sqrt{\sigma^2 + 4\tau^2}\right]$$

and the maximum shear stress on the pin

$$= \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

The value of maximum principal stress varies from 28 to 42 MPa.

Note: After designing the pins and rubber bush, the hub, key and flange may be designed in the similar way as discussed for flange coupling.

Pbm:1 Design a bushed-pin type of flexible coupling to connect a pump shaft to a motor shaft transmitting 32 kW at 960 r.p.m. The overall torque is 20 percent more than mean torque. The material properties are as follows :

(a) The allowable shear and crushing stress for shaft and key material is 40 MPa and 80 MPa respectively.

(b) The allowable shear stress for cast iron is 15 MPa.

(c) The allowable bearing pressure for rubber bush is 0.8 N/mm2.

(d) The material of the pin is same as that of shaft and key.

Draw neat sketch of the coupling.

Given data :

P = 32 kW = 32 × 103 W ; N = 960 r.p.m. ;

Tmax = 1.2 Tmean ;

τs = τk = 40 Mpa = 40 N/mm2 ;

σcs = σck = 80 MPa = 80 N/mm2 ;

τc = 15 MPa = 15 N/mm2 ; pb = 0.8 N/mm2

1. *Design for pins and rubber bush*

First of all, let us find the diameter of the shaft (d). We know that the mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{32 \times 10^3 \times 60}{2\pi \times 960} = 318.3 \text{ N-m}$$

and the maximum or overall torque transmitted,

$$T_{max} = 1.2\, T_{mean} = 1.2 \times 318.3 = 382 \text{ N-m} = 382 \times 10^3 \text{ N-mm}$$

We also know that the maximum torque transmitted by the shaft (T_{max}),

$$382 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 382 \times 10^3 / 7.86 = 48.6 \times 10^3 \text{ or } d = 36.5 \text{ say } 40 \text{ mm}$$

We have discussed in rigid type of flange coupling that the number of bolts for 40 mm diameter shaft are 3. In the flexible coupling, we shall use the number of pins (n) as 6.

$$\therefore \quad \text{Diameter of pins, } d_1 = \frac{0.5\,d}{\sqrt{n}} = \frac{0.5 \times 40}{\sqrt{6}} = 8.2 \text{ mm}$$

Design of Shaft

A shaft is a rotating member usually of circular cross-section (solid or hollow), which transmits power and rotational motion.

Machine elements such as gears, pulleys (sheaves), flywheels, clutches, and sprockets are mounted on the shaft and are used to transmit power from the driving device (motor or engine) through a machine.

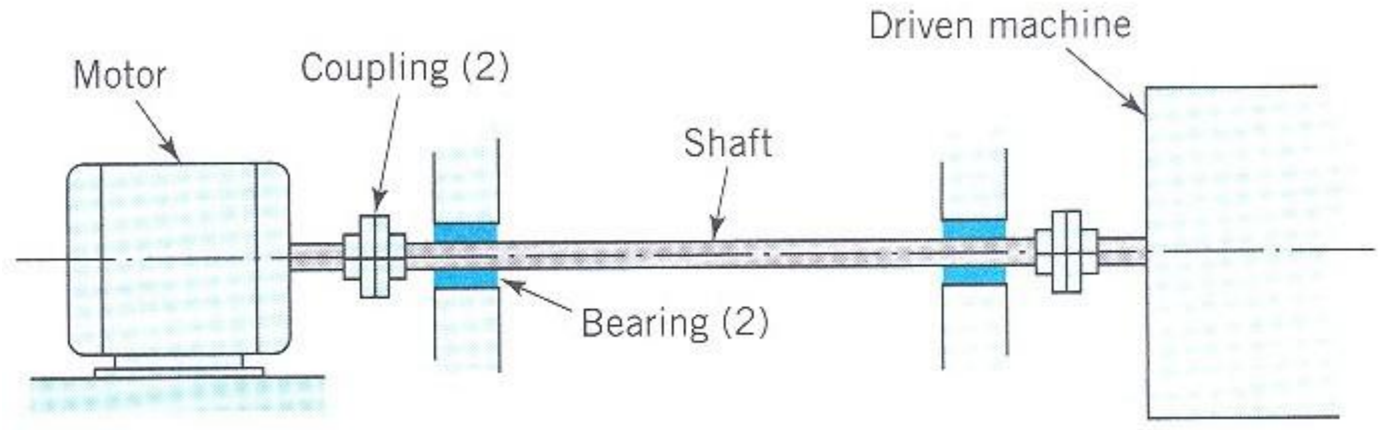
Press fit, keys, dowel, pins and splines are used to attach these machine elements on the shaft.

The shaft rotates on rolling contact bearings or bush bearings.

Various types of retaining rings, thrust bearings, grooves and steps in the shaft are used to take up axial loads and locate the rotating elements.

Couplings are used to transmit power from drive shaft (e.g., motor) to the driven shaft (e.g. gearbox, wheels).

The connecting shaft is loaded primarily in torsion.



(*a*) Connecting shaft

Introduction

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft.

In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it.

In other words, we may say that ***a shaft is used for the transmission of torque and bending moment***. The various members are mounted on the shaft by means of keys or splines.

Material Used for Shafts

The material used for shafts should have the following properties :

1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12.

The mechanical properties of these grades of carbon steel are given in the following table.

Table 14.1. Mechanical properties of steels used for shafts.

Indian standard designation	*Ultimate tensile strength, MPa*	*Yield strength, MPa*
40 C 8	560 - 670	320
45 C 8	610 - 700	350
50 C 4	640 - 760	370
50 C 12	700 Min.	390

When a shaft of high strength is required, then an alloy steel such as nickel, nickel-chromium

Manufacturing of Shafts

Shafts are generally manufactured by hot rolling and finished to size by cold drawing or turning and grinding. The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses.

The residual stresses may cause distortion of the shaft when it is machined, especially when slots or keyways are cut. Shafts of larger diameter are usually forged and turned to size in a lathe.

Types of Shafts

The following two types of shafts are important from the subject point of view :

1.Transmission shafts. These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2.Machine shafts. These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

Standard Sizes of Transmission Shafts

The standard sizes of transmission shafts are :

25 mm to 60 mm with 5 mm steps;

60 mm to 110 mm with 10 mm steps ;

110 mm to 140 mm with 15 mm steps ;

140 mm to 500 mm with 20 mm steps.

The standard length of the shafts are 5 m, 6 m and 7 m.

Stresses in Shafts

The following stresses are induced in the shafts :

1.Shear stresses due to the transmission of torque (*i.e. due to torsional load).*

2.Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.

3.Stresses due to combined torsional and bending loads.

Maximum Permissible Working Stresses for Transmission Shafts

According to American Society of Mechanical Engineers (ASME) code for the design of transmission shafts, the maximum permissible working stresses in tension or compression may be taken as

(a)112 MPa for shafts without allowance for keyways.

(b)84 MPa for shafts with allowance for keyways.

For shafts purchased under definite physical specifications, the permissible tensile stress (σ_t) may be taken as 60 percent of the elastic limit in tension (σ_{el}), *but not more than 36 percent of the* ultimate tensile strength (σ_u). *In other words, the permissible tensile stress,*

$\sigma_t = 0.6\ \sigma_{el}$ *or* $0.36\ \sigma_u$, *whichever is less.*

The maximum permissible shear stress may be taken as

(a)56 MPa for shafts without allowance for key ways.

(b)42 MPa for shafts with allowance for keyways.

For shafts purchased under definite physical specifications, the permissible shear stress (τ) may be taken as 30 per cent of the elastic limit in tension (σ_{el}) *but not more than 18 percent of the ultimate* tensile strength (σ_u). *In other words, the permissible shear stress,*

$\tau = 0.3\ \sigma_{el}$ *or* $0.18\ \sigma_u$, *whichever is less.*

Design of Shafts

The shafts may be designed on the basis of

1. Strength **2. Rigidity and stiffness.**

In designing shafts on the basis of strength, the following cases may be considered :

(a)Shafts subjected to twisting moment or torque only,

(b)Shafts subjected to bending moment only,

(c)Shafts subjected to combined twisting and bending moments, and

(d)Shafts subjected to axial loads in addition to combined torsional and bending loads.

We shall now discuss the above cases, in detail, in the following pages.

Shafts Subjected to Twisting Moment Only

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that

$$\frac{T}{J} = \frac{\tau}{r} \qquad \ldots(i)$$

where

T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ = Torsional shear stress, and

r = Distance from neutral axis to the outer most fibre = d / 2; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation (*i*) may now be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \qquad \ldots(ii)$$

From this equation, we may determine the diameter of round solid shaft (d).

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

where d_o and d_i = Outside and inside diameter of the shaft, and $r = d_o / 2$.

Substituting these values in equation (*i*), we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] \qquad \ldots(iii)$$

Let k = Ratio of inside diameter and outside diameter of the shaft

$= d_i / d_o$

Now the equation (*iii*) may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \qquad \ldots(iv)$$

From the equations *(iii)* or *(iv)*, the outside and inside diameter of a hollow shaft may be determined.

It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft.

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o} \right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \qquad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3 (1 - k^4) = d^3$$

2. The twisting moment (T) may be obtained by using the following relation :

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2 \pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2 \pi N}$$

where T = Twisting moment in N-m, and

N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$$T = (T_1 - T_2) R$$

where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and

R = Radius of the pulley.

pbm:1

A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.

Solution.

Given : N = 200 r.p.m. ; P = 20 kW = 20 × 103 W; τ = 42 MPa = 42 N/mm2

Let d = Diameter of the shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the shaft (*T*),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$$\therefore \quad d^3 = 955 \times 10^3 / 8.25 = 115\ 733 \text{ or } d = 48.7 \text{ say } 50 \text{ mm} \textbf{ Ans.}$$

Pbm: 2.

A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the *shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.*

Solution.

Given : ***P = 1 MW = 1 × 106 W ; N = 240 r.p.m. ; Tmax = 1.2 Tmean ; τ = 60 MPa = 60 N/mm²***

Let $\quad d$ = *Diameter of the shaft.*

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\ 784 \text{ N-m} = 39\ 784 \times 10^3 \text{ N-mm}$$

Maximum torque transmitted,

$T_{max} = 1.2\ T_{mean} = 1.2 \times 39\ 784 \times 10^3 = 47\ 741 \times 10^3$ N-mm

We know that maximum torque transmitted (T_{max}),

$$47\ 741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\ d^3$$

$$d^3 = 47\ 741 \times 10^3 / 11.78 = 4053 \times 10^3$$

$$d = 159.4 \text{ say } 160 \text{ mm } \textbf{Ans.}$$

Pbm: 3.

Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8. If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Solution.

Given : $P = 20\ kW = 20 \times 10^3\ W$; $N = 200$ r.p.m. ; $\tau u = 360\ MPa = 360\ N/mm^2$;
$F.S. = 8$; $k = di / do = 0.5$

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{360}{8} = 45\ \text{N/mm}^2$$

Diameter of the solid shaft

Let d = *Diameter of the solid shaft.*

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955\ \text{N-m} = 955 \times 10^3\ \text{N-mm}$$

We also know that torque transmitted by the solid shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\ d^3$$

$$d^3 = 955 \times 10^3 / 8.84 = 108\ 032 \quad \text{or} \quad d = 47.6 \text{ say } 50 \text{ mm } \textbf{Ans.}$$

Diamater of hollow shaft

Let *di = Inside diameter, and do = Outside diameter.*

We know that the torque transmitted by the hollow shaft (*T*),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

$$= \frac{\pi}{16} \times 45\,(d_o)^3\,[1 - (0.5)^4] = 8.3\,(d_o)^3$$

$$(d_o)^3 = 955 \times 10^3 / 8.3 = 115\,060 \quad \text{or} \quad d_o = 48.6 \text{ say } 50 \text{ mm } \textbf{Ans.}$$

$$d_i = 0.5\,d_o = 0.5 \times 50 = 25 \text{ mm } \textbf{Ans.}$$

Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

where *M = Bending moment,*
I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,
σb = Bending stress, and
y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation (i), we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained. We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4\ (1 - k^4) \qquad \text{...(where } k = d_i / d_o)$$

$$y = d_o / 2$$

Again substituting these values in equation ***(i), we have***

$$\frac{M}{\frac{\pi}{64}(d_o)^4\ (1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \qquad \text{or} \qquad M = \frac{\pi}{32} \times \sigma_b\ (d_o)^3\ (1 - k^4)$$

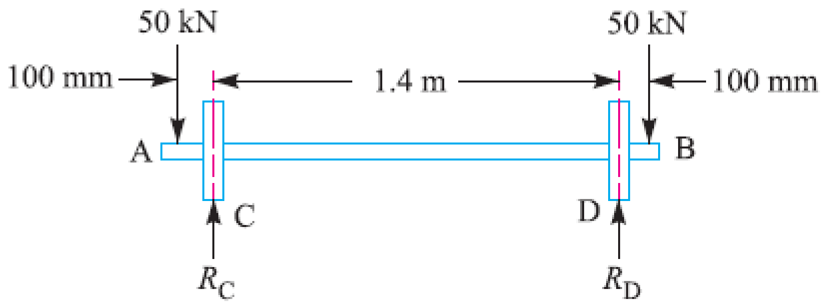
From this equation, the outside diameter of the shaft (do) may be obtained.

Pbm:1

A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.

Solution.

Given : $W = 50\ kN = 50 \times 10^3\ N$; $L = 100\ mm$; $x = 1.4\ m$; $\sigma b = 100\ MPa = 100\ N/mm^2$



The axle with wheels is shown in Fig. 1.

A little consideration will show that the maximum bending moment acts on the wheels at C and D. Therefore maximum bending moment,

*$M = W.L = 50 \times 10^3 \times 100 = 5 \times 10^6$ N-mm

The maximum B.M. may be obtained as follows :

$RC = RD = 50\ kN = 50 \times 10^3\ N$

B.M. at A, $MA = 0$

B.M. at C, $MC = 50 \times 10^3 \times 100 = 5 \times 10^6\ N\text{-}mm$

B.M. at D, $MD = 50 \times 10^3 \times 1500 - 50 \times 10^3 \times 1400 = 5 \times 10^6\ N\text{-}mm$

B.M. at B, $MB = 0$

Let d = Diameter of the axle.
We know that the maximum bending moment (M),

$$5 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\, d^3$$

$$d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6 \quad \text{or} \quad d = 79.8 \text{ say } 80 \text{ mm } \textbf{Ans.}$$

Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1.Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.

2.Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = *Bending stress (tensile or compressive) induced due to bending* moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

Substituting the values of τ and σb *from Art. 14.9 and Art. 14.10, we have*

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi\,d^3}\right)^2 + 4\left(\frac{16\,T}{\pi\,d^3}\right)^2} = \frac{16}{\pi\,d^3}\left[\sqrt{M^2 + T^2}\right]$$

$$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2}$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation (*i*) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \qquad \text{...}(ii)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} \qquad \text{...}(iii)$$

$$= \frac{1}{2} \times \frac{32\,M}{\pi\,d^3} + \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi\,d^3}\right)^2 + 4\left(\frac{16\,T}{\pi\,d^3}\right)^2}$$

$$= \frac{32}{\pi\,d^3}\left[\frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)\right]$$

or

$$\frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] \qquad \text{...}(iv)$$

The expression $\frac{1}{2}\left[(M+\sqrt{M^2+T^2})\right]$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as **that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment.** By limiting the maximum normal stress [$\sigma_{b(max)}$] equal to the allowable bending stress (σ_b), then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M+\sqrt{M^2+T^2}\right] = \frac{\pi}{32}\times\sigma_b\times d^3 \qquad \text{...}(v)$$

From this expression, diameter of the shaft (d) may be evaluated.

Notes: 1. In case of a hollow shaft, the equations (*ii*) and (*v*) may be written as

$$T_e = \sqrt{M^2+T^2} = \frac{\pi}{16}\times\tau\,(d_o)^3\,(1-k^4)$$

and

$$M_e = \tfrac{1}{2}\left(M+\sqrt{M^2+T^2}\right) = \frac{\pi}{32}\times\sigma_b\,(d_o)^3\,(1-k^4)$$

2. It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.

Pbm:1.

A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.

Solution. Given : $M = 3000$ N-m $= 3 \times 10^6$ N-mm ; $T = 10\ 000$ N-m $= 10 \times 10^6$ N-mm ; $\sigma_{tu} = 700$ MPa $= 700$ N/mm^2 ; $\tau_u = 500$ MPa $= 500$ N/mm^2

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let d = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36\ d^3$$

$$\therefore \quad d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6 \quad \text{or} \quad d = 86 \text{ mm}$$

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \tfrac{1}{2}\,(M + T_e)$$

$$= \tfrac{1}{2}\,(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\, d^3$$

$\therefore$ $$d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6 \quad \text{or} \quad d = 83.7 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 86 \text{ say } 90 \text{ mm}$$ **Ans.**

Pbm:2. *A line shaft is driven by means of a motor placed vertically below it. The pulley on the line shaft is 1.5 metre in diameter and has belt tensions 5.4 kN and 1.8 kN on the tight side and slack side of the belt respectively. Both these tensions may be assumed to be vertical. If the pulley be overhang from the shaft, the distance of the centre line of the pulley from the centre line of the bearing being 400 mm, find the diameter of the shaft. Assuming maximum allowable shear stress of 42 MPa.*

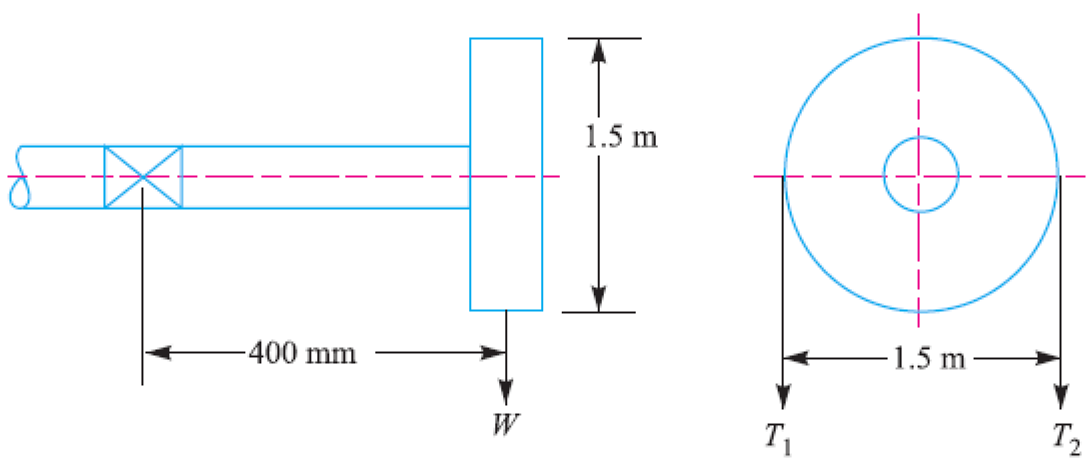
Given : *D = 1.5 m or R = 0.75 m; T1 = 5.4 kN = 5400 N ; T2 = 1.8 kN = 1800 N ;*
L = 400 mm ; τ = 42 MPa = 42 N/mm2

Solution :

We know that torque transmitted by the shaft,

T = (T1 – T2) R = (5400 – 1800) 0.75 = 2700 N-m

= 2700 × 103 N-mm



Neglecting the weight of shaft, total vertical load acting on the pulley,

$$W = T_1 + T_2 = 5400 + 1800 = 7200 \text{ N}$$

∴ Bending moment, $M = W \times L = 7200 \times 400 = 2880 \times 10^3$ N-mm

Let d = Diameter of the shaft in mm.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(2880 \times 10^3)^2 + (2700 \times 10^3)^2}$$

$$= 3950 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$3950 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

∴ $d^3 = 3950 \times 10^3 / 8.25 = 479 \times 10^3$ or $d = 78$ say 80 mm **Ans.**

Pbm:3.

A shaft supported at the ends in ball bearings carries a straight tooth spur gear at its mid span and is to transmit 7.5 kW at 300 r.p.m. The pitch circle diameter of the gear is 150 mm. The distances between the centre line of bearings and gear are 100 mm each. If the shaft is made of steel and the allowable shear stress is 45 MPa, determine the diameter of the shaft. Show in a sketch how the gear will be mounted on the shaft; also indicate the ends where the bearings will be mounted?

The pressure angle of the gear may be taken as 20°.

Solution. Given : P = 7.5 kW = 7500 W ; N = 300 r.p.m. ; D = 150 mm = 0.15 m ; L = 200 mm = 0.2 m ; τ = 45 MPa = 45 N/mm^2 ; α = 20°

Fig. 14.2 shows a shaft with a gear mounted on the bearings.

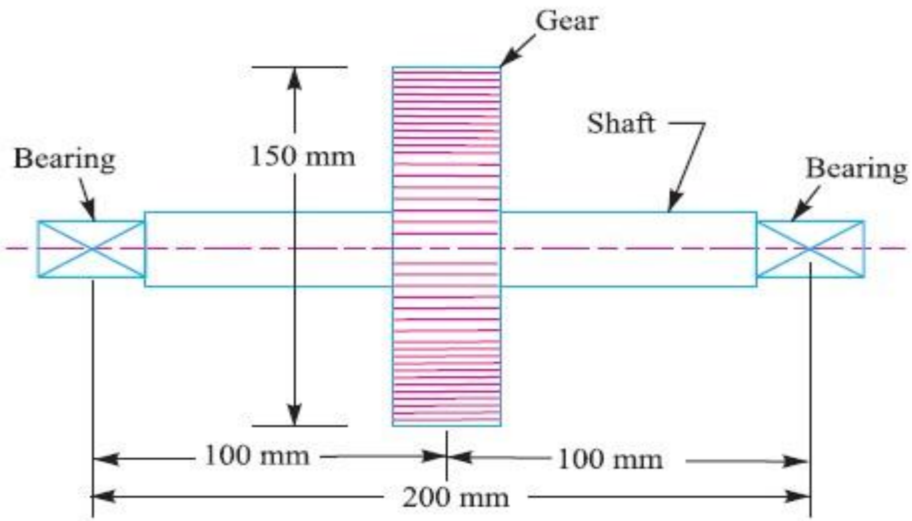


Fig. 14.2

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{7500 \times 60}{2\pi \times 300} = 238.7 \text{ N-m}$$

∴ Tangential force on the gear,

$$F_t = \frac{2T}{D} = \frac{2 \times 238.7}{0.15} = 3182.7 \text{ N}$$

and the normal load acting on the tooth of the gear,

$$W = \frac{F_t}{\cos\alpha} = \frac{3182.7}{\cos 20^\circ} = \frac{3182.7}{0.9397} = 3387 \text{ N}$$

Since the gear is mounted at the middle of the shaft, therefore maximum bending moment at the centre of the gear,

$$M = \frac{W.L}{4} = \frac{3387 \times 0.2}{4} = 169.4 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(169.4)^2 + (238.7)^2} = 292.7 \text{ N-m}$$

$$= 292.7 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$292.7 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84 \ d^3$$

∴ $d^3 = 292.7 \times 10^3 / 8.84 = 33 \times 10^3$ or $d = 32$ say 35 mm **Ans.**

Example 14.7. *A shaft made of mild steel is required to transmit 100 kW at 300 r.p.m. The supported length of the shaft is 3 metres. It carries two pulleys each weighing 1500 N supported at a distance of 1 metre from the ends respectively. Assuming the safe value of stress, determine the diameter of the shaft.*

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 300$ r.p.m. ; $L = 3$ m ; $W = 1500$ N

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2 \pi N} = \frac{100 \times 10^3 \times 60}{2 \pi \times 300} = 3183 \text{ N-m}$$

The shaft carrying the two pulleys is like a simply supported beam as shown in Fig. 14.3. The reaction at each support will be 1500 N, *i.e.*

$$R_A = R_B = 1500 \text{ N}$$

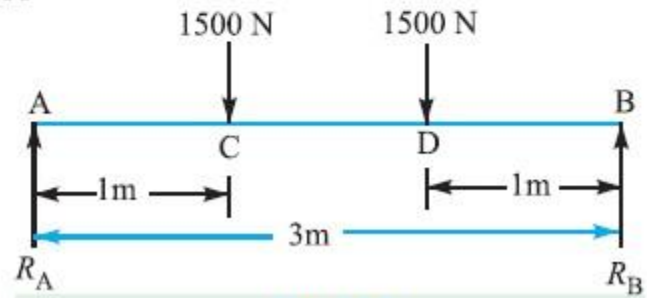


Fig. 14.3

A little consideration will show that the maximum bending moment lies at each pulley *i.e.* at *C* and *D*.

∴ Maximum bending moment,

$$M = 1500 \times 1 = 1500 \text{ N-m}$$

Let d = Diameter of the shaft in mm.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(1500)^2 + (3183)^2} = 3519 \text{ N-m}$$
$$= 3519 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$3519 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.8\, d^3 \quad \text{...(Assuming } \tau = 60 \text{ N/mm}^2\text{)}$$

∴ $d^3 = 3519 \times 10^3 / 11.8 = 298 \times 10^3$ or $d = 66.8$ say 70 mm **Ans.**

Pbm:5

shaft is supported by two bearings placed 1 m apart. A 600 mm diameter
pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25 kN. Another pulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and μ = 0.24. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.

Given :

AB = 1 m ; DC = 600 mm or RC = 300 mm = 0.3 m ;

AC = 300 mm = 0.3 m ; T1 = 2.25 kN = 2250 N ;

DD = 400 mm or RD = 200 mm = 0.2 m ;

BD = 200 mm = 0.2 m ;

θ = 180° = π rad ; μ = 0.24 ; σ*b = 63 MPa = 63 N/mm2 ;*

τ = 42 MPa = 42 N/mm2

We know that

$$2.3 \log \left(\frac{T_1}{T_2} \right) = \mu.\theta = 0.24 \times \pi = 0.754$$

$$\therefore \quad \log \left(\frac{T_1}{T_2} \right) = \frac{0.754}{2.3} = 0.3278 \quad \text{or} \quad \frac{T_1}{T_2} = 2.127 \quad \text{...(Taking antilog of 0.3278)}$$

and

$$T_2 = \frac{T_1}{2.127} = \frac{2250}{2.127} = 1058 \text{ N}$$

$\therefore$ Vertical load acting on the shaft at C,

$$W_C = T_1 + T_2 = 2250 + 1058 = 3308 \text{ N}$$

and vertical load on the shaft at D

$$= 0$$

The vertical load diagram is shown in Fig. 14.5 (*c*).

We know that torque acting on the pulley C,

$$T = (T_1 - T_2)\, R_C = (2250 - 1058)\, 0.3 = 357.6 \text{ N-m}$$

We know that torque acting on the pulley C,

$$T = (T_1 - T_2)\, R_C = (2250 - 1058)\, 0.3 = 357.6 \text{ N-m}$$

The torque diagram is shown in Fig. 14.5 (*b*).

Let T_3 = Tension in the tight side of the belt on pulley D, and

T_4 = Tension in the slack side of the belt on pulley D.

Since the torque on both the pulleys (*i.e.* C and D) is same, therefore

$$(T_3 - T_4)\, R_D = T = 357.6 \text{ N-m} \quad \text{or} \quad T_3 - T_4 = \frac{357.6}{R_D} = \frac{357.6}{0.2} = 1788 \text{ N}$$

We know that $$= \frac{T_3}{T_4} = \frac{T_1}{T_2} = 2.127 \quad \text{or} \quad T_3 = 2.127\, T_4$$

$$T_3 = 3376 \text{ N, and } T_4 = 1588 \text{ N}$$

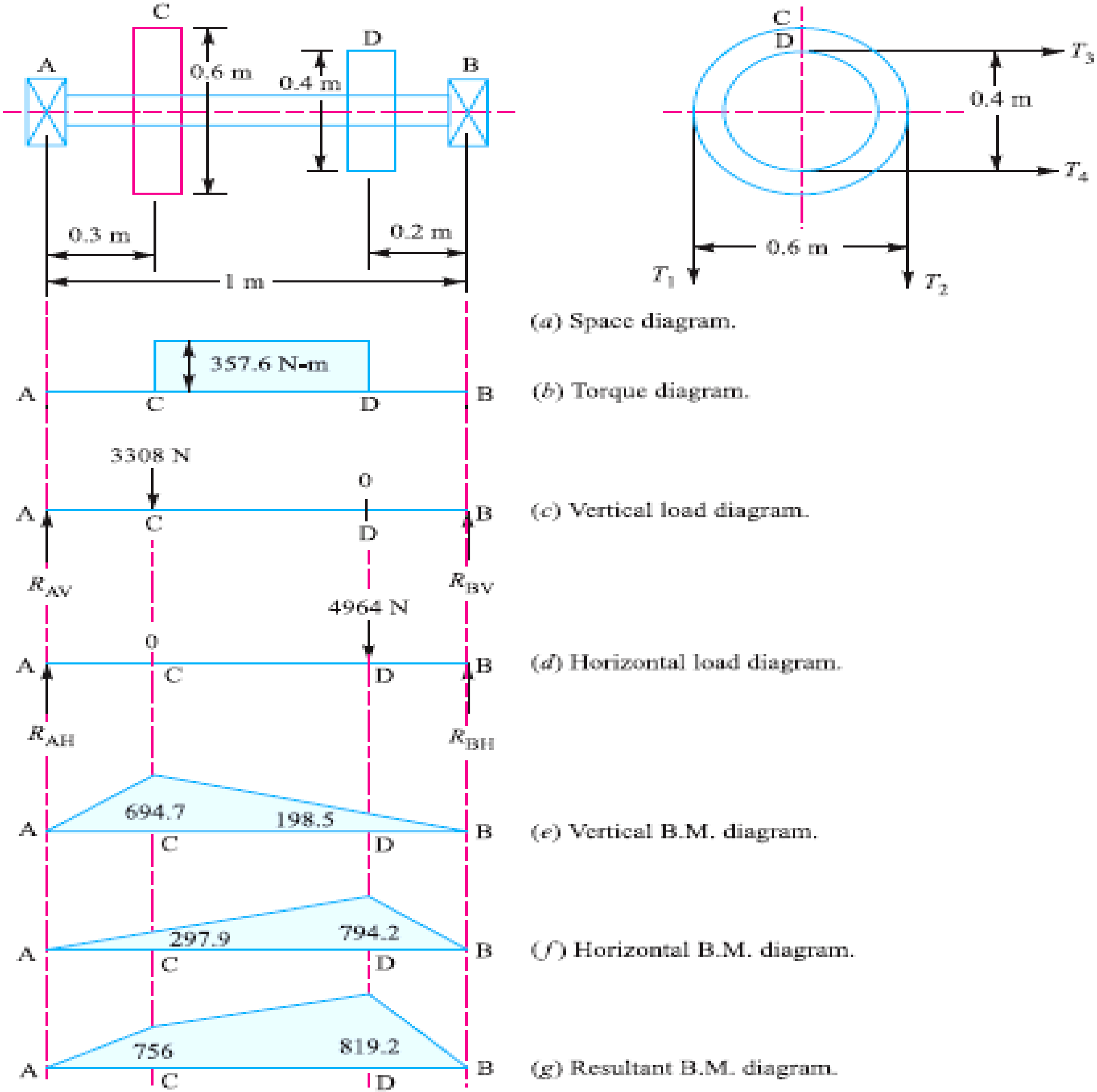
$\therefore$ Horizontal load acting on the shaft at D,

$$W_D = T_3 + T_4 = 3376 + 1588 = 4964 \text{ N}$$

and horizontal load on the shaft at $C = 0$

The horizontal load diagram is shown in Fig. 14.5 (*d*).

Now let us find the maximum bending moment for vertical and horizontal loading.



(a) Space diagram.

(b) Torque diagram.

(c) Vertical load diagram.

(d) Horizontal load diagram.

(e) Vertical B.M. diagram.

(f) Horizontal B.M. diagram.

(g) Resultant B.M. diagram.

First of all, considering the vertical loading at *C*. Let R_{AV} and R_{BV} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AV} + R_{BV} = 3308 \text{ N}$$

Taking moments about *A*,

$$R_{BV} \times 1 = 3308 \times 0.3 \quad \text{or} \quad R_{BV} = 992.4 \text{ N}$$

and $$R_{AV} = 3308 - 992.4 = 2315.6 \text{ N}$$

We know that B.M. at *A* and *B*,

$$M_{AV} = M_{BV} = 0$$

B.M. at *C*, $$M_{CV} = R_{AV} \times 0.3 = 2315.6 \times 0.3 = 694.7 \text{ N-m}$$

B.M. at *D*, $$M_{DV} = R_{BV} \times 0.2 = 992.4 \times 0.2 = 198.5 \text{ N-m}$$

The bending moment diagram for vertical loading in shown in Fig. 14.5 (*e*).

Now considering horizontal loading at *D*. Let R_{AH} and R_{BH} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AH} + R_{BH} = 4964 \text{ N}$$

Taking moments about *A*,

$$R_{BH} \times 1 = 4964 \times 0.8 \quad \text{or} \quad R_{BH} = 3971 \text{ N}$$

and $$R_{AH} = 4964 - 3971 = 993 \text{ N}$$

We know that B.M. at *A* and *B*,

$$M_{AH} = M_{BH} = 0$$

B.M. at *C*, $$M_{CH} = R_{AH} \times 0.3 = 993 \times 0.3 = 297.9 \text{ N-m}$$

B.M. at *D*, $$M_{DH} = R_{BH} \times 0.2 = 3971 \times 0.2 = 794.2 \text{ N-m}$$

Resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(694.7)^2 + (297.9)^2} = 756 \text{ N-m}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(198.5)^2 + (794.2)^2} = 819.2 \text{ N-m}$$

The resultant bending moment diagram is shown in Fig. 14.5 (g).

We see that bending moment is maximum at D.

$\therefore$ Maximum bending moment,

$$M = M_D = 819.2 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(819.2)^2 + (357.6)^2} = 894 \text{ N-m}$$
$$= 894 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$894 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$$\therefore \quad d^3 = 894 \times 10^3 / 8.25 = 108 \times 10^3 \text{ or } d = 47.6 \text{ mm}$$

Example 14.11. *A steel solid shaft transmitting 15 kW at 200 r.p.m. is supported on two bearings 750 mm apart and has two gears keyed to it. The pinion having 30 teeth of 5 mm module is located 100 mm to the left of the right hand bearing and delivers power horizontally to the right. The gear having 100 teeth of 5 mm module is located 150 mm to the right of the left hand bearing and receives power in a vertical direction from below. Using an allowable stress of 54 MPa in shear, determine the diameter of the shaft.*

Solution. Given : P = 15 kW = 15×10^3 W ; N = 200 r.p.m. ; AB = 750 mm ; $T_D = 30$; m_D = 5 mm ; BD = 100 mm ; $T_C = 100$; m_C = 5 mm ; AC = 150 mm ; τ = 54 MPa = 54 N/mm²

The space diagram of the shaft is shown in Fig. 14.8 (*a*).

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 200} = 716 \text{ N-m} = 716 \times 10^3 \text{ N-mm}$$

The torque diagram is shown in Fig. 14.8 (*b*).

We know that diameter of gear

= No. of teeth on the gear × module

∴ Radius of gear C,

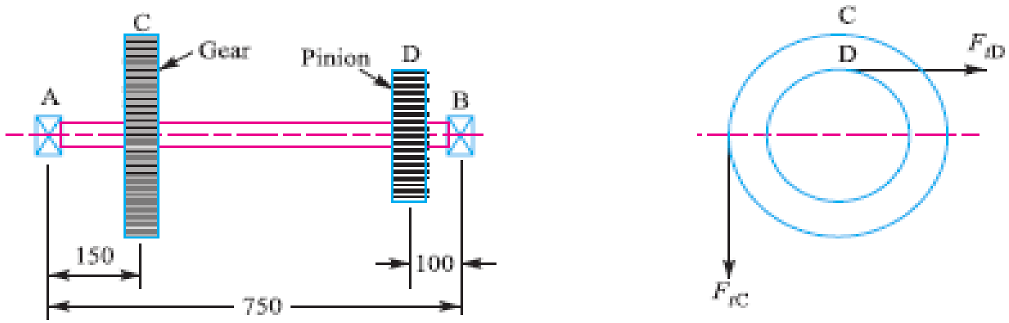
$$R_C = \frac{T_C \times m_C}{2} = \frac{100 \times 5}{2} = 250 \text{ mm}$$

and radius of pinion D,

$$R_D = \frac{T_D \times m_D}{2} = \frac{30 \times 5}{2} = 75 \text{ mm}$$

Assuming that the torque at C and D is same (*i.e.* 716×10^3 N-mm), therefore tangential force on the gear C, acting downward,

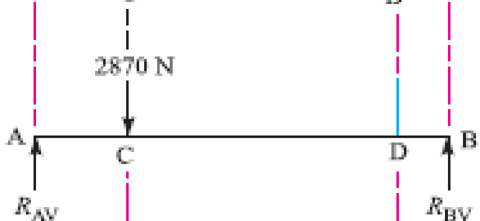
$$T \quad 716 \times 10^3 \qquad\qquad T \quad 716 \times 10^3$$



All dimensions in mm.

(*a*) Space diagram.

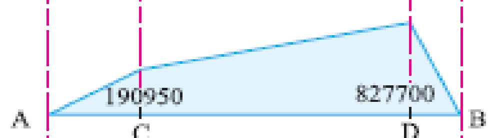
(*b*) Torque diagram.



(*c*) Vertical load diagram.

(*d*) Horizontal load diagram.

(*e*) Vertical B.M. diagram.



(*f*) Horizontal B.M. diagram.

(*g*) Resultant B.M. diagram.

First of all, considering the vertical loading at C. Let R_{AV} and R_{BV} be the reactions at the bearings A and B respectively. We know that

$$R_{AV} + R_{BV} = 2870 \text{ N}$$

Taking moments about A, we get

$$R_{BV} \times 750 = 2870 \times 150$$

B.M. at C, $\quad M_{CV} = R_{AV} \times 150 = 2296 \times 150 = 344\,400$ N-mm

B.M. at D, $\quad M_{DV} = R_{BV} \times 100 = 574 \times 100 = 57\,400$ N-mm

The B.M. diagram for vertical loading is shown in Fig. 14.8 (*e*).

Now considering horizontal loading at D. Let R_{AH} and R_{BH} be the reactions at the bearings A and B respectively. We know that

$$R_{AH} + R_{BH} = 9550 \text{ N}$$

Taking moments about A, we get

$$R_{BH} \times 750 = 9550\,(750 - 100) = 9550 \times 650$$

$\therefore \quad R_{BH} = 9550 \times 650 / 750 = 8277$ N

and $\quad R_{AH} = 9550 - 8277 = 1273$ N

We know that B.M. at A and B,

$$M_{AH} = M_{BH} = 0$$

B.M. at C, $\quad M_{CH} = R_{AH} \times 150 = 1273 \times 150 = 190\,950$ N-mm

B.M. at D, $\quad M_{DH} = R_{BH} \times 100 = 8277 \times 100 = 827\,700$ N-mm

The B.M. diagram for horizontal loading is shown in Fig. 14.8 (*f*).

We know that resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(344\ 400)^2 + (190\ 950)^2}$$

$$= 393\ 790 \text{ N-mm}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(57\ 400)^2 + (827\ 700)^2}$$

$$= 829\ 690 \text{ N-mm}$$

The resultant B.M. diagram is shown in Fig. 14.8 (*g*). We see that the bending moment is maximum at *D*.

