

MOHAMED SATHAK A J COLLEGE OF ENGINEERING

Kinematics of Machinery

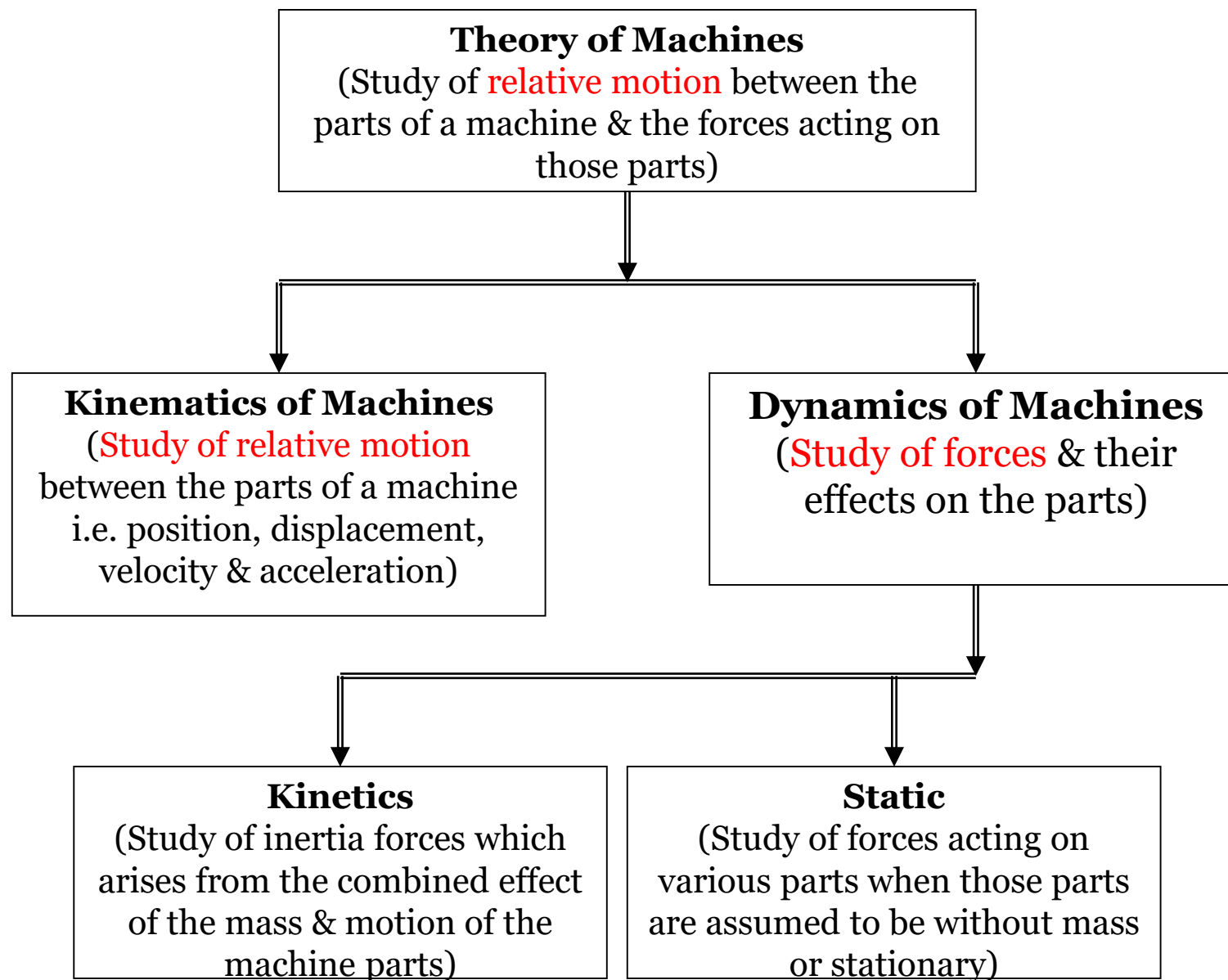
By

Dr.G.RAMESH

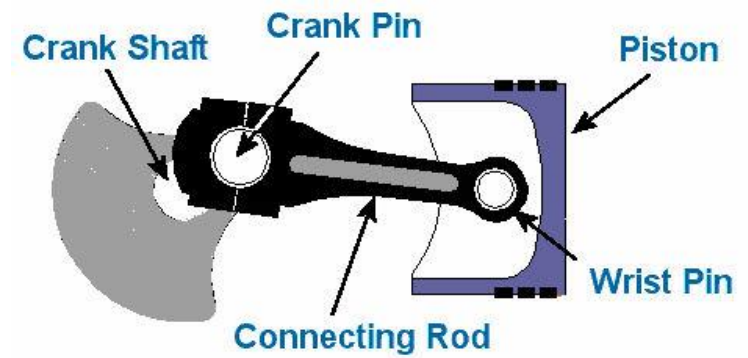
- **Introduction:** Definitions: Link, kinematic pairs, kinematic chain, mechanism, structure, degrees of freedom, Classification links, Classification of pairs based on type of relative motion, Grubler's criterion, mobility of mechanism, Groshoff's criteria, inversions of Grashoff's chain.
- **Mechanisms:** Quick return motion mechanisms-Drag link mechanism, Whitworth mechanism and Crank and slotted lever Mechanism. Oldham's coupling, Straight line motion mechanisms Peaucellier's mechanism and Robert's mechanism. Intermittent Motion mechanisms: Geneva wheel mechanism, Ratchet and Pawl mechanism, toggle mechanism, pantograph, condition for correct steering, Ackerman steering gear mechanism.

INTRODUCTION

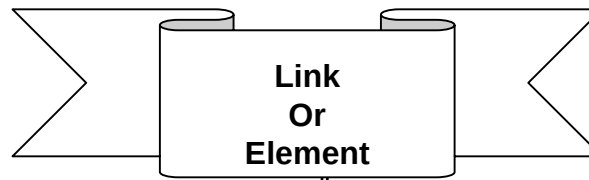
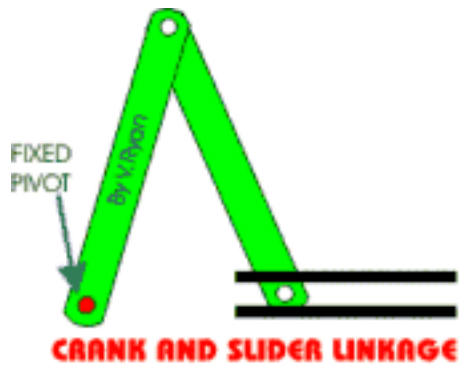




LINKS:

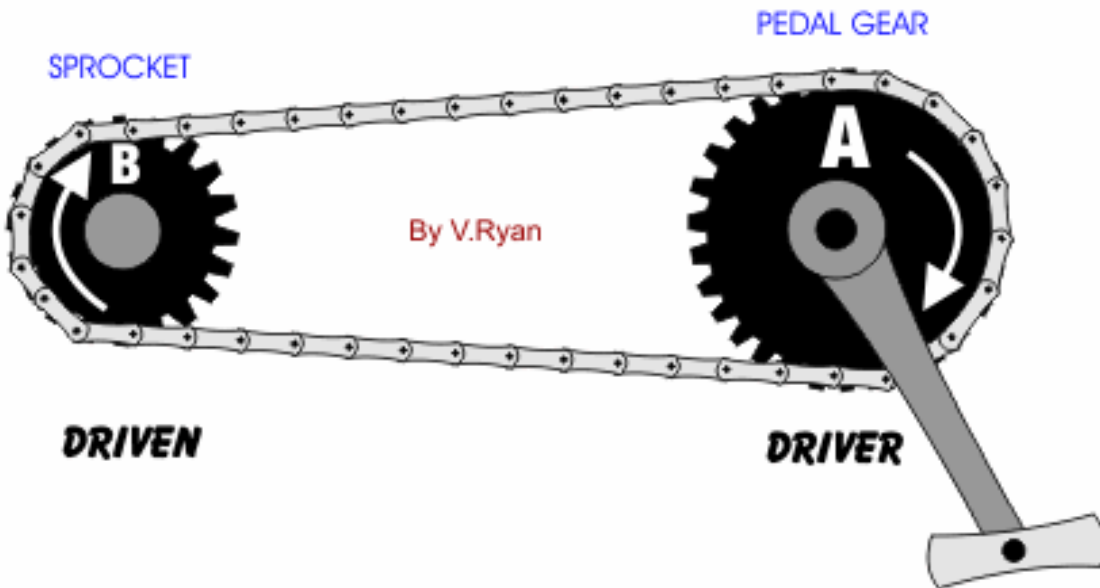
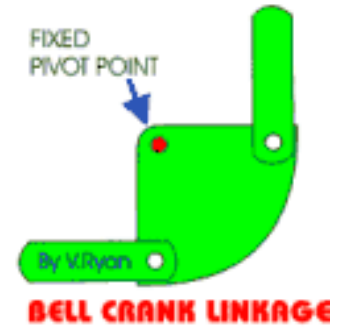


- A link (or element or kinematic link) is a **resistance body** (or assembly of resistance bodies) that constitute the part (parts) of the machine **connecting other parts** which have **motion relative** to it.
- Characteristics:
 - It should have **relative motion**
 - It need not necessarily be rigid body, but it must be a **resistance body** (a body capable of transmitting the required forces with negligible deformation).
- Ex:
 - Liquids which are resistance to compressive forces
 - Chains, belts & ropes, which are resistance to tensile forces
- A link which is stationary and which supports the moving members is called **frame**.



Depending upon the **ends** on which revolute or turning pairs can be placed

Depending upon the **effect** on the link



Rigid link

One which **does not undergo any deformation** while transmitting motion.
Ex: connecting rod, crank etc

Flexible link

One which **partly deforms** in a manner not to affect the transmission of motion.
Ex: belts, ropes, chains etc

Fluid link

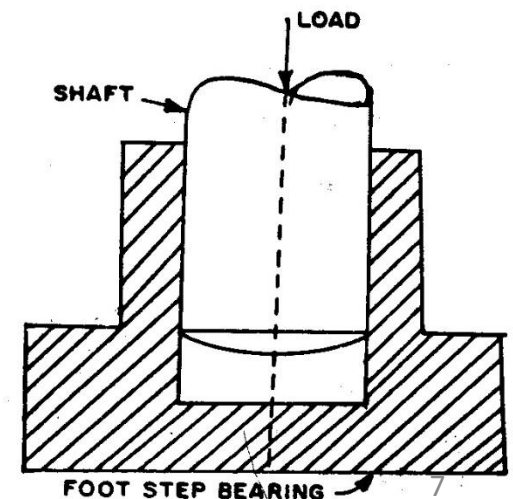
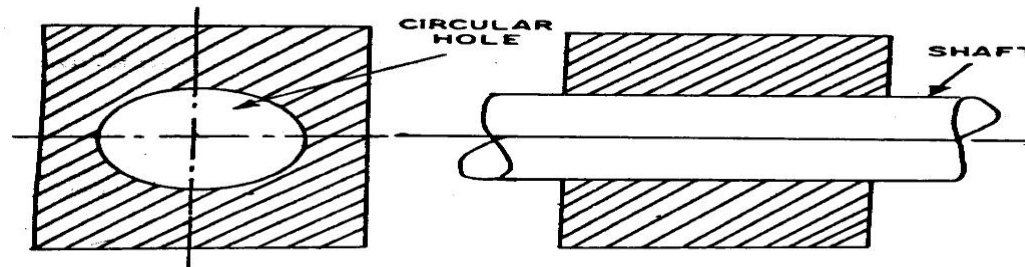
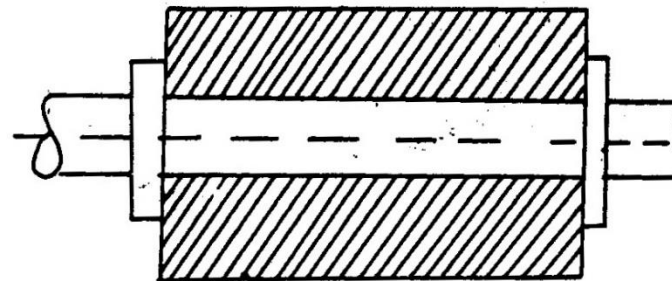
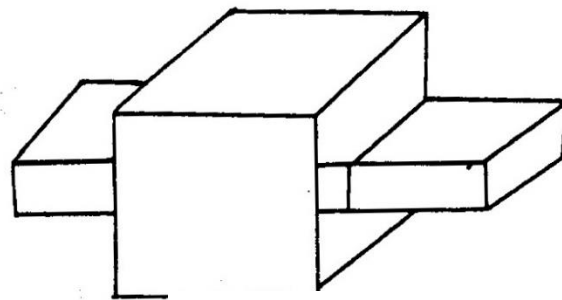
One which is formed by having a fluid in a receptacle & **motion is transmitted through the fluid** by pressure or compression only
Ex: hydraulic presses, jacks, brakes etc