$\therefore$ Maximum bending moment,

$$M = M_D = 829\ 690 \text{ N-mm}$$

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(829\ 690)^2 + (716 \times 10^3)^2} = 1096 \times 10^3 \text{ N-mm}$$

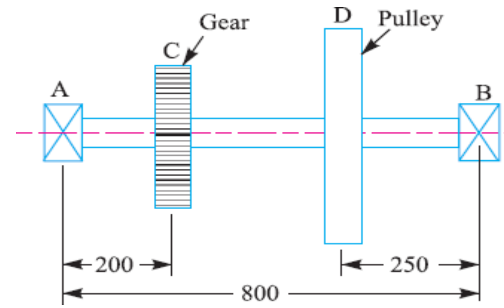
We also know that equivalent twisting moment (T_e),

$$1096 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 54 \times d^3 = 10.6\ d^3$$

$$\therefore \quad d^3 = 1096 \times 10^3/10.6 = 103.4 \times 10^3$$

or d = 47 say 50 mm **Ans.**

Example 14.9. *A shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25 kN. Another pulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and μ = 0.24. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.*

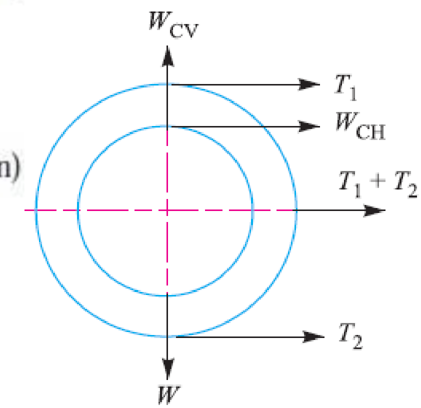


Solution. Given : $AB = 1$ m ; $D_C = 600$ mm or $R_C = 300$ mm $= 0.3$ m ; $AC = 300$ mm $= 0.3$ m ; $T_1 = 2.25$ kN $= 2250$ N ; $D_D = 400$ mm or $R_D = 200$ mm $= 0.2$ m ; $BD = 200$ mm $= 0.2$ m ; $\theta = 180° = \pi$ rad ; $\mu = 0.24$; $\sigma_b = 63$ MPa $= 63$ N/mm^2 ; $\tau = 42$ MPa $= 42$ N/mm^2

The space diagram of the shaft is shown in Fig. 14.5 (*a*).

Let T_1 = Tension in the tight side of the belt on pulley $C = 2250$ N ...(Given)

T_2 = Tension in the slack side of the belt on pulley C.



We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.24 \times \pi = 0.754$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.754}{2.3} = 0.3278 \quad \text{or} \quad \frac{T_1}{T_2} = 2.127 \qquad \text{...(Taking antilog of 0.3278)}$$

and

$$T_2 = \frac{T_1}{2.127} = \frac{2250}{2.127} = 1058 \text{ N}$$

$\therefore$ Vertical load acting on the shaft at C,

$$W_C = T_1 + T_2 = 2250 + 1058 = 3308 \text{ N}$$

and vertical load on the shaft at D

$$= 0$$

The vertical load diagram is shown in Fig. 14.5 (*c*).

We know that torque acting on the pulley *C*,

$$T = (T_1 - T_2)\, R_C = (2250 - 1058)\, 0.3 = 357.6 \text{ N-m}$$

The torque diagram is shown in Fig. 14.5 (*b*).

Let T_3 = Tension in the tight side of the belt on pulley *D*, and

T_4 = Tension in the slack side of the belt on pulley *D*.

Since the torque on both the pulleys (*i.e.* *C* and *D*) is same, therefore

$$(T_3 - T_4)\, R_D = T = 357.6 \text{ N-m} \quad \text{or} \quad T_3 - T_4 = \frac{357.6}{R_D} = \frac{357.6}{0.2} = 1788 \text{ N} \qquad \text{...(i)}$$

We know that

$$= \frac{T_3}{T_4} = \frac{T_1}{T_2} = 2.127 \quad \text{or} \quad T_3 = 2.127\, T_4 \qquad \text{...(ii)}$$

From equations (*i*) and (*ii*), we find that

$$T_3 = 3376 \text{ N, and } T_4 = 1588 \text{ N}$$

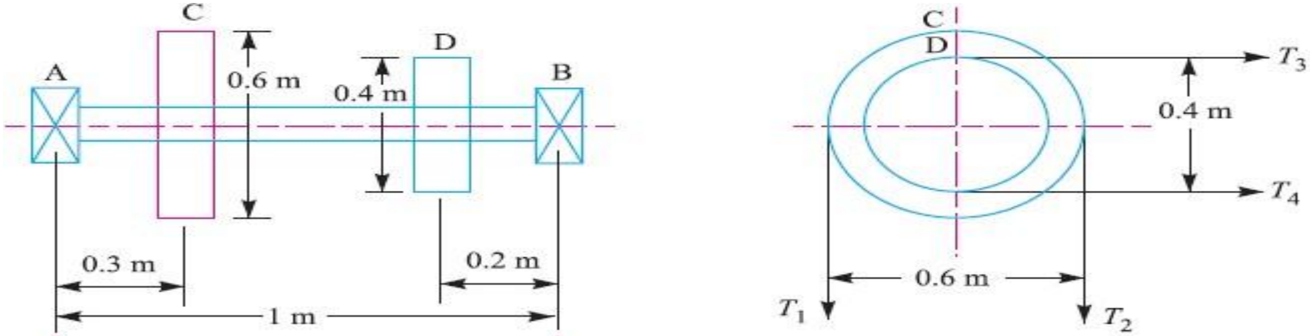
∴ Horizontal load acting on the shaft at *D*,

$$W_D = T_3 + T_4 = 3376 + 1588 = 4964 \text{ N}$$

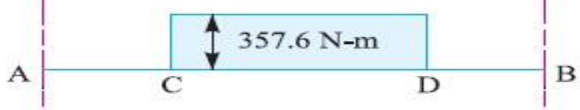
and horizontal load on the shaft at $C = 0$

The horizontal load diagram is shown in Fig. 14.5 (*d*).

Now let us find the maximum bending moment for vertical and horizontal loading.



(*a*) Space diagram.



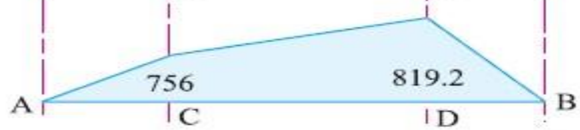
(*b*) Torque diagram.

(*c*) Vertical load diagram.

(*d*) Horizontal load diagram.

(*e*) Vertical B.M. diagram.

(*f*) Horizontal B.M. diagram.



(*g*) Resultant B.M. diagram.

First of all, considering the vertical loading at C. Let R_{AV} and R_{BV} be the reactions at the bearings A and B respectively. We know that

$$R_{AV} + R_{BV} = 3308 \text{ N}$$

Taking moments about A,

$$R_{BV} \times 1 = 3308 \times 0.3 \quad \text{or} \quad R_{BV} = 992.4 \text{ N}$$

and $$R_{AV} = 3308 - 992.4 = 2315.6 \text{ N}$$

We know that B.M. at A and B,

$$M_{AV} = M_{BV} = 0$$

B.M. at C, $$M_{CV} = R_{AV} \times 0.3 = 2315.6 \times 0.3 = 694.7 \text{ N-m}$$

B.M. at D, $$M_{DV} = R_{BV} \times 0.2 = 992.4 \times 0.2 = 198.5 \text{ N-m}$$

The bending moment diagram for vertical loading in shown in Fig. 14.5 (*e*).

Now considering horizontal loading at D. Let R_{AH} and R_{BH} be the reactions at the bearings A and B respectively. We know that

$$R_{AH} + R_{BH} = 4964 \text{ N}$$

Taking moments about A,

$$R_{BH} \times 1 = 4964 \times 0.8 \quad \text{or} \quad R_{BH} = 3971 \text{ N}$$

and $$R_{AH} = 4964 - 3971 = 993 \text{ N}$$

We know that B.M. at A and B,

$$M_{AH} = M_{BH} = 0$$

B.M. at C, $$M_{CH} = R_{AH} \times 0.3 = 993 \times 0.3 = 297.9 \text{ N-m}$$

B.M. at D, $$M_{DH} = R_{BH} \times 0.2 = 3971 \times 0.2 = 794.2 \text{ N-m}$$

The bending moment diagram for horizontal loading is shown in Fig. 14.5 (*f*).

Resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(694.7)^2 + (297.9)^2} = 756 \text{ N-m}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(198.5)^2 + (794.2)^2} = 819.2 \text{ N-m}$$

The resultant bending moment diagram is shown in Fig. 14.5 (g).

We see that bending moment is maximum at D.

$\therefore$ Maximum bending moment,

$$M = M_D = 819.2 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(819.2)^2 + (357.6)^2} = 894 \text{ N-m}$$

$$= 894 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$894 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$$\therefore \quad d^3 = 894 \times 10^3 / 8.25 = 108 \times 10^3 \text{ or } d = 47.6 \text{ mm}$$

Again we know that equivalent bending moment,

$$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \tfrac{1}{2}\,(M + T_e)$$

$$= \tfrac{1}{2}\,(819.2 + 894) = 856.6 \text{ N-m} = 856.6 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$856.6 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 63 \times d^3 = 6.2\ d^3$$

$$\therefore \quad d^3 = 856.6 \times 10^3 / 6.2 = 138.2 \times 10^3 \text{ or } d = 51.7 \text{ mm}$$

Taking larger of the two values, we have

$$d = 51.7 \text{ say } 55 \text{ mm}$$ **Ans.**

Shafts Subjected to Fluctuating Loads

In the previous articles we have assumed that the shaft is subjected to constant torque and bending moment. But in actual practice, the shafts are subjected to fluctuating torque and bending moments. In order to design such shafts like line shafts and counter shafts, the combined shock and fatigue factors must be taken into account for the computed *twisting moment* (*T*) *and bending moment (M).* Thus for a shaftsubjected to combined bending and torsion, the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t + T)^2}$$

and equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right]$$

where *Km = Combined shock and fatigue factor for bending, and*

Kt = Combined shock and fatigue factor for torsion.

The following table shows the recommended values for *Km and Kt.*

Table 14.2. Recommended values for K_m and K_t

Nature of load	K_m	K_t
1. Stationary shafts		
(*a*) Gradually applied load	1.0	1.0
(*b*) Suddenly applied load	1.5 to 2.0	1.5 to 2.0
2. Rotating shafts		
(*a*) Gradually applied or steady load	1.5	1.0
(*b*) Suddenly applied load with minor shocks only	1.5 to 2.0	1.5 to 2.0
(*c*) Suddenly applied load with heavy shocks	2.0 to 3.0	1.5 to 3.0

Example 14.12. *A mild steel shaft transmits 20 kW at 200 r.p.m. It carries a central load of 900 N and is simply supported between the bearings 2.5 metres apart. Determine the size of the shaft, if the allowable shear stress is 42 MPa and the maximum tensile or compressive stress is not to exceed 56 MPa. What size of the shaft will be required, if it is subjected to gradually applied loads?*

Solution. Given : $P = 20$ kW $= 20 \times 10^3$ W ; $N = 200$ r.p.m. ; $W = 900$ N ; $L = 2.5$ m ; $\tau = 42$ MPa $= 42$ N/mm^2 ; $\sigma_b = 56$ MPa $= 56$ N/mm^2

Size of the shaft

Let d = Diameter of the shaft, in mm.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

and maximum bending moment of a simply supported shaft carrying a central load,

$$M = \frac{W \times L}{4} = \frac{900 \times 2.5}{4} = 562.5 \text{ N-m} = 562.5 \times 10^3 \text{ N-mm}$$

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(562.5 \times 10^3)^2 + (955 \times 10^3)^2}$$

$$= 1108 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1108 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$$\therefore \quad d^3 = 1108 \times 10^3 / 8.25 = 134.3 \times 10^3 \text{ or } d = 51.2 \text{ mm}$$

We know that the equivalent bending moment,

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(562.5 \times 10^3 + 1108 \times 10^3) = 835.25 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$835.25 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 56 \times d^3 = 5.5\, d^3$$

$$\therefore \quad d^3 = 835.25 \times 10^3 / 5.5 = 152 \times 10^3 \text{ or } d = 53.4 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 53.4 \text{ say } 55 \text{ mm}$$ **Ans.**

Size of the shaft when subjected to gradually applied load

Let d = Diameter of the shaft.

From Table 14.2, for rotating shafts with gradually applied loads,

$$K_m = 1.5 \text{ and } K_t = 1$$

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(1.5 \times 562.5 \times 10^3)^2 + (1 \times 955 \times 10^3)^2} = 1274 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1274 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$\therefore$ $$d^3 = 1274 \times 10^3 / 8.25 = 154.6 \times 10^3 \text{ or } d = 53.6 \text{ mm}$$

We know that the equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \frac{1}{2}\left[K_m \times M + T_e\right]$$

$$= \frac{1}{2}\left[1.5 \times 562.5 \times 10^3 + 1274 \times 10^3\right] = 1059 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$1059 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 56 \times d^3 = 5.5\, d^3$$

$\therefore$ $$d^3 = 1059 \times 10^3 / 5.5 = 192.5 \times 10^3 = 57.7 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 57.7 \text{ say } 60 \text{ mm}$$ **Ans.**

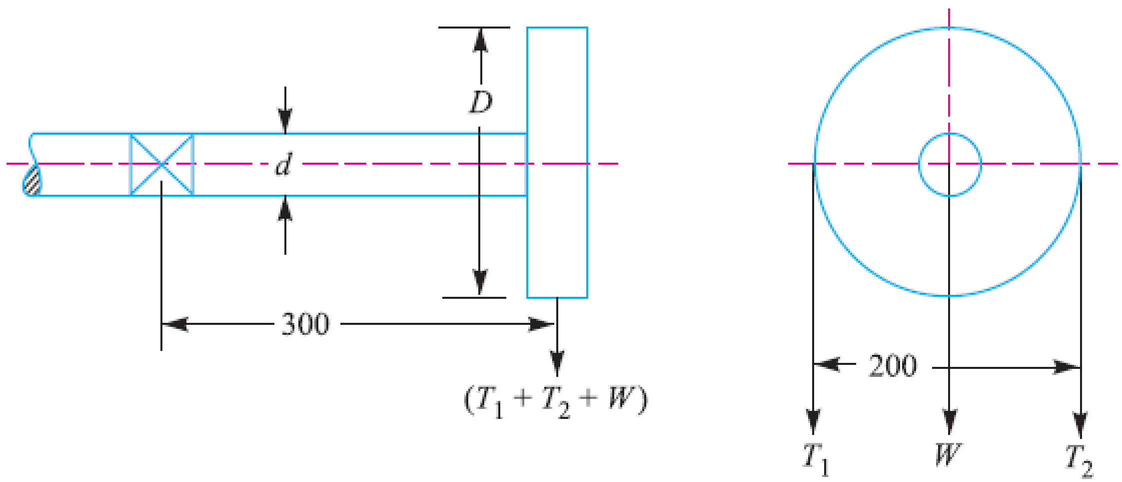
Example 14.13. *Design a shaft to transmit power from an electric motor to a lathe head stock through a pulley by means of a belt drive. The pulley weighs 200 N and is located at 300 mm from the centre of the bearing. The diameter of the pulley is 200 mm and the maximum power transmitted is 1 kW at 120 r.p.m. The angle of lap of the belt is 180° and coefficient of friction between the belt and the pulley is 0.3. The shock and fatigue factors for bending and twisting are 1.5 and 2.0 respectively. The allowable shear stress in the shaft may be taken as 35 MPa.*

Solution. Given : $W = 200$ N ; $L = 300$ mm ; $D = 200$ mm or $R = 100$ mm ; $P = 1$ kW $= 1000$ W ; $N = 120$ r.p.m. ; $\theta = 180° = \pi$ rad ; $\mu = 0.3$; $K_m = 1.5$; $K_t = 2$; $\tau = 35$ MPa $= 35$ N/mm²

The shaft with pulley is shown in Fig. 14.9.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{1000 \times 60}{2\pi \times 120} = 79.6 \text{ N-m} = 79.6 \times 10^3 \text{ N-mm}$$



All dimensions in mm.

Let T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively in newtons.

$\therefore$ Torque transmitted (T),

$$79.6 \times 10^3 = (T_1 - T_2)\,R = (T_1 - T_2)\,100$$

$$\therefore \quad T_1 - T_2 = 79.6 \times 10^3 / 100 = 796 \text{ N} \qquad \ldots(i)$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3\,\pi = 0.9426$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.9426}{2.3} = 0.4098 \text{ or } \frac{T_1}{T_2} = 2.57 \qquad \ldots(ii)$$

...(Taking antilog of 0.4098)

From equations (*i*) and (*ii*), we get,

$$T_1 = 1303 \text{ N, and } T_2 = 507 \text{ N}$$

We know that the total vertical load acting on the pulley,

$$W_T = T_1 + T_2 + W = 1303 + 507 + 200 = 2010 \text{ N}$$

$\therefore$ Bending moment acting on the shaft,

$$M = W_T \times L = 2010 \times 300 = 603 \times 10^3 \text{ N-mm}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t + T)^2}$$

$$= \sqrt{(1.5 \times 603 \times 10^3)^2 + (2 \times 79.6 \times 10^3)^2} = 918 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$918 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 35 \times d^3 = 6.87\,d^3$$

$$\therefore \quad d^3 = 918 \times 10^3 / 6.87 = 133.6 \times 10^3 \text{ or } d = 51.1 \text{ say } 55 \text{ mm}$$ **Ans.**

Shafts Subjected to Axial Load in addition to Combined Torsion and Bending Loads

When the shaft is subjected to an axial load (F) in addition to torsion and bending loads as in propeller shafts of ships and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (σ_b). We know that bending equation is

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \text{or} \quad \sigma_b = \frac{M \cdot y}{I} = \frac{M \times d/2}{\frac{\pi}{64} \times d^4} = \frac{32\,M}{\pi\, d^3}$$

and stress due to axial load

$$= \frac{F}{\frac{\pi}{4} \times d^2} = \frac{4\,F}{\pi\, d^2} \qquad \text{...(For round solid shaft)}$$

$$= \frac{F}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]} = \frac{4\,F}{\pi\left[(d_o)^2 - (d_i)^2\right]} \qquad \text{...(For hollow shaft)}$$

$$= \frac{F}{\pi\,(d_o)^2\,(1 - k^2)} \qquad \text{... } (\because\ k = d_i/d_o)$$

$\therefore$ Resultant stress (tensile or compressive) for solid shaft,

$$\sigma_1 = \frac{32\,M}{\pi\, d^3} + \frac{4\,F}{\pi\, d^2} = \frac{32}{\pi\, d^3}\left(M + \frac{F \times d}{8}\right) \qquad \text{...(i)}$$

$$= \frac{32\,M_1}{\pi\, d^3} \qquad \text{...}\left(\text{Substituting } M_1 = M + \frac{F \times d}{8}\right)$$

In case of a hollow shaft, the resultant stress,

$$\sigma_1 = \frac{32\,M}{\pi\,(d_o)^3\,(1-k^4)} + \frac{4\,F}{\pi\,(d_o)^2\,(1-k^2)}$$

$$= \frac{32}{\pi\,(d_o)^3\,(1-k^4)}\left[M + \frac{F\,d_o\,(1+k^2)}{8}\right] = \frac{32\,M_1}{\pi\,(d_o)^3\,(1-k^4)}$$

$$\ldots\left[\text{Substituting for hollow shaft, } M_1 = M + \frac{F\,d_o\,(1+k^2)}{8}\right]$$

In case of long shafts (slender shafts) subjected to compressive loads, a factor known as ***column factor*** (α) must be introduced to take the column effect into account.

$\therefore$ Stress due to the compressive load,

$$\sigma_c = \frac{\alpha \times 4\,F}{\pi\,d^2} \qquad \text{...(For round solid shaft)}$$

$$= \frac{\alpha \times 4F}{\pi (d_o)^2 \, (1 - k^2)}$$ ...(For hollow shaft)

The value of column factor (α) for compressive loads* may be obtained from the following relation :

Column factor, $$\alpha = \frac{1}{1 - 0.0044 \, (L / K)}$$

This expression is used when the slenderness ratio (L / K) is less than 115. When the slenderness ratio (L / K) is more than 115, then the value of column factor may be obtained from the following relation :

**Column factor, $$\alpha = \frac{\sigma_y \, (L / K)^2}{C \pi^2 E}$$

where L = Length of shaft between the bearings,

K = Least radius of gyration,

σ_y = Compressive yield point stress of shaft material, and

C = Coefficient in Euler's formula depending upon the end conditions.

The following are the different values of C depending upon the end conditions.

C = 1, for hinged ends,

= 2.25, for fixed ends,

= 1.6, for ends that are partly restrained as in bearings.

Note: In general, for a hollow shaft subjected to fluctuating torsional and bending load, along with an axial load, the equations for equivalent twisting moment (T_e) and equivalent bending moment (M_e) may be written as

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha\, F\, d_o\,(1+k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \frac{\pi}{16} \times \tau\,(d_o)^3\,(1-k^4)$$

and

$$M_e = \frac{1}{2}\left[K_m \times M + \frac{\alpha F d_o\,(1+k^2)}{8} + \sqrt{\left\{K_m \times M + \frac{\alpha F d_o\,(1+k^2)}{8}\right\}^2 + (K_t \times T)^2}\right]$$

$$= \frac{\pi}{32} \times \sigma_b\,(d_o)^3\,(1-k^4)$$

It may be noted that for a solid shaft, $k = 0$ and $d_0 = d$. When the shaft carries no axial load, then $F = 0$ and when the shaft carries axial tensile load, then $\alpha = 1$.

Example 14.18. *A hollow shaft is subjected to a maximum torque of 1.5 kN-m and a maximum bending moment of 3 kN-m. It is subjected, at the same time, to an axial load of 10 kN. Assume that the load is applied gradually and the ratio of the inner diameter to the outer diameter is 0.5. If the outer diameter of the shaft is 80 mm, find the shear stress induced in the shaft.*

Solution. Given : $T = 1.5$ kN-m $= 1.5 \times 10^3$ N-m ; $M = 3$ kN-m $= 3 \times 10^3$ N-m ; $F = 10$ kN $= 10 \times 10^3$ N ; $k = d_i / d_o = 0.5$; $d_o = 80$ mm $= 0.08$ m

Let τ = Shear stress induced in the shaft.

Since the load is applied gradually, therefore from Table 14.2, we find that

$$K_m = 1.5 \text{ ; and } K_t = 1.0$$

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o (1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 3 \times 10^3 + \frac{1 \times 10 \times 10^3 \times 0.08 (1 + 0.5^2)}{8}\right]^2 + (1 \times 1.5 \times 10^3)^2}$$

... ($\because$ $\alpha = 1$, for axial tensile loading)

$$= \sqrt{(4500 + 125)^2 + (1500)^2} = 4862 \text{ N-m} = 4862 \times 10^3 \text{ N-mm}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$4862 \times 10^3 = \frac{\pi}{16} \times \tau \, (d_o)^3 \, (1 - k^4) = \frac{\pi}{16} \times \tau \, (80)^3 \, (1 - 0.5^4) = 94\,260 \, \tau$$

$\therefore$ $\tau = 4862 \times 10^3 / 94\,260 = 51.6 \text{ N/mm}^2 = 51.6$ MPa **Ans.**

Example 14.19. *A hollow shaft of 0.5 m outside diameter and 0.3 m inside diameter is used to drive a propeller of a marine vessel. The shaft is mounted on bearings 6 metre apart and it transmits 5600 kW at 150 r.p.m. The maximum axial propeller thrust is 500 kN and the shaft weighs 70 kN. Determine :*

1. *The maximum shear stress developed in the shaft, and*
2. *The angular twist between the bearings.*

Solution. Given : $d_o = 0.5$ m ; $d_i = 0.3$ m ; $P = 5600$ kW $= 5600 \times 10^3$ W ; $L = 6$ m ; $N = 150$ r.p.m. ; $F = 500$ kN $= 500 \times 10^3$ N ; $W = 70$ kN $= 70 \times 10^3$ N

1. *Maximum shear stress developed in the shaft*

Let τ = Maximum shear stress developed in the shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2 \pi N} = \frac{5600 \times 10^3 \times 60}{2 \pi \times 150} = 356\ 460 \text{ N-m}$$

and the maximum bending moment,

$$M = \frac{W \times L}{8} = \frac{70 \times 10^3 \times 6}{8} = 52\ 500 \text{ N-m}$$

Now let us find out the column factor α. We know that least radius of gyration,

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right]}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]}}$$

$$= \sqrt{\frac{[(d_o)^2 + (d_i)^2]\,[(d_o)^2 - (d_i)^2]}{16\,[(d_o)^2 - (d_i)^2]}}$$

$$= \frac{1}{4}\sqrt{(d_o)^2 + (d_i)^2} = \frac{1}{4}\sqrt{(0.5)^2 + (0.3)^2} = 0.1458 \text{ m}$$

$\therefore$ Slenderness ratio,

$$L / K = 6 / 0.1458 = 41.15$$

and column factor,

$$\alpha = \frac{1}{1 - 0.0044\left(\frac{L}{K}\right)} \qquad \ldots\left(\because \frac{L}{K} < 115\right)$$

$$= \frac{1}{1 - 0.0044 \times 41.15} = \frac{1}{1 - 0.18} = 1.22$$

Assuming that the load is applied gradually, therefore from Table 14.2, we find that

$$K_m = 1.5 \text{ and } K_t = 1.0$$

Also

$$k = d_i / d_o = 0.3 / 0.5 = 0.6$$

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o \ (1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 52\ 500 + \frac{1.22 \times 500 \times 10^3 \times 0.5\ (1 + 0.6^2)}{8}\right]^2 + (1 \times 356\ 460)^2}$$

$$= \sqrt{(78\ 750 + 51\ 850)^2 + (356\ 460)^2} = 380 \times 10^3 \text{ N-m}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$380 \times 10^3 = \frac{\pi}{16} \times \tau\ (d_o)^3\ (1 - k^4) = \frac{\pi}{16} \times \tau\ (0.5)^3\ [1 - (0.6)^4] = 0.02\ \tau$$

$\therefore$

$$\tau = 380 \times 10^3 / 0.02 = 19 \times 10^6 \text{ N/m}^2 = 19 \text{ MPa}$$ **Ans.**

2. *Angular twist between the bearings*

Let θ = Angular twist between the bearings in radians.

We know that the polar moment of inertia for a hollow shaft,

$$J = \frac{\pi}{32}[(d_o)^4 - (d_i)^4] = \frac{\pi}{32}[(0.5)^4 - (0.3)^4] = 0.005\ 34\ \text{m}^4$$

From the torsion equation,

$$\frac{T}{J} = \frac{G \times \theta}{L}, \text{ we have}$$

$$\theta = \frac{T \times L}{G \times J} = \frac{356\ 460 \times 6}{84 \times 10^9 \times 0.00\ 534} = 0.0048 \text{ rad}$$

... (Taking $G = 84$ GPa $= 84 \times 10^9$ N/m^2)

$$= 0.0048 \times \frac{180}{\pi} = 0.275° \textbf{ Ans.}$$

Design of Shafts on the basis of Rigidity

1. *Torsional rigidity.* The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be effected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft.

The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G.\theta}{L} \quad \text{or} \quad \theta = \frac{T.L}{J.G}$$

where θ = Torsional deflection or angle of twist in radians,

T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$$= \frac{\pi}{32} \times d^4 \quad \text{...(For solid shaft)}$$

$$= \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \quad \text{...(For hollow shaft)}$$

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

2. *Lateral rigidity*. It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, *i.e.*

$$\frac{d^2 y}{dx^2} = \frac{M}{EI}$$

Example 14.21. *A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25° per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84 GPa, find the diameter of the spindle and the shear stress induced in the spindle.*

Solution. Given : $P = 4$ kW = 4000 W ; $N = 800$ r.p.m. ; $\theta = 0.25° = 0.25 \times \frac{\pi}{180} = 0.0044$ rad ; $L = 1$ m = 1000 mm ; $G = 84$ GPa $= 84 \times 10^9$ N/m² $= 84 \times 10^3$ N/mm²

Diameter of the spindle

Let d = Diameter of the spindle in mm.

We know that the torque transmitted by the spindle,

$$T = \frac{P \times 60}{2\pi N} = \frac{4000 \times 60}{2\pi \times 800} = 47.74 \text{ N-m} = 47\,740 \text{ N-mm}$$

We also know that $\frac{T}{J} = \frac{G \times \theta}{L}$ or $J = \frac{T \times l}{G \times \theta}$

or $$\frac{\pi}{32} \times d^4 = \frac{47\,740 \times 1000}{84 \times 10^3 \times 0.0044} = 129\,167$$

$\therefore$ $d^4 = 129\,167 \times 32/\pi = 1.3 \times 10^6$ or $d = 33.87$ say 35 mm **Ans.**

Shear stress induced in the spindle

Let τ = Shear stress induced in the spindle.

We know that the torque transmitted by the spindle (T),

$$47\,740 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times \tau\,(35)^3 = 8420\,\tau$$

$\therefore$ $\tau = 47\,740 / 8420 = 5.67$ N/mm² $= 5.67$ MPa **Ans.**

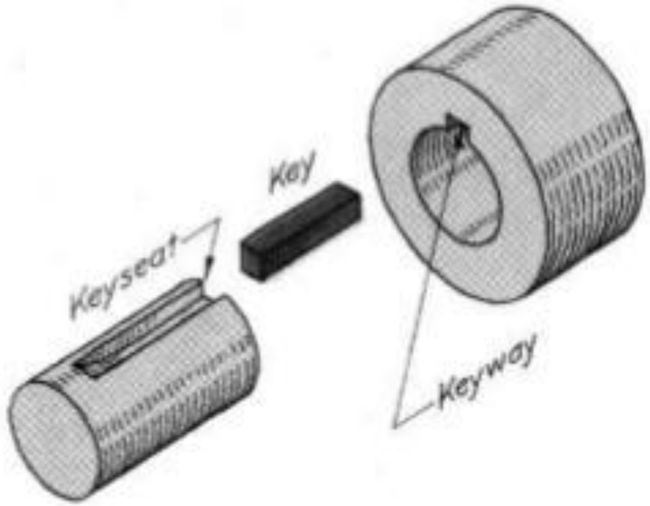
Design of key

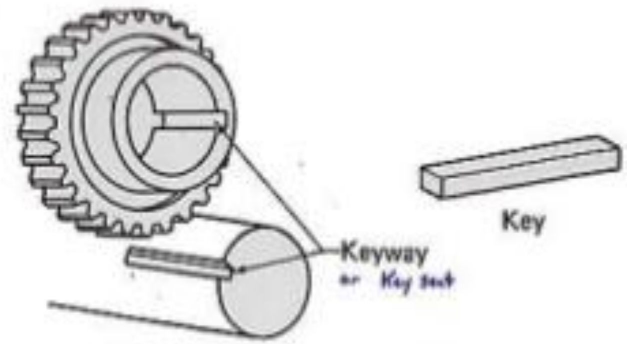
What is key

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them.

It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

Definition & Nomenclature





A key and the keyways in a gear and shaft

Key is a machine element which provides a torque transmitting link between two power transmitting elements Key prevents the relative rotary motion between two components to be connected with positive lacking

Types of Keys

- The following types of keys are important from the

1. **Sunk keys,**
2. **Saddle keys**
3. **Tangent keys,**
4. **Round keys,**
5. **Splines.**

13.3 Sunk Keys

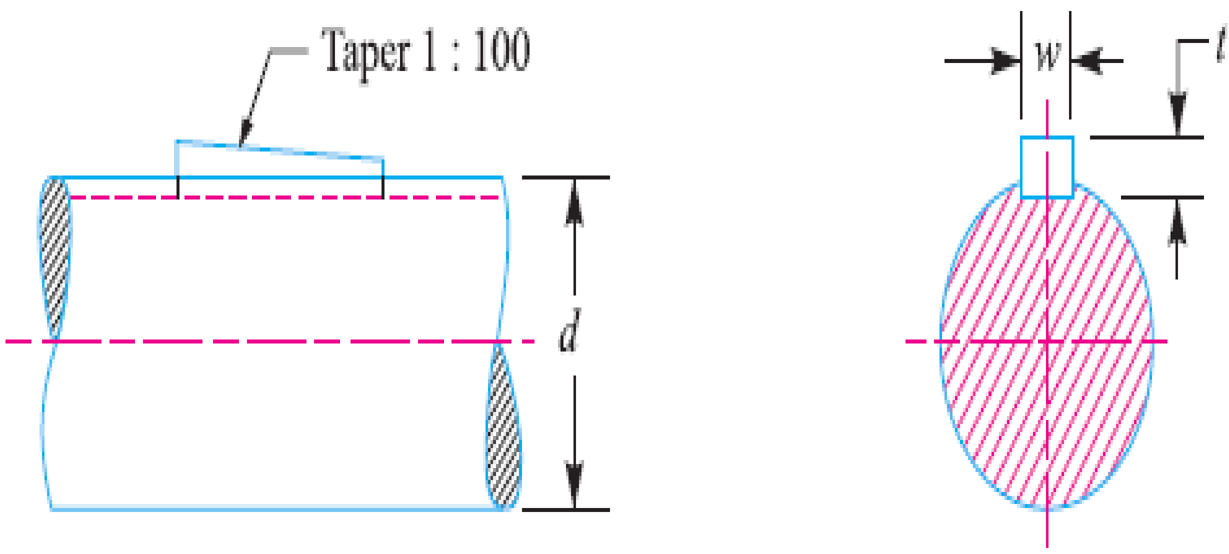
The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types :

1. *Rectangular sunk key.* A rectangular sunk key is shown in Fig. 13.1. The usual proportions of this key are :

Width of key, $w = d / 4$; and thickness of key, $t = 2w / 3 = d / 6$

where d = Diameter of the shaft or diameter of the hole in the hub.

The key has taper 1 in 100 on the top side only.



Square sunk key.

The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal,

i.e. $w = t = d / 4$

Parallel sunk key.

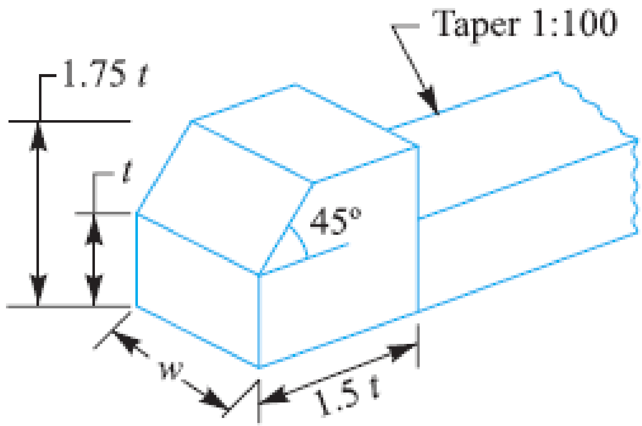
The parallel sunk keys may be of rectangular or square sectionuniform in width and thickness throughout. It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

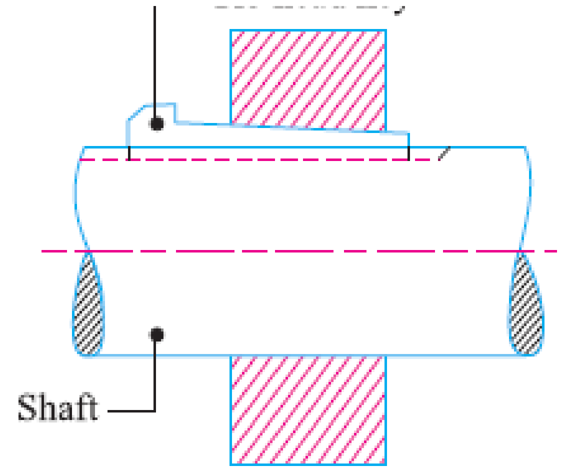
Gib-head key

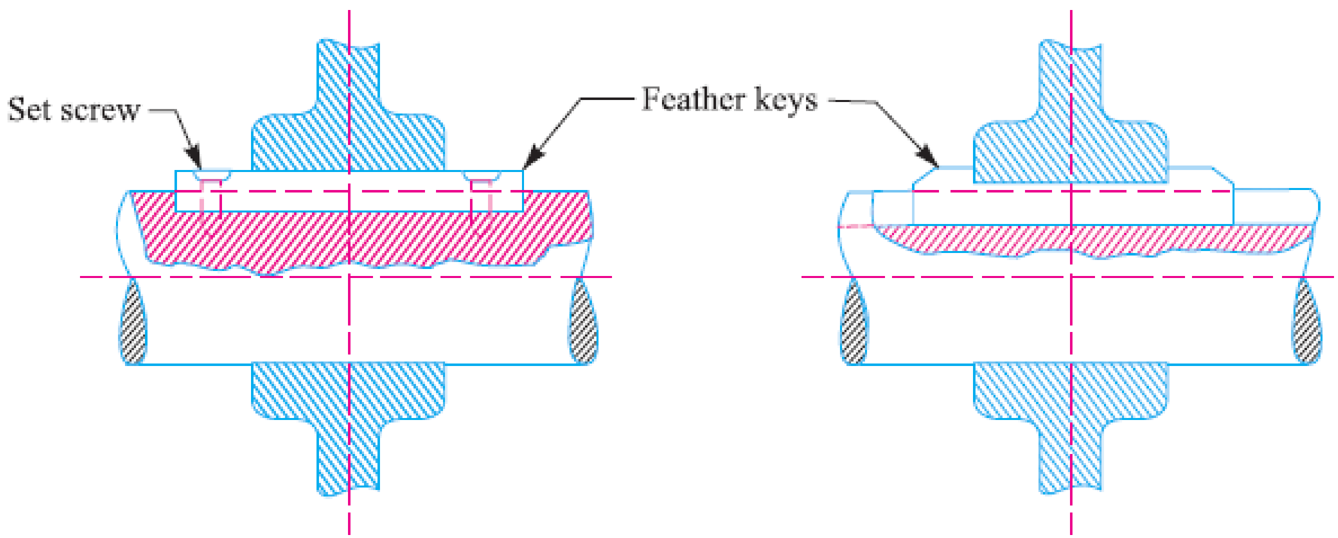
It is a rectangular sunk key with a head at one end known as gib head. It is usually provided to facilitate the removal of key.

Feather key

A key attached to one member of a pair and which permits relative axial movement is known as feather key. It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.


Taper 1:100
1.75 t
t
45°
w
1.5 t


Shaft


Set screw
Feather keys

Example 13.1. *Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MPa and 70 MPa.*

Solution. Given : $d = 50$ mm ; $\tau = 42$ MPa $= 42$ N/mm^2 ; $\sigma_c = 70$ MPa $= 70$ N/mm^2

The rectangular key is designed as discussed below:

From Table 13.1, we find that for a shaft of 50 mm diameter,

Width of key, $w = 16$ mm **Ans.**

and thickness of key, $t = 10$ mm **Ans.**

The length of key is obtained by considering the key in shearing and crushing.

Let l = Length of key.