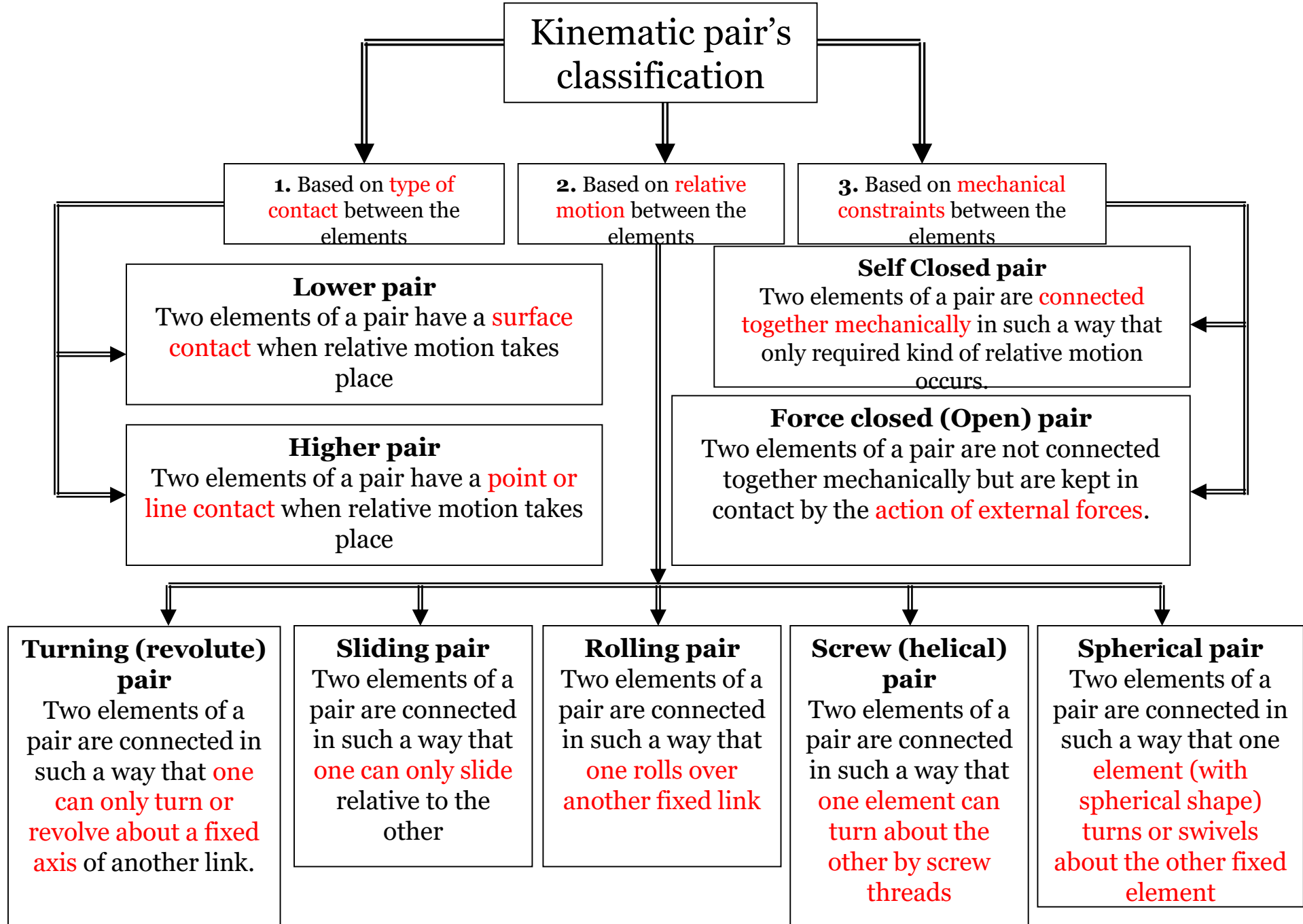
Floating link

One which connected to the **frame**

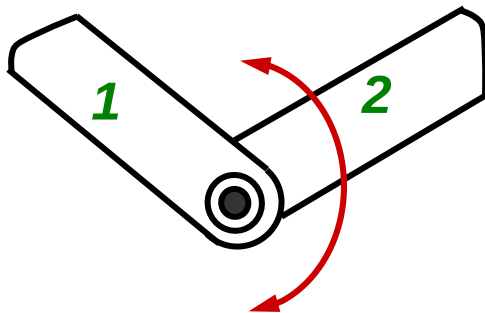
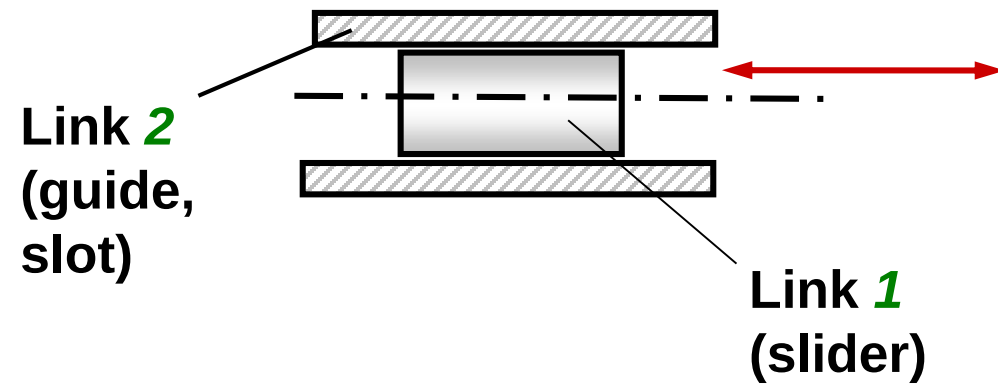
Kinematic pairs:

- When **two elements or links are connected** together in such a way that their **relative motion is completely constrained or successfully constrained**, form a kinematic pair.
 - Completely constrained: Motion between a pair of links is limited to a **definite direction**. Ex: motion of a shaft with collars at each end in a circular hole, motion of a square bar in a square hole, piston & cylinder
 - Incompletely constrained: Motion between a pair of links is **not confined to a definite direction**. Ex: shaft in a circular hole
 - Successfully constrained: Motion in a definite direction is **not brought about by itself**, but by some other means. Ex: shaft in foot step bearing

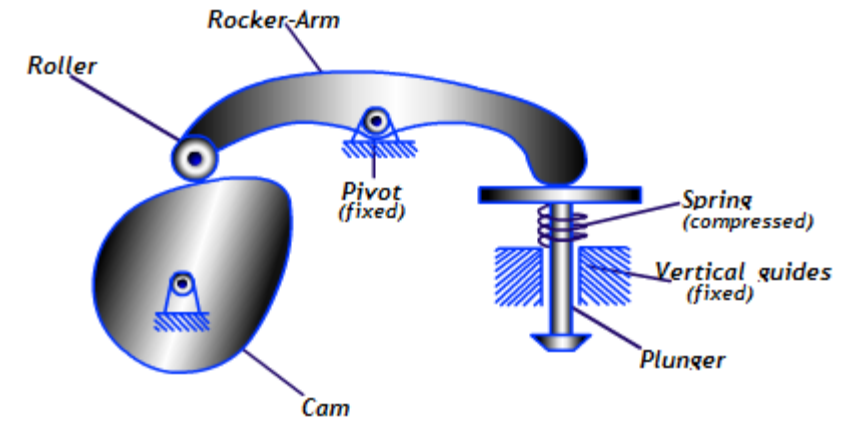




Lower pairs

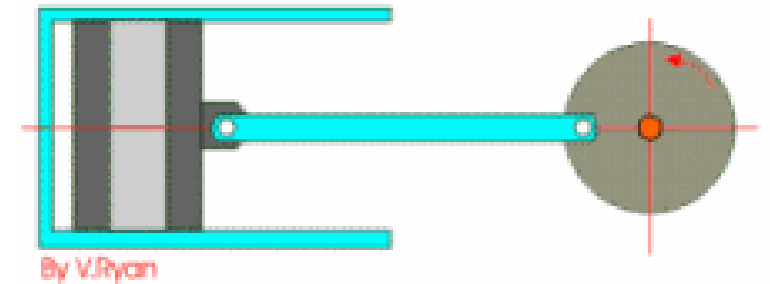
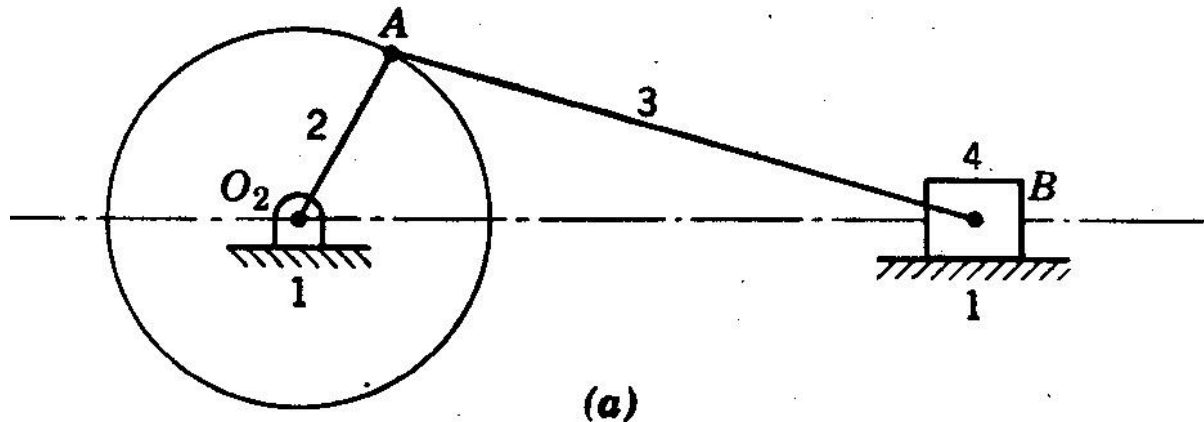


Higher pairs



Kinematic chain

- **Combination of kinematic pairs**, joined in such a way that each link forms a part of two pairs and the relative motion between the links (elements) is **completely or successfully constrained**.
- They are coupled in such a way that the last link is always joined to the first link to transmit definite motion.
- Example: Slider Crank Mechanism



If each link is assumed to **form two pairs with two adjacent links**, then the relation between the number of pairs (p) forming a kinematic chain and the number of links (l) may be expressed in the form of an equation:

$$l = 2p - 4 \dots\dots\dots (1)$$

Another relation between the number of links (l) and the number of joints (j) which constitute a kinematic chain is given by the expression:

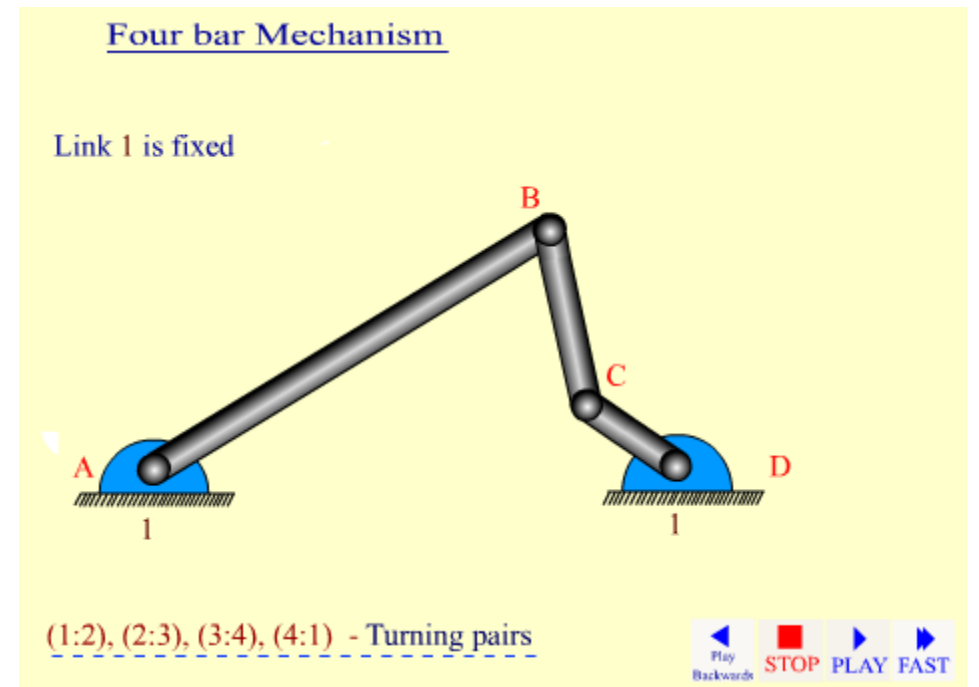
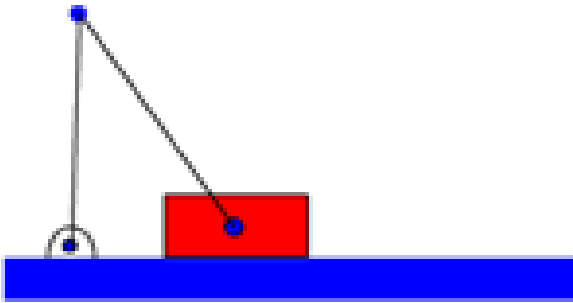
$$j = (3/2) l - 2 \dots\dots\dots (2)$$

Note:

1. These two equations are applicable only to kinematic chains, in which **lower pairs are used**. These equations may also be applied to kinematic chains, in which higher pairs are used. In that case **each higher pair may be taken as equivalent to two lower pairs** with an additional element or link.
2. If **L.H.S > R.H.S**. then the chain is **locked**
3. If **L.H.S = R.H.S**. then the chain is **constrained**
4. If **L.H.S < R.H.S**. then the chain is **unconstrained**

Mechanism

- It is a **constrained kinematic chain**, with **one link fixed**, which is used to transmit or transform motion.

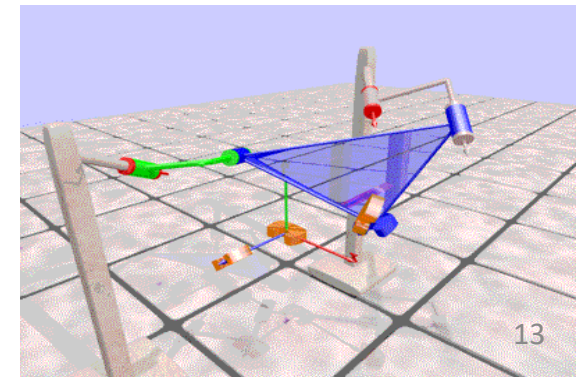
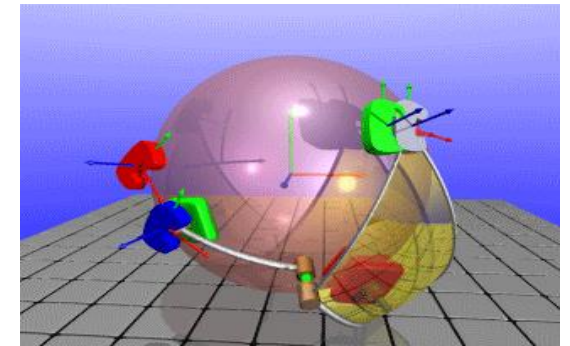
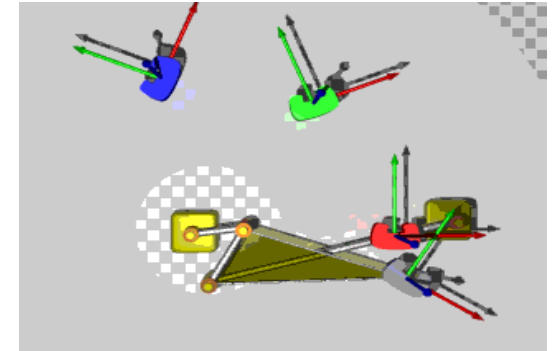
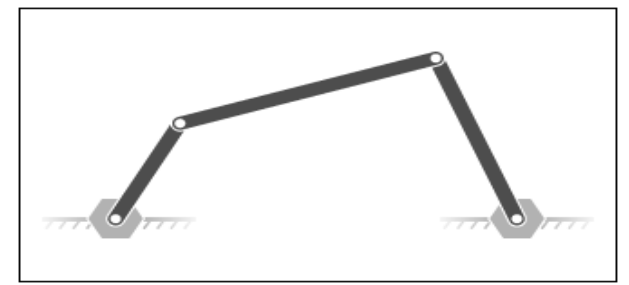


TYPES OF MECHANISM:

- 1) **Simple mechanism:** It has **four** links.
- 2) **Compound mechanism:** has **more than four** links.
- 3) **Complex mechanism:** formed by the inclusion of **ternary or higher order** floating link to a simple mechanism.
- 4) **Planar mechanism:** formed when all **links** of the mechanism **lie in the same plane**.
- 5) **Spatial mechanism:** formed when all **links** of the mechanism **lie in the different plane**.
- 6) **Equivalent mechanism:** formed when **one pairs** is **replaced** by **other type of pairs** and the new mechanism obtained must have the **same number of degrees of freedom** as the original mechanism.