Considering shearing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times w \times \tau \times \frac{d}{2} = l \times 16 \times 42 \times \frac{50}{2} = 16\ 800\ l \text{ N-mm} \qquad \text{...}(i)$$

and torsional shearing strength (or torque transmitted) of the shaft,

$$T = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42\ (50)^3 = 1.03 \times 10^6 \text{ N-mm} \qquad \text{...}(ii)$$

From equations *(i)* and *(ii)*, we have

$$l = 1.03 \times 10^6 / 16\ 800 = 61.31 \text{ mm}$$

Now considering crushing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} = l \times \frac{10}{2} \times 70 \times \frac{50}{2} = 8750\ l \text{ N-mm} \qquad \text{...}(iii)$$

From equations *(ii)* and *(iii)*, we have

$$l = 1.03 \times 10^6 / 8750 = 117.7 \text{ mm}$$

Taking larger of the two values, we have length of key,

$l = 117.7$ say 120 mm **Ans.**

UNIT-III

TEMPORARY AND PERMANENT JOINTS

Threaded fasteners - Bolted joints including eccentric loading, Knuckle joints, Cotter joints

Welded joints, riveted joints for structures - theory of bonded joints.

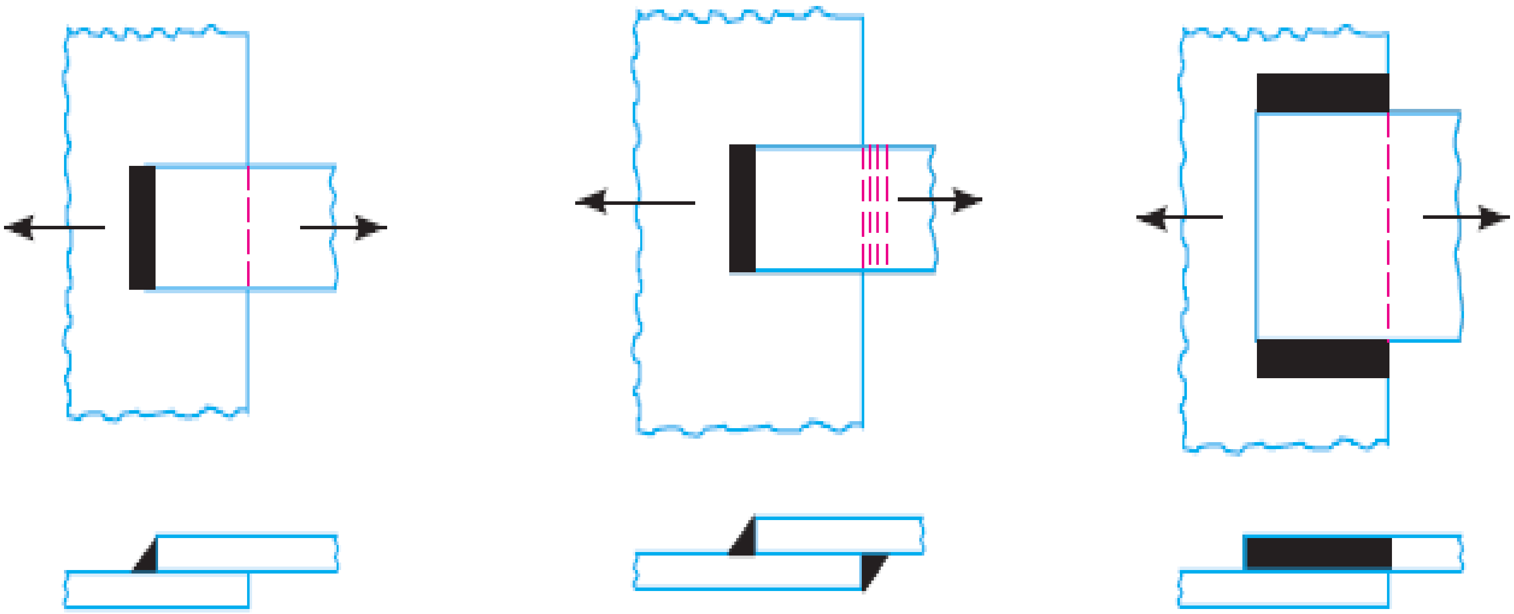
Welded joints

- A welded joint is a permanent joint which is obtained by the fusion of the edges of the two parts to be joined together, with or without the application of pressure and a filler material.
- The heat required for the fusion of the material may be obtained by burning of gas (in case of gas welding) or by an electric arc (in case of electric arc welding).
- Welding is extensively used in fabrication as an alternative method for casting or forging and as a replacement for bolted and riveted joints.

Types of Welded Joints

Following two types of welded joints are important from the subject point of view:

1. Lap joint or fillet joint, and 2. Butt joint.



(*a*) Single transverse. (*b*) Double transverse. (*c*) Parallel fillet.

Welding Processes

The welding processes may be broadly classified into the following two groups:

1. Welding processes that use heat alone

   e.g. fusion welding.

2. Welding processes that use a combination of heat and pressure

   e.g. forge welding.

Fusion Welding

- In case of fusion welding, the parts to be jointed are held in position while the molten metal is supplied to the joint.
- The molten metal may come from the parts themselves (i.e. parent metal) or filler metal which normally have the composition of the parent metal.
- The fusion welding, according to the method of heat generated, may be classified as:

  1. Thermit welding,

  2. Gas welding, and

  3. Electric arc welding.

Thermit Welding

- In thermit welding, a mixture of iron oxide and aluminium called ***thermit is ignited and the iron*** oxide is reduced to molten iron.
- The molten iron is poured into a mould made around the joint and fuses with the parts to be welded.
- A major advantage of the thermit welding is that all parts of weld section are molten at the same time and the weld cools almost uniformly.
- This results in a minimum problem with residual stresses.

Thermit Welding (continued)

- The thermit welding is often used in joining iron and steel parts that are too large to be manufactured in one piece, such as rails, truck frames, locomotive frames, other large sections used on steam and rail roads, for stern frames, rudder frames etc.
- In steel mills, thermit electric welding is employed to replace broken gear teeth, to weld new necks on rolls and pinions, and to repair broken shears.

Gas Welding

- A gas welding is made by applying the flame of an oxy-acetylene or hydrogen gas from a welding torch upon the surfaces of the repared joint.
- The intense heat at the white cone of the flame heats up the local surfaces to fusion point while the operator manipulates a welding rod to supply the metal for the weld. A flux is being used to remove the slag.
- Since the heating rate in gas welding is slow, therefore it can be used on thinner materials.

Electric Arc Welding

- In electric arc welding, the work is prepared in the same manner as for gas welding. In this case
- The filler metal is supplied by metal welding electrode.
- A small depression is formed in the base metal and the molten metal is deposited around the edge of this depression, which is called the ***arc crater.***

Lap Joint

- The lap joint or the fillet joint is obtained by overlapping the plates and then welding the edges of the plates. The cross-section of the fillet is approximately triangular.

The fillet joints may be

1. **Single transverse fillet,**
2. **Double transverse fillet, and**
3. **Parallel fillet joints.**

- A single transverse fillet joint has the disadvantage that the edge of the plate which is not welded can buckle or warp out of shape.

Butt Joint

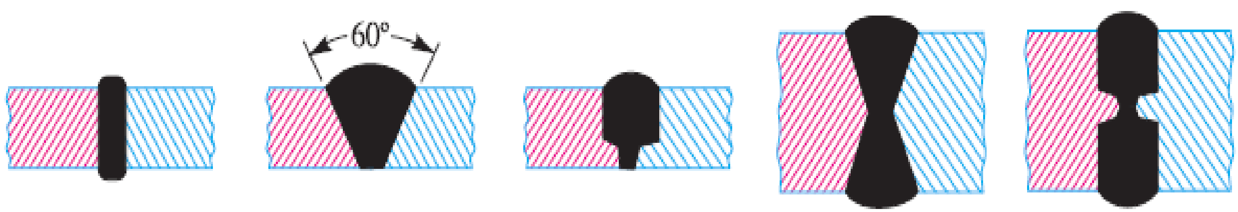
- The butt joint is obtained by placing the plates edge to edge
- .In butt welds the plate edges do not require bevelling if the thickness of plate is less than 5 mm.
- On the other hand, if the plate thickness is 5 mm to 12.5 mm, the edges should be bevelled to V or U-groove on both sides.

TYPES

The butt joints may be

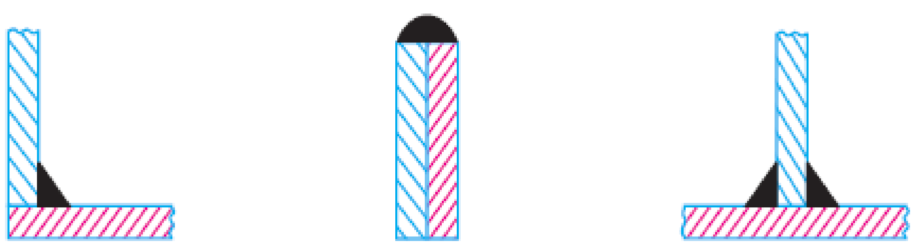
1.Square butt joint, 2. Single V-butt joint 3. Single U-butt joint,

4. Double V-butt joint, 5. Double U-butt joint.



(*a*) Square butt joint. (*b*) Single *V*-butt joint. (*c*) Single *U*-butt joint. (*d*) Double *V*-butt joint. (*e*) Double *U*-butt joint.

The other type of welded joints are corner joint, edge joint and T-joint



(*a*) Corner joint. (*b*) Edge joint. (*c*) *T*-joint.

Strength of Transverse Fillet Welded Joints

- We have already discussed that the fillet or lap joint is obtained by overlapping the plates and then welding the edges of the plates.
- The transverse fillet welds are designed for tensile strength.
- Let us consider a single and double transverse fillet welds *respectively.*

In order to determine the strength of the fillet joint, it is assumed that the section of fillet is a right angled triangle *ABC* with hypotenuse *AC* making equal angles with other two sides *AB* and *BC*. The enlarged view of the fillet is shown in Fig. 10.7. The length of each side is known as ***leg*** or ***size of the weld*** and the perpendicular distance of the hypotenuse from the intersection of legs (*i.e. BD*) is known as ***throat thickness***. The minimum area of the weld is obtained at the throat *BD*, which is given by the product of the throat thickness and length of weld.

Let t = Throat thickness (*BD*),

s = Leg or size of weld,

= Thickness of plate, and

l = Length of weld,

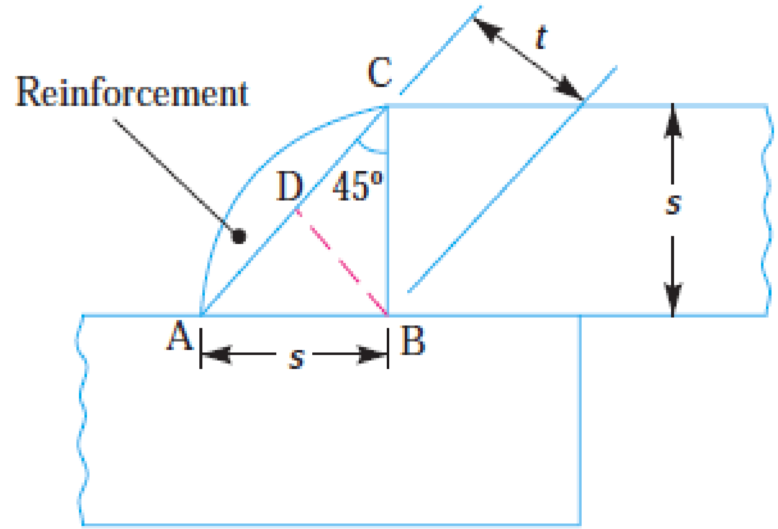


Fig. 10.7. Enlarged view of a fillet weld.

From Fig. 10.7, we find that the throat thickness,

$$t = s \times \sin 45° = 0.707\ s$$

∴ *Minimum area of the weld or throat area,

$$A = \text{Throat thickness} \times \text{Length of weld} = t \times l = 0.707\ s \times l$$

If σ_t is the allowable tensile stress for the weld metal, then the tensile strength of the joint for single fillet weld,

$$P = \text{Throat area} \times \text{Allowable tensile stress} = 0.707\ s \times l \times \sigma_t$$

and tensile strength of the joint for double fillet weld,

$$P = 2 \times 0.707\ s \times l \times \sigma_t = 1.414\ s \times l \times \sigma_t$$

Strength of Parallel Fillet Welded Joints

- The parallel fillet welded joints are designed for shear strength.
- Consider a double parallel fillet welded joint We have already discussed in the previous that the minimum area of weld or the throat area,

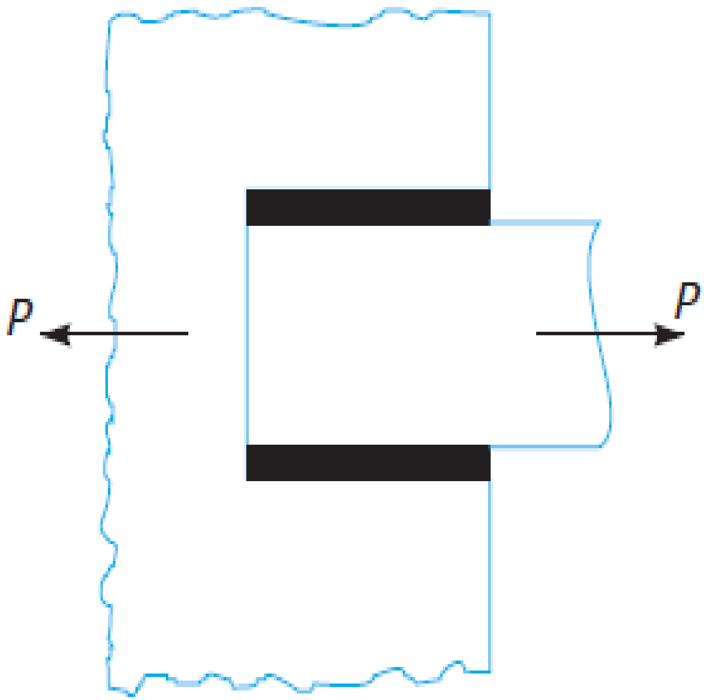
$$A = 0.707\ s \times l$$

If τ is the allowable shear stress for the weld metal, then the shear strength of the joint for single parallel fillet weld,

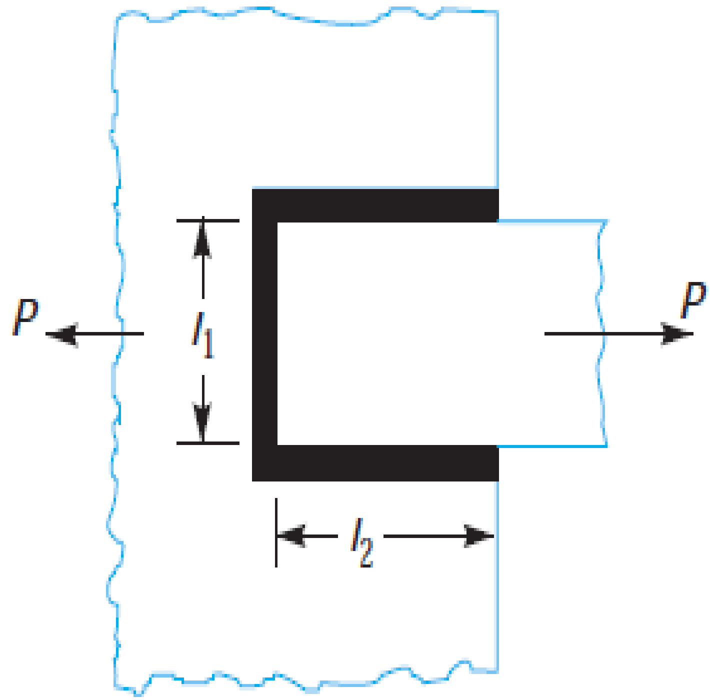
$$P = \text{Throat area} \times \text{Allowable shear stress} = 0.707\, s \times l \times \tau$$

and shear strength of the joint for double parallel fillet weld,

$$P = 2 \times 0.707 \times s \times l \times \tau = 1.414\, s \times l \times \tau$$

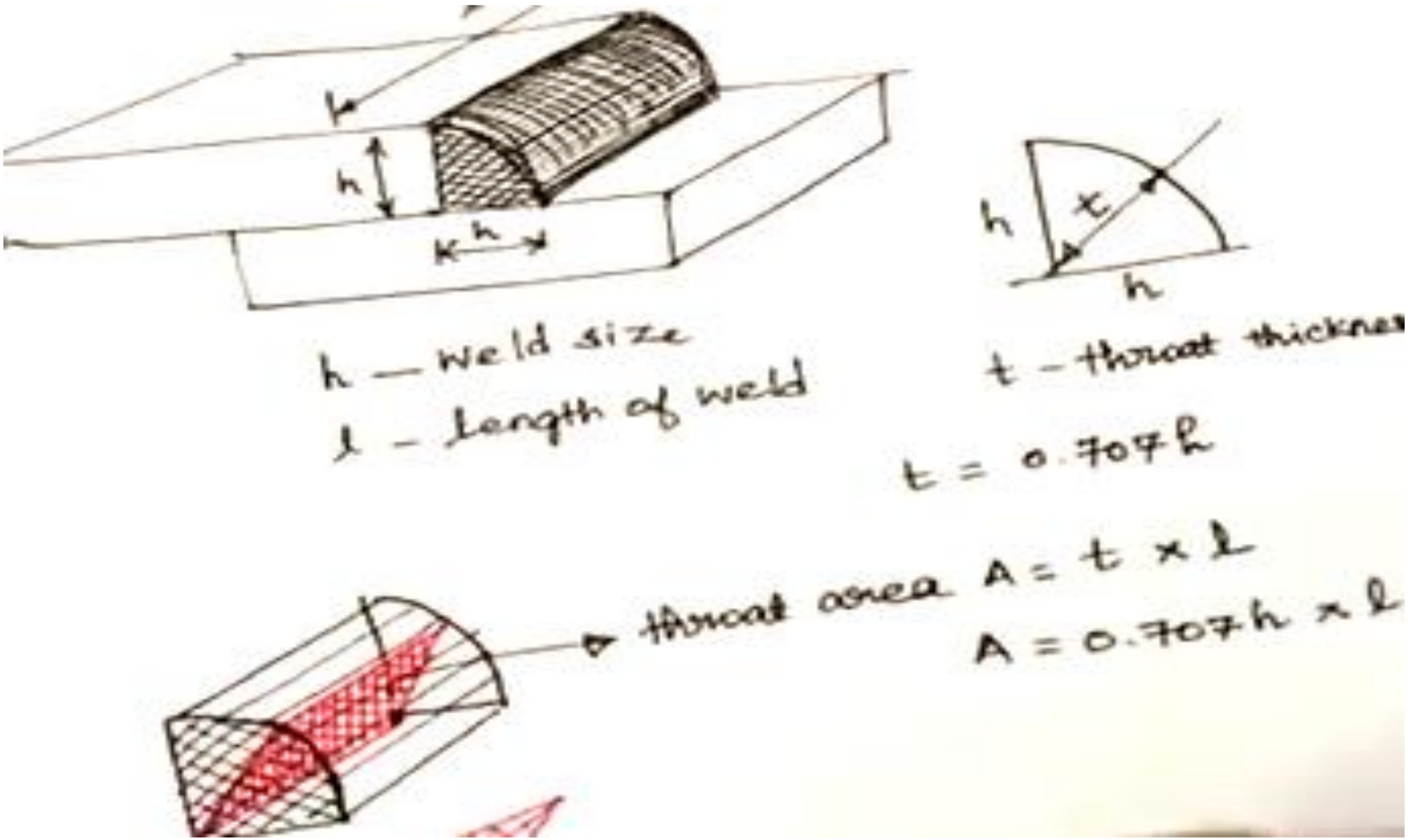


(a) Double parallel fillet weld.



(b) Combination of transverse and parallel fillet weld.

Single transverse fillet welds



Pbm: 1 A plate 100 mm wide and 10 mm thick is to be welded to another plate by means of double parallel fillets. The plates are subjected to a static load of 80 kN. Find the length of weld if the permissible shear stress in the weld does not exceed 55 MPa.

Given data:

*Width = 100 mm ;

Thickness = 10 mm ;

$P = 80\ kN = 80 \times 103\ N$;

τ = 55 MPa = 55 N/mm2

Solution

Let l =*Length of weld, and*

s = Size of weld = Plate thickness = 10 mm

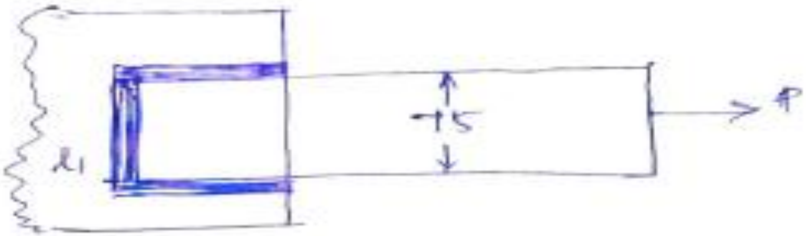
We know that maximum load which the plates can carry for double parallel fillet weld (P).

$$80 \times 10^3 = 1.414 \times s \times l \times \tau = 1.414 \times 10 \times l \times 55 = 778\, l$$

$$\therefore \qquad l = 80 \times 10^3 / 778 = 103 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l = 103 + 12.5 = 115.5 \text{ mm} \quad \textbf{Ans.}$$



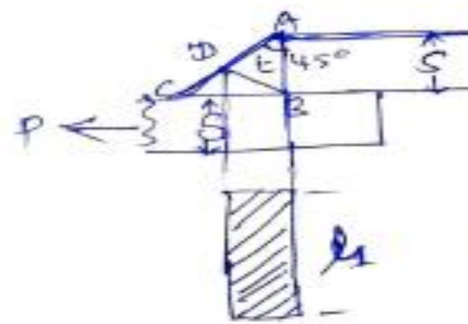
length
$l_1 = w - t$

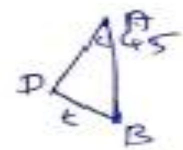
Total load : $\sigma_t \times A$ ——— Area of plate

$P = \sigma_t \times w \times t$

Load carried by single transverse load :-

$P_1 = A \times \sigma_1$





where :-
A – minimum Area of transfer fillet

$P_1 = A \times \sigma_1$
$= 0.707 S \times l_1 \times \sigma_1$

$P_1 =$ ______

$\sin 45^\circ = \frac{t}{S}$

$t = 0.707 S$

$A = l \times t$ – minimum Area throat

Load carried by Double parallel fillet :-

$P_2 = A \times \tau$

$P_2 = 1.414 S \times l_2 \times \tau$

Considering Fatigue loading :-

(i) Stress concentration factor for:

transvers fillet weld = 1.5

Parallel " " = 2.7

Tensile stress for fatigue loading :-

$$\sigma_{t_1} = \frac{\sigma_t}{1.5} =$$

$\parallel^{ly}$ Shear stress for fatigue loading.

$$\tau_1 = \frac{\tau}{2.7}$$

Load on transver fillet weld because of fatigue load:-

$$P_1 = A \times \sigma_{t_1}$$

lly Load on parallel fillet weld because of fatigue load:

$$P_2 = A \times \tau_1$$

$$P_2 = l_2 \text{ (in the form)}$$

∴ Total load carried by weld:

$$P = P_1 + P_2$$

from this we can get l_2 ⇒

Consider for starting and stopping of weld:

pbm:2 A plate 75 mm wide and 12.5 mm thick is joined with another plate by a single transverse weld and a double parallel fillet weld as shown in Fig. 10.15. The maximum tensile and shear stresses are 70 MPa and 56 MPa respectively. Find the length of each parallel fillet weld, if the joint is subjected to both static and fatigue loading

Given data :

Width = 75 mm ;

Thickness = 12.5 mm ;

$\sigma\tau$ = 70 MPa = 70 N/mm2 ;

τ = 56 MPa = 56 N/mm2.

Solution:

$l_1 = 75 – 12.5 = 62.5$ mm

the maximum load which the plate can carry is

P = Area × Stress = 75 × 12.5 × 70 = 65 625 N

Load carried by single transverse weld,

$$P_1 = 0.707\, s \times l_1 \times \sigma_t = 0.707 \times 12.5 \times 62.5 \times 70 = 38\,664 \text{ N}$$

and the load carried by double parallel fillet weld,

$$P_2 = 1.414\, s \times l_2 \times \tau = 1.414 \times 12.5 \times l_2 \times 56 = 990\, l_2 \text{ N}$$

$\therefore$ Load carried by the joint (P),

$$65\,625 = P_1 + P_2 = 38\,664 + 990\, l_2 \quad \text{or} \quad l_2 = 27.2 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l_2 = 27.2 + 12.5 = 39.7 \text{ say } 40 \text{ mm}$$ **Ans.**

Length of each parallel fillet for fatigue loading

From Table 10.6, we find that the stress concentration factor for transverse welds is 1.5 and for parallel fillet welds is 2.7.

$\therefore$ Permissible tensile stress,

$$\sigma_t = 70 / 1.5 = 46.7 \text{ N/mm}^2$$

and permissible shear stress,

$$\tau = 56 / 2.7 = 20.74 \text{ N/mm}^2$$

Load carried by single transverse weld,

$$P_1 = 0.707\, s \times l_1 \times \sigma_t = 0.707 \times 12.5 \times 62.5 \times 46.7 = 25\,795 \text{ N}$$

and load carried by double parallel fillet weld,

$$P_2 = 1.414\, s \times l_2 \times \tau = 1.414 \times 12.5\, l_2 \times 20.74 = 366\, l_2 \text{ N}$$

$\therefore$ Load carried by the joint (P),

$$65\,625 = P_1 + P_2 = 25\,795 + 366\, l_2 \quad \text{or} \quad l_2 = 108.8 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l_2 = 108.8 + 12.5 = 121.3 \text{ mm}$$ **Ans.**

Example 10.6. *Determine the length of the weld run for a plate of size 120 mm wide and 15 mm thick to be welded to another plate by means of*

1. *A single transverse weld; and*

2. *Double parallel fillet welds when the joint is subjected to variable loads.*

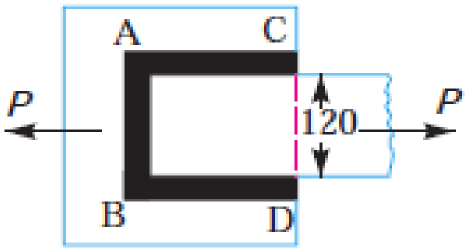


Fig. 10.16

Solution. Given : Width = 120 mm ; Thickness = 15 mm

In Fig. 10.16, *AB* represents the single transverse weld and *AC* and *BD* represents double parallel fillet welds.

1. *Length of the weld run for a single transverse weld*

The effective length of the weld run (l_1) for a single transverse weld may be obtained by subtracting 12.5 mm from the width of the plate.

$$\therefore \quad l_1 = 120 - 12.5 = 107.5 \text{ mm} \quad \textbf{Ans.}$$

2. *Length of the weld run for a double parallel fillet weld subjected to variable loads*

Let l_2 = Length of weld run for each parallel fillet, and

s = Size of weld = Thickness of plate = 15 mm

Assuming the tensile stress as 70 MPa or N/mm² and shear stress as 56 MPa or N/mm² for static loading. We know that the maximum load which the plate can carry is

$$P = \text{Area} \times \text{Stress} = 120 \times 15 \times 70 = 126 \times 10^3 \text{ N}$$

From Table 10.6, we find that the stress concentration factor for transverse weld is 1.5 and for parallel fillet welds is 2.7.

∴ Permissible tensile stress,

$$\sigma_t = 70 / 1.5 = 46.7 \text{ N/mm}^2$$

and permissible shear stress,

$$\tau = 56 / 2.7 = 20.74 \text{ N/mm}^2$$

$\therefore$ Load carried by single transverse weld,

$$P_1 = 0.707 s \times l_1 \times \sigma_t = 0.707 \times 15 \times 107.5 \times 46.7 = 53\,240 \text{ N}$$

and load carried by double parallel fillet weld,

$$P_2 = 1.414 s \times l_2 \times \tau = 1.414 \times 15 \times l_2 \times 20.74 = 440\, l_2 \text{ N}$$

$\therefore$ Load carried by the joint (P),

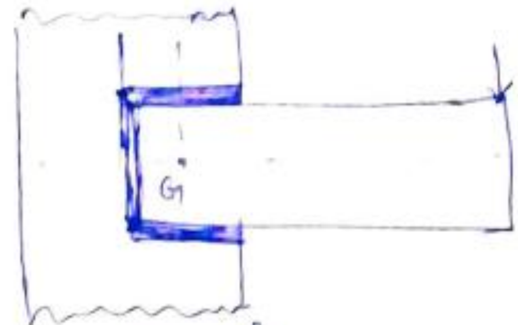
$$126 \times 10^3 = P_1 + P_2 = 53\,240 + 440\, l_2 \quad \text{or} \quad l_2 = 165.4 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l_2 = 165.4 + 12.5 = 177.9 \text{ say } 178 \text{ mm}$$ **Ans.**

Eccentric loading

Welding



direct shear $= \frac{F}{A}$

$\tau_2 = h_1$

$\tau_1 = h_1 \Rightarrow$

$\tau_1 = \frac{P}{A} \Rightarrow \quad \tau_2 = \frac{M_t \times r}{J} \Rightarrow \frac{P \times e \times r}{J} \Rightarrow P = 11.5$

Resultant $\Rightarrow \tau = \sqrt{\tau_1^2 + \tau^2 + 2\tau_1\tau_2 \cos\theta}$

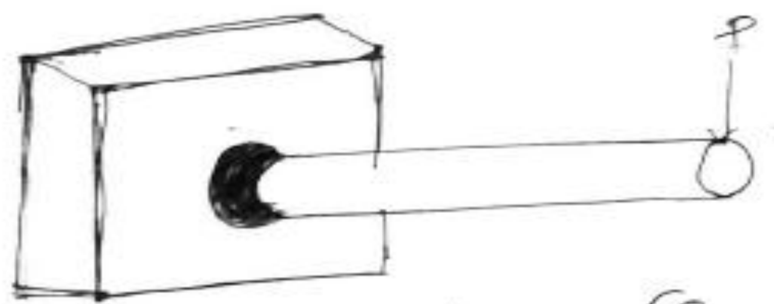
h

Given $= 140 \Rightarrow 20.22$

$\bar{x}y = CG = \frac{b^2}{2b+d}$

$J = \left[\frac{(2b \times d)^3}{12} - \frac{b^2(b+d)^2}{2b \times d}\right] \times t$

Eccentric loading different plan:-



$A = \pi D \times t$

$0.707t$

Direct Stress:-

$\tau = \frac{P}{A} = \tau = \text{?}$

$\sigma = \frac{M_b \times y}{I} = \frac{M_b}{Z} = \frac{P \times e}{Z} \longrightarrow 115$ $\quad Z = \frac{y}{I}$

$\sigma = \text{?}$ $\quad Z = \left[\frac{\pi d^2}{4}\right] \times t$

given $\rightarrow \sigma_t = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{\sigma_b^2 + 4\tau^2}$

Example 10.9. *A welded joint as shown in Fig. 10.24, is subjected to an eccentric load of 2 kN. Find the size of weld, if the maximum shear stress in the weld is 25 MPa.*

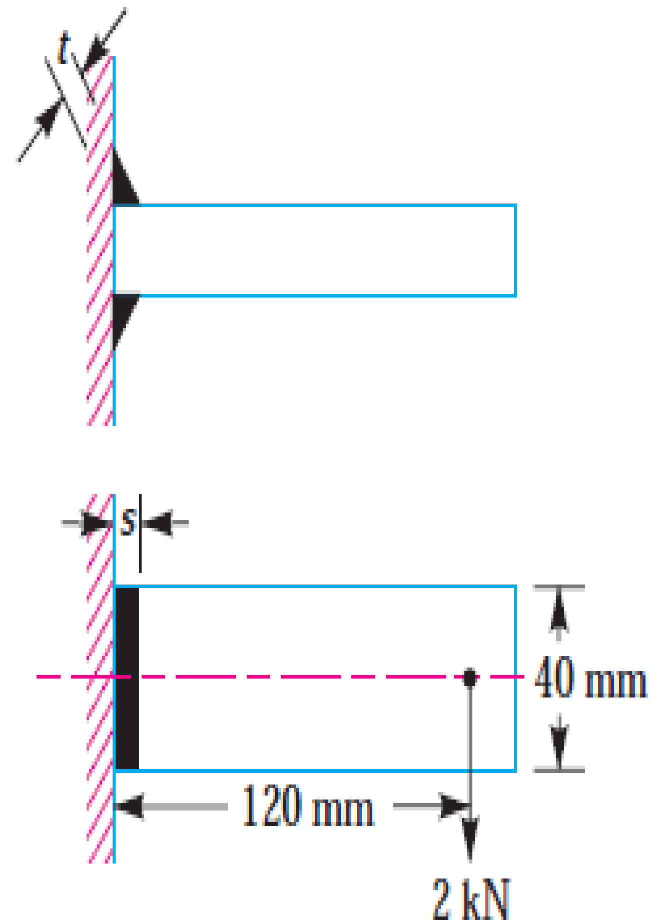


Fig. 10.24

Solution. Given: $P = 2\text{kN} = 2000$ N ; $e = 120$ mm ; $l = 40$ mm ; $\tau_{max} = 25$ MPa $= 25$ N/mm²

Let s = Size of weld in mm, and

t = Throat thickness.

The joint, as shown in Fig. 10.24, will be subjected to direct shear stress due to the shear force, $P = 2000$ N and bending stress due to the bending moment of $P \times e$.

We know that area at the throat,

$$A = 2t \times l = 2 \times 0.707\, s \times l$$

$$= 1.414\, s \times l$$

$$= 1.414\, s \times 40 = 56.56 \times s \text{ mm}^2$$

$\therefore$ Shear stress, $\tau = \dfrac{P}{A} = \dfrac{2000}{56.56 \times s} = \dfrac{35.4}{s}$ N/mm^2

Bending moment, $M = P \times e = 2000 \times 120 = 240 \times 10^3$ N-mm

Section modulus of the weld through the throat,

$$Z = \frac{s \times l^2}{4.242} = \frac{s\,(40)^2}{4.242} = 377 \times s \text{ mm}^3$$

$\therefore$ Bending stress, $\sigma_b = \dfrac{M}{Z} = \dfrac{240 \times 10^3}{377 \times s} = \dfrac{636.6}{s}$ N/mm^2

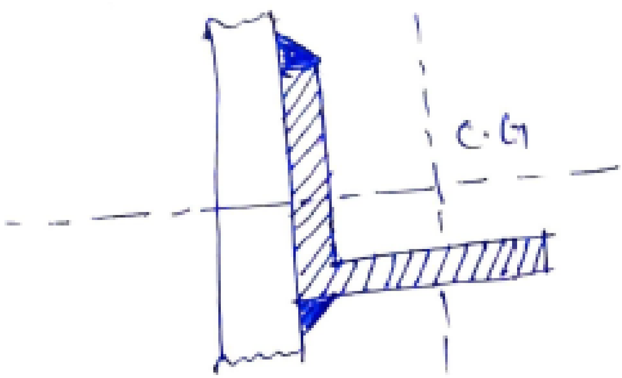
We know that maximum shear stress (τ_{max}),

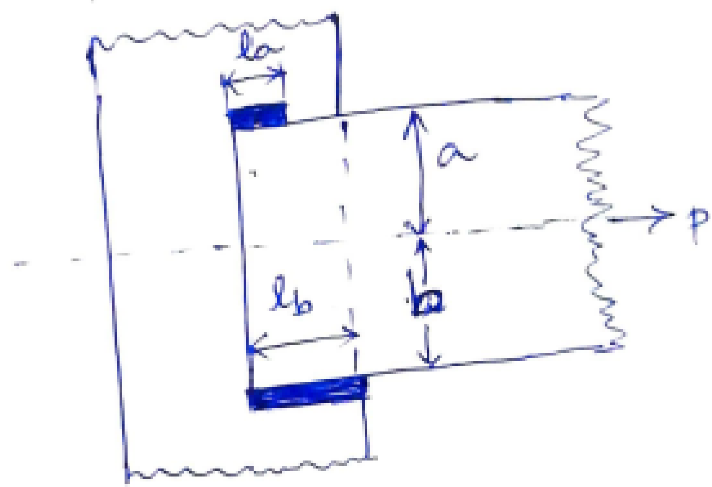
$$25 = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{\left(\frac{636.6}{s}\right)^2 + 4\left(\frac{35.4}{s}\right)^2} = \frac{320.3}{s}$$

$\therefore$ $s = 320.3 / 25 = 12.8$ mm **Ans.**

Design of axially loaded Unsymmetric weld:-

Sometimes Unsymmetrical Sections such as angles Channels T-Sections etc.





Moment of the top weld about gravity axis:-

$= l_a \times f \times a$

Bottom weld about gravity:-

$= l_b \times f \times b$

Sum of the moments of weld about the gravity axis must be zero.

$$l_a \times f \times a - l_b \times f \times b = 0$$

$$l_a \times a = l_b \times b \quad \text{--- (1)}$$

WKT

$$l = l_a + l_b \quad \text{--- (2)} \quad \text{(or)} \quad l = l_1 + l_2$$

From equation (1) & (2)

$$\therefore \quad l = l_1 + l_2$$

$$= l_1 + \frac{l_1 \times a}{b}$$

$$= \frac{bl_1 + l_1 \times a}{b} = \frac{l_1\,(a+b)}{b}$$

$$\therefore \quad l_1 = \frac{b \times l}{a+b}$$

Substituting the value of l_1 in equation (ii), we get

$$l = \frac{b \times l}{a+b} + l_2 = \frac{b \times l + l_2\,(a+b)}{(a+b)}$$

$$l(a+b) = b \times l + l_2(a+b)$$

$$l \times a + l \times b = b \times l + l_2\,(a+b)$$

$$l \times a + l \times b - b \times l = l_2(a+b)$$

$$l \times a = l_2(a+b)$$

$$l_2 = \frac{a \times l}{(a+b)}$$

Example 10.8. *A 200 × 150 × 10 mm angle is to be welded to a steel plate by fillet welds as shown in Fig. 10.21. If the angle is subjected to a static load of 200 kN, find the length of weld at the top and bottom. The allowable shear stress for static loading may be taken as 75 MPa.*

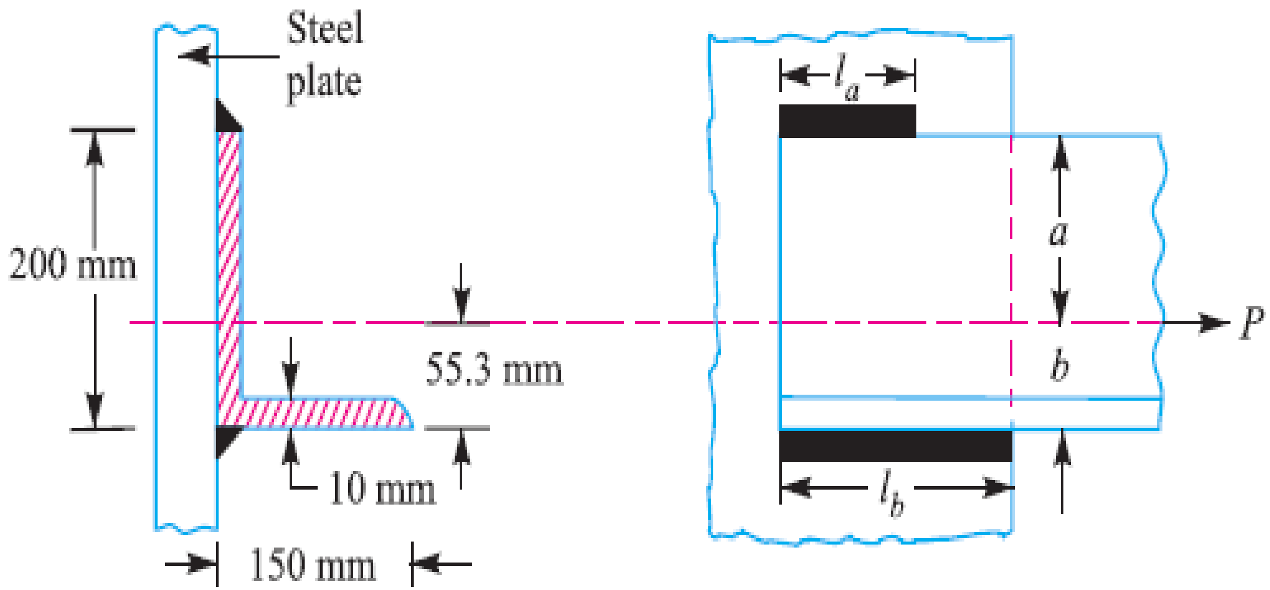


Fig. 10.21

Solution. Given : $a + b = 200$ mm ; $P = 200$ kN $= 200 \times 10^3$ N ; $\tau = 75$ MPa $= 75$ N/mm^2

Let l_a = Length of weld at the top,

l_b = Length of weld at the bottom, and

l = Total length of the weld $= l_a + l_b$

Since the thickness of the angle is 10 mm, therefore size of weld,

$$s = 10 \text{ mm}$$

We know that for a single parallel fillet weld, the maximum load (P),

$$200 \times 10^3 = 0.707\, s \times l \times \tau = 0.707 \times 10 \times l \times 75 = 530.25\, l$$

$$\therefore \quad l = 200 \times 10^3 / 530.25 = 377 \text{ mm}$$

or $$l_a + l_b = 377 \text{ mm}$$

Now let us find out the position of the centroidal axis.

Let b = Distance of centroidal axis from the bottom of the angle.

$$\therefore \quad b = \frac{(200 - 10)\, 10 \times 95 + 150 \times 10 \times 5}{190 \times 10 + 150 \times 10} = 55.3 \text{ mm}$$

and $$a = 200 - 55.3 = 144.7 \text{ mm}$$

We know that $$l_a = \frac{l \times b}{a + b} = \frac{377 \times 55.3}{200} = 104.2 \text{ mm}$$ **Ans.**

and $$l_b = l - l_a = 377 - 104.2 = 272.8 \text{ mm}$$ **Ans.**

INTRODUCTION

- Mechanical joints or fasteners are used for making connections between different elements of machine or structure.
- A machine or a structure is made of a large number of parts and they need be joined suitably for the machine to operate satisfactorily.
- In manufacturing industries, joining of two or more components is necessary for assembly purposes.
- Joining makes the production system more reliable, efficient and profitable. In fact, joining can be defined as one of the manufacturing processes by which two or more solid components can be assembled together.

Types of joints

Mechanical joints are broadly classified into following two categories:

1. Permanent joints

- Permanent joints can not be easily disassembled without damaging the connecting

elements.

- Different types of permanent joints are welded joints, brazed joints, soldered

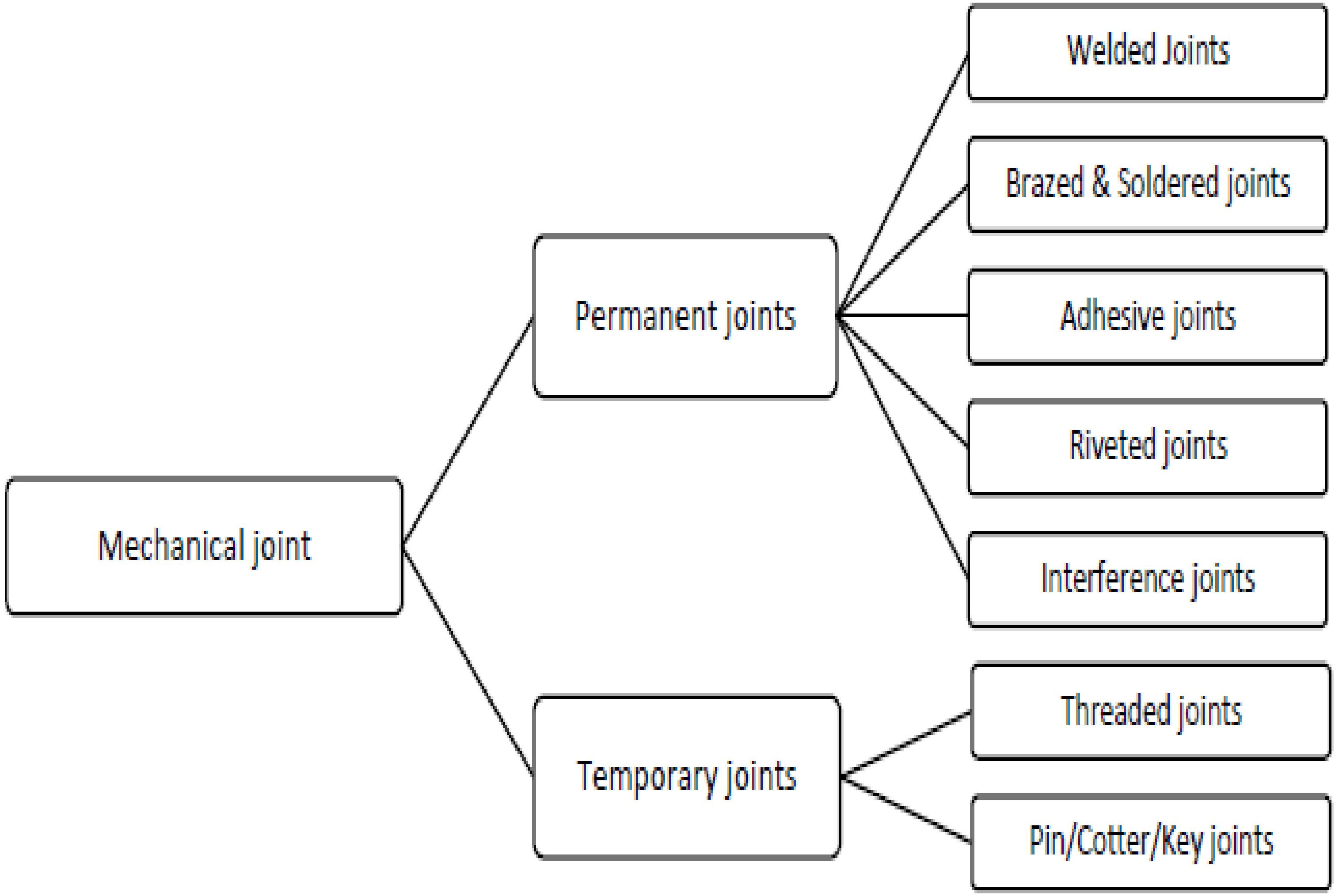
joints, adhesive joints, riveted joints and interference joints.

2. Temporary or detachable joints

- Temporary joints can be easily disassembled without damaging the connecting

elements.

- Different types of joints are threaded joints, pin joints, cotter joints and key joints.

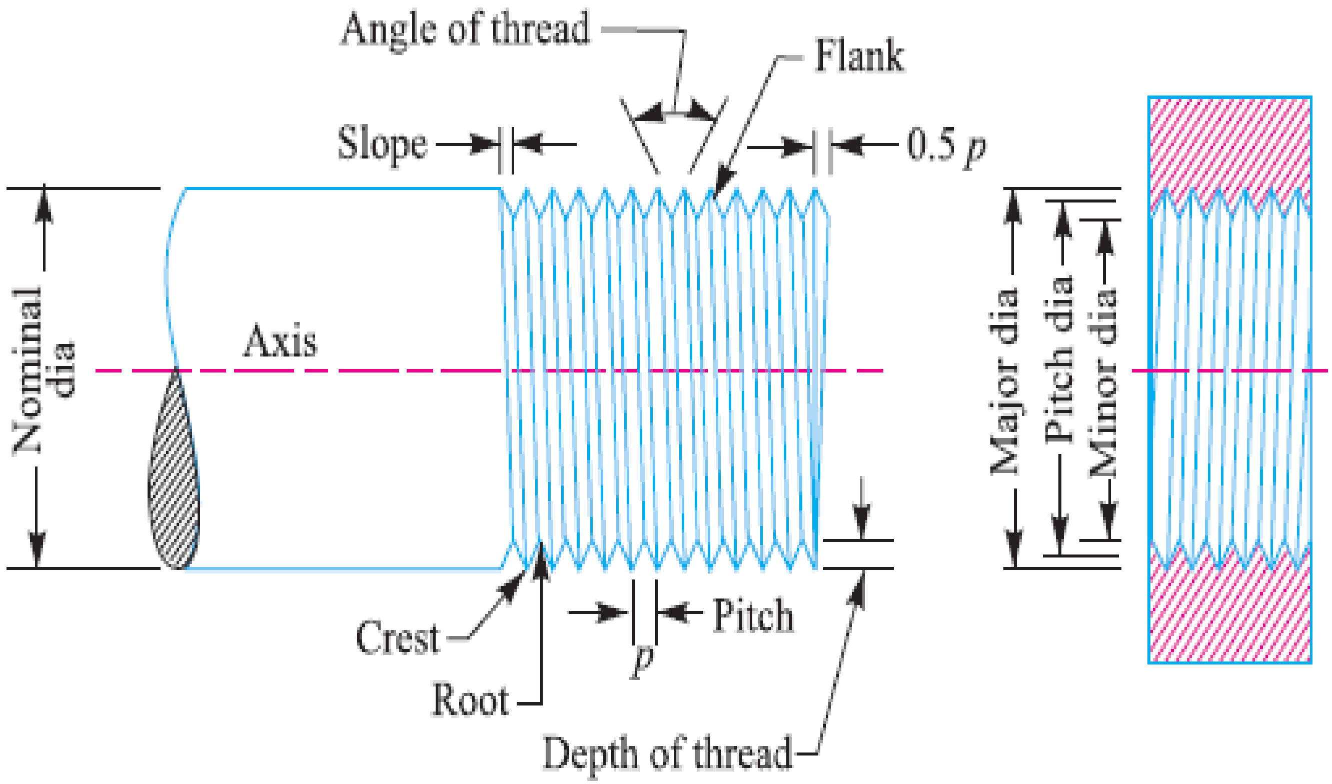

Mechanical joint
Permanent joints
Temporary joints
Welded Joints
Brazed & Soldered joints
Adhesive joints
Riveted joints
Interference joints
Threaded joints
Pin/Cotter/Key joints

Introduction

- A screw thread is formed by cutting a continuous helical groove on a cylindrical surface. A screw made by cutting a single helical groove on the cylinder is known as single threaded
- A screwed joint is mainly composed of two elements i.e. a bolt and nut. The screwed joints are widely used where the machine parts are required to be readily connected or disconnected without damage to the machine or the fastening.

Important Terms Used in Screw Threads

The following terms used in screw threads, as shown in Fig are important from the subject
point of view :



1. *Major diameter*. It is the largest diameter of an external or internal screw thread. The screw is specified by this diameter. It is also known as ***outside*** or ***nominal diameter***.

2. *Minor diameter*. It is the smallest diameter of an external or internal screw thread. It is also known as ***core*** or ***root diameter***.

3. *Pitch diameter*. It is the diameter of an imaginary cylinder, on a cylindrical screw thread, the surface of which would pass through the thread at such points as to make equal the width of the thread and the width of the spaces between the threads. It is also called an ***effective diameter***. In a nut and bolt assembly, it is the diameter at which the ridges on the bolt are in complete touch with the ridges of the corresponding nut.

4. *Pitch*. It is the distance from a point on one thread to the corresponding point on the next. This is measured in an axial direction between corresponding points in the same axial plane. Mathematically,

$$\text{Pitch} = \frac{1}{\text{No. of threads per unit length of screw}}$$

5. *Lead*. It is the distance between two corresponding points on the same helix. It may also be defined as the distance which a screw thread advances axially in one rotation of the nut. Lead is equal to the pitch in case of single start threads, it is twice the pitch in double start, thrice the pitch in triple start and so on.

6. *Crest*. It is the top surface of the thread.

7. *Root*. It is the bottom surface created by the two adjacent flanks of the thread.

8. *Depth of thread*. It is the perpendicular distance between the crest and root.

9. *Flank*. It is the surface joining the crest and root.

10. *Angle of thread*. It is the angle included by the flanks of the thread.

11. *Slope*. It is half the pitch of the thread.

Stresses in Screwed Fastening due to Static Loading

- The following stresses in screwed fastening due to static loading are important from the subject
- point of view :
- **1. Internal stresses due to screwing up forces,**
- **2. Stresses due to external forces, and**
- **3. Stress due to combination of stresses at (1) and (2).**

Initial Stresses due to Screwing up Forces

1. *Tensile stress due to stretching of bolt.* Since none of the above mentioned stresses are accurately determined, therefore bolts are designed on the basis of direct tensile stress with a large factor of safety in order to account for the indeterminate stresses. The initial tension in a bolt, based on experiments, may be found by the relation

$$P_i = 2840\, d \text{ N}$$

where P_i = Initial tension in a bolt, and

d = Nominal diameter of bolt, in mm.

The above relation is used for making a joint fluid tight like steam engine cylinder cover joints etc. When the joint is not required as tight as fluid-tight joint, then the initial tension in a bolt may be reduced to half of the above value. In such cases

$$P_i = 1420\, d \text{ N}$$

The small diameter bolts may fail during tightening, therefore bolts of smaller diameter (less than M 16 or M 18) are not permitted in making fluid tight joints.

If the bolt is not initially stressed, then the maximum safe axial load which may be applied to it, is given by

P = Permissible stress × Cross-sectional area at bottom of the thread (*i.e.* stress area)

The stress area may be obtained from Table 11.1 or it may be found by using the relation

$$\text{Stress area} = \frac{\pi}{4}\left(\frac{d_p + d_c}{2}\right)^2$$

where d_p = Pitch diameter, and

d_c = Core or minor diameter.

2. ***Torsional shear stress caused by the frictional resistance of the threads during its tightening.*** The torsional shear stress caused by the frictional resistance of the threads during its tightening may be obtained by using the torsion equation. We know that

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\therefore \qquad \tau = \frac{T}{J} \times r = \frac{T}{\frac{\pi}{32}(d_c)^4} \times \frac{d_c}{2} = \frac{16\,T}{\pi\,(d_c)^3}$$

where τ = Torsional shear stress,

T = Torque applied, and

d_c = Minor or core diameter of the thread.

It has been shown during experiments that due to repeated unscrewing and tightening of the nut, there is a gradual scoring of the threads, which increases the torsional twisting moment (T).

3. ***Shear stress across the threads.*** The average thread shearing stress for the screw (τ_s) is obtained by using the relation :

$$\tau_s = \frac{P}{\pi d_c \times b \times n}$$

where b = Width of the thread section at the root.

The average thread shearing stress for the nut is

$$\tau_n = \frac{P}{\pi d \times b \times n}$$

where d = Major diameter.

4. *Compression or crushing stress on threads.* The compression or crushing stress between the threads (σ_c) may be obtained by using the relation :

$$\sigma_c = \frac{P}{\pi\,[d^2 - (d_c)^2]\,n}$$

where d = Major diameter,

d_c = Minor diameter, and

n = Number of threads in engagement.

5. *Bending stress if the surfaces under the head or nut are not perfectly parallel to the bolt axis.* When the outside surfaces of the parts to be connected are not parallel to each other, then the bolt will be subjected to bending action. The bending stress (σ_b) induced in the shank of the bolt is given by

$$\sigma_b = \frac{x \,.\, E}{2l}$$

where x = Difference in height between the extreme corners of the nut or head,

l = Length of the shank of the bolt, and

E = Young's modulus for the material of the bolt.

Example 11.2. *Two machine parts are fastened together tightly by means of a 24 mm tap bolt. If the load tending to separate these parts is neglected, find the stress that is set up in the bolt by the initial tightening.*

Solution. Given : $d = 24$ mm

From Table 11.1 (coarse series), we find that the core diameter of the thread corresponding to M 24 is $d_c = 20.32$ mm.

Let σ_t = Stress set up in the bolt.

We know that initial tension in the bolt,

$$P = 2840\, d = 2840 \times 24 = 68\,160 \text{ N}$$

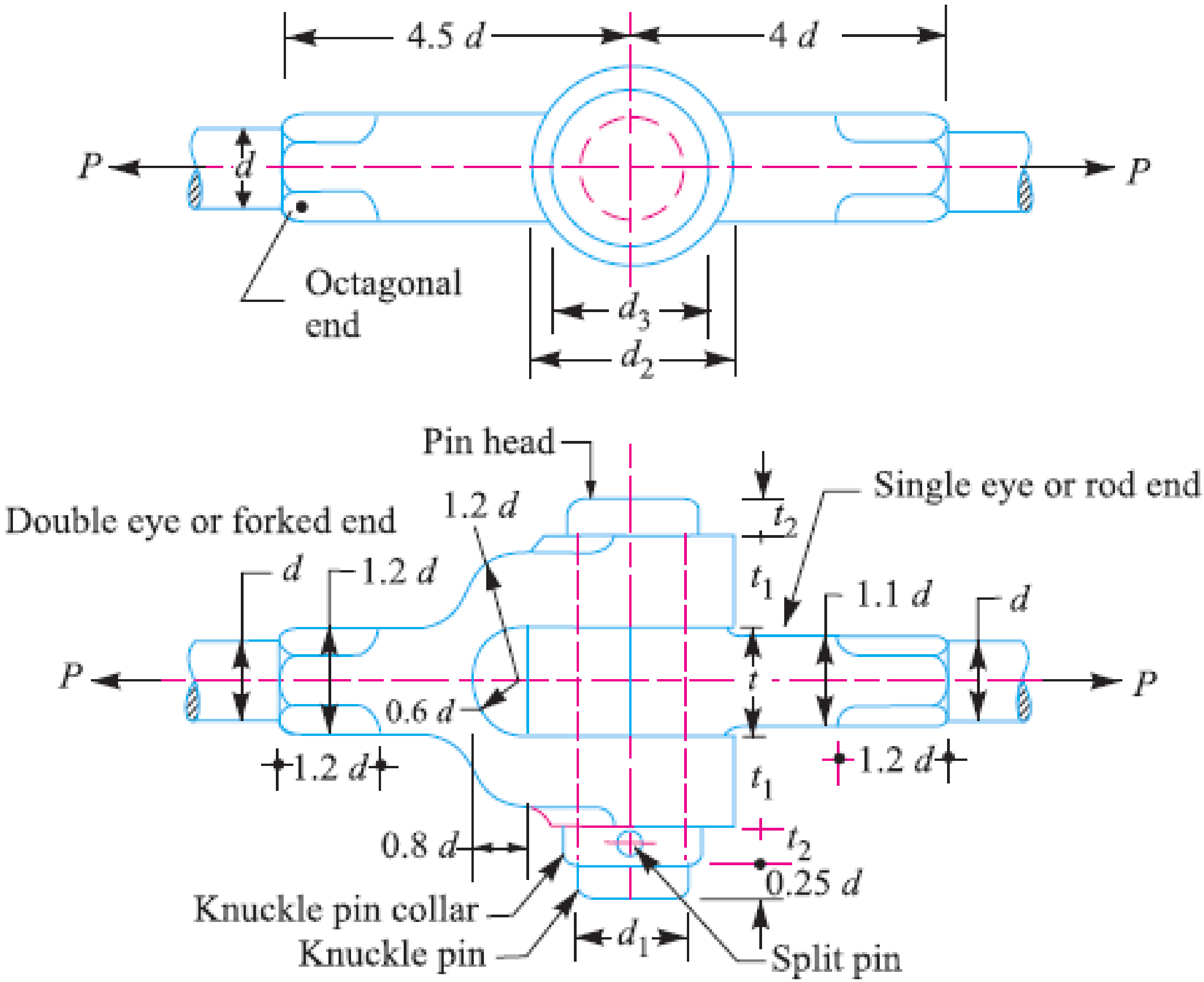
We also know that initial tension in the bolt (P),

$$68\,160 = \frac{\pi}{4} (d_c)^2\, \sigma_t = \frac{\pi}{4} (20.30)^2\, \sigma_t = 324\, \sigma_t$$

$\therefore$ $\sigma_t = 68\,160 / 324 = 210 \text{ N/mm}^2 = 210$ MPa **Ans.**

Knuckle Joint

- A knuckle joint is used to connect two rods which are under the action of tensile loads.
- However, if the joint is guided, the rods may support a compressive load. A knuckle joint may be readily disconnected for adjustments or repairs.


4.5 d
4 d
P
d
P
Octagonal end
d3
d2
Pin head
Single eye or rod end
1.2 d
t2
Double eye or forked end
d
1.2 d
t1
1.1 d
d
P
t
P
0.6 d
1.2 d
1.2 d
t1
0.8 d
t2
0.25 d
Knuckle pin collar
Knuckle pin
d1
Split pin

Dimensions of Various Parts Knuckle Joint

- The dimensions of various parts of the knuckle joint are fixed by empirical relations as given below.
- It may be noted that all the parts should be made of the same material i.e. mild steel or wrought iron.
- If d is the diameter of rod, then diameter of pin

$$d1 = d$$

- Outer diameter of eye,

$$d2 = 2\ d$$

Diameter of knuckle pin head and collar,

$d3 = 1.5\ d$

Thickness of single eye or rod end,

$t = 1.25\ d$

Thickness of fork, $t1 = 0.75\ d$

Thickness of pin head, $t2 = 0.5\ d$

Other dimensions of the joint are shown in Fig.

Methods of Failure of Knuckle Joint

In determining the strength of the joint for the various methods of failure, it is assumed that

1. There is no stress concentration, and

2. The load is uniformly distributed over each part of the joint.

1. *Failure of solid rod in tension:*

Due to the tensile load, the rods are subjected to tensile stresses, and may fail if this stress exceeds the limit. For a 'Just safe design', we have

$$P = \frac{\pi}{4}.d^2.\sigma_t$$

2. *Failure of knuckle pin in double shear:*

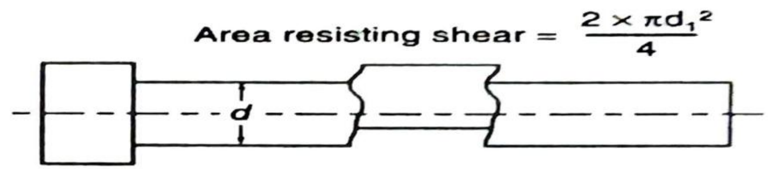


Figure 3.23

There are two surfaces of the pin that resist shearing shown in Figure 3.23. So, the pin is in double shear for which the equation is given by

$$P = 2 \times \frac{\pi}{4}.d_1^2.\tau$$

3. *Failure of the single eye or rod end in tension:*

The rod end may 'tear' along a plane passing through the pin axis shown in Figure 3.24. It gives a raise to a stress σ_t, given by

$$P = (d_2 - d_1)t.\sigma_t$$

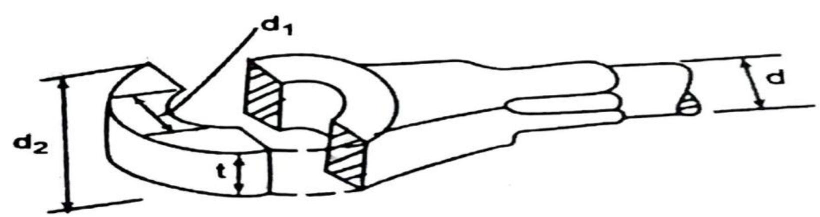


Figure 3.24

4. *Failure of single eye (or) rod end in double shear*

The eye end will get sheared off shown in Figure 3.25. The resisting stress is given by

$$P = 2\,\frac{(d_2 - d_1)\,t}{2}.\tau = (d_2 - d_1)\,t.\tau$$

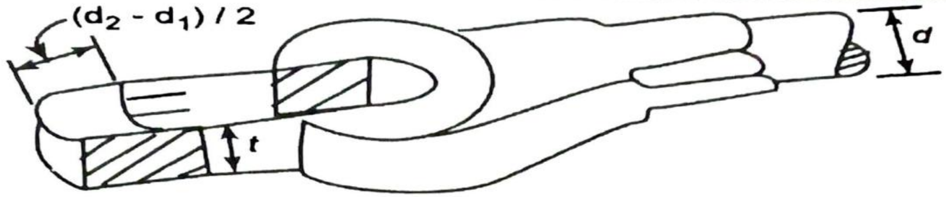


Figure 3.25

5. *Failure of single eye or rod end in crushing:*

Due to the tensile load, the eye end may be crushed to failure. The area resisting crushing is taken as the projected area and the corresponding stress is related to the tensile load given by

$$P = d_1\, t\, \sigma_c$$

where σ_c is allowable stress in crushing

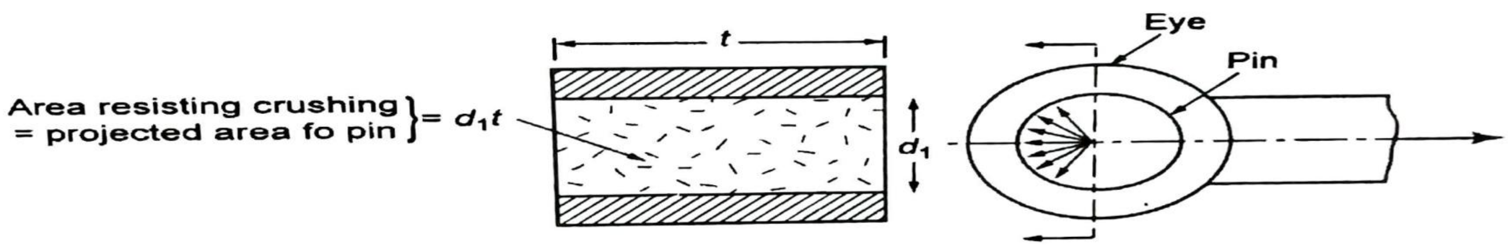


Figure 3.26

6. *Failure of forked end in tension*

The fork end may fail in tension across the pin hole shown in Figure 3.27. The relation is given by

$$P = (d_2 - d_1) t_1 \times 2 \times \sigma_t$$

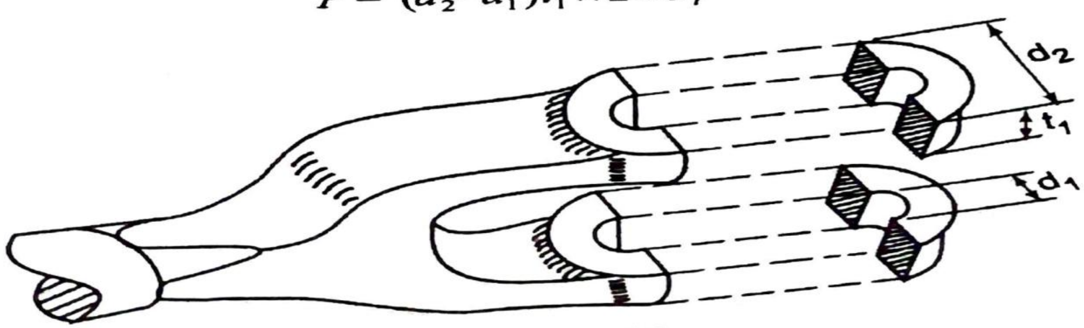


Figure 3.27

7. Failure of forked ends in double shear (as shear of eye) (Figure 3.27)

The fork end may fail by shear with the relation being

$$P = (d_2 - d_1) t \times 2 \times \tau$$

8. Failure of forked end in crushing (As crushing of eye) (Figure 3.27)

If the forked end is weak in crushing than in other modes, it will result in crushing failure. In that case, the equation is given by

$$P = d_1 \, t_1 \times 2 \times \sigma_c$$

9. Failure of knuckle pin by bending

A loosely fitted knuckle pin experiences bending. The load on the pin uniformly varies over the pin area in the fork. Maximum bending is at the mid span of the pin. The bending stress is given by

$$\sigma_b = \frac{\frac{P}{2}\left(\frac{t_1}{3} + \frac{t}{4}\right)}{\frac{\pi d_1^3}{32}}$$

3.5.11. Design Of Riveted Joint For Pressure Vessels

In a pressure vessel i.e., Boiler of cylindrical shape, there are two types of joints.

(i) Longitudinal joint:

It is used to join the ends of the plate to get the required diameter of a boiler, shown in Figure 3.82. Generally, the double-strap butt joint is used for this purpose.

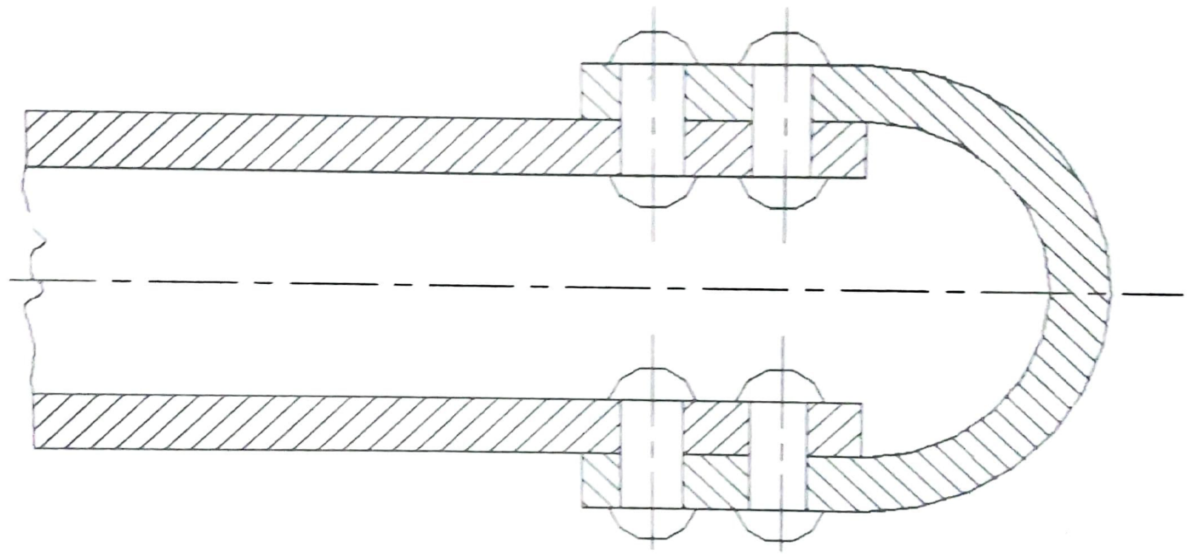


Figure 3.82 Longitudinal joint

(ii) Circumferential joint:

It is used to get the required length of the shell and to close its ends. A lap joint is most widely used for this purpose. Refer Figure 3.83.

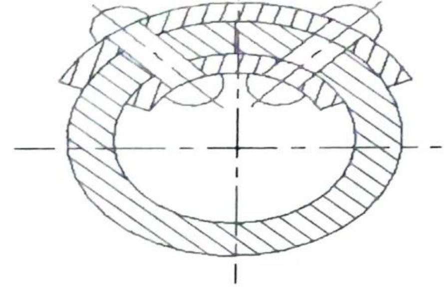


Figure 3.83 Circumferential joint

3.5.12. Design Of Longitudinal Butt Joint For Pressure Vessels

According to I.B.R the following procedure should be adopted for design of longitudinal butt joint for a pressure vessel.

1. Select a suitable butt joint according to the diameter of the boiler shell and desired efficiency from table 3.5.3.

Table 3.5.3

Diameter of shell, D (*mm*)	Thickness of shell (*mm*)	Type of joint	Efficiency (%)
600 – 1200	6.25 to 12.5	Double riveted butt joint	72 – 82
900 – 2000	7.5 to 25	Triple riveted butt joint	80 – 90
1500 – 2500	9 to 31.25	Quadruple riveted butt joint	85 - 95

2. The thickness of the boiler shell plate is determined by the following formula

$$t = \frac{pD}{2\sigma_t \eta_l} + 1 \text{ to } 2\ mm \text{ as corrosion allowable}$$

where

p = stream pressure, N/mm^2.

D = diameter of the shell, mm.

σ_t = Allowable tensile strength of the material, N/mm^2.

η_l = Efficiency of the joint (refer table 3.5.3).

The thickness of the boiler shell should not be less than 7 *mm*.

3. Calculate diameter of the rivet by the *Unwin's formula*

$d = 6.05 \sqrt{t}$ -------- when $t > 8$ *mm*.

If $t < 8$ *mm*, then the diameter of the rivet may be calculated by equating the shearing and crushing strength of the rivet. The diameter of rivet should not be less than the thickness of the plate.

The pitch of the riveted joint is determined by equating the shear strength to the tearing strength of the joint. It may be noted that

For leak proof joint --- $p_{max} \leq 6\ d$

$p_{min} \geq (2.25 \text{ to } 2.5)\ d$

According to I.B.R.

$p_{max} = C\ t + 41.$

Where

C = constant. It may be selected from Table 3.5.4.

t = thickness of the shell plate

Table 3.5.4

Number of rivets per pitch	***Lap joint***	***Butt joint***	
		Single cover	***Double cover***
1	1.31	1.53	1.75
2	2.62	3.06	3.5
3	3.47	4.05	4.63
4	4.17	-	5.52
5	-	-	6.0

4. The transverse or back pitch may be selected from the equation given under topic 3.5.9.

5. Thickness of the cover plate for butt joint can be calculated from the relations given under the topic 3.5.8.

6. The margin or marginal pitch, $m = 1.5\ d$.

3.5.13. Design Of Circumferential Joint For Pressure Vessels

The multi row lap joint is commonly used for circumferential joint of the boiler. According to I.B.R, the following steps should be followed for the design of circumferential joint.

The diameter of the rivet and thickness of the plate (t) should be same as that of longitudinal joint. Refer step 2 and step 3 of longitudinal joint.

1. Total number of rivets required for the circumferential joint,

$$i = \left(\frac{D}{d}\right)^2 \frac{p}{\tau}$$

where D and d = diameter of the boiler shell and rivet.

p = steam pressure.

τ = Permissible shear strength of the rivet.

2. The pitch of the circumferential joint is calculated from the tearing efficiency of the joint.

$$\eta_c = \frac{p-d}{p}$$

The value of η_c is given in table 3.5.5 for various joints.

Table 3.5.5

Type of joint	η_c
Lap joint	
- Single riveted	0.45 to 0.65
- Double riveted	0.63 to 0.77
- Triple riveted	0.75 to 0.85

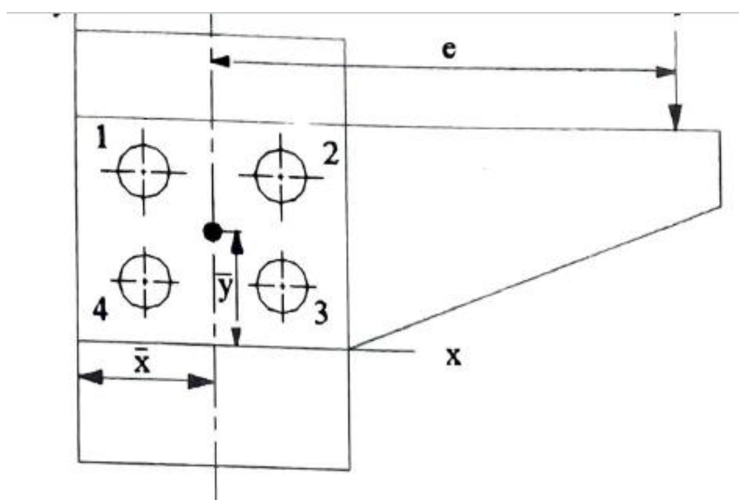
3. Number of rows required can be calculated by

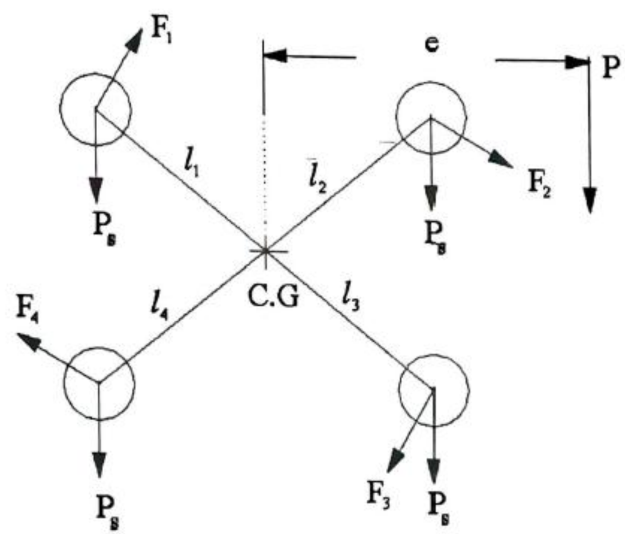
$$\text{No. of row, } r_n = \frac{ip}{\pi(D+t)}$$

4. The transverse pitch is selected in a similar way as in longitudinal joint. Refer topic 3.5.9.

Eccentrically Loaded Riveted Joints

In the previous cases of riveted joints, it is designed with the assumption that all rivets of the joints are subjected to shear force i.e., line of action of force passes through the centre of gravity. But in some practical cases of structural joints, the line of action of force does not pass through the centre of gravity. It is offset by a distance '*e*' shown in Figure 3.80 and Figure 3.81. Such type of joint is called *eccentrically loaded riveted joint*.





1. Determine centre of gravity of the joint by taking moment about OX and OY

$$\bar{X} = \frac{x_1 + x_2 + x_3 + \ldots\ldots x_n}{n}$$

$$\bar{Y} = \frac{y_1 + y_2 + y_3 + \ldots\ldots y_n}{n}$$

2. The eccentric load will produce two types of stresses on the joint.
   (i) Direct shear stress due to external for P
   (ii) Moment $P \times e$ which rises to rotate the joint about $C.G.$

3. Primary or Direct shear force, $P_s = \frac{P}{n}$ .... (3.32) acting parallel to the load P.

4. The secondary shear force on each rivet can be determined from the fact that the amount of shear force shared by each rivet is proportional to the radial distance between the rivet hole centers.

Let, F_1, F_2, ... F_n are the secondary loads on rivets 1, 2, ... n

l_1, l_2, l_3, ... l_n are the radial distance of rivets 1, 2, ... n from C.G.

Since the shear force is proportional to the radial distance between the rivet hole centers.