Example:

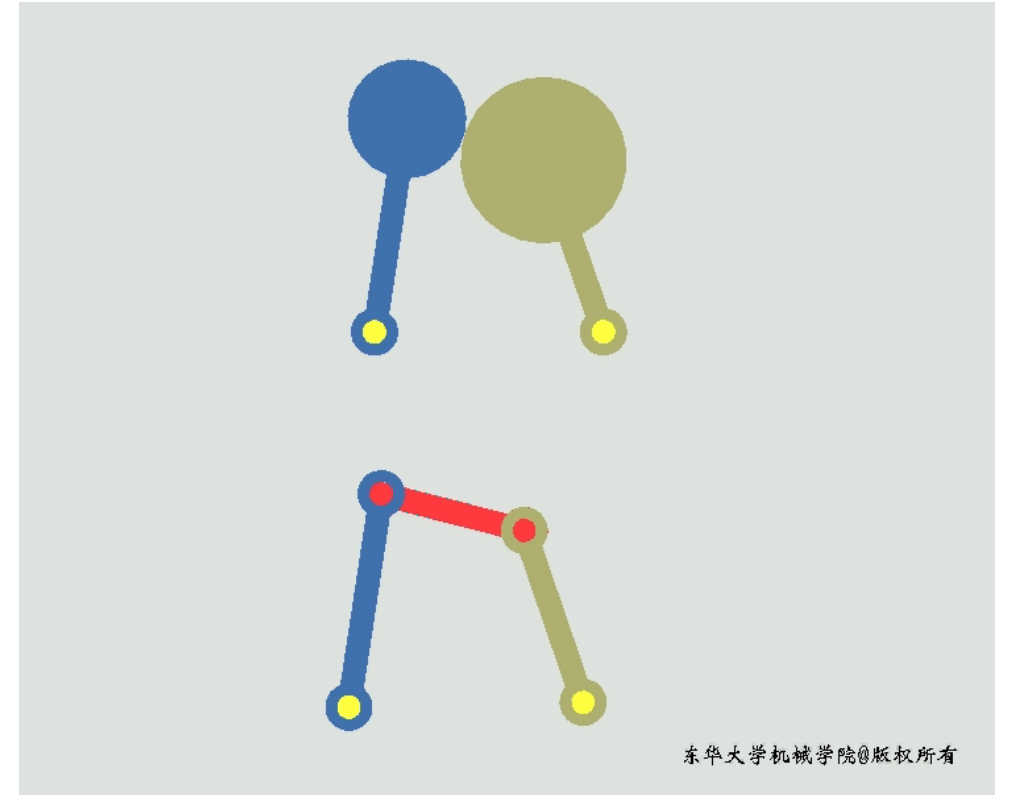
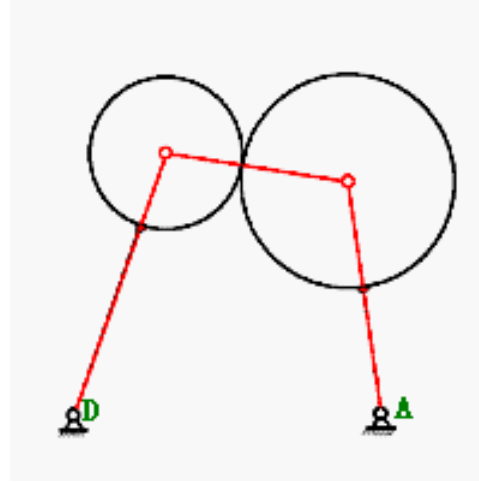
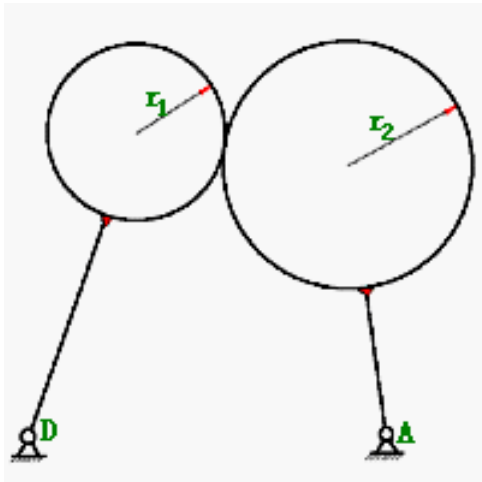
- 1) A turning pair can be replaced by a sliding pair
- 2) A spring can be replaced by two binary links
- 3) A cam pair can be replaced by one binary link with two turning pairs at each end.



7. Replace higher pairs with lower pairs

Example 1:

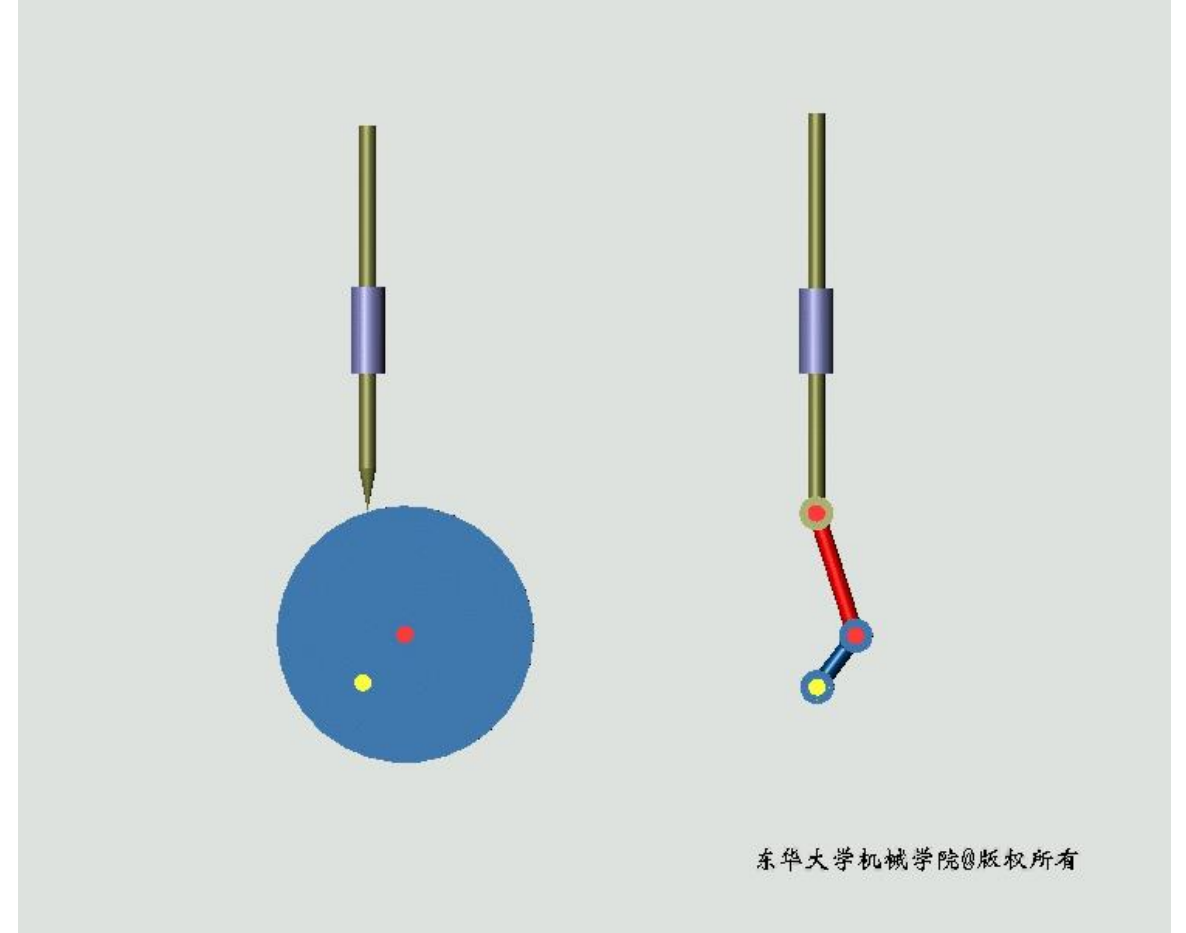
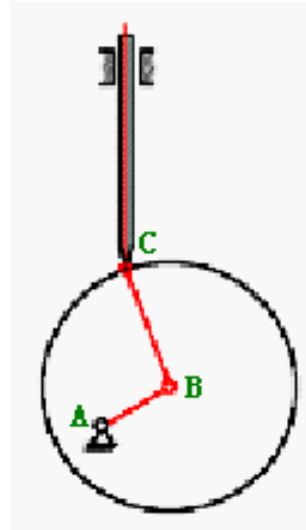
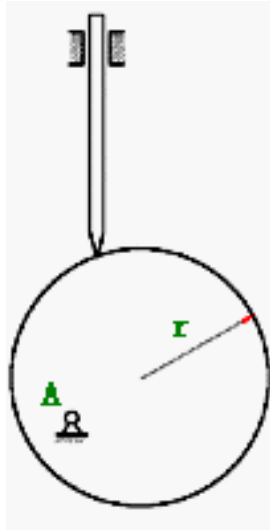
Both of the link are circles