$F_1 \propto l_1$, $F_2 \propto l_2$ and so on.

$$\frac{F_1}{l_1} = \frac{F_2}{l_2} = \frac{F_3}{l_3} = \ldots\ldots = \frac{F_n}{l_n} = \text{Constant.}$$

$$F_2 = F_1 \frac{l_2}{l_1} \text{ and } F_3 = F_2 \frac{l_3}{l_1}$$

For equilibrium of the system the sum of the moments should be equal to zero. Thus

$$P \times e = F_1 l_1 + F_2 l_2 + F_3 l_3 + \ldots + F_n l_n$$

$$= F_1 l_1 + F_1 \frac{l_2}{l_1} \times l_2 + F_3 \frac{l_3}{l_1} \times l_3 + \ldots$$

$$P \times e = \frac{F_1}{l_1}\left(l_1^2 + l_2^2 + ------ + l_n^2\right) \qquad \ldots (3.33)$$

5. Calculate the resultant force on all rivets. The resultant force on the rivet is given by

$$R = \sqrt{p_s^2 + F^2 + 2P_s \times F \times Cos\theta}$$

where θ = Angle between the direct and the secondary shear force.

6. Determine the diameter of the rivet by considering the shear failure of the rivet

Maximum resultant shear load, $R = \frac{\pi}{4} d^2 \tau$

From Table 3.5.2, the standard diameter of the rivet hole (d) and rivet diameter may be selected.

Pbm:1 A double riveted lap joint with zig-zag riveting is to be designed for 13 mm thick plates. Assume $\sigma t = 80$ MPa ; $\tau = 60$ MPa ; and $\sigma c = 120$ Mpa State how the joint will fail and find the efficiency of the joint.

Given data :

$t = 13$ mm

$\sigma t = 80$ MPa = 80 N/mm2

$\tau = 60$ MPa = 60 N/mm2

$\sigma c = 120$ MPa = 120 N/mm2

1. *Diameter of rivet*

Since the thickness of plate is greater than 8 mm, therefore diameter of rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{13} = 21.6 \text{ mm}$$

From Table 9.3, we find that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard size of the rivet hole (d) is 23 mm and the corresponding diameter of the rivet is 22 mm. **Ans.**

2. *Pitch of rivets*

Let p = Pitch of the rivets.

Since the joint is a double riveted lap joint with zig-zag riveting [See Fig. 9.6 (c)], therefore there are two rivets per pitch length, *i.e.* $n = 2$. Also, in a lap joint, the rivets are in single shear.

We know that tearing resistance of the plate,

$$P_t = (p - d)t \times \sigma_t = (p - 23)\ 13 \times 80 = (p - 23)\ 1040 \text{ N} \quad ...(i)$$

and shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4}\ (23)^2\ 60 = 49\ 864 \text{ N} \quad ...(ii)$$

...($\because$ There are two rivets in single shear)

From equations (i) and (ii), we get

$$p - 23 = 49864 / 1040 = 48 \quad \text{or} \quad p = 48 + 23 = 71 \text{ mm}$$

The maximum pitch is given by,

$$p_{max} = C \times t + 41.28 \text{ mm}$$

From Table 9.5, we find that for 2 rivets per pitch length, the value of C is 2.62.

$$\therefore \quad p_{max} = 2.62 \times 13 + 41.28 = 75.28 \text{ mm}$$

Since p_{max} is more than p, therefore we shall adopt

$$p = 71 \text{ mm}$$ **Ans.**

3. *Distance between the rows of rivets*

We know that the distance between the rows of rivets (for zig-zag riveting),

$$p_b = 0.33\,p + 0.67\,d = 0.33 \times 71 + 0.67 \times 23 \text{ mm}$$

$$= 38.8 \text{ say } 40 \text{ mm} \quad \textbf{Ans.}$$

4. *Margin*

We know that the margin,

$$m = 1.5\,d = 1.5 \times 23 = 34.5 \text{ say } 35 \text{ mm} \quad \textbf{Ans.}$$

Failure of the joint

Now let us find the tearing resistance of the plate, shearing resistance and crushing resistance of the rivets.

We know that tearing resistance of the plate,

$$P_t = (p - d)\,t \times \sigma_t = (71 - 23)13 \times 80 = 49\,920 \text{ N}$$

Shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4}\,(23)^2\,60 = 49\,864 \text{ N}$$

and crushing resistance of the rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 23 \times 13 \times 120 = 71\,760 \text{ N}$$

The least of P_t, P_s and P_c is $P_s = 49\,864$ N. Hence the joint will fail due to shearing of the rivets. **Ans.**

Efficiency of the joint

We know that strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 71 \times 13 \times 80 = 73\,840 \text{ N}$$

$\therefore$ Efficiency of the joint,

$$\eta = \frac{P_s}{P} = \frac{49\,864}{73\,840} = 0.675 \text{ or } 67.5\% \quad \textbf{Ans.}$$

Example 9.6. *Two plates of 10 mm thickness each are to be joined by means of a single riveted double strap butt joint. Determine the rivet diameter, rivet pitch, strap thickness and efficiency of the joint. Take the working stresses in tension and shearing as 80 MPa and 60 MPa respectively.*

Solution. Given : $t = 10$ mm ; $\sigma_t = 80$ MPa $= 80$ N/mm^2 ; $\tau = 60$ MPa $= 60$ N/mm^2

1. ***Diameter of rivet***

Since the thickness of plate is greater than 8 mm, therefore diameter of rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{10} = 18.97 \text{ mm}$$

From Table 9.3, we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of rivet hole (d) is 19 mm and the corresponding diameter of the rivet is 18 mm. **Ans.**

2. ***Pitch of rivets***

Let p = Pitch of rivets.

Since the joint is a single riveted double strap butt joint as shown in Fig. 9.8, therefore there is one rivet per pitch length (*i.e.* $n = 1$) and the rivets are in double shear.

We know that tearing resistance of the plate,

$$P_t = (p - d)\, t \times \sigma_t = (p - 19)10 \times 80 = 800\,(p - 19) \text{ N} \qquad \ldots(i)$$

and shearing resistance of the rivets,

$$P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau \qquad \ldots(\because \text{ Rivets are in double shear})$$

$$= 1 \times 1.875 \times \frac{\pi}{4}\,(19)^2\, 60 = 31\,900 \text{ N} \qquad \ldots(\because n = 1) \quad \ldots(ii)$$

From equations (*i*) and (*ii*), we get

$$800\,(p - 19) = 31\,900$$

$$\therefore \quad p - 19 = 31\,900 / 800 = 39.87 \text{ or } p = 39.87 + 19 = 58.87 \text{ say } 60 \text{ mm}$$

According to I.B.R., the maximum pitch of rivets,

$$p_{max} = C.t + 41.28 \text{ mm}$$

From Table 9.5, we find that for double strap butt joint and 1 rivet per pitch length, the value of C is 1.75.

$$\therefore \qquad p_{max} = 1.75 \times 10 + 41.28 = 58.78 \text{ say } 60 \text{ mm}$$

From above we see that $p = p_{max} = 60$ mm **Ans.**

3. *Thickness of cover plates*

We know that thickness of cover plates,

$$t_1 = 0.625\, t = 0.625 \times 10 = 6.25 \text{ mm} \qquad \textbf{Ans.}$$

Efficiency of the joint

We know that tearing resistance of the plate,

$$P_t = (p - d)\, t \times \sigma_t = (60 - 19)\, 10 \times 80 = 32\,800 \text{ N}$$

and shearing resistance of the rivets,

$$P_s = n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau = 1 \times 1.875 \times \frac{\pi}{4} (19)^2\, 60 = 31\,900 \text{ N}$$

$\therefore$ Strength of the joint

$$= \text{Least of } P_t \text{ and } P_s = 31\,900 \text{ N}$$

Strength of the unriveted plate per pitch length

$$P = p \times t \times \sigma_t = 60 \times 10 \times 80 = 48\,000 \text{ N}$$

$\therefore$ Efficiency of the joint,

$$\eta = \frac{\text{Least of } P_t \text{ and } P_s}{P} = \frac{31\,900}{48\,000} = 0.665 \text{ or } 66.5\% \qquad \textbf{Ans.}$$

Pbm:3 A steam boiler is to be designed for a working pressure of 2.5 N/mm2 with its inside diameter 1.6 m. Give the design calculations for the longitudinal and circumferential joints for the following working stresses for steel plates and rivets : In tension = 75 MPa ; In shear = 60 MPa; In crushing = 125 MPa. Draw the joints to a suitable scale.

Given data :

P = 2.5 N/mm2

D = 1.6 m = 1600 mm

σt = 75 MPa = 75 N/mm2

τ = 60 MPa = 60 N/mm2

σc = 125 MPa = 125 N/mm2

Design of longitudinal joint

The longitudinal joint for a steam boiler may be designed as follows :

1. *Thickness of boiler shell*

We know that the thickness of boiler shell,

$$t = \frac{P.D}{2\,\sigma_t} + 1\text{ mm} = \frac{2.5 \times 1600}{2 \times 75} + 1\text{ mm}$$

$$= 27.6 \text{ say } 28 \text{ mm} \quad \textbf{Ans.}$$

2. *Diameter of rivet*

Since the thickness of the plate is more than 8 mm, therefore diameter of rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{28} = 31.75 \text{ mm}$$

From Table 9.3, we see that according to IS : 1928 – 1961 (Reaffirmed 1996), the standard diameter of rivet hole (d) is 34.5 mm and the corresponding diameter of the rivet is 33 mm. **Ans.**

3. *Pitch of rivets*

Assume the joint to be triple riveted double strap butt joint with unequal cover straps, as shown in Fig. 9.11.

Let p = Pitch of the rivet in the outer most row.

$\therefore$ Tearing resistance of the plate per pitch length,

$$P_t = (p - d)\, t \times \sigma_t = (p - 34.5)\, 28 \times 75 \text{ N}$$

$$= 2100\, (p - 34.5) \text{ N} \quad \text{...}(i)$$

Since the joint is triple riveted with two unequal cover straps, therefore there are 5 rivets per pitch length. Out of these five rivets, four are in double shear and one is in single shear. Assuming the strength of rivets in double shear as 1.875 times that of single shear, therefore

Shearing resistance of the rivets per pitch length,

$$P_s = 4 \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau + \frac{\pi}{4} \times d^2 \times \tau$$

$$= 8.5 \times \frac{\pi}{4} \times d^2 \times \tau$$

$$= 8.5 \times \frac{\pi}{4} (34.5)^2\, 60 = 476\ 820 \text{ N} \quad \text{...}(ii)$$

Equating equations *(i)* and *(ii)*, we get

$$2100\, (p - 34.5) = 476\ 820$$

$$\therefore \quad p - 34.5 = 476\ 820 / 2100 = 227 \quad \text{or} \quad p = 227 + 34.5 = 261.5 \text{ mm}$$

According to I.B.R., the maximum pitch,

$$p_{max} = C.t + 41.28 \text{ mm}$$

From Table 9.5, we find that for double strap butt joint with 5 rivets per pitch length, the value of C is 6.

$$\therefore \quad p_{max} = 6 \times 28 + 41.28 = 209.28 \text{ say } 220 \text{ mm}$$

Since p_{max} is less than p, therefore we shall adopt

$$p = p_{max} = 220 \text{ mm} \quad \textbf{Ans.}$$

$\therefore$ Pitch of rivets in the inner row,

$$p' = 220 / 2 = 110 \text{ mm} \quad \textbf{Ans.}$$

4. ***Distance between the rows of rivets***

According to I.B.R., the distance between the outer row and the next row

$$= 0.2\,p + 1.15\,d = 0.2 \times 220 + 1.15 \times 34.5 \text{ mm}$$

$$= 83.7 \text{ say } 85 \text{ mm} \quad \textbf{Ans.}$$

and the distance between the inner rows for zig-zig riveting

$$= 0.165\,p + 0.67\,d = 0.165 \times 220 + 0.67 \times 34.5 \text{ mm}$$

$$= 59.4 \text{ say } 60 \text{ mm} \quad \textbf{Ans.}$$

5. ***Thickness of butt straps***

We know that for unequal width of butt straps, the thicknesses are :

For wide butt strap, $t_1 = 0.75\,t = 0.75 \times 28 = 21$ mm **Ans.**

and for narrow butt strap, $t_2 = 0.625\,t = 0.625 \times 28 = 17.5$ say 18 mm **Ans.**

It may be noted that the wide and narrow butt straps are placed on the inside and outside of the shell respectively.

6. *Margin*

We know that the margin,

$$m = 1.5\,d = 1.5 \times 34.5 = 51.75 \text{ say } 52 \text{ mm} \quad \textbf{Ans.}$$

Let us now check the efficiency of the designed joint.

Tearing resistance of the plate in the outer row,

$$P_t = (p - d)\,t \times \sigma_t = (220 - 34.5)\,28 \times 75 = 389\ 550 \text{ N}$$

Shearing resistance of the rivets,

$$P_s = 4 \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau + \frac{\pi}{4} \times d^2 \times \tau = 8.5 \times \frac{\pi}{4} \times d^2 \times \tau$$

$$= 8.5 \times \frac{\pi}{4}\,(34.5)^2\,60 = 476\ 820 \text{ N}$$

and crushing resistance of the rivets,

$$P_c = n \times d \times t \times \sigma_c = 5 \times 34.5 \times 28 \times 125 = 603\ 750 \text{ N}$$

The joint may also fail by tearing off the plate between the rivets in the second row. This is only possible if the rivets in the outermost row gives way (*i.e.* shears). Since there are two rivet holes per pitch length in the second row and one rivet in the outermost row, therefore

Combined tearing and shearing resistance

$$= (p - 2d)\,t \times \sigma_t + \frac{\pi}{4} \times d^2 \times \tau$$

$$= (220 - 2 \times 34.5)\,28 \times 75 + \frac{\pi}{4}\,(34.5)^2\,60$$

$$= 317\ 100 + 56\ 096 = 373\ 196 \text{ N}$$

From above, we see that the strength of the joint

$$= 373\ 196 \text{ N}$$

Strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 220 \times 28 \times 75 = 462\ 000 \text{ N}$$

$\therefore$ Efficiency of the designed joint,

$$\eta = \frac{373\ 196}{462\ 000} = 0.808 \text{ or } 80.8\% \quad \textbf{Ans.}$$

Design of circumferential joint

The circumferential joint for a steam boiler may be designed as follows :

1. The thickness of the boiler shell (t) and diameter of rivet hole (d) will be same as for longitudinal joint, *i.e.*

$$t = 28 \text{ mm} \; ; \text{ and } d = 34.5 \text{ mm}$$

2. *Number of rivets*

Let n = Number of rivets.

We know that shearing resistance of the rivets

$$= n \times \frac{\pi}{4} \times d^2 \times \tau \qquad \text{...}(i)$$

and total shearing load acting on the circumferential joint

$$= \frac{\pi}{4} \times D^2 \times P \qquad \text{...}(ii)$$

From equations (*i*) and (*ii*), we get

$$n \times \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times D^2 \times P$$

$$\therefore \qquad n = \frac{D^2 \times P}{d^2 \times \tau} = \frac{(1600)^2 \; 2.5}{(34.5)^2 \; 60} = 89.6 \text{ say } 90 \quad \textbf{Ans.}$$

3. ***Pitch of rivets***

Assuming the joint to be double riveted lap joint with zig-zag riveting, therefore number of rivets per row

$$= 90 / 2 = 45$$

We know that the pitch of the rivets,

$$p_1 = \frac{\pi (D + t)}{\text{Number of rivets per row}} = \frac{\pi (1600 + 28)}{45} = 113.7 \text{ mm}$$

Let us take pitch of the rivets, $p_1 = 140$ mm **Ans.**

4. ***Efficiency of the joint***

We know that the efficiency of the circumferential joint,

$$\eta_c = \frac{p_1 - d}{p_1} = \frac{140 - 34.5}{140} = 0.753 \text{ or } 75.3\%$$

5. ***Distance between the rows of rivets***

We know that the distance between the rows of rivets for zig-zag riveting,

$$= 0.33\, p_1 + 0.67\, d = 0.33 \times 140 + 0.67 \times 34.5 \text{ mm}$$

$$= 69.3 \text{ say } 70 \text{ mm}$$ **Ans.**

6. ***Margin***

We know that the margin,

$$m = 1.5\, d = 1.5 \times 34.5$$

$$= 51.75 \text{ say } 52 \text{ mm}$$ **Ans.**

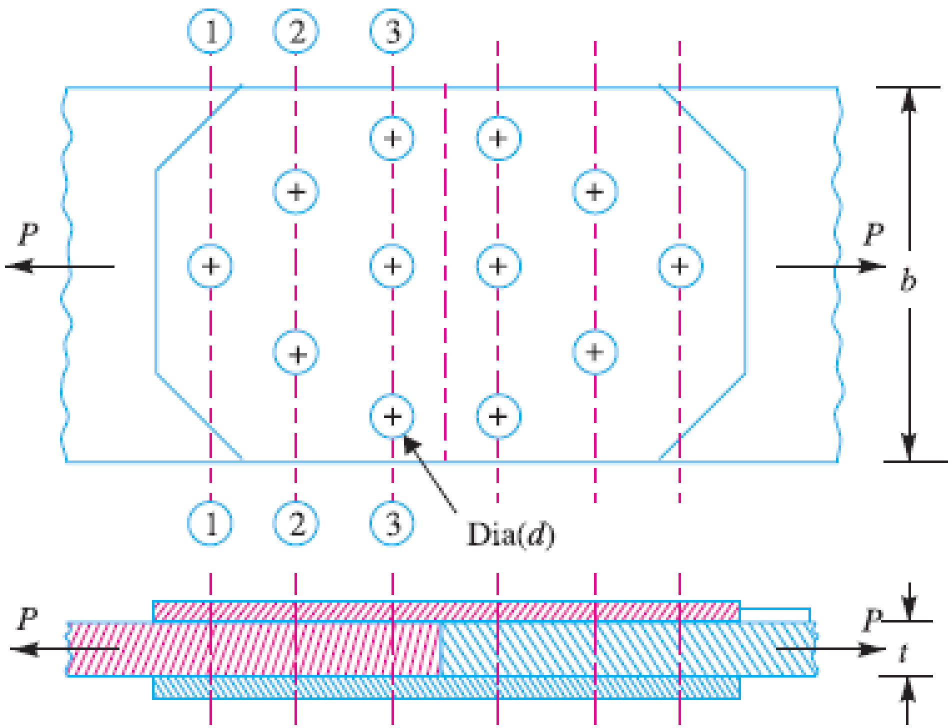
Riveted Joint for Structural Use–Joints of Uniform Strength

A riveted joint known as Lozenge joint used for roof, bridge work

1. *Diameter of rivet*

The diameter of the rivet hole is obtained by using Unwin's formula, *i.e.*

$$d = 6\sqrt{t}$$



2. *Number of rivets*

The number of rivets required for the joint may be obtained by the shearing or crushing resistance of the rivets.

Let P_t = Maximum pull acting on the joint. This is the tearing resistance of the plate at the outer row which has only one rivet.

$$= (b - d)\, t \times \sigma_t$$

∴ Shearing resistance of one rivet,

$$P_s = 1.75 \times \frac{\pi}{4} \times d^2 \times \tau$$

and crushing resistance of one rivet,

$$P_c = d \times t \times \sigma_c$$

∴ Number of rivets required for the joint,

$$n = \frac{P_t}{\text{Least of } P_s \text{ or } P_c}$$

3. From the number of rivets, the number of rows and the number of rivets in each row is decided.

4. *Thickness of the butt straps*

The thickness of the butt strap,

$$t_1 = 1.25\, t, \text{ for single cover strap}$$

$$= 0.75\, t, \text{ for double cover strap}$$

5. *Efficiency of the joint*

First of all, calculate the resistances along the sections 1-1, 2-2 and 3-3.

At section 1-1, there is only one rivet hole.

∴ Resistance of the joint in tearing along 1-1,

$$P_{t1} = (b - d)\, t \times \sigma_t$$

At section 2-2, there are two rivet holes.

∴ Resistance of the joint in tearing along 2-2,

$$P_{t2} = (b - 2d)\, t \times \sigma_t + \text{Strength of one rivet in front of section 2-2}$$

(This is due to the fact that for tearing off the plate at section 2-2, the rivet in front of section 2-2 *i.e.* at section 1-1 must first fracture).

Similarly at section 3-3 there are three rivet holes.

∴ Resistance of the joint in tearing along 3-3,

$$P_{t3} = (b - 3d)\, t \times \sigma_t + \text{Strength of 3 rivets in front of section 3-3}$$

The least value of P_{t1}, P_{t2}, P_{t3}, P_s or P_c is the strength of the joint.

We know that the strength of unriveted plate,

$$P = b \times t \times \sigma_t$$

∴ Efficiency of the joint,

$$\eta = \frac{\text{Least of } P_{t1},\, P_{t2},\, P_{t3},\, P_s \text{ or } P_c}{P}$$

6. The pitch of the rivets is obtained by equating the strength of the joint in tension to the strength of the rivets in shear.
7. The marginal pitch (m) should not be less than 1.5 d.
8. The distance between the rows of rivets is 2.5 d to 3 d.

Example 9.11. *Two lengths of mild steel tie rod having width 200 mm and thickness 12.5 mm are to be connected by means of a butt joint with double cover plates. Design the joint if the permissible stresses are 80 MPa in tension, 65 MPa in shear and 160 MPa in crushing. Make a sketch of the joint.*

Solution. Given : $b = 200$ mm ; $t = 12.5$ mm ; $\sigma_t = 80$ MPa $= 80$ N/mm^2 ; $\tau = 65$ MPa $= 65$ N/mm^2 ; $\sigma_c = 160$ MPa $= 160$ N/mm^2

1. *Diameter of rivet*

We know that the diameter of rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{12.5} = 21.2 \text{ mm}$$

From Table 9.7, we see that according to IS : 1929 – 1982 (Reaffirmed 1996), the standard diameter of the rivet hole (d) is 21.5 mm and the corresponding diameter of rivet is 20 mm. **Ans.**

2. *Number of rivets*

Let n = Number of rivets.

We know that maximum pull acting on the joint,

$$P_t = (b - d)\, t \times \sigma_t = (200 - 21.5)\, 12.5 \times 80 = 178\ 500 \text{ N}$$

Since the joint is a butt joint with double cover plates as shown in Fig. 9.20, therefore the rivets are in double shear. Assume that the resistance of the rivet in double shear is 1.75 times than in single shear.

∴ Shearing resistance of one rivet,

$$P_s = 1.75 \times \frac{\pi}{4} \times d^2 \times \tau = 1.75 \times \frac{\pi}{4}\, (21.5)^2\, 65 = 41\ 300 \text{ N}$$

and crushing resistance of one rivet,

$$P_c = d \times t \times \sigma_c = 21.5 \times 12.5 \times 160 = 43\ 000 \text{ N}$$

Since the shearing resistance is less than the crushing resistance, therefore number of rivets required for the joint,

$$n = \frac{P_t}{P_s} = \frac{178\ 500}{41\ 300} = 4.32 \text{ say } 5 \qquad \textbf{Ans.}$$

3. The arrangement of the rivets is shown in Fig. 9.20.

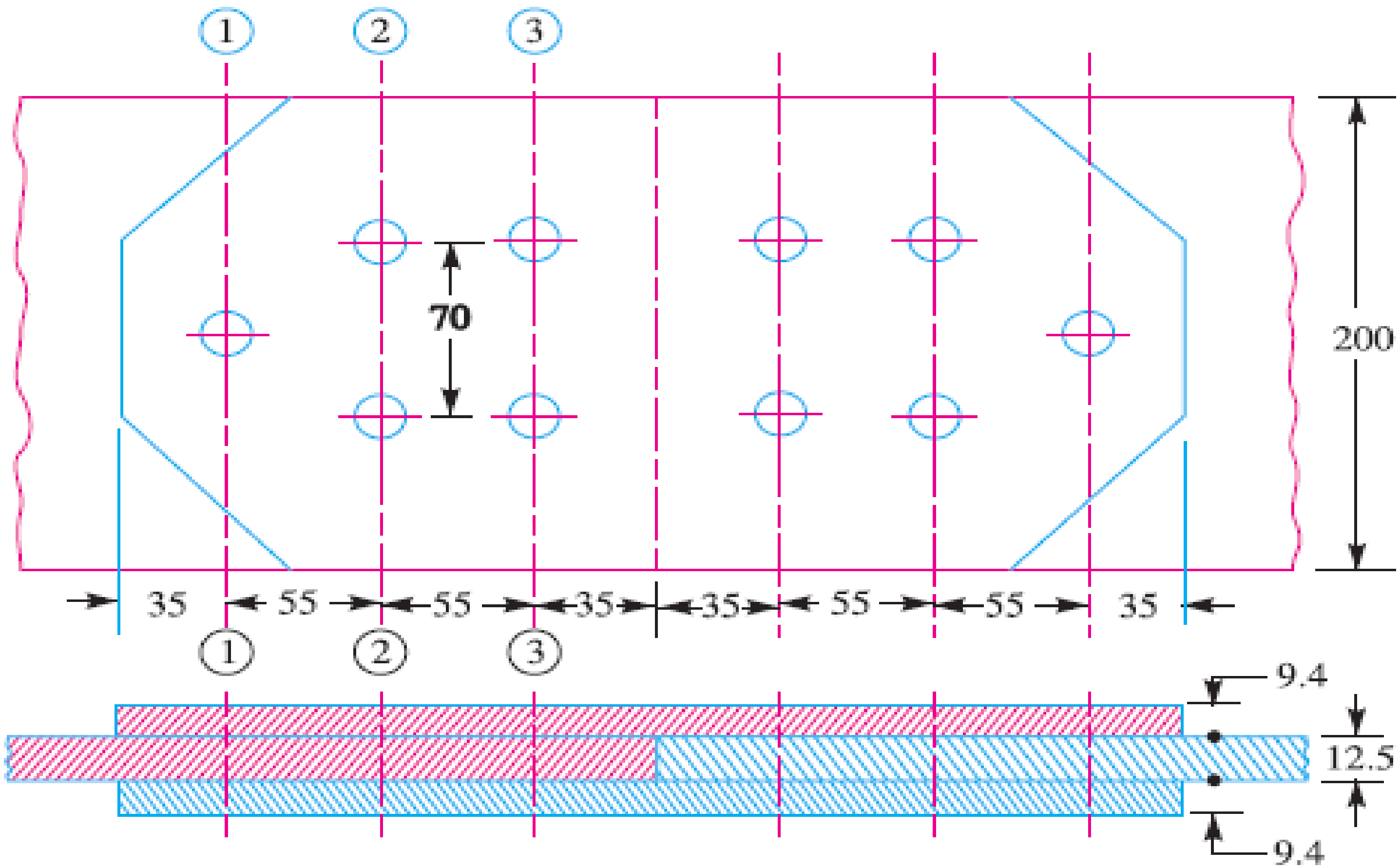


Fig. 9.20. All dimensions in mm.

4. ***Thickness of butt straps***

We know that thickness of butt straps,

$$t_1 = 0.75\,t = 0.75 \times 12.5 = 9.375 \text{ say } 9.4 \text{ mm} \quad \textbf{Ans.}$$

5. *Efficiency of the joint*

First of all, let us find the resistances along the sections 1-1, 2-2 and 3-3.

At section 1-1, there is only one rivet hole.

∴ Resistance of the joint in tearing along section 1-1,

$$P_{t1} = (b - d)\, t \times \sigma_t = (200 - 21.5)\, 12.5 \times 80 = 178\,500 \text{ N}$$

At section 2-2, there are two rivet holes. In this case, the tearing of the plate will only take place if the rivet at section 1-1 (in front of section 2-2) gives way (*i.e.* shears).

∴ Resistance of the joint in tearing along section 2-2,

$$P_{t2} = (b - 2d)\, t \times \sigma_t + \text{Shearing resistance of one rivet}$$

$$= (200 - 2 \times 21.5)\, 12.5 \times 80 + 41\,300 = 198\,300 \text{ N}$$

At section 3-3, there are two rivet holes. The tearing of the plate will only take place if one rivet at section 1-1 and two rivets at section 2-2 gives way (*i.e.* shears).

∴ Resistance of the joint in tearing along section 3-3,

$$P_{t3} = (b - 2d)\, t \times \sigma_t + \text{Shearing resistance of 3 rivets}$$

$$= (200 - 2 \times 21.5)\, 12.5 \times 80 + 3 \times 41\,300 = 280\,900 \text{ N}$$

Shearing resistance of all the 5 rivets

$$P_s = 5 \times 41\,300 = 206\,500 \text{ N}$$

and crushing resistance of all the 5 rivets,

$$P_c = 5 \times 43\,000 = 215\,000 \text{ N}$$

Since the strength of the joint is the least value of P_{t1}, P_{t2}, P_{t3}, P_s and P_c, therefore strength of the joint

$$= 178\ 500 \text{ N along section 1-1}$$

We know that strength of the un-riveted plate,

$$= b \times t \times \sigma_t = 20 \times 12.5 \times 80 = 200\ 000 \text{ N}$$

∴ Efficiency of the joint,

$$\eta = \frac{\text{Strength of the joint}}{\text{Strength of the unriveted plate}} = \frac{178\ 500}{200\ 000}$$

$$= 0.8925 \quad \text{or} \quad 89.25\% \quad \textbf{Ans.}$$

6. Pitch of rivets, $p = 3\,d + 5 \text{ mm} = (3 \times 21.5) + 5 = 69.5$ say 70 mm **Ans.**

7. Marginal pitch, $m = 1.5\,d = 1.5 \times 21.5 = 33.25$ say 35 mm **Ans.**

8. Distance between the rows of rivets

$$= 2.5\,d = 2.5 \times 21.5 = 53.75 \text{ say } 55 \text{ mm} \quad \textbf{Ans.}$$

ECCENTRIC LOADED RIVETED JOINT

- When the line of action of the load does not pass through the centroid of the rivet system and thus all rivets are not equally loaded, then the joint is said to be an eccentric loaded riveted joint, as
- The eccentric loading results in secondary shear caused by the tendency of force to twist the joint about the centre of gravity in addition to direct shear or primary shear.

Example 9.14. *An eccentrically loaded lap riveted joint is to be designed for a steel bracket as shown in Fig. 9.24.*

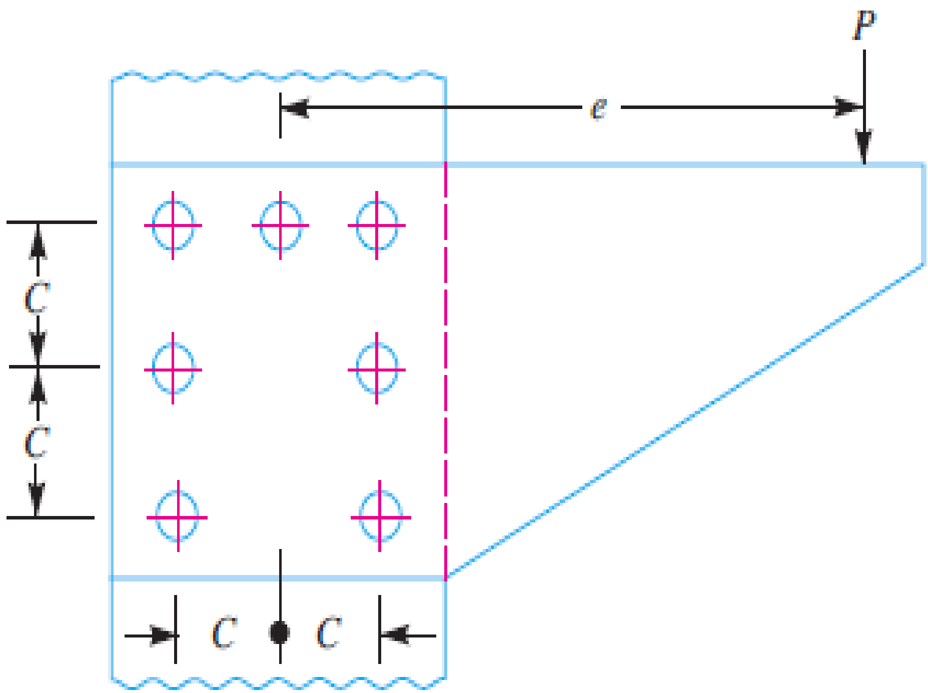


Fig. 9.24

The bracket plate is 25 mm thick. All rivets are to be of the same size. Load on the bracket, P = 50 kN ; rivet spacing, C = 100 mm; load arm, e = 400 mm.

Permissible shear stress is 65 MPa and crushing stress is 120 MPa. Determine the size of the rivets to be used for the joint.

Solution. Given : $t = 25$ mm ; $P = 50$ kN $= 50 \times 10^3$ N ; $e = 400$ mm ; $n = 7$; $\tau = 65$ MPa $= 65$ N/mm^2 ; $\sigma_c = 120$ MPa $= 120$ N/mm^2

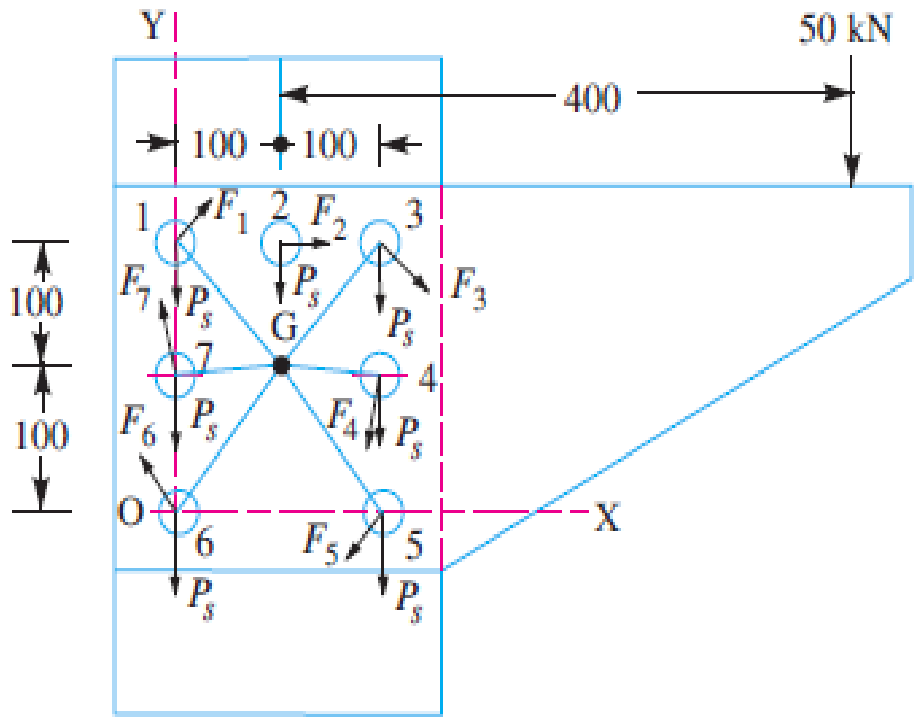


Fig. 9.25

First of all, let us find the centre of gravity (G) of the rivet system.

Let $\bar{x}$ = Distance of centre of gravity from OY,

$\bar{y}$ = Distance of centre of gravity from OX,

$x_1, x_2, x_3 \ldots$ = Distances of centre of gravity of each rivet from OY, and

$y_1, y_2, y_3 \ldots$ = Distances of centre of gravity of each rivet from OX.

We know that

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7}{n}$$

$$= \frac{100 + 200 + 200 + 200}{7} = 100 \text{ mm} \qquad \ldots(\because x_1 = x_6 = x_7 = 0)$$

and

$$\bar{y} = \frac{y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7}{n}$$

$$= \frac{200 + 200 + 200 + 100 + 100}{7} = 114.3 \text{ mm} \qquad \ldots(\because y_5 = y_6 = 0)$$

$\therefore$ The centre of gravity (G) of the rivet system lies at a distance of 100 mm from OY and 114.3 mm from OX, as shown in Fig. 9.25.

We know that direct shear load on each rivet,

$$P_s = \frac{P}{n} = \frac{50 \times 10^3}{7} = 7143 \text{ N}$$

The direct shear load acts parallel to the direction of load P *i.e.* vertically downward as shown in Fig. 9.25.

Turning moment produced by the load P due to eccentricity (e)

$$= P \times e = 50 \times 10^3 \times 400 = 20 \times 10^6 \text{ N-mm}$$

From the geometry of the figure, we find that

$$l_1 = l_3 = \sqrt{(100)^2 + (200 - 114.3)^2} = 131.7 \text{ mm}$$

$$l_2 = 200 - 114.3 = 85.7 \text{ mm}$$

$$l_4 = l_7 = \sqrt{(100)^2 + (114.3 - 100)^2} = 101 \text{ mm}$$

and
$$l_5 = l_6 = \sqrt{(100)^2 + (114.3)^2} = 152 \text{ mm}$$

Now equating the turning moment due to eccentricity of the load to the resisting moment of the rivets, we have

$$P \times e = \frac{F_1}{l_1}\left[(l_1)^2 + (l_2)^2 + (l_3)^2 + (l_4)^2 + (l_5)^2 + (l_6)^2 + (l_7)^2\right]$$

$$= \frac{F_1}{l_1}\left[2(l_1)^2 + (l_2)^2 + 2(l_4)^2 + 2(l_5)^2\right]$$

....($\because$ $l_1 = l_3$; $l_4 = l_7$ and $l_5 = l_6$)

$$50 \times 10^3 \times 400 = \frac{F_1}{131.7}\left[2(131.7)^2 + (85.7)^2 + 2(101)^2 + 2(152)^2\right]$$

$$20 \times 10^6 \times 131.7 = F_1(34\,690 + 7345 + 20\,402 + 46\,208) = 108\,645\, F_1$$

$\therefore$
$$F_1 = 20 \times 10^6 \times 131.7 / 108\,645 = 24\,244 \text{ N}$$

Since the secondary shear loads are proportional to their radial distances from the centre of gravity, therefore

$$F_2 = F_1 \times \frac{l_2}{l_1} = 24\ 244 \times \frac{85.7}{131.7} = 15\ 776 \text{ N}$$

$$F_3 = F_1 \times \frac{l_3}{l_1} = F_1 = 24\ 244 \text{ N} \qquad \ldots(\because l_1 = l_3)$$

$$F_4 = F_1 \times \frac{l_4}{l_1} = 24\ 244 \times \frac{101}{131.7} = 18\ 593 \text{ N}$$

$$F_5 = F_1 \times \frac{l_5}{l_1} = 24\ 244 \times \frac{152}{131.7} = 27\ 981 \text{ N}$$

$$F_6 = F_1 \times \frac{l_6}{l_1} = F_5 = 27\ 981 \text{ N} \qquad \ldots(\because l_6 = l_5)$$

$$F_7 = F_1 \times \frac{l_7}{l_1} = F_4 = 18\ 593 \text{ N} \qquad \ldots(\because l_7 = l_4)$$

By drawing the direct and secondary shear loads on each rivet, we see that the rivets 3, 4 and 5 are heavily loaded. Let us now find the angles between the direct and secondary shear load for these three rivets. From the geometry of Fig. 9.26, we find that

$$\cos\theta_3 = \frac{100}{l_3} = \frac{100}{131.7} = 0.76$$

$$\cos\theta_4 = \frac{100}{l_4} = \frac{100}{101} = 0.99$$

and

$$\cos\theta_5 = \frac{100}{l_5} = \frac{100}{152} = 0.658$$

Now resultant shear load on rivet 3,

$$R_3 = \sqrt{(P_s)^2 + (F_3)^2 + 2P_s \times F_3 \times \cos\theta_3}$$

$$= \sqrt{(7143)^2 + (24\ 244)^2 + 2 \times 7143 \times 24\ 244 \times 0.76} = 30\ 033 \text{ N}$$

Resultant shear load on rivet 4,

$$R_4 = \sqrt{(P_s)^2 + (F_4)^2 + 2\ P_s \times F_4 \times \cos\theta_4}$$

$$= \sqrt{(7143)^2 + (18\ 593)^2 + 2 \times 7143 \times 18\ 593 \times 0.99} = 25\ 684 \text{ N}$$

and resultant shear load on rivet 5,

$$R_5 = \sqrt{(P_s)^2 + (F_5)^2 + 2\ P_s \times F_5 \times \cos\theta_5}$$

$$= \sqrt{(7143)^2 + (27\ 981)^2 + 2 \times 7143 \times 27\ 981 \times 0.658} = 33\ 121 \text{ N}$$

The resultant shear load may be determined graphically, as shown in Fig. 9.26.

From above we see that the maximum resultant shear load is on rivet 5. If d is the diameter of rivet hole, then maximum resultant shear load (R_5),

$$33\,121 = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times d^2 \times 65 = 51\,d^2$$

$$\therefore \qquad d^2 = 33\,121 / 51 = 649.4 \quad \text{or} \quad d = 25.5 \text{ mm}$$

From Table 9.7, we see that according to IS : 1929–1982 (Reaffirmed 1996), the standard diameter of the rivet hole (d) is 25.5 mm and the corresponding diameter of rivet is 24 mm.

Let us now check the joint for crushing stress. We know that

$$\text{Crushing stress} = \frac{\text{Max. load}}{\text{Crushing area}} = \frac{R_5}{d \times t} = \frac{33\,121}{25.5 \times 25}$$

$$= 51.95 \text{ N/mm}^2 = 51.95 \text{ MPa}$$

Since this stress is well below the given crushing stress of 120 MPa, therefore the design is satisfactory.

UNIT IV
ENERGY STORING ELEMENTS AND ENGINE COMPONENTS

Various types of springs, optimization of helical springs - rubber springs

Flywheels considering stresses in rims and arms for engines and punching machines- Connecting Rods and crank shafts.

Introduction

- A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed.

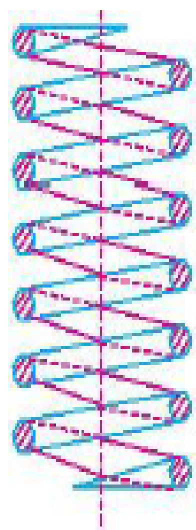
APPLICATIONS OF SPRINGS

1. To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.
2. To apply forces, as in brakes, clutches and spring-loaded valves.
3. To control motion by maintaining contact between two elements as in cams and followers.
4. To measure forces, as in spring balances and engine indicators etc.

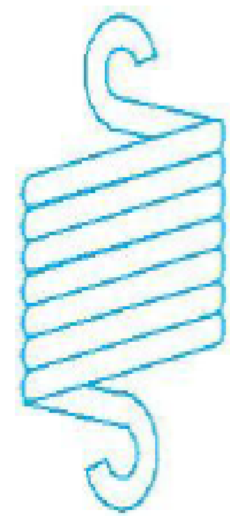
Types of Springs

Though there are many types of the springs, yet the following, according to their shape, are important from the subject point of view.

1. *Helical springs.* The helical springs are made up of a wire coiled in the form of a helix and are primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are ***compression helical spring*** as shown in Fig (*a*) and ***tension helical spring*** as shown in Fig (*b*).

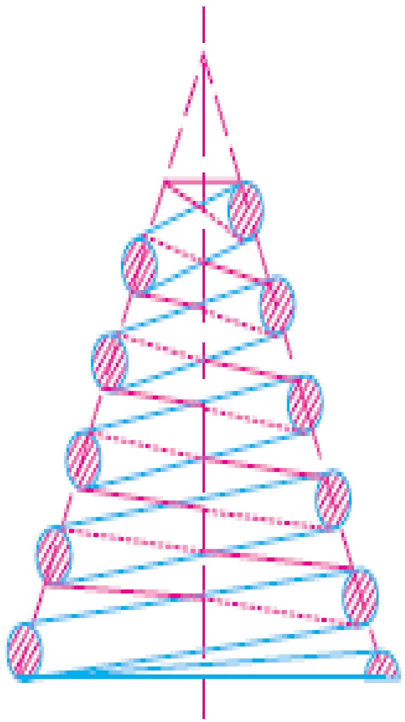


(*a*) Compression helical spring.

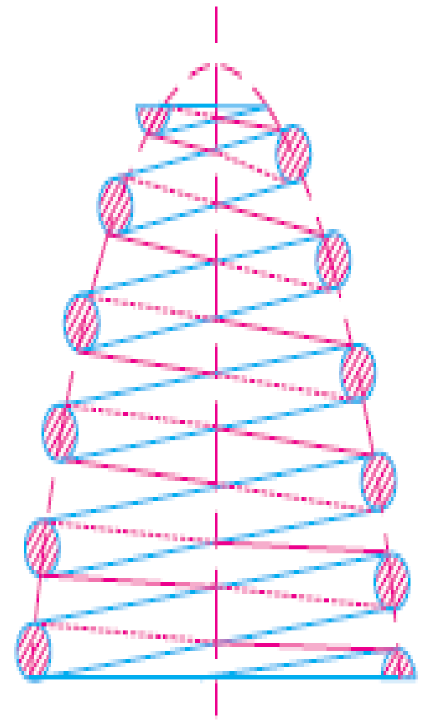


(*b*) Tension helical spring.

2. ***Conical and volute springs.*** The conical and volute springs, as shown in Fig. 23.2, are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired. The conical spring, as shown in Fig. 23.2 (*a*), is wound with a uniform pitch whereas the volute springs, as shown in Fig. 23.2 (*b*), are wound in the form of paraboloid with constant pitch



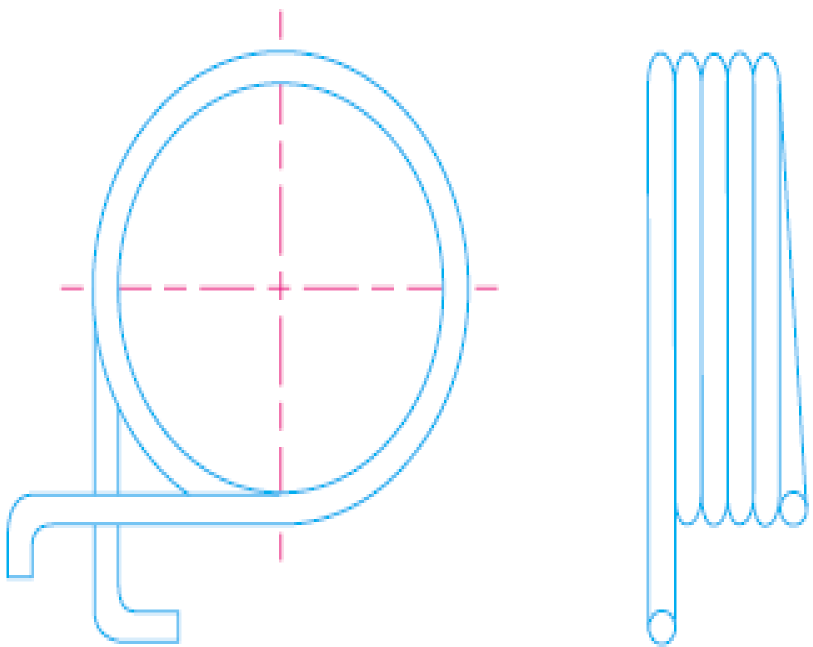
(*a*) Conical spring.



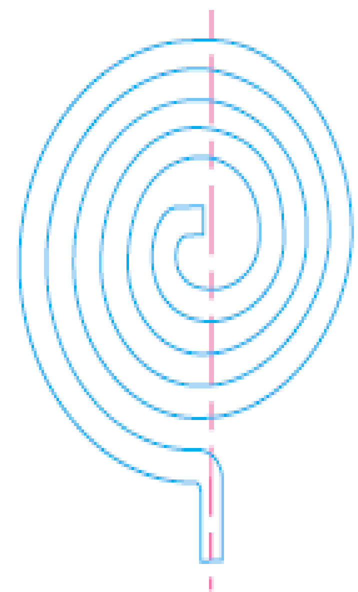
(*b*) Volute spring.

3. *Torsion springs*. These springs may be of *helical* or *spiral* type as shown in Fig. 23.3. The **helical type** may be used only in applications where the load tends to wind up the spring and are used in various electrical mechanisms. The **spiral type** is also used where the load tends to increase the number of coils and when made of flat strip are used in watches and clocks.

The major stresses produced in torsion springs are tensile and compressive due to bending.



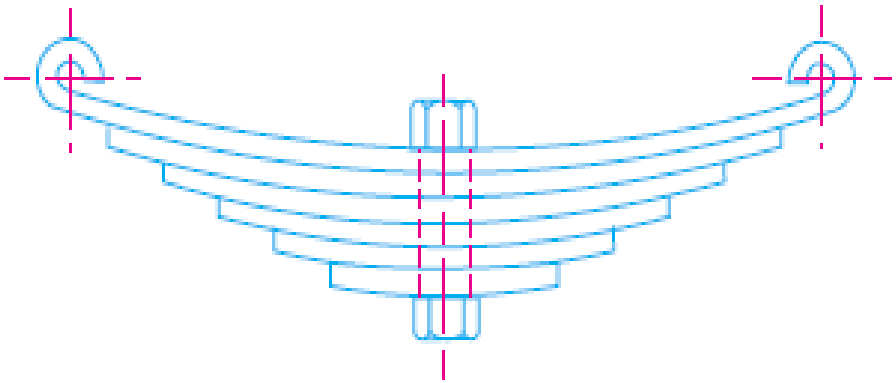
(*a*) Helical torsion spring.



(*b*) Spiral torsion spring.

4. *Laminated or leaf springs.* The laminated or leaf spring (also known as *flat spring* or *carriage spring*) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts, as shown in Fig. 23.4. These are mostly used in automobiles.

The major stresses produced in leaf springs are tensile and compressive stresses.



5. Disc or bellevile springs. These springs consist of a number of conical discs held together against slipping by a central bolt or tube as shown in Fig. 23.5. These springs are used in applications where high spring rates and compact spring units are required.

The major stresses produced in disc or bellevile springs are tensile and compressive stresses.

6. Special purpose springs. These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or liquid) can behave as a compression spring. These springs are used for special types of application only.

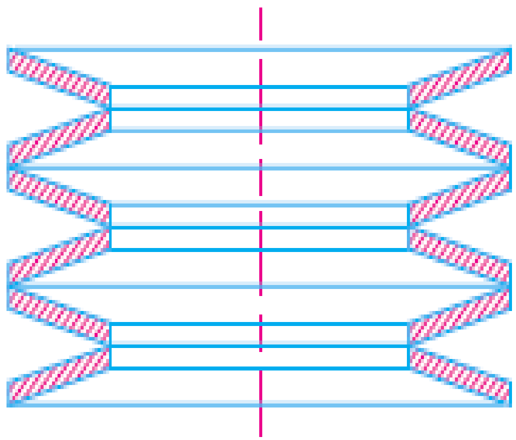


Fig. 23.5. Disc or bellevile springs.

23.5 Terms used in Compression Springs

The following terms used in connection with compression springs are important from the subject point of view.

1. *Solid length*. When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be *solid*. The solid length of a spring is the product of total number of coils and the diameter of the wire. Mathematically,

Solid length of the spring,

$$L_S = n'.d$$

where n' = Total number of coils, and

d = Diameter of the wire.

2. *Free length*. The free length of a compression spring, as shown in Fig. 23.6, is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed). Mathematically,

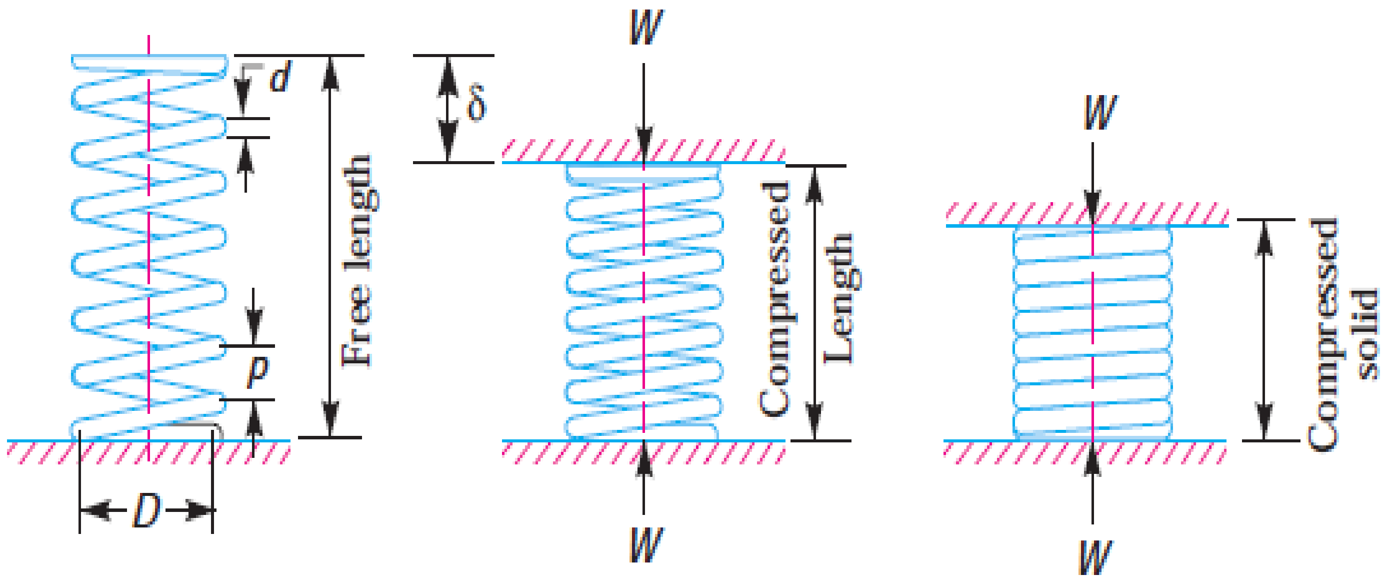


Fig. 23.6. Compression spring nomenclature.

Free length of the spring,

L_F = Solid length + Maximum compression + *Clearance between adjacent coils (or clash allowance)

$$= n'.d + \delta_{max} + 0.15\ \delta_{max}$$

The following relation may also be used to find the free length of the spring, *i.e.*

$$L_F = n'.d + \delta_{max} + (n' - 1) \times 1 \text{ mm}$$

In this expression, the clearance between the two adjacent coils is taken as 1 mm.

3. ***Spring index.*** The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Mathematically,

Spring index, $C = D / d$

where D = Mean diameter of the coil, and

d = Diameter of the wire.

4. ***Spring rate.*** The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,

Spring rate, $k = W / \delta$

where W = Load, and

δ = Deflection of the spring.

5. *Pitch*. The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically,

Pitch of the coil, $$p = \frac{\text{Free length}}{n' - 1}$$

The pitch of the coil may also be obtained by using the following relation, *i.e.*

Pitch of the coil, $$p = \frac{L_F - L_S}{n'} + d$$

where L_F = Free length of the spring,

L_S = Solid length of the spring,

n' = Total number of coils, and

d = Diameter of the wire.

In choosing the pitch of the coils, the following points should be noted :

(*a*) The pitch of the coils should be such that if the spring is accidently or carelessly compressed, the stress does not increase the yield point stress in torsion.

(*b*) The spring should not close up before the maximum service load is reached.

Note : In designing a tension spring (See Example 23.8), the minimum gap between two coils when the spring is in the free state is taken as 1 mm. Thus the free length of the spring,

$$L_F = n.d + (n - 1)$$

and pitch of the coil, $$p = \frac{L_F}{n - 1}$$

23.8 Stresses in Helical Springs of Circular Wire

Consider a helical compression spring made of circular wire and subjected to an axial load W, as shown in Fig. 23.10 (a).

Let

D = Mean diameter of the spring coil,

d = Diameter of the spring wire,

n = Number of active coils,

G = Modulus of rigidity for the spring material,

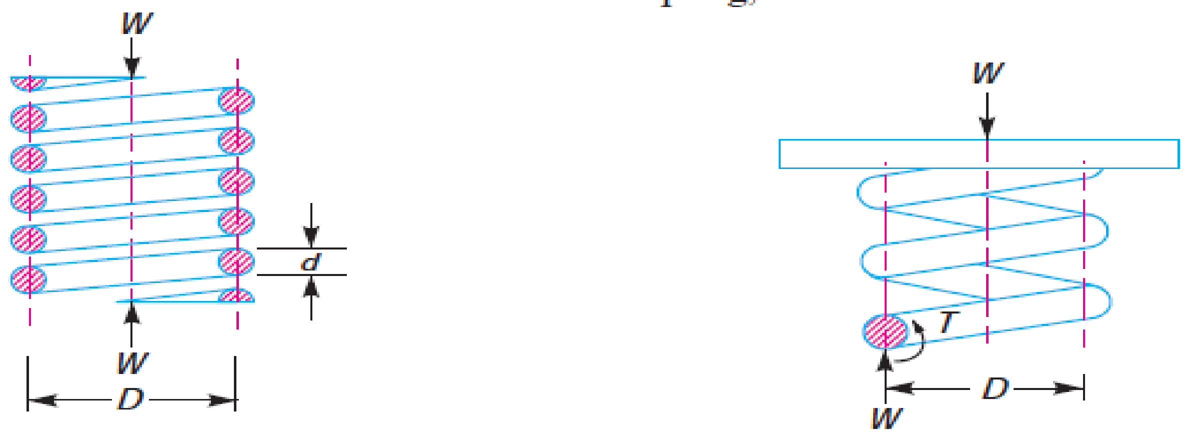
W = Axial load on the spring,

τ = Maximum shear stress induced in the wire,

C = Spring index = D/d,

p = Pitch of the coils, and

δ = Deflection of the spring, as a result of an axial load W.



(a) Axially loaded helical spring.

(b) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Example 23.2. *A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm², find the axial load which the spring can carry and the deflection per active turn.*

Solution. Given : $d = 6$ mm ; $D_o = 75$ mm ; $\tau = 350$ MPa $= 350$ N/mm² ; $G = 84$ kN/mm² $= 84 \times 10^3$ N/mm²

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

∴ Spring index,

$$C = \frac{D}{d} = \frac{69}{6} = 11.5$$

Let W = Axial load, and

δ / n = Deflection per active turn.

1. *Neglecting the effect of curvature*

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire (τ),

$$350 = K_S \times \frac{8\, W.D}{\pi\, d^3} = 1.043 \times \frac{8\, W \times 69}{\pi \times 6^3} = 0.848\, W$$

∴ $W = 350 / 0.848 = 412.7$ N **Ans.**

We know that deflection of the spring,

$$\delta = \frac{8\, W.D^3.n}{G.d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\, W.D^3}{G.d^4} = \frac{8 \times 412.7\,(69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm } \textbf{Ans.}$$

2. *Considering the effect of curvature*

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire (τ),

$$350 = K \times \frac{8W.C}{\pi\, d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913\, W$$

$\therefore$ $$W = 350 / 0.913 = 383.4 \text{ N} \quad \textbf{Ans.}$$

and deflection of the spring,

$$\delta = \frac{8\, W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\, W.D^3}{G.d^4} = \frac{8 \times 383.4\, (69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm} \quad \textbf{Ans.}$$

Example 23.6. *Design a close coiled helical compression spring for a service load ranging from 2250 N to 2750 N. The axial deflection of the spring for the load range is 6 mm. Assume a spring index of 5. The permissible shear stress intensity is 420 MPa and modulus of rigidity, G = 84 kN/mm².*

Neglect the effect of stress concentration. Draw a fully dimensioned sketch of the spring, showing details of the finish of the end coils.

Solution. Given : $W_1 = 2250$ N ; $W_2 = 2750$ N ; $\delta = 6$ mm ; $C = D/d = 5$; $\tau = 420$ MPa $= 420$ N/mm² ; $G = 84$ kN/mm² $= 84 \times 10^3$ N/mm²

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil for a maximum load of $W_2 = 2750$ N, and

d = Diameter of the spring wire.

We know that twisting moment on the spring,

$$T = W_2 \times \frac{D}{2} = 2750 \times \frac{5d}{2} = 6875\ d \qquad \ldots\left(\because C = \frac{D}{d} = 5\right)$$

We also know that twisting moment (T),

$$6875\ d = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 420 \times d^3 = 82.48\ d^3$$

$\therefore$ $d^2 = 6875 / 82.48 = 83.35$ or $d = 9.13$ mm

From Table 23.2, we shall take a standard wire of size *SWG* 3/0 having diameter (d) = 9.49 mm.

$\therefore$ Mean diameter of the spring coil,

$$D = 5d = 5 \times 9.49 = 47.45 \text{ mm} \quad \textbf{Ans.}$$

We know that outer diameter of the spring coil,

$$D_o = D + d = 47.45 + 9.49 = 56.94 \text{ mm} \quad \textbf{Ans.}$$

and inner diameter of the spring coil,

$$D_i = D - d = 47.45 - 9.49 = 37.96 \text{ mm} \quad \textbf{Ans.}$$

2. *Number of turns of the spring coil*

Let $\qquad n$ = Number of active turns.

It is given that the axial deflection (δ) for the load range from 2250 N to 2750 N (*i.e.* for W = 500 N) is 6 mm.

We know that the deflection of the spring (δ),

$$6 = \frac{8\,W.C^3.n}{G.d} = \frac{8 \times 500\,(5)^3\,n}{84 \times 10^3 \times 9.49} = 0.63\,n$$

$$\therefore \qquad n = 6 / 0.63 = 9.5 \text{ say } 10 \textbf{ Ans.}$$

For squared and ground ends, the total number of turns,

$$n' = 10 + 2 = 12 \textbf{ Ans.}$$

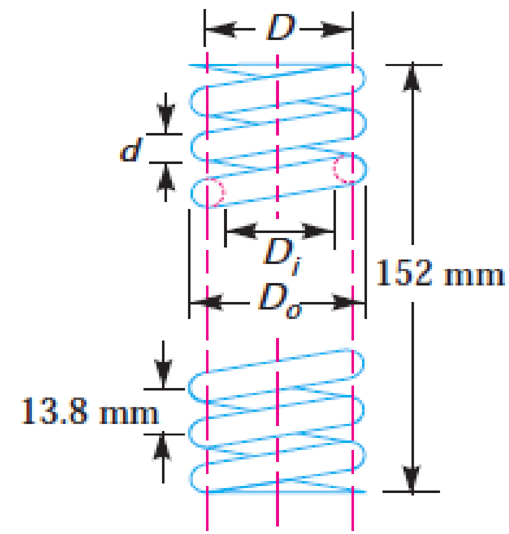


Fig. 23.14

3. *Free length of the spring*

Since the compression produced under 500 N is 6 mm, therefore maximum compression produced under the maximum load of 2750 N is

$$\delta_{max} = \frac{6}{500} \times 2750 = 33 \text{ mm}$$

We know that free length of the spring,

$$L_F = n'.d + \delta_{max} + 0.15\,\delta_{max}$$
$$= 12 \times 9.49 + 33 + 0.15 \times 33$$
$$= 151.83 \text{ say } 152 \text{ mm } \textbf{Ans.}$$

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{152}{12 - 1} = 13.73 \text{ say } 13.8 \text{ mm } \textbf{Ans.}$$

Leaf springs problems

Pbm:1

Design a leaf spring with maximum load 140 kN

Number of springs =4

Material for spring-Chromium Vanadium Steel

Permissible tensile stress = 600N/mm2

Maximum number of leaves =10

Span of spring =1000 mm

Permissible deflection = 80 mm

Young's modulus of the spring =200 KN/mm2

Given data:

Maximum load on the spring, $2P = 140\ kN = 140 \times 10^3\ N$

Number of spring, $n_s = 4$

Permissible tensile stress, $\sigma = 600\ N/mm^2$

Maximum number of leaves = 10

Span of spring, $2L = 1000\ mm$, $L = 500\ mm$

Permissible deflection, $y = 80\ mm$

Young's modulus of the spring, $E = 200\ kN/mm^2 = 200 \times 10^3\ N/mm^2$

To find:

(i) Width of the leaf spring, b

(ii) Thickness of the leaf, t

Solution:

Load on the spring, $2P = 140 \times 10^3$

$$2P = \frac{140 \times 10^3}{\text{Number of spring}}$$

$$2P = \frac{140 \times 10^3}{4}$$

$$P = 17500\ N$$

$\therefore$ Permissible stress, $\sigma = \dfrac{6PL}{nbt^2}$

$$600 = \frac{6 \times 17500 \times 500}{10 \times bt^2}$$

Deflection of the spring, $y = \dfrac{6 P L^3}{E n b t^3}$

$$80 = \frac{6 \times 17500 \times 500^3}{200 \times 10^3 \times 10 \times b t^3}$$

$$b t^3 = 82031.25$$

$$b t^2 \,.\, t = 82031.25$$

From equations (4.90) and (4.91),

$$8750 t = 82031.25$$

$$t = \frac{8203125}{8750} = 9.375\ mm$$

Say thickness of the leaves, $t = \mathbf{10}\ \boldsymbol{mm}$

$$b = \frac{8750}{t^2} = \frac{8750}{10^2} = 87.5\ mm$$

Width of the leaves, $b = 87.5\ mm$

$\therefore$ The nearest standard size, $b = \mathbf{90}\ \boldsymbol{mm}$

Result:

(i) Width of the leaf spring, $b = 90\ mm$

(ii) Thickness of the load, $t = 10\ mm$.

Design a cantilever leaf spring to absorb 600 N-m energy without exceeding a deflection of 150 mm and a stress of 800 N/mm². The length of the spring is 600 mm. The material of the spring is steel. ***[MU-Oct'02]***

Given data:

Cantilever leaf spring energy absorbed, $E_1 = 600\ N\text{-}m = 600 \times 10^3\ N\text{-}mm$

Deflection of the spring, $y = 150\ mm$

Bending stress of the spring, $\sigma_b = 800\ N/mm^2$

Length of the spring, $L = 600\ mm$

Solution:

$$\text{Energy, } E_1 = \frac{1}{2} \times P \times y$$

$$600 \times 10^3 = \frac{1}{2} \times P \times 150$$

Maximum load, $P = 8000\ N$

The maximum permissible stress in leaf spring is

$$\sigma_b = \frac{6PL}{nbt^2}$$

$$\therefore \quad 800 = \frac{6 \times 8000 \times 600}{nbt^2} \qquad [\because n = 1]$$

$$nbt^2 = \frac{6 \times 8000 \times 600}{800}$$

$$nbt^2 = 36000$$

Deflection of the spring,

$$y = \frac{12 P L^3}{E n b t^3}$$

For steel spring, Young's modulus,

$$E = 2 \times 10^5 \, N/mm^2$$

$$150 = \frac{6 \times 8000 \times 600^3}{2 \times 10^5 \times n b t^3} \qquad [\because n = 1]$$

$$nbt^3 = 345600$$

$$nbt^2 \cdot t = 345600$$

From equations (4.92) and (4.93)

$$36000 \, t = 345600$$

$$t = \frac{345600}{36000} = 9.6 \, mm$$

Select standard size, $t = \mathbf{10 \, mm}$

$$nb = \frac{345600}{t^3}$$

$$nb = \frac{345600}{(9.6)^3} = 390.625$$

Select the width appropriate to, $t = 10\ mm$

The standard size of the width is **80 *mm***

$\therefore$ Number of leaves,

$$n = \frac{390.625}{80} = \mathbf{4.88 \approx say\ 5}$$

Result:

(i) Width of the leaf, $b = 80\ mm$

(ii) Thickness of the leaf, $t = 10\ mm$

(iii) Number of leaves, $n = 5$.

4.2. DESIGN OF FLYWHEELS

A *flywheel* is a heavy rotating mass which is placed between power source and driven member to act as a reservoir of energy. The primary function of flywheel is to act as an "*energy accumulator*". It will absorb energy when the demand is less than the supply of energy and it will release it when the demand is more than the energy being supplied. Depending on the source of power and type of driven machinery, there are two distinct applications of the flywheel.

(i) In some cases, the power is supplied at a uniform rate while the requirement of power from the driven machinery is variable. Example: Punching press driven by an electric motor, rolling mill driven by an electric motor. In these cases, the flywheel stores energy during the idle portion of the work cycle by increasing its speed and it delivers this energy during the peak load period of punching.

(ii) In other applications, the availability of energy is at a fluctuating rate but the requirement of it for the driven machinery is at uniform rate. Example: Machinery driven by an I.C. engine.

Flywheels are made in the form of three different types.

1. Disc type flywheel:

This type of flywheel is of solid disc type shown in Figure 4.31. It is used in automobiles engines.

2. Web type flywheel:

It consists of a heavy rim connected to the hub by a disc shaped plate called *web* shown in Figure 4.32. This flywheel is mostly used with small power vertical I.C. engines.

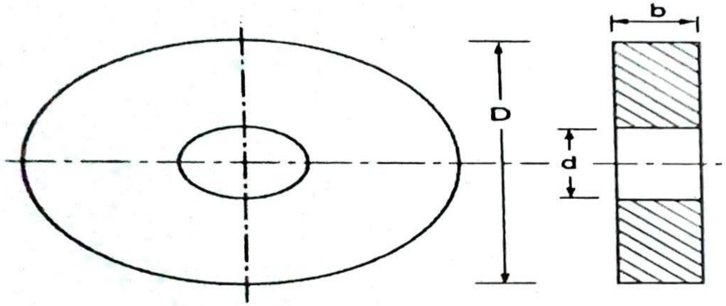


Figure 4.31 Solid disc type flywheel

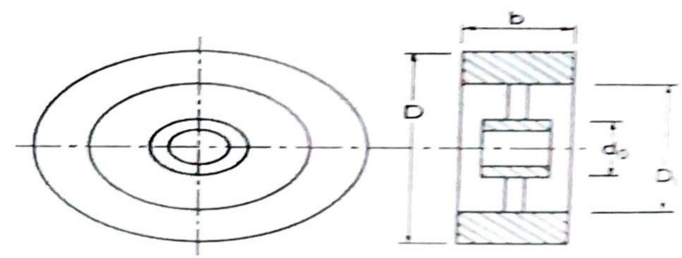


Figure 4.32 Web-type flywheel

3. Arm type flywheel:

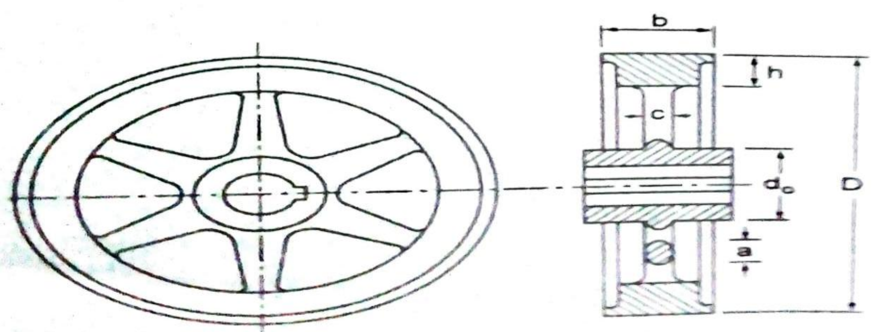


Figure 4.33 Arm type flywheel

4.2.1. Flywheel Effect And Co-Efficient Of Fluctuation Of Speed

Energy stored in a flywheel is in the form of kinetic energy of rotating mass. The kinetic energy of a revolving mass is given by the expression,

$$E = \frac{1}{2} m v^2 = \frac{1}{2} m\omega^2 k^2 = \frac{1}{2} I\omega^2$$

where

$I \Rightarrow$ Mass moment of inertia $= mk^2$

$m \Rightarrow$ Mass of the flywheel

$k \Rightarrow$ Radius of gyration of the flywheel

$\omega \Rightarrow$ Angular speed

The mass moment of inertia required for the flywheel is termed as *flywheel effect*.

When the speed of flywheel changes from ω_{max} to ω_{min}, the change in kinetic energy of the flywheel is given by

$$\Delta K.E = \frac{1}{2} I\left[\omega^2_{max} - \omega^2_{min}\right]$$

In order to keep the variation of speed within the permissible range, the fluctuation of energy (ΔE) of the combined driver/driven system should be equal to change in kinetic energy.

$$\Delta E = \frac{1}{2} I\left[\omega^2_{max} - \omega^2_{min}\right]$$

$$\boxed{\Delta E = I\,\omega^2\,K_s} \quad \dots (4.94)$$

where K_s is the co efficient of fluctuation of speed $= \dfrac{\omega^2_{max} - \omega^2_{min}}{\omega}$

ω is the mean angular speed $= \dfrac{\omega_{max} + \omega_{min}}{2}$

4.2.2. Coefficient of fluctuation of energy (k_e)

The difference between maximum and minimum energy during the cycle is called *fluctuation of energy* (ΔE).

$$\Delta E = E_{max} - E_{min}$$

The ratio of fluctuation of energy to the mean energy is called *coefficient of fluctuation of energy*.

$$K_E = \frac{E_{max} - E_{min}}{E} = \frac{\Delta E}{E}$$

The fluctuation of energy can be determined by the turning moment diagram for one complete cycle of operation. Mean energy of the cycle may be assumed as equal to the work done per cycle. The work done per cycle may be obtained by the following relation:

(i) Work done per cycle $= (M_t)_{mean} \times \theta$

where

$$(M_t)_{mean} \Rightarrow \text{mean torque} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

$P \Rightarrow$ Power to be transmitted

$\theta \Rightarrow$ angle turned in radians per revolution

$\theta = 2\pi$ (for 2 stroke I.C engines)

$= 4\pi$ (for 4 stroke I.C engines)

(ii) Work done per cycle $= \dfrac{P \times 60}{n}$

where

$P \Rightarrow$ Power

$n \Rightarrow$ Number of working stroke per minute

2.3. Stresses in Flywheel Rim

arious stresses in the flywheel rim are induced due to the following reasons.

(i) Tensile stress due to centrifugal force.

(ii) Bending stress produced due to restraining effect of arms.

(iii) Shrinkage stress due to solidification of casting and stress due to variations of load and speed.

nsile stress due to centrifugal force is given by

$$\sigma_t = \frac{\gamma V^2}{g} = \rho V^2$$

where

$\gamma \Rightarrow$ Specific weight of the rim material

$$V \Rightarrow \text{Mean rim speed} = \frac{\pi D N}{60}$$

$$\rho \Rightarrow \text{Mass density of the rim material}$$

$$\rho = 7200\ kg/m^3 \text{ for cast iron}$$

$$= 7800\ kg/m^3 \text{ for cast steel}$$

$$\text{Bending stress, } \sigma_b = \frac{\pi^2 V^2 D\rho}{n^2 h}$$

where

$$D \Rightarrow \text{Diameter of the flywheel rim}$$

$$h \Rightarrow \text{thickness of the rim}$$

$$n \Rightarrow \text{number of arms} = 6, 8 \text{ or } 10$$

The resultant stress in the rim at the junctions with the arms may be taken as

$$\sigma_{total} = \frac{3}{4}\sigma_t + \frac{1}{4}\sigma_b$$

$$= 0.75 \times \rho V^2 + \frac{0.25 \times \pi^2 V^2 D\rho}{n^2 h}$$

$$= \rho V^2 \left[0.75 + \frac{4.935R}{n^2 h}\right]$$

For the safe design of the flywheel rim, the resultant stress should be less than or equal to allowable strength of the flywheel. Generally, $\sigma_{total} \leq 40 MN/m^2$ for cast iron flywheel.

4.2.4. Stresses in Arms

The arms of the flywheel are subjected to following types of stresses.

1. Tensile stress due to centrifugal force
2. Bending stress due to torque
3. Stress due to belt tension.

Tensile stress due to centrifugal force is given by the equation

$$\sigma_{t_1} = \frac{3}{4}\rho V^2$$

4.2.5. Design Of Flywheel Shaft, Hub and Key

Shaft:

The diameter of flywheel shaft (d) is calculated on the basis of maximum torque to transmitted

$$(M_t)_{max} = \frac{\pi}{16} \times \tau \times d_1^3$$

where d_1 is the diameter of shaft

Hub:

The inside diameter of the hub is equal to the diameter of the shaft. The outside diameter generally taken as twice the diameter of the shaft.

$$d_o = 2d$$

Otherwise, it may also be calculated by

$$(M_t)_{max} = \frac{\pi}{16} \times \tau \times \left(\frac{d_o^4 - d_1^4}{d_o} \right)$$

Length of the hub, $L_h = 2d_1$ to $2.5d_1$

Key:

A rectangular or square sunk key may be used.

For square key,

Width of the key, w = thickness of key $= \dfrac{d_1}{4}$

Length of the key can be found out on the basis of maximum torque.

$$(M_t)_{max} = L_k \times w \times \tau \times \frac{d_1}{2}$$

A single cylinder single acting four-stroke oil engine develops 18 kW indicated power at 275rpm. The work done by gases during the expansion stroke is 2.3 times the work done on the gases during the compression stroke and work done during the suction and exhaust strokes is negligible. If the speed is to be maintained within ± 1% of the mean speed, find the flywheel effect.

Given data:

Indicated power, $P = 18\ kW$

Speed, $N = 275\ rpm$

Work done during expansion, $W_E = 2.3$ times work done during compression.