7. Replace higher pairs with lower pairs

Example 2

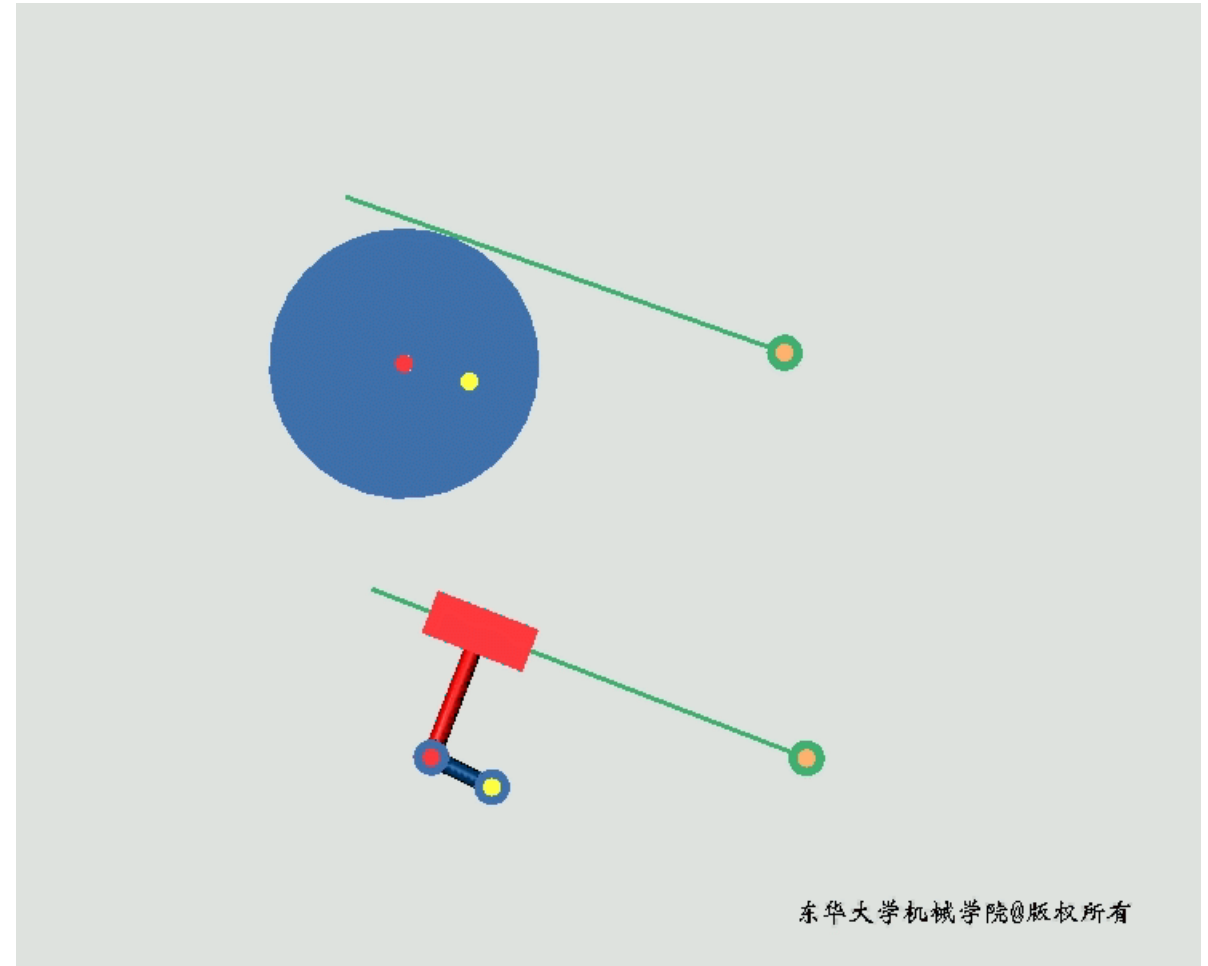
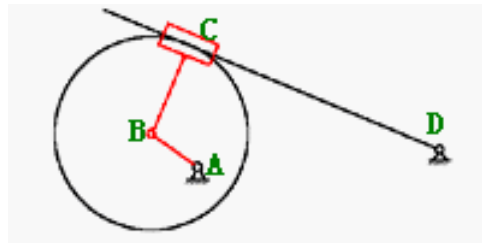
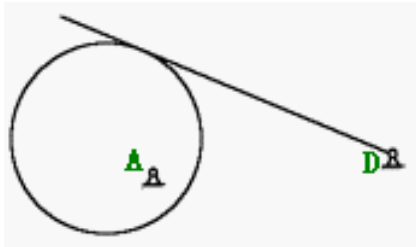
One is a circle, the other is a point.