Fluctuation of speed, $= \pm\ 1\%$

To find:

Flywheel effect

☺ ***Solution:***

$$\text{Mean torque, } (M_t)_{mean} = \frac{P \times 60}{2\pi N} = \frac{18 \times 10^3 \times 60}{2\pi \times 275} = 625\ N\text{-}m$$

Indicated work done per cycle $= (M_t)_{mean} \times 4\pi = 625 \times 4\pi = 7853.98\ N\text{-}m$

Since the work done during suction and exhaust strokes is negligible, then

$$\text{Work done per cycle} = W_E - W_C = W_E - \frac{W_E}{2.3} = 0.565 W_E \qquad (\because\ W_E = 2.3 W_C)$$

$$7853.98 = 0.565 \times W_E$$

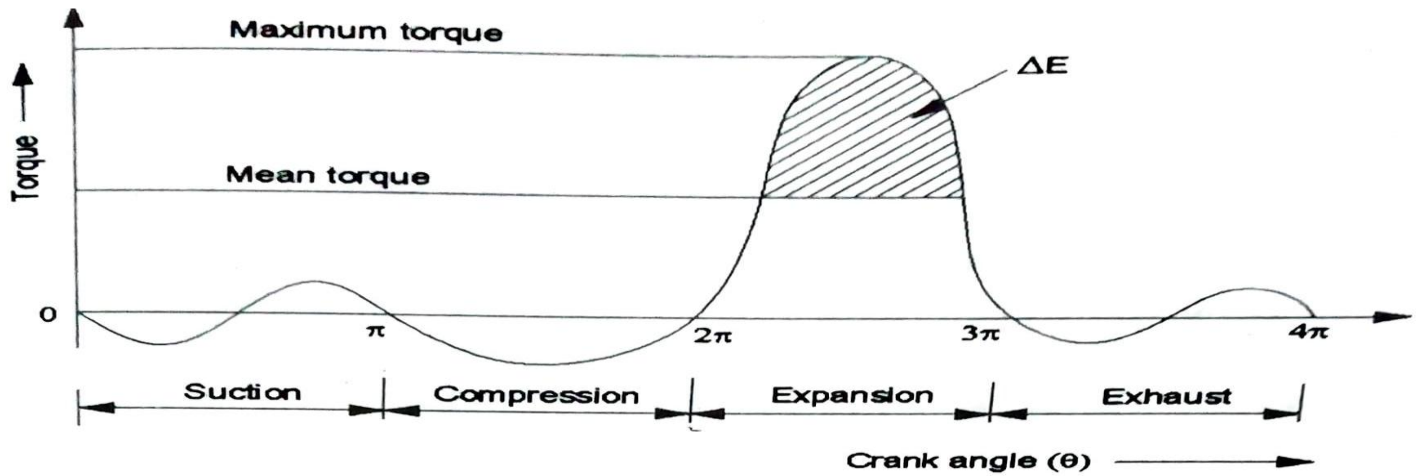


Figure 4.35

Work done during expansion,

$$W_E = 13900.85\ N\text{-}m$$

Indicated work done per stroke $= 625 \times \pi = 1963.49\ N\text{-}m$

Maximum fluctuation of energy (ΔE),

$$= \text{Work done during expansion} - \text{Indicated W.D. per stroke.}$$

$$\Delta E = 13900.85 - 1963.79 = 11937.06\ N\text{-}m$$

Co-efficient of fluctuation of energy = 0.02

Mean velocity of rim,

$$V = \frac{\pi D N}{60} = \frac{\pi \times D \times 275}{60} = 14.4D$$

We know that $\quad \Delta E = mV^2 K_S$

$$11937.06 = m\,(14.4D)^2 \times 0.02$$

$$m\,D^2 = 2878.34$$

$$m\,(2R)^2 = 2878.34$$

$$mR^2 = \mathbf{719.59\ kg\text{-}m^2} \qquad \textbf{Ans.}$$

Result:

Flywheel effect, $I = mR^2 = 719.59\ kg\text{-}m^2$.

A single cylinder I.C. engine working on four-stroke cycle develops 75 kW at 360 rpm. The maximum fluctuation of energy can be assumed to be 0.9 times the energy developed/cycle. If the total fluctuation of speed is not to exceed 1% and the maximum centrifugal stress in the flywheel is to be 5.5 MN/m², estimate the mean diameter and the cross-sectional area of the rim. Flywheel is made of cast iron. ***[M.U.-Oct.'97]***

Given data:

Power $= 75\ kW = 75 \times 10^3\ W$

Speed, $N = 360\ rpm$

$\Delta E = 0.9E$

Centrifugal stress $= 5.5\ MN/m^2$

Total fluctuation of speed, $K_s = 1\% = 0.01$

To find:

1. Mean diameter
2. Cross sectional area of the rim.

Solution:

We know that the centrifugal stress

$$\sigma_t = \rho\, V^2$$

$$5.5 \times 10^6 = 7200 \times V^2 \quad (\because \text{ for C.I., } \rho = 7200\ kg/m^3)$$

$$V = 27.6\ m/s$$

$$V = \frac{\pi\, D\, N}{60}$$

$$27.6 = \frac{\pi \times D \times 600}{60}$$

$$\mathbf{D = 0.87976\ m}$$ **Ans.**

Energy developed per cycle,

$$E = \frac{P \times 60}{N} = \frac{75 \times 10^3 \times 60}{600} = 7500\ N\text{-}m$$

Maximum fluctuation of energy,

$$\Delta E = 0.9 \times E = 0.9 \times 7500 = 6750\ N\text{-}m$$

We know that $\Delta E = m\, R^2\, \omega^2\, K_S$

$$6750 = m \times \left(\frac{0.879}{2}\right)^2 \times \left(\frac{2\pi \times 360}{60}\right)^2 \times 0.01$$

$$\therefore\ m = 458.8\ kg$$

Mass of the flywheel rim, $m = 2\,\pi\, R\, A\, \rho$

$$458.8 = 2\pi \times \left(\frac{0.879}{2}\right) \times A \times 7200$$

Cross sectional area of the rim, $A = \mathbf{0.023\ m^2}$

Result:

(i) Diameter of the rim, $D = 0.879m = 879.76\ mm$

(ii) Cross sectional area of the rim, $A = 0.023\ m^2$.

Design and sketch a C.I. flywheel for four-stroke C.I. engine developing 50 kW at 150 rpm with 75 explosions per minute. The total fluctuation of speed is limited to 0.5% of the mean speed on either side. The work done during working stroke is 1.4 times the work done during the cycle. Assume flywheel stores 90% of energy on the rim, hoop stress on the rim should not increase more than 4 N/mm² and density of the material is 7200 kg/m³.

Given data: ***[M.U.-Apr.'2003]***

Power, $P = 50\ kW = 50 \times 10^3 W$

Speed, $N = 150\ rpm$

For 4 stroke engine, $n = N/2 = 75$

W.D during working stroke = 1.4 × work done during the cycle

Hoop stress, $\sigma_t = 4\ N/mm^2$

$\rho = 7200\ kg/m^3$

To find:

Design and sketch of the cast iron flywheel

☺ ***Solution:***

First of all, let us find the mean torque (M_t) transmitted by the engine or flywheel. We know that the power transmitted (P),

$$50 \times 10^3 = \frac{2\pi N \times (M_t)_{mean}}{60} = \frac{2\pi \times 150 \times M_{t_{mean}}}{60} = 15.71\ M_{t_{mean}}$$

$$(M_t)_{mean} = \frac{50\times10^3}{15.71} = 3183.01\ N\text{-}m = 3183.01\ N\text{-}m$$

Since the explosions per minute are equal to $N/2$, the engine is a four-stroke cycle engine. The turning moment diagram of four-stroke engine is shown in Figure 4.36.

We know that the work done per cycle $= (M_t)_{mean} \times \theta = 3183 \times 4\pi = 40000\ N\text{-}m$

$\therefore$ Work done during power (or) Working stroke $= 1.4 \times 40000 = 56000\ N\text{-}m$ .... (4.95)

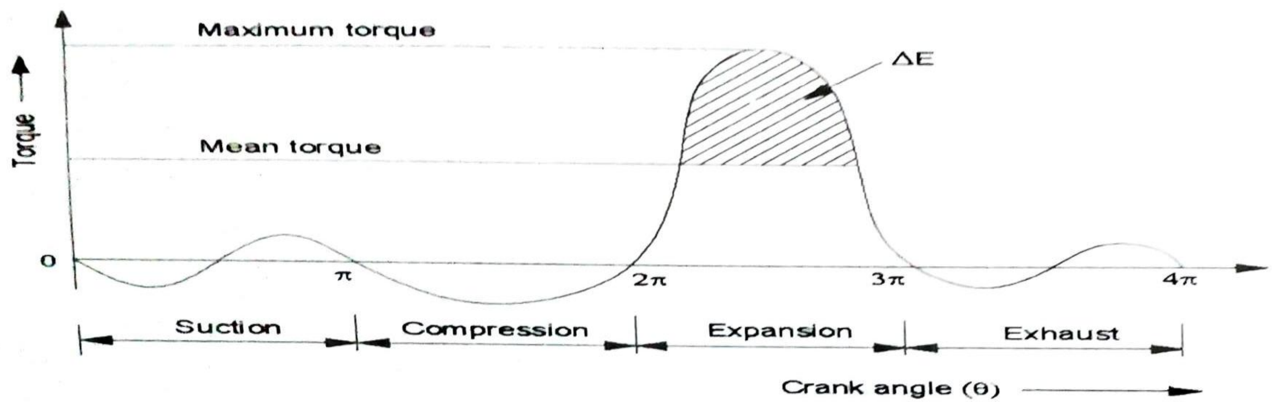


Figure 4.36

Since the hatched area above the mean torque line represents the maximum fluctuation of energy (ΔE), therefore

$$\text{Hatched area, } \Delta E = \text{Work done during power stroke} - (M_t)_{mean}$$

$$= 56000 - 3183 \times \pi = 46000\ N\text{-}m$$

We know that the hoop stress $[\sigma_t]$, $\sigma_t = \rho\ V^2$

$$4 \times 10^6 = 7200 \times V^2$$

$$V^2 = \frac{4\times10^6}{7200}$$

$$V = 23.57\ m/s$$

We also know that peripheral velocity (V),

$$V = \frac{\pi D N}{60}$$

$$23.57 = \frac{\pi\times D\times150}{60}$$

$$D = \frac{23.58}{7.855} = \mathbf{3\ m}$$ **Ans.**

Cross sectional dimensions of rim, t = Thickness of rim in meters

b = Width of rim in meters = $4h$

$\therefore$ Cross sectional area of rim, $A = b \times h = 4h \times h = 4h^2$

First of all, Let us find the mass of the flywheel rim.

m = Mass of the flywheel rim, and

E = Total energy of the flywheel.

Since the fluctuation of speed is 0.5% of the mean speed on either side. Therefore, total fluctuation of speed,

$N_1 - N_2$ = 1% of mean speed = $0.01N$

and co-efficient of fluctuation of speed,

$$K_s = \frac{N_1 - N_2}{N} = 0.01$$

We know that the maximum fluctuation of energy (ΔE),

$$46000 = E \times 2\ K_s = E \times 2 \times 0.01 = 0.02E$$

$$E = \frac{46000}{0.02} = 2300 \times 10^3\ N\text{-}m$$

Since the flywheel stores 90% of energy on rim,

$$E_{rim} = 0.9 \times E = 0.9 \times 2300 \times 10^3 = 2070 \times 10^3 N\text{-}m$$

We know that the energy in the rim (E_{rim}),

$$2070 \times 10^3 = \frac{1}{2} \times m \times V^2 = \frac{1}{2} \times m \times (23.58)^2$$

$$m = 7445.8\ kg$$

We also know that mass of flywheel rim (m),

$$7445.8 = A \times \pi D \times \rho = 4h^2 \times 3 \times 7200 \times \pi = 271433.6h^2$$

$$h^2 = \frac{74458}{2714336} = 0.027$$

$h = 0.1656\ m = \mathbf{165.6\ mm}$ **Ans.**

$b = 4t = 4 \times 165.6 = \mathbf{662.5\ mm}$ **Ans.**

Design of shaft:

Assume maximum torque $= 2\,(M_t)_{mean} = 2 \times 3183 = 6366\ N\text{-}m = 6366\times10^3\ N\text{-}mm$

$$\text{Maximum torque} = \frac{\pi}{16} \times \tau \times d_1^{\,3}$$

$$6366\times10^3 = \frac{\pi}{16} \times 45 \times d_1^{\,3} \qquad (\because \text{ Assume } \tau = 45N/mm^2)$$

$$d_1 = 89.65\ mm$$

Take standard diameter, $d_1 = \mathbf{90}\ \boldsymbol{mm}$ **Ans.**

Diameter and length of hub:

The diameter of hub is made equal to twice the diameter of the shaft and length of hub is equal to width of the rim.

$\therefore$ Diameter of hub, $d = 2d_1 = 2 \times 90 = \mathbf{180}\ \boldsymbol{mm}$ **Ans.**

Length of hub, $l = b = \mathbf{662.5}\ \boldsymbol{mm}$ **Ans.**

Cross sectional dimensions of the elliptical arms:

Assume Minor axis = 0.5 × major axis

$$c = 0.5a$$

Bending moment in the arm at the hub end,

$$M = \frac{(M_t)}{D\times n}(D-d) = \frac{3183\times10^3}{3000\times6}(3000-180) = 498670\ N\text{-}mm$$

Section modulus for cross section of the arm,

$$Z = \frac{\pi}{32}\times c\times a^2 = \frac{\pi}{32}\times0.5a\times a^2 = 0.049a^3$$

Let us assume bending stress,

$$\sigma_{b_1} = 15\ N/mm^2$$

Bending stress, $$15 = \frac{M}{Z} = \frac{498670}{0.049a^3}$$

$a = 87.87\ mm$ say **88** $\boldsymbol{mm}$ **Ans.**

$c = 0.5 \times 88 = \mathbf{44}\ \boldsymbol{mm}$ **Ans.**

Tensile stress due to centrifugal force,

$$\sigma_{t_1} = \frac{3}{4}\rho V^2 = \frac{3}{4} \times 7250 \times (23.58)^2$$

$$= 3.02 \times 10^6 \ N/m^2 = 3.02 \ N/mm^2$$

$$\text{Total stress, } \sigma_{max} = \sigma_{t_1} + \sigma_{b_1} = 3.02 + 15 = 18.02 \ N/mm^2$$

which is less than allowable strength (20 N/mm^2) of flywheel material. Hence, the design of arms is safe.

Dimensions of key:

The standard dimensions of rectangular sunk key for a shaft of diameter 90*mm* are as follows.

From PSGDB 5.16, Corresponding to $d = 90 \ mm$

Width of key, $w = 12 \ mm$ **Ans.**

Thickness of key, $t = 8 \ mm$ **Ans.**

Length of key may be taken as equal to length of hub.

So, length of key = **662.5** *mm* **Ans.**

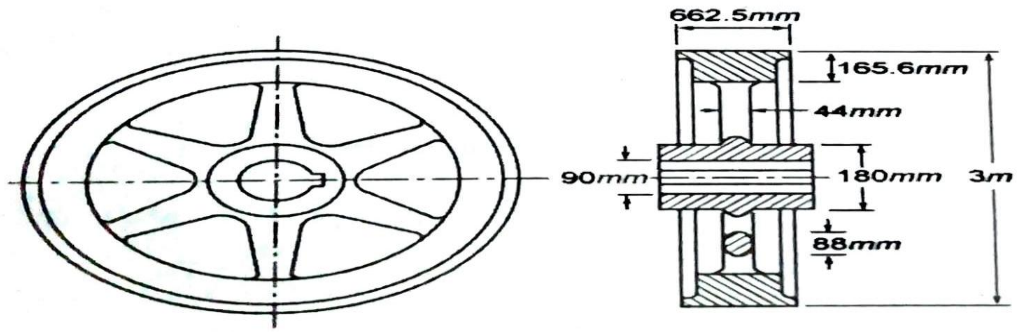


Figure 4.37 Arm type flywheel

Result:

Diameter of the flywheel rim, $D = 3 \ m$

Thickness of the flywheel rim, $h = 165.6 \ mm$

Width of the flywheel rim, $b = 662.5 \ mm$

Diameter of shaft, $d_1 = 90 \ mm$

Diameter of hub, $d = 180 \ mm$

Length of hub, $l = b = 662.5 \ mm$

4.3. DESIGN OF CONNECTING ROD

The connecting rod is an intermediate link between piston and crankshaft of an I.C. engine. It transmits force from piston to crankshaft. It also carries the lubricating oil from the crank pin end to the piston pin end and it provides lubrication to the piston cylinder assembly. The connecting rod converts the reciprocating motion of the piston to rotary motion of the crankshaft. The main parts of the connecting rod of an I.C engine are shown in Figure 4.40.

It has the following parts.

(i) An eye at the small end to accommodate piston pin bearing

(ii) A long shank usually of I-section, and

(iii) A big end opening which is usually split to take the crank pin bearing shells. The length of the connecting rod is usually kept 3 to 4.5 times the crank radius.

The materials for connecting rod range from mild or medium carbon steels to alloy steels. In industrial engines, carbon steel with ultimate tensile strength 550 to 670 N/mm^2 is used. In transport engines, alloy steel having strength of about 780 N/mm^2 to 940 N/mm^2 is used.

Example: Manganese steel. In aero engines, nickel chrome steel having ultimate tensile strength of about 940 N/mm^2 to 1350 N/mm^2 is used. Connecting rods are mostly manufactured by drop forging.

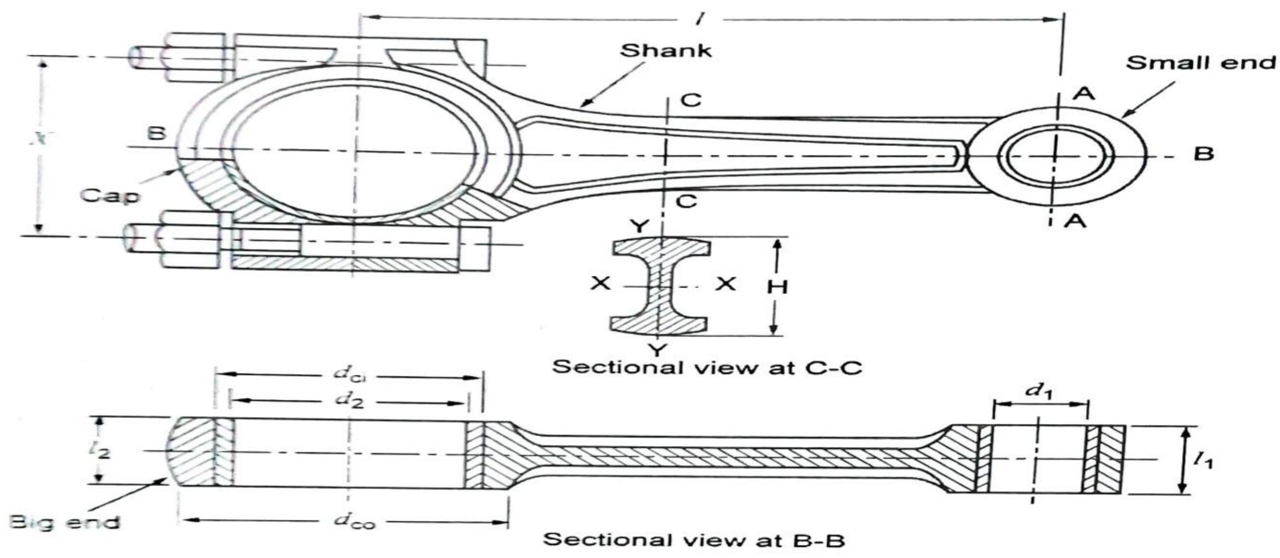


Figure 4.40 Connecting rod

4.3.1. Stresses in Connecting Rod

A connecting rod is subjected to alternating tension and compression. The compressive stress being much greater than tensile stress and it is therefore mainly designed as a strut. The stresses in the connecting rod are set up by a combination of forces. The various forces acting on the connecting rod are as follows.

1. The combined effects of gas pressure on the piston and the inertia of the reciprocating parts.
2. Inertia of the connecting rod.
3. Friction of the piston rings and of the piston.
4. The friction of the two-end bearings.

4.3.1.1. Force Due to Gas Pressure and Piston Inertia

The direct load on the piston due to gas pressure is transformed to the connecting rod. This force can be calculated with the help of an indicator diagram. Otherwise, empirically

Force due to gas pressure, $F_G = \frac{\pi}{4} d^2 \times p$ [Refer also PSGDB 7.122]

Force due to inertia of reciprocating parts, $F_I = \frac{R}{g} \omega^2 \left[\cos\theta + \frac{\cos 2\theta}{(l/r)} \right]$ [ref. PSGDB 7.122]

where d = Diameter of piston

p = Gas pressure

R = Weight of the reciprocating parts

r = Radius of the crank

ω = Angular velocity

l = Length of the connecting rod.

The net force acting on the piston will be algebraic sum of the gas force and the inertia force.

$F = F_G \pm F_I$ [Refer also PSGDB 7.122]

4.3.1.2. Force Due to Inertia of Connecting Rod

The small end of the connecting rod has a motion of pure translation and the big end a rotary motion. The inertia of connecting rod produces two types of forces namely longitudinal force along the length of the rod and a force normal to connecting rod called *normal force*. The longitudinal component is taken into account by considering the one third portion of the connecting rod on the small end side as reciprocating. The remaining two third can be assumed as rotating with crank which produces a normal force and due to normal load a centrifugal force will act on connecting rod shown in Figure 4.41. This force will try to bend the connecting rod. The inertia forces act opposite to the direction of the centrifugal force. This action is termed as *whipping action* and the stress induced due to the whipping action is called ***whipping stress***.

Inertia force on the connecting rod at $\frac{1}{3} l$ from the crank pin,

$P = \frac{\gamma}{2g} a l \omega^2 r$ for $\alpha = 90°$ [Refer also PSGDB 7.122]

$\therefore$ Maximum bending stress, $\sigma_{bmax} = \dfrac{M_{\max}}{Z_{xx}} = \dfrac{2l}{9\sqrt{3}} \times \dfrac{\gamma}{2g} a\, l\, \omega^2\, r \times \dfrac{1}{Z_{xx}}$

$$\sigma_{bmax} = \frac{\gamma\, a\, l^2\, \omega^2\, r}{9\sqrt{3}\, g\, Z_{xx}}$$ [Refer also PSGDB 7.122]

where Z_{xx} = Section modulus.

4.3.1.3. Force Due to Friction of Piston and Piston Rings

The friction force due to piston and piston rings may be determined by the following relation

$$P_f = h\, \pi\, d\, z\, p_r\, \mu$$

where

z = number of rings

d = diameter of cylinder or piston

h = axial width of the ring

p_r = pressure of rings = 0.0245 to 0.042 N/mm^2

μ = Coefficient of friction = 0.01 (about)

This force is quite small and can be neglected.

4.3.1.4. Friction of the Two End Bearings

The effect of the friction at the two end bearings is to bend the connecting rod and it tries to increase the compressive stress on the connecting rod due to the direct load. The maximum bending moment will be at sections where the shank of the connecting rod joints the piston pin and the crank pin ends.

4.3.2. Design Of Shank Of The Connecting Rod

The shank of the connecting rod may be of circular section, rectangular section or I-section. Connecting rods of circular and rectangular sections are generally used in low speed engines where as in high speed engines connecting rod of I-section is preferred. In high-speed engines, the weight of the connecting rod should be as small as possible without sacrificing strength so that the inertia forces remain small. Considering these two aspects, the most suitable section for the shank is the I-section. The usual proportions chosen for the I-section are shown in Figure 4.42 [Refer also PSGDB 7.122].

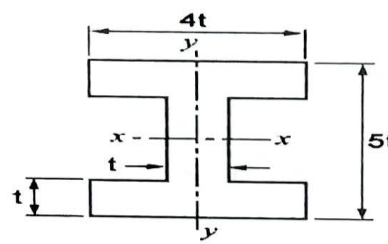


Figure 4.42 I-section

When the gas force acts on the connecting rod, it behaves like a strut with both ends hinged. Therefore, a connecting rod can be equally strong if it satisfies the condition,

$$I_{xx} = 3 \text{ to } 3.5\ I_{yy}$$

where I_{xx} and I_{yy} are the moment of inertia about the x-x and y-y axis respectively.

For the usual proportion shown in Figure 4.42 [Refer also PSGDB 7.122],

Area of cross section, $a = 11t^2$

$$I_{xx} = \frac{419}{12} t^4$$

$$I_{yy} = \frac{131}{12} t^4$$

$$K^2_{xx} = 3.18t^2$$

The crippling stress induced in the rod can be computed by the Rankine's formula.

$$F_{cr} = \frac{\sigma_C . a}{1 + C\left(\frac{L}{K_{xx}}\right)^2}$$

where σ_C = Crippling stress

F_{cr} = Crippling load = $F_G \times FOS$

FOS = Factor of safety = 6 to 15 for heavy shock

= 4 to 7 for light shock

= 3 to 5 for steady load

C = Rankine constant = $\frac{1}{1600}$ for Cast iron

= $\frac{1}{7500}$ for Mild steel

4.3.3. Design Of Pin For Small End (i.e. Piston pin)

Let L_1 – Length of the small end pin

d_1 – Diameter of the small end pin

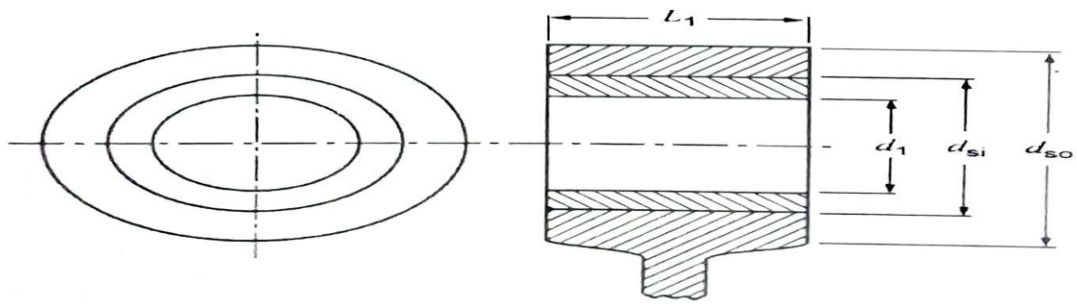


Figure 4.43 Small end of the connecting rod

The usual proportion of these two is given by

$$\frac{L_1}{d_1} = 1.5 \text{ to } 2$$

The design of piston pin is based on bearing pressure and load due to gas pressure

$$F_G = L_1 \times d_1 \times p_{b_1}$$

where p_{b_1} = Bearing pressure = 10.5 to 15.0MPa for small end pin

The other dimension of the small end of the connecting rod is given by

(i) Inner diameter of the small end, $d_{si} = (1.1 \text{ to } 1.25)\, d_1$

(ii) Outer diameter of the small end, $d_{so} = (1.25 \text{ to } 1.65)\, d_1$.

4.3.4. Design Of Pin For Big End

Let L_2 – Length of the big end pin

d_2 – Diameter of the big end pin

The usual proportion of these two is

$$\frac{L_2}{d_2} = 1.25 \text{ to } 1.5$$

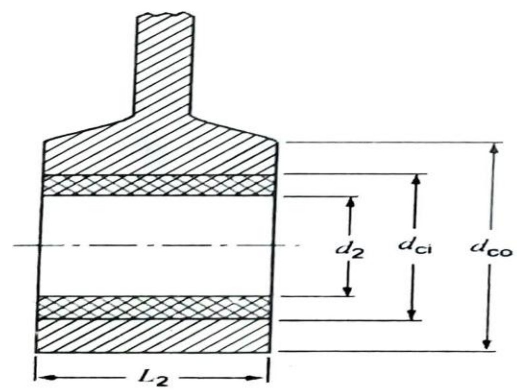


Figure 4.44 Big end of the connecting rod

The design of big end pin is also based on the load due to gas pressure and bearing pressure of the crank pin end.

$$F_G = L_2 \times d_2 \times p_{b_2}$$

where p_{b_2} = Bearing pressure at the crank pin end = 5 to 11 N/mm^2

Bush thickness,

$$t_{bush} = (0.03 \text{ to } 0.1)\, d_2$$

Distance between the centers of bolts (refer Figure 4.44),

$$X = (1.3 \text{ to } 1.75)\, d_2$$

The other dimension of the big end of the connecting rod is given by

(i) Inner diameter of the big end, $d_{ci} = d_2 + 2 \times t_{bush}$

(ii) Outer diameter of the big end, $d_{co} = (1.25 \text{ to } 1.65)\, d_2$

4.3.5. Design Of Bolts For Big End

Let n be the number of bolts used to fasten the big end cap. The core diameter (d_c) of the bolt can be computed by using the following formula.

$$F_i = n \times \frac{\pi}{4} \times d_c^2 \times \tau$$

where

F_i = Inertia force due to reciprocating parts

n = number of bolts

τ = Shear strength of the bolt material

= 100 N/mm^2 for Mild steel

The nominal diameter (d_b) of the bolt,

$$d_b = \frac{d_c}{0.84}$$

4.3.6. Design Of Cap Of The Big End

The cap of the big end is designed as a beam supported at the bolt center and may be assumed to be loaded with concentrated load at centre.

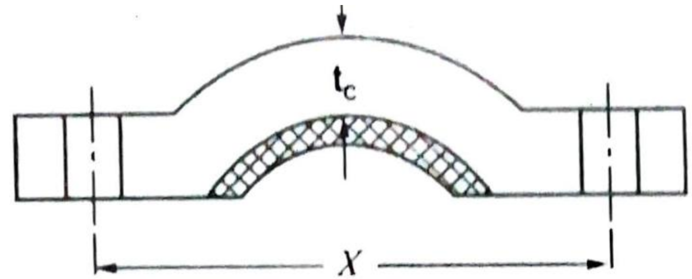


Figure 4.45 Cap of the big end of the connecting rod

The thickness of the cap is computed by the relation,

Bending moment, $M_c = \dfrac{F_i \times X}{6}$

where X = Distance between bolt centers = (1.3 to 1.75) d_2

We also know that $M_c = \sigma_b \times Z$

where Z = section modulus = $\dfrac{b\,t_c^2}{6}$

σ_b may be assumed as $120 N/mm^2$

t_c = thickness of the cap.

Design a suitable connecting rod for a petrol engine for the following details.

Diameter of the piston = 100 mm

Weight of reciprocating parts per cylinder = 20 N

Connecting rod length = 300 mm

Compression ratio = 7:1

Maximum explosion pressure = 3 N/mm²

Stroke = 140 mm

Speed of the engine = 2000 rpm. ***[Madras University-2002]***

Given data:

Diameter of the piston, $d = 100\ mm$

Weight of reciprocating parts per cylinder, $R = 20\ N$

Connecting rod length, $l = 300\ mm$

Compression ratio = 7:1

Maximum explosion pressure, $p = 3\ N/mm^2$

Stroke = 140 mm

Speed of the engine, $N = 2000\ rpm$

☺ ***Solution:***

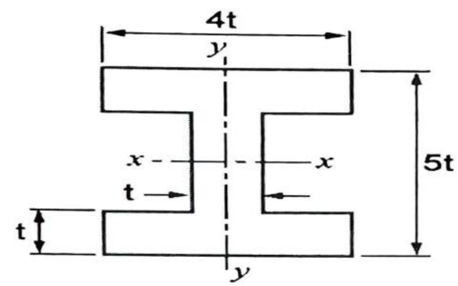


Figure 4.46

The connecting rod used in I.C. Engine is mostly of I-section with the proportions shown in Figure 4.46.

For the selected I-section, from PSGDB 7.122

Area of the section, $a = 11t^2$

Moment of inertia of the section about X-axis,

$$I_{xx} = \frac{419}{12} t^4$$

Moment of inertia of the section about Y-axis, $I_{yy} = \dfrac{131}{12} t^4$

The ratio $\dfrac{I_{xx}}{I_{yy}}$ should be 3 to 3.5 for safety design.

For the selected I-section, $\dfrac{I_{xx}}{I_{yy}} = 3.2$ which is satisfactory.

(i) Dimensions of I-section:

Load due to maximum explosion pressure,

$$F_G = \frac{\pi}{4} \times d^2 \times p$$

$$F_G = \frac{\pi}{4} \times 100^2 \times 3 = 23561.95\ N$$

We know that radius of gyration about the x-axis

$$K^2_{xx} = 3.18t^2$$

Rankine's formula for buckling load (F_{cr})

$$F_{cr} = \frac{\sigma_c . a}{1 + C\left(\frac{l}{K_{xx}}\right)^2} \quad \ldots (4.98)$$

$$F_{cr} = \text{Factor of safety} \times F_G$$

$$F_{cr} = 6 \times 23561.95 = 141371.7\ N \quad [\text{Assume } FOS = 6]$$

For cast iron, $C = \frac{1}{1600}$

For mild steel, $C = \frac{1}{7500}$

We assume that the connecting rod is made of mild steel,

Assume the stress value for mild steel, $\sigma_c = 330\ N/mm^2$

$$141371.7 = \frac{330 \times 11t^2}{1 + \frac{1}{7500}\left(\frac{(300)^2}{3.18t^2}\right)}$$

$$141371.7 = \frac{330 \times 11t^4}{t^2 + 3.77}$$

$$t^4 - 38.94\ t^2 - 3.92 \times 10^{-3} = 0$$

$$\therefore \quad t = 4.68\ mm$$

Thickness of I-section, $t = 5\ mm$ **Ans.**

Height of the I-section, $H = 5t = 5 \times 5 = 25\ mm$ **Ans.**

Width of the I-section, $B = 4t = 4 \times 5 = 20\ mm$ **Ans.**

(ii) Design of pin for small end:

Length of small end pin $= L_1$

Diameter of small end pin $= d_1$

We know that $\dfrac{L_1}{d_1} = 1.5 \text{ to } 2$

$$\text{Let, } \frac{L_1}{d_1} = 1.75$$

$$L_1 = 1.75 d_1$$

Load due to steam pressure,

$$F_G = L_1 \times d_1 \times p_{b_1}$$

Assume, bearing pressure for small end,

$$p_{b_1} = 13 \ N/mm^2$$

$$23561.95 = 1.75\, d_1 \times d_1 \times 13$$

$$\therefore d_1 = 32.18 \ mm$$

Say, diameter of small endpin,

$$d_1 = \mathbf{33 \ mm}$$

$$L_1 = 1.75 \times 33 = 57.75 \ mm$$

Say, Length of small endpin,

$$L_1 = \mathbf{58 \ mm}$$

(iii) Design of pin for big end:

$$\frac{L_2}{d_2} = 1.375 \qquad \left[\because \frac{L_2}{d_2} = 1.25 \text{ to } 1.5\right]$$

$$L_2 = 1.375 d_2$$

Assume, bearing pressure for big end, $p_{b_2} = \mathbf{8} \ N/mm^2$

Load due to steam pressure, $F_G = L_2 \times d_2 \times p_{b_2}$

$$23561.95 = 1.375 d_2 \times d_2 \times \mathbf{8}$$

$$\therefore d_2 = 46.28 \ mm$$

Say, diameter of big end pin, $d_2 = \mathbf{47 \ mm}$

$$L_2 = 1.375 \times 47 = 64.62 mm$$

Say, Length of big end pin, $L_2 = 65\ mm$ **Ans.**

(iv) Diameter of bolts:

Angular velocity,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 2000}{60} = 209.439\ rad/s$$

Radius of the crank, $r = \frac{stroke}{2} = \frac{140}{2} = 70\ mm$

Inertia force (F_i) due to reciprocating parts will be maximum when $\theta = 0$,

$$F_i = \frac{R}{g} \times \omega^2 \times r \left[\cos\theta + \frac{\cos 2\theta}{l}\right]$$

$$F_i = \frac{20}{9.81} \times (209.439^2) \times 0.07\left[1 + \frac{0.07}{0.3}\right] = 7720.66\ N$$

Inertia force, $\boxed{F_i = n \times \frac{\pi}{4} \times d_c^{\ 2} \times \tau}$

$$7720 = 4 \times \frac{\pi}{4} \times d_c^{\ 2} \times 100 \times 10^6 \qquad [\because \tau = 100 N/mm^2 ;\ n = 4]$$

$$\therefore \qquad d_c = 4.957 \times 10^{-3}\ m = 4.957\ mm$$

Say core diameter of bolt, $d_c = \mathbf{5\ mm}$ **Ans.**

Nominal diameter of bolt, $d_b = \frac{d_c}{0.84} = \frac{5}{0.84} = 5.95\ mm$

Say diameter of bolt, $d_b = \mathbf{6\ mm}$ **Ans.**

(v) Thickness of big end cap, t_c :

Distance between the bolt centers, $X = (1.3 \text{ to } 1.75)\ d_2$

$$X = 1.5\ d_2 = 1.5 \times 47 = 70.5\ mm$$

Maximum bending moment acting on the cap,

$$M_c = \frac{F_i \times X}{6} = \frac{772066 \times 70.5}{6} = 90717.755\ mm$$

Section modulus, $Z = \frac{bt_c^2}{6}$

Width of cap, $b = L_2 = 65\ mm$

$$Z = \frac{65 \times t_c^2}{6} = 10.8\, t_c^2$$

$$\boxed{M_c = \sigma_b \times Z}$$

$$90717.755 = 120 \times 10.8\, t_c^2 \qquad (\text{Assume } \because \sigma_b = 120\ N/mm^2)$$

$$\therefore \quad t_c = 8.366\ mm$$

Say thickness of big end cap,

$$\boldsymbol{t_c = 8.5\ mm}$$ **Ans.** ☞

UNIT V
BEARINGS

Sliding contact and rolling contact bearings - Hydrodynamic journal bearings, Sommerfeld Number, Raimondi and Boyd graphs, -- Selection of Rolling Contact bearings.

Introduction

- A bearing is a machine element which support another moving machine element (known as journal).
- It permits a relative motion between the contact surfaces of the members, while carrying the load.
- In order to reduce frictional resistance and wear and in some cases to carry away the heat generated, a layer of fluid (known as lubricant) may be provided.

Classification of Bearings

The bearings may be classified in many yet the following are:

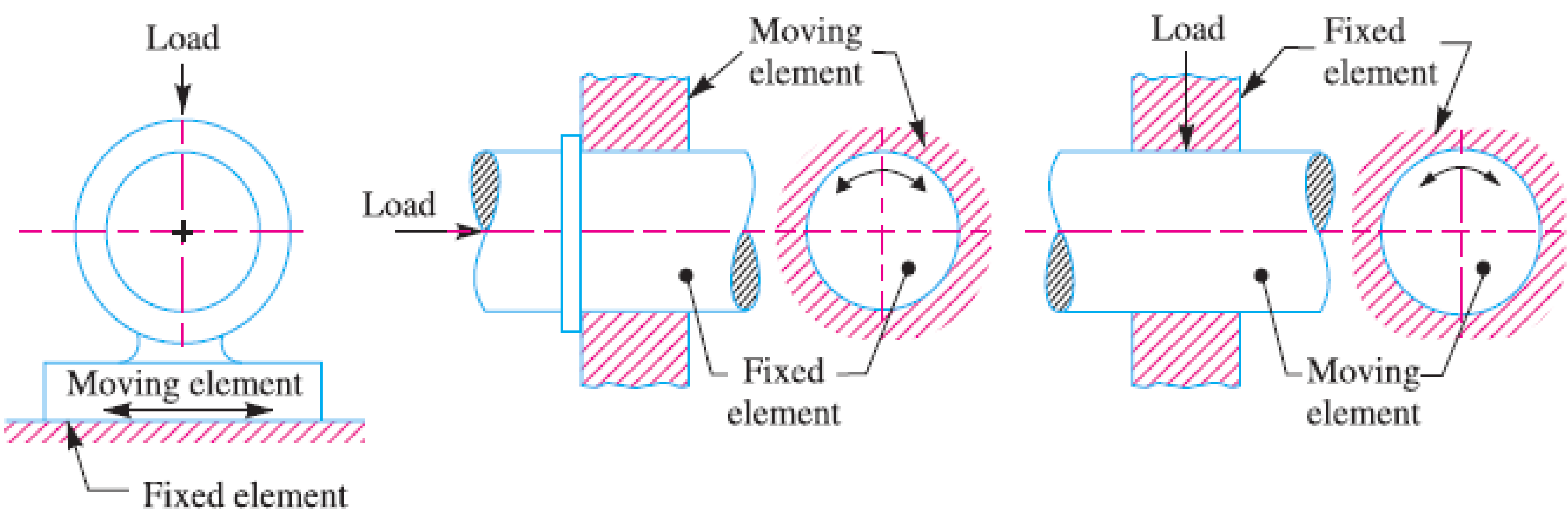
1. *Depending upon the direction of load to be supported.* The bearings under this group are classified as:

(*a*) Radial bearings, and (*b*) Thrust bearings.

In ***radial bearings***, the load acts perpendicular to the direction of motion of the moving element as shown in Fig. 26.1 (*a*) and (*b*).

In ***thrust bearings***, the load acts along the axis of rotation as shown in Fig. 26.1 (*c*).

Note : These bearings may move in either of the directions as shown in Fig. 26.1.

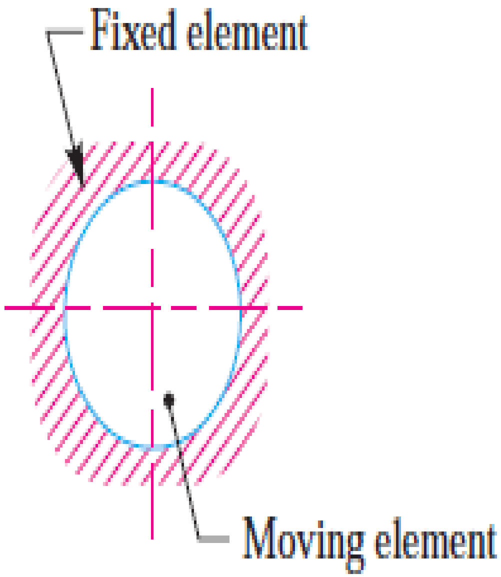


(*a*) Radial bearing. (*b*) Radial bearing. (*c*) Thrust bearing.

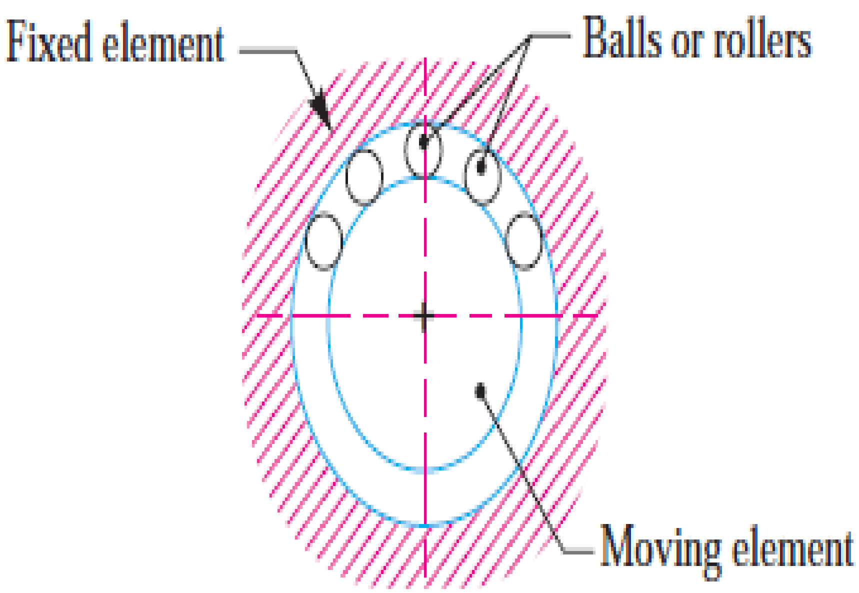
2. ***Depending upon the nature of contact.*** The bearings under this group are classified as :

(*a*) Sliding contact bearings, and (*b*) Rolling contact bearings.

In *sliding contact bearings*, as shown in Fig. 26.2 (*a*), the sliding takes place along the surfaces of contact between the moving element and the fixed element. The sliding contact bearings are also known as *plain bearings.*



(*a*) Sliding contact bearing.



(*b*) Rolling contact bearings.

26.13 Terms used in Hydrodynamic Journal Bearing

A hydrodynamic journal bearing is shown in Fig. 26.7, in which O is the centre of the journal and O' is the centre of the bearing.

Let D = Diameter of the bearing,

d = Diameter of the journal, and

l = Length of the bearing.

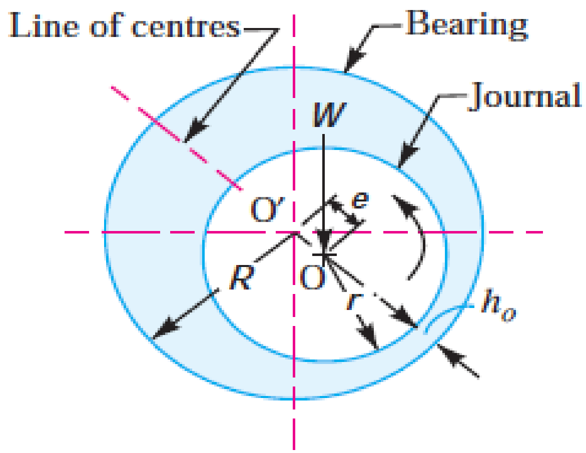


Fig. 26.7. Hydrodynamic journal bearing.

The following terms used in hydrodynamic journal bearing are important from the subject point of view :

1. *Diametral clearance.* It the difference between the diameters of the bearing and the journal. Mathematically, diametral clearance,

$$c = D - d$$

Note : The diametral clearance (c) in a bearing should be small enough to produce the necessary velocity gradient, so that the pressure built up will support the load. Also the small clearance has the advantage of decreasing side leakage. However, the allowance must be made for manufacturing tolerances in the journal and bushing. A commonly used clearance in industrial machines is 0.025 mm per cm of journal diameter.

2. *Radial clearance.* It is the difference between the radii of the bearing and the journal. Mathematically, radial clearance,

$$c_1 = R - r = \frac{D - d}{2} = \frac{c}{2}$$

3. *Diametral clearance ratio.* It is the ratio of the diametral clearance to the diameter of the journal. Mathematically, diametral clearance ratio

$$= \frac{c}{d} = \frac{D - d}{d}$$

4. ***Eccentricity.*** It is the radial distance between the centre (O) of the bearing and the displaced centre (O') of the bearing under load. It is denoted by e.

5. ***Minimum oil film thickness.*** It is the minimum distance between the bearing and the journal, under complete lubrication condition. It is denoted by h_0 and occurs at the line of centres as shown in Fig. 26.7. Its value may be assumed as $c / 4$.

6. ***Attitude or eccentricity ratio.*** It is the ratio of the eccentricity to the radial clearance. Mathematically, attitude or eccentricity ratio,

$$\varepsilon = \frac{e}{c_1} = \frac{c_1 - h_0}{c_1} = 1 - \frac{h_0}{c_1} = 1 - \frac{2h_0}{c} \qquad \ldots (\because\ c_1 = c / 2)$$

7. ***Short and long bearing.*** If the ratio of the length to the diameter of the journal (*i.e.* l / d) is less than 1, then the bearing is said to be **short bearing.** On the other hand, if l / d is greater than 1, then the bearing is known as ***long bearing.***

Axle bearings

Notes : 1. When the length of the journal (l) is equal to the diameter of the journal (d), then the bearing is called ***square bearing.***

2. Because of the side leakage of the lubricant from the bearing, the pressure in the film is atmospheric at the ends of the bearing. The average pressure will be higher for a long bearing than for a short or square bearing. Therefore, from the stand point of side leakage, a bearing with a large l / d ratio is preferable. However, space requirements, manufacturing, tolerances and shaft deflections are better met with a short bearing. The value of l / d may be taken as 1 to 2 for general industrial machinery. In crank shaft bearings, the l / d ratio is frequently less than 1.

Rolling Contact Bearings

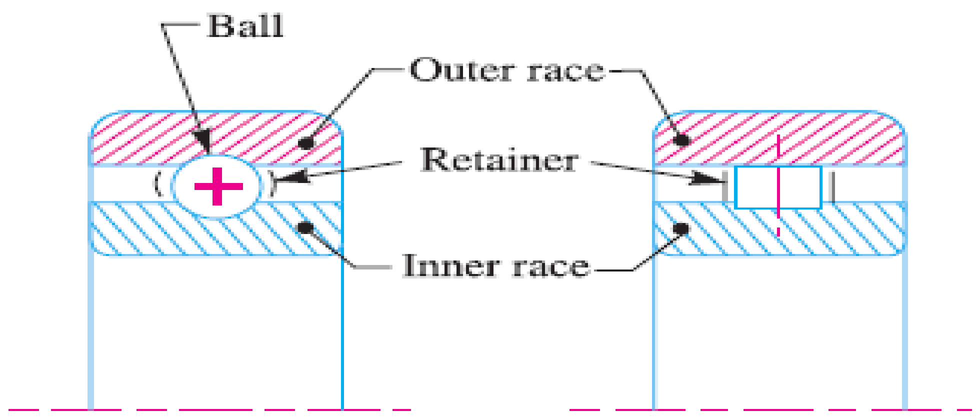
- In rolling contact bearings, the contact between the bearing surfaces is rolling instead of sliding as in sliding contact bearings.
- We have already discussed that the ordinary sliding bearing starts from rest with practically metal-to-metal contact and has a high coefficient of friction.
- It is an outstanding advantage of a rolling contact bearing over a sliding bearing that it has a low starting friction.
- Due to this low friction offered by rolling contact bearings, these are called antifriction bearings.

Types of Rolling Contact Bearings

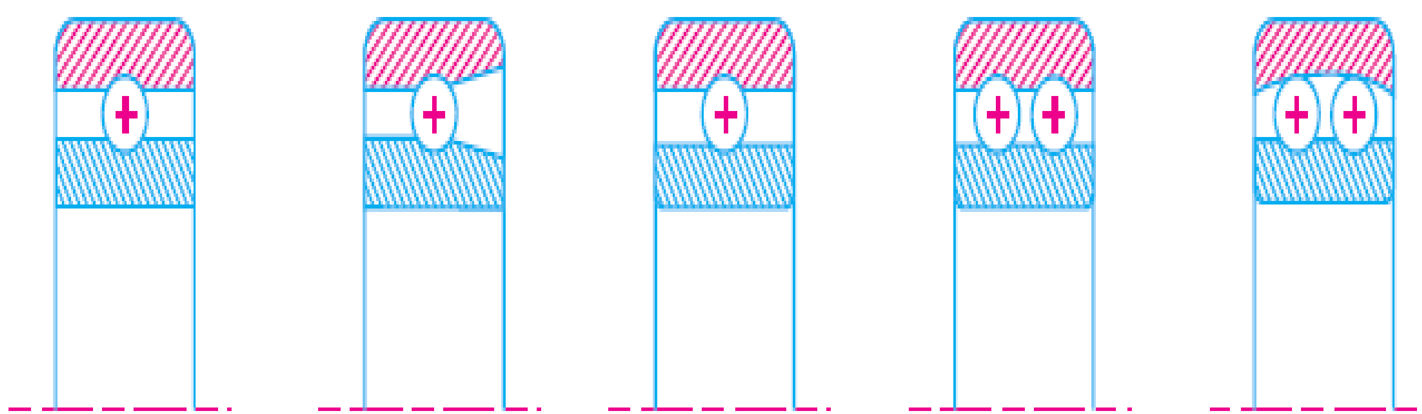
- Following are the two types of rolling contact bearings:

1. Ball bearings; and 2. Roller bearings.

- The ball and roller bearings consist of an inner race which is mounted on the shaft or journal and an outer race which is carried by the housing or casing. In between the inner and outer race, there are balls or rollers
- A number of balls or rollers are used and these are held at proper distances by retainers so that they do not touch each other. The retainers are thin strips and is usually in two parts which are assembled after the balls have been properly spaced.
- The ball bearings are used for light loads and the roller bearings are used for heavier loads.



(*a*) Ball bearing. (*b*) Roller bearing.



(*a*) Single row deep groove. (*b*) Filling notch. (*c*) Angular contact. (*d*) Double row. (*e*) Self-aligning.

The rolling contact bearings, depending upon the load to be carried, are classified as :

(a) Radial bearings, and (b) Thrust bearings.

- When a ball bearing supports only a radial load (WR), the plane of rotation of the ball is normal to the centre line of the bearing,
- The action of thrust load (WA) is to shift the plane of rotation of the balls The radial and thrust loads both may be carried simultaneously.

Types of Radial Ball Bearings

- Following are the various types of radial ball bearings:

1. Single row deep groove bearing.

- **A single row deep groove bearing** During assembly of this bearing, the races are offset and the maximum number of balls are placed
- between the races. The races are then centred and the balls are symmetrically located by the use of a retainer or cage. The deep groove ball bearings are used due to their high load carrying capacity and suitability for high running speeds.
- The load carrying capacity of a ball bearing is related to the size and number of the balls.

2. Filling notch bearing.

- These bearings have notches in the inner and outer races which permit more balls to be inserted than in a deep groove ball bearings.
- The notches do not extend to the bottom of the race way and therefore the balls inserted through the notches must be forced in position.
- Since this type of bearing contains larger number of balls than a corresponding unnotched one, therefore it has a larger bearing load capacity.

3. *Angular contact bearing*.

- have one side of the outer race cut away to permit the insertion of more balls than in a deep groove bearing but without having a notch cut into both races. This permits the bearing to carry a relatively large axial load in one direction while also carrying a relatively large radial load.
- The angular contact bearings are usually used in pairs so that thrust loads may be carried in either direction.

4. *Double row bearing*.

- These bearings may be made with radial or angular contact between the balls and races. The double row bearing is appreciably narrower than two single row bearings.
- The load capacity of such bearings is slightly less than twice that of a single row bearing.

5. Self-aligning bearing.

- These bearings permit shaft deflections within 2-3 degrees. It may be noted that normal clearance in a ball bearing are too small to accommodate any appreciable misalignment of the shaft relative to the housing.
- If the unit is assembled with shaft misalignment present, then the bearing will be subjected to a load that may be in excess of the design value and premature failure may occur. Following are the two types of

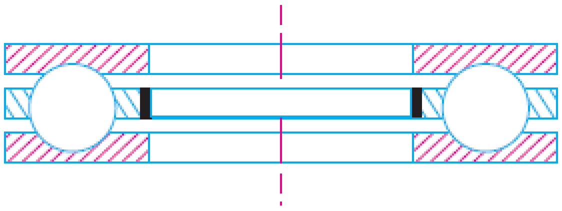
self-aligning bearings :

(a) Externally self-aligning bearing,

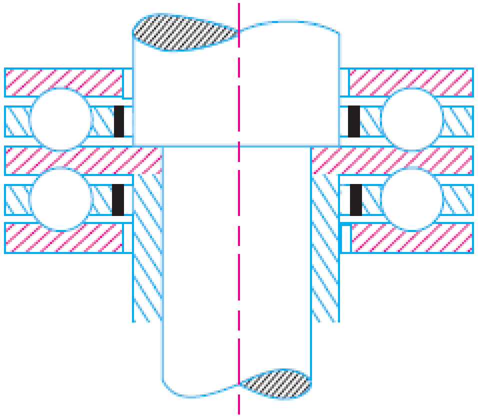
(b) Internally self-aligning bearing.

Thrust Ball Bearings

- The thrust ball bearings are used for carrying thrust loads exclusively and at speeds below 2000 r.p.m. At high speeds, centrifugal force causes the balls to be forced out of the races. Therefore at high speeds, it is recommended that angular contact ball bearings should be used in place of thrust ball bearings.



(*a*) Single direction thrust ball bearing.



(*b*) Double direction thrust ball bearing.

Types of Roller Bearings

- Following are the principal types of roller bearings :

1. Cylindrical roller bearings.

- These bearings have short rollers guided in a cage. These bearings are relatively rigid against radial motion and have the lowest coefficient of friction of any form of heavy duty rolling-contact bearings. Such type of bearings are used in high speed service.

2. Spherical roller bearings.

- These bearings are self-aligning bearings. The self-aligning feature is achieved by grinding one of the races in the form of sphere.
- These bearings can normally tolerate angular misalignment in the order of $\pm 1.5^{\circ}$ and when used with a double row of rollers, these can carry thrust loads in either direction.

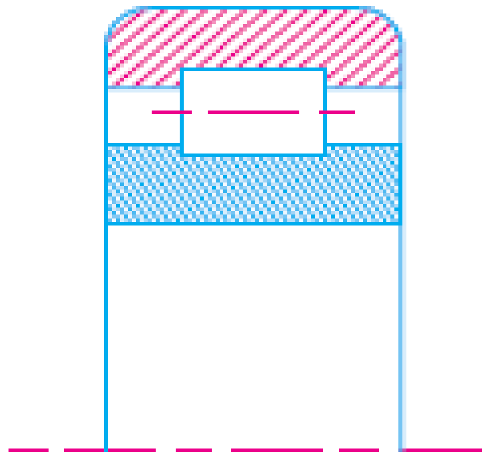
3. Needle roller bearings.

- These bearings are relatively slender and completely fill the space so that neither a cage nor a retainer is needed. These bearings are used when heavy loads are to be carried with an oscillatory motion, e.g. piston pin bearings in heavy duty diesel engines, where the reversal of motion tends to keep the rollers in correct alignment.

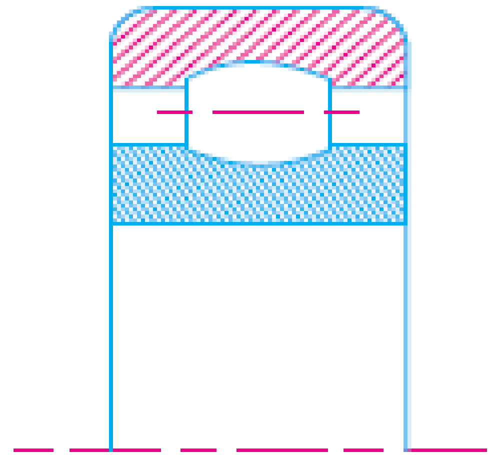
4. Tapered roller bearings.

The rollers and race ways of these bearings are truncated cones whose elements intersect at a common point. Such type of bearings can carry both radial and thrust loads.

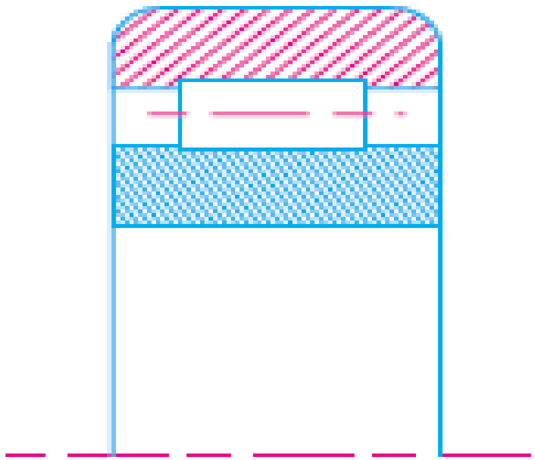
These bearings are available in various combinations as double row bearings and with different cone angles for use with different relative magnitudes of radial and thrust loads.



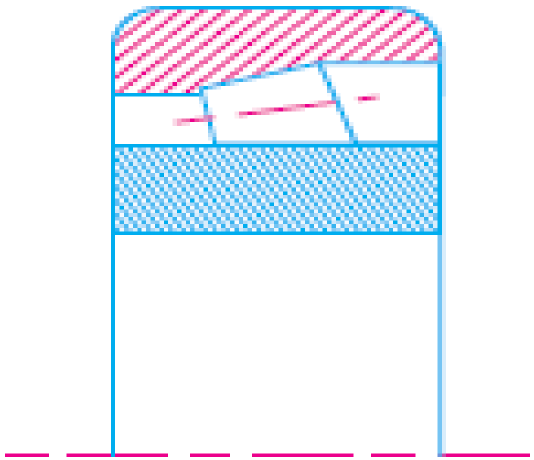
(*a*) Cylindrical roller.



(*b*) Spherical roller.



(*c*) Needle roller.



(*d*) Tapered roller.

Advantages

1. Low starting and running friction except at very high speeds.
2. Ability to withstand momentary shock loads.
3. Accuracy of shaft alignment.
4. Low cost of maintenance, as no lubrication is required while in service.
5. Small overall dimensions.
6. Reliability of service.
7. Easy to mount and erect.
8. Cleanliness.

Disadvantages

1. More noisy at very high speeds.
2. Low resistance to shock loading.
3. More initial cost.
4. Design of bearing housing complicated.

5.3.8. Load Rating

The load carrying capacity of a rolling element bearing is called *load rating*. There are two types of load rating.

5.3.8.1. Basic Static Load Rating

It is defined as the load acting on a non-rotating bearing under which permanent deformation of 0.0001 times the ball or roller diameter. The basic static load rating C_o is used in calculation when bearings are to rotate at very slow speed or stationary under the load for extended periods of time. Generally, the values of C_o are given in catalogues.

5.3.8.2. Dynamic Load Rating

Dynamic load rating is defined as the radial load in radial bearings (or thrust load in thrust bearings) which can be carried for a minimum life of one million revolutions. The minimum life in this definition is the life at which 90% of the bearings will reach or exceed before fatigue failure. The dynamic load rating is based on the assumption that the inner race is rotating while the outer race is stationary. The dynamic load rating is usually provided by the bearing manufacturer.

5.3.9. Equivalent Bearing Load

If a bearing is to be chosen to withstand a combination of radial load (F_r) and thrust load (F_a), on equivalent static load (P) which causes the square deformation as the combined loads, can be computed by the formula.

$$P = (XF_r + YF_a)\,S$$

where

X – Radial factor

Y – Thrust factor

S – Service factor.

Type of bearing	Series (SKF)		$\frac{F_a}{F_r} \le e$		$\frac{F_a}{F_r} > e$		e
			X	Y	X	Y	
Deep groove ball bearing	Series EL, R 160, 60, 62, 63, 64, EE, RLS, RMS	$\frac{F_a}{C_o} = 0.025$				2	0.22
		= 0.04	1	0	0.56	1.8	0.24
		= 0.07				1.6	0.27
		= 0.13				1.4	0.31
		= 0.25				1.2	0.37
		= 0.5				1.0	0.44
Angular contact ball bearing	72B, 73B		1	0	0.35	0.57	1.14
	72BG, 73BG⊕		1	0	0.35	0.57	1.14
	72BG, 73BG♁		1	0.55	0.57	0.93	1.14
	32, 33		1	0.73	0.62	1.17	0.86
Self aligning ball bearing	2200 – 2204			1.3		2	0.5
	2205 – 2207			1.7		2.6	0.37
	2208 – 2209		1	2	0.65	3.1	0.31
	2210 – 2213			2.3		3.5	0.28
	2214 – 2220			2.4		3.8	0.26
	2221 – 2222			2.3		3.5	0.28
Spherical roller bearing	22205C – 22207C			2.1		3.1	0.32
	22208C – 22209C		1	2.5	0.67	3.7	0.27
	22210C – 22220C			2.9		4.4	0.23
	22222C – 22244C			2.6		3.9	0.26
Taper roller bearing	32206 – 32208					1.6	0.37
	32209 – 32222		1	0	0.4	1.45	0.41
	32224 – 32230					1.35	0.44

Type of machinery	*Service factor*
Rotary machine with no impact	1.1 — 1.5
Reciprocating machine	1.3 — 1.9
Machine with pronounced impact, hammer mills etc.	1.6 - 4

The values of factors *X*, *Y* and *S* can be obtained from Table 5.3.2 and Table 5.3.3 (also refer PSGDB 4.4 and PSGDB 4.2).

⊕ - A pair of bearings mounted in tandem

♰ - A pair of bearings mounted back to back or face to face.

5.3.10. Load-Life Relationship

The relationship between the dynamic load carrying capacity (C), the equivalent dynamic load (P) and the bearing life is given by

$$L = \left(\frac{C}{P}\right)^b$$

where

b = Constant

= 3 for ball bearings

= 10/3 for roller bearings

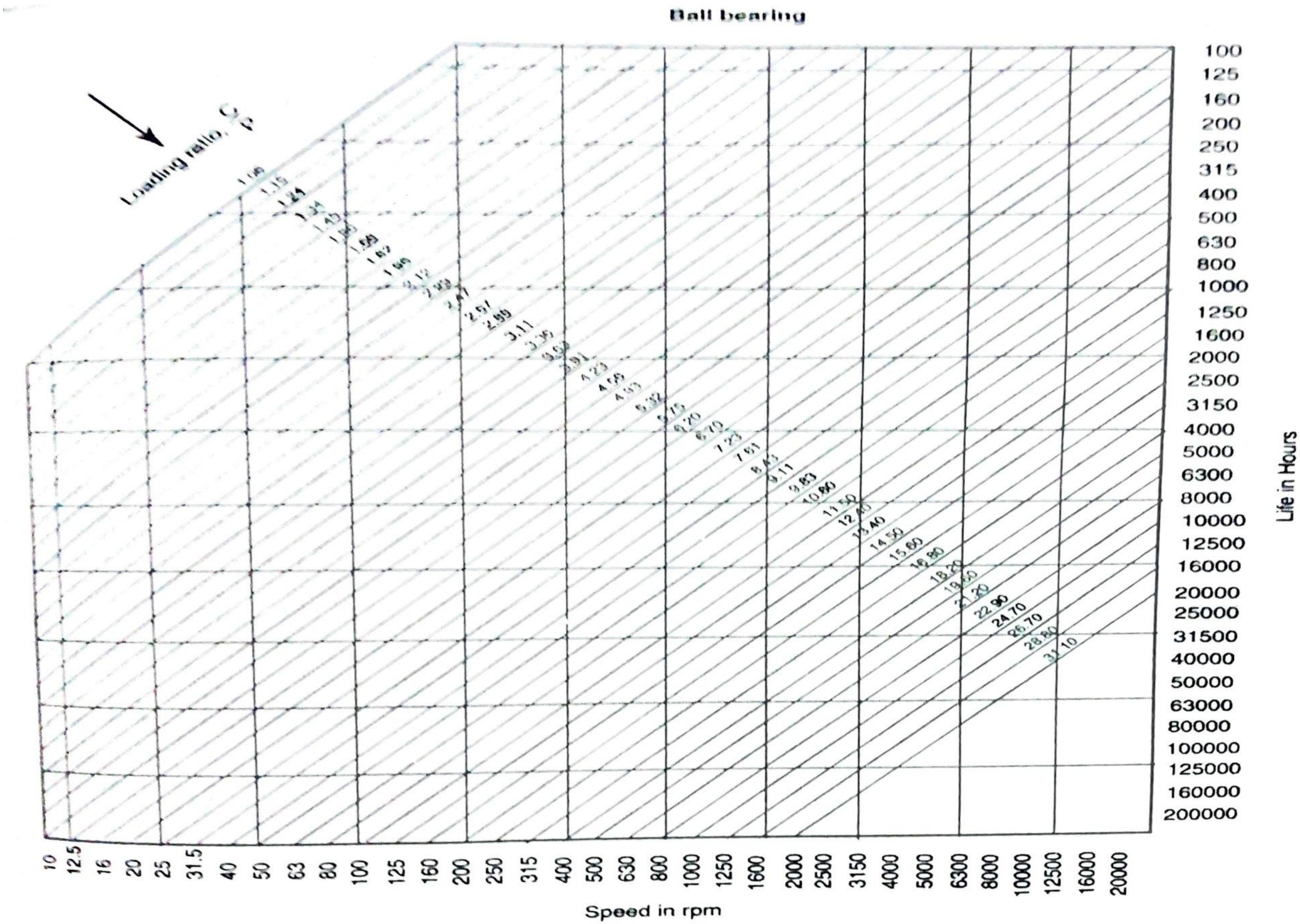
The C/P ratio for corresponding life and speed can be determined by using the graphs in Figure 5.18 and Figure 5.19 (also refer PSGDB 4.6 and 4.7.)

The relationship between life in million revolutions and life in working hours is given by

$$L = \frac{60\, n\, L_h}{10^6}$$

where L_h = Bearing life in hours

n = Speed of rotation in *rpm*



Ball bearing
Loading ratio, C/P
Life in Hours
Speed in rpm
100
125
160
200
250
315
400
500
630
800
1000
1250
1600
2000
2500
3150
4000
5000
6300
8000
10000
12500
16000
20000
25000
31500
40000
50000
63000
80000
100000
125000
160000
200000
10
12.5
16
20
25
31.5
40
50
63
80
100
125
160
200
250
315
400
500
630
800
1000
1250
1600
2000
2500
3150
4000
5000
6300
8000
10000
12500
16000
20000

Figure 5.18

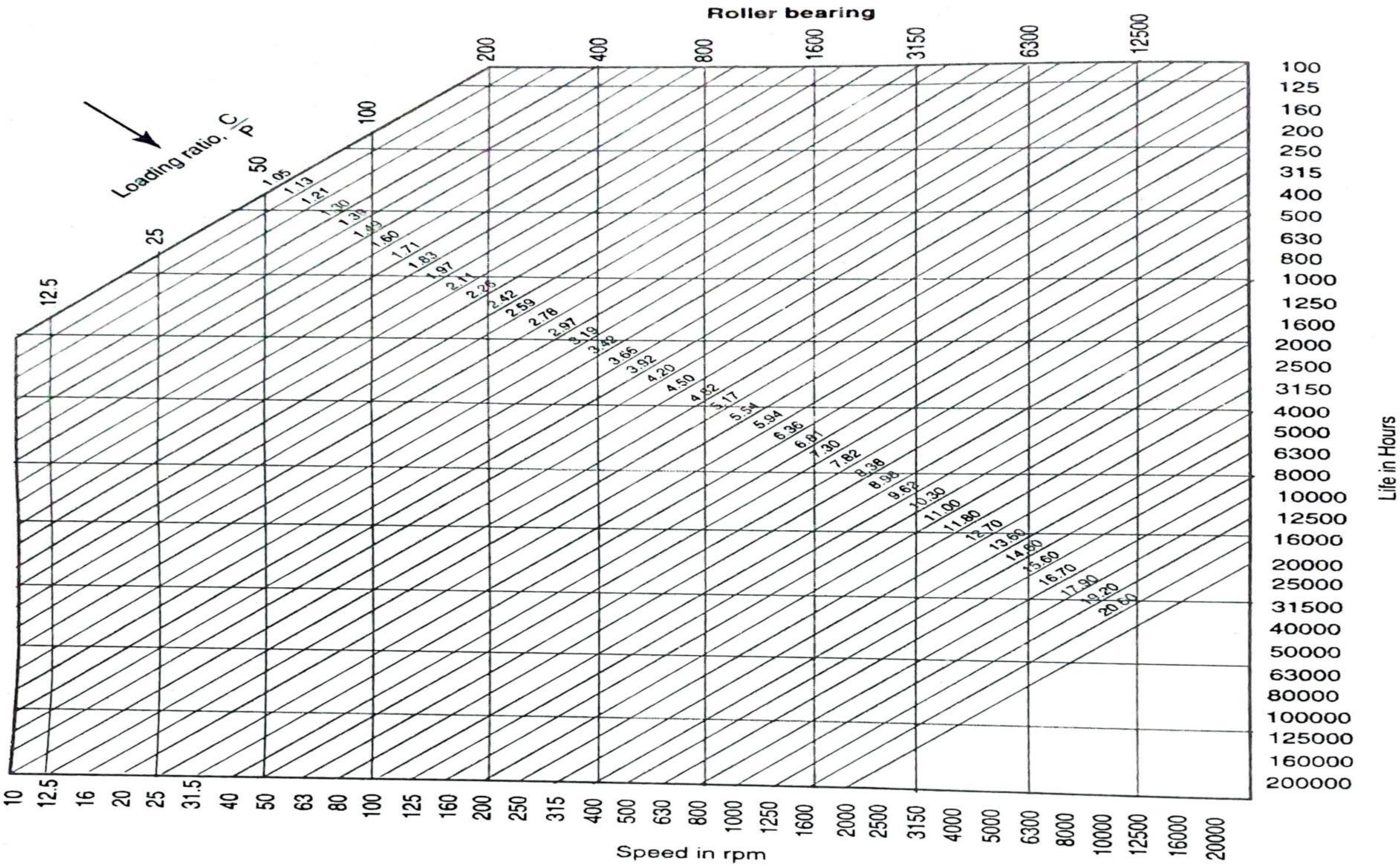


Figure 5.19

5.3.11. Selection Of Rolling Contact Bearings

The general procedure for the selection of bearings from the manufacture's catalogue is given as follows.

(*i*) Calculate the radial and axial forces acting on the bearing.

(*ii*) Calculate the shaft diameter.

(iii) Determine the radial load factor (X) and thrust load factor (Y) from the manufacturer's catalogue. The values of X and Y for ball and roller bearings are given Table 5.3.2 (or) in PSGDB 4.4. The values depend upon two ratios $\left(\frac{F_a}{F_r}\right)$ and $\left(\frac{F_a}{C_o}\right)$, where C_o is the static load capacity. Select the series (60, 62, 63 ...) for the given diameter of the shaft and the value of C_o found. It is in tables in PSGDB 4.12 to 4.20.

(iv) Calculate the equivalent dynamic load from the equation

$$P = (XF_r + YF_a)\, S$$

where S = Service factor. It can be obtained from Table 5.3.3 (or in PSGDB 4.2)

(v) Decide the expected life of the bearing. Convert the expected life in hours into millions of revolutions.

(vi) Calculate the dynamic load capacity from the equation,

$$L = \left(\frac{C}{P}\right)^b$$

(vii) Check whether the selected bearing has the required dynamic load capacity. If yes, the selected bearing is suitable for this purpose. Otherwise, select another bearing from the next series and go back to step (iii) and continue.

Select a bearing for a 40 mm diameter shaft rotating at 400 rpm. Due to a bevel gear mounted on the shaft, the bearing will have to withstand a 5000 N radial load and a 3000 N thrust load. The life of the bearing is expected to be at least 1000 hrs.

Given data:

Diameter of shaft = 40 mm

Speed, $N = 400\ rpm$

Radial load, $F_r = 5000\ N$

Thrust load, $F_a = 3000\ N$

Life, $L_h = 1000\ hrs$

☺ ***Solution:***

$$\frac{F_a}{F_r} = \frac{3000}{5000} = 0.6$$

Since $\frac{F_a}{F_r} < 0.7$, a single row deep groove ball bearing may be suitable.

For the given diameter of shaft $40mm$, from PSGDB 4.13, select SKF6208 bearing. Static load rating, $C_o = 1600kgf = 16000N$

Dynamic load rating, $C = 2280\ kgf = 22800\ N$

$$\frac{F_a}{C_o} = \frac{3000}{16000} = 0.1875$$

From Table 5.3.2 or PSGDB 4.4, corresponding to $\frac{F_a}{C_o} = 0.1875$

$$e = 0.33875 \text{ (by interpolation)}$$

Since $\frac{F_a}{F_r} = \frac{3000}{5000} = 0.6 > e$, from Table 5.3.2 or PSGDB 4.4, the radial load factor

$$X = 0.56 \text{ and } Y = 1.23 \text{ (by interpolation).}$$

The service factor is selected from Table 5.3.3 or PSGDB 4.2 as 1.2

Equivalent load,

$$P = (X\,F_r + Y\,F_a)\,S = (0.56 \times 5000 + 1.23 \times 3000)\ 1.2$$

$$P = 7788\ N$$

From graph in Figure 5.18 or PSGDB 4.6, corresponding to $400rpm$ and $1000hrs$ of life, loading ratio,

$$\frac{C}{P} = 2.9$$

$$\therefore \quad C = 2.9 \times P = 2.9 \times 7788 = 22588.2\ N$$

Since the dynamic load rating of the **SKF6208 bearing** is more than the required dynamic load capacity, the selected bearing is suitable. **Ans.**

A 30BC03 deep groove ball bearing is to operate at 1600 rpm and carries 8 kN radial load and 6 kN thrust load. The bearing is subjected to a light shock load. Determine the rating life of the bearing.

Given data:

Bearing type: *30BC03*

Speed = 1600 *rpm*

Radial load, F_r = 8 kN = 8000 N

Thrust load, F_a = 6 kN = 6000 N

Solution:

From PSGDB 4.14, for *30BC03* (SKF 6306) bearing

$$C_o = 1460\ kgf = 14600\ N$$

$$C = 2200\ kgf = 22000\ N$$

$$\frac{F_a}{C_o} = \frac{6000}{14600} = 0.411$$

From Table 5.3.2 or PSGDB 4.4, corresponding to

$$\frac{F_a}{C_o} = 0.411, \text{ the value of } e = 0.415 \text{ (by interpolation).}$$

Since $\frac{F_a}{F_r} = \frac{6000}{8000} = 0.75 > e$, from Table 5.3.2 or PSGDB 4.4., the radial load factor X = 0.56 and thrust load factor Y = 1.083 (by interpolation). The service factor is selected from Table 5.3.3 or PSGDB 4.2.

$$S = 1.3 \text{ to } 1.9 \text{ say } 1.5$$

Equivalent load, $P = (X F_r + Y F_a)\, S = (0.56 \times 8000 + 1.083 \times 6000)\ 1.5$

$$P = 16467\ N$$

From graph in Figure 5.18 or PSGDB 4.6, corresponding to 1600*rpm* and $\frac{C}{P} = \frac{22000}{16467}$ = 1.34, the rating life of the bearing is 25 *hrs*.

We know that

$$L = \frac{60 n L_h}{10^6} = \frac{60 \times 1600 \times 25}{10^6} = \textbf{2.4 million revolutions} \quad \textbf{Ans.}$$

A ball bearing for drilling machine spindle is rotating at 3000 rpm. It is subjected to a radial load of 2500 N and an axial thrust of 1500 N. It is to work 50 hours per week for one year. Design a suitable bearing if the diameter of the spindle is 40 mm.

Given data:

Diameter of shaft, $d = 40\ mm$

Radial load, $F_r = 2500\ N$

Thrust load, $F_a = 1500\ N$

Speed, $n = 3000\ rpm$

Life, $L_h = 50\ hrs$ per week for one year $= 2600\ hrs$

☺ *Solution:*

From PSGDB 4.13, for $d = 40\ mm$, select a bearing SKF6208. The value of $C_o = 16000\ N$ and $C = 22800\ N$

$$\text{The ratio}\quad \frac{F_a}{C_o} = \frac{1500}{16000} = 0.09375$$

From PSGDB 4.4, corresponding to $F_a/C_o = 0.09375$, $e = 0.30$ (by interpolation)

$$\frac{F_a}{F_r} = \frac{1.5}{2.5} = 0.6 > e$$

$\therefore$ $X = 0.56$ and $Y = 1.425$ (by interpolation)

Assume service factor, $S = 1.2$

Equivalent load, $P = (X F_r + Y F_a) S = (0.56 \times 2500 + 1.425 \times 1500)\ 1.2 = 4245\ N$

From PSGDB 4.6, corresponding to 3000 *rpm* and 2600 *hrs*

Loading ratio,

$$\frac{C}{P} = 7.8$$

$$\therefore \qquad C = 7.8 \times 4245 = 33111\ N$$

which is more than the load capacity of SKF6208. Hence, select another series of bearing. Let us select SKF6408 which has $C_o = 38000 N$, $C = 50000\ N$

$$\frac{F_a}{C_o} = \frac{1500}{38000} = 0.03947$$

$$e = 0.24$$

$$\frac{F_a}{F_r} = 0.6 > e$$

$$\therefore \qquad X = 0.56 \text{ and } Y = 1.8$$

Equivalent load, $P = (0.56 \times 2000 + 1.8 \times 1500)\ 1.2 = 4584\ N$

We know that $\frac{C}{P} = 7.8$

$$\therefore \qquad C = 7.8 \times 4584 = 35755.2\ N$$

which is less than the value of C for **SKF6408**. Hence, it is suitable. **Ans.**