7. Replace higher pairs with lower pairs

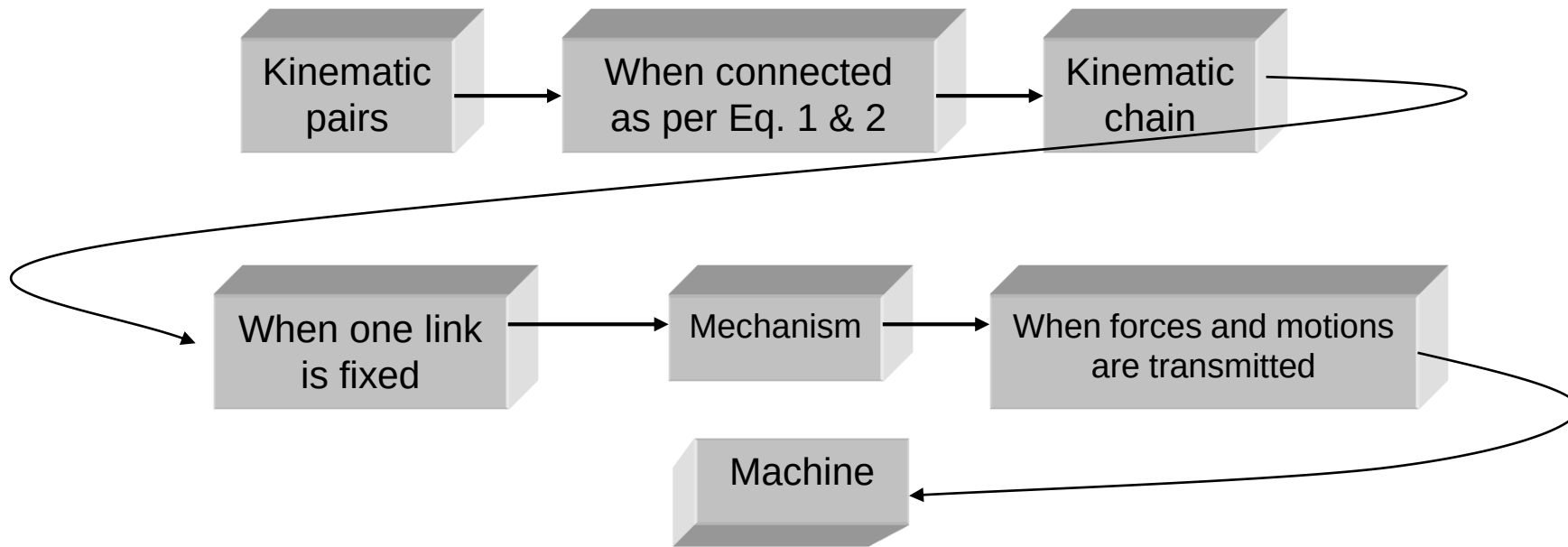
Example 3:

One is a line, the other is a circle.



Machine:

- A machine is a mechanism or group of mechanisms used to perform useful work.
- Its chief function is to adopt a source of power to some specific work requirements.



Structure

- It is an assemblage of a number of resistance bodies having **no relative motion** between them.
- These are meant for **taking up loads**.
- There is **only straining action** due to forces acting on them.

Difference between machine and structure

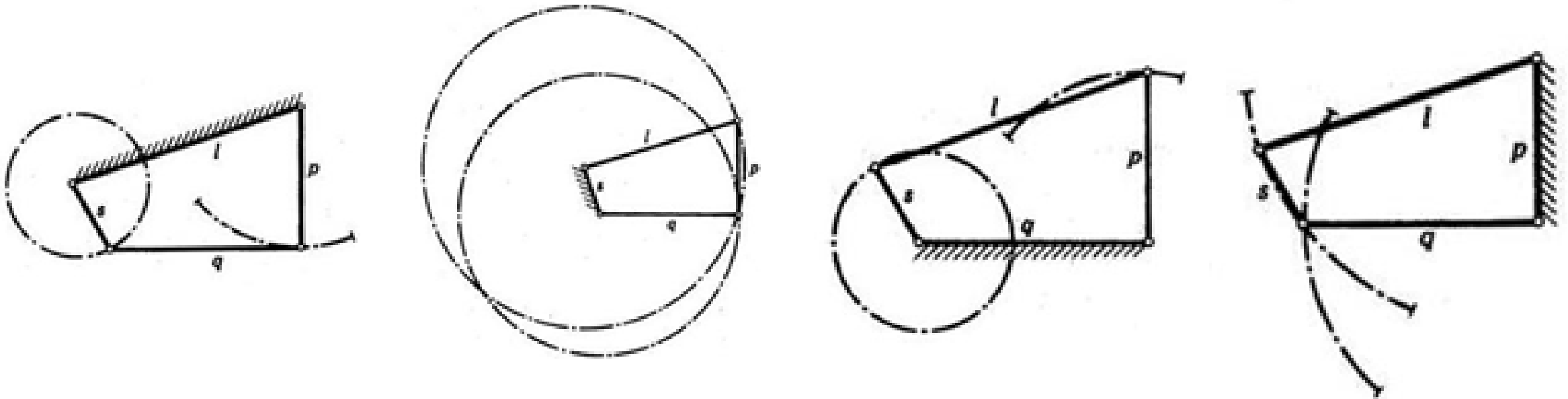
Sl. No.	Particulars	Machine	Structure
1	Definition	A machine is a mechanism or group of mechanisms used to perform useful work	It is an assemblage of a number of resistance bodies having no relative motion between them
2	Work	Modifies or transmit energy to do some kind of work	Modifies & transmit force only
3	Relative motion	Exists between its members	Not exists between its members
4	Energy	Transmits useful energy	No energy transmission
5	Examples	Steam engine, shaper etc	Roof truss, railway bridges, machine frames etc

Difference between machine & mechanism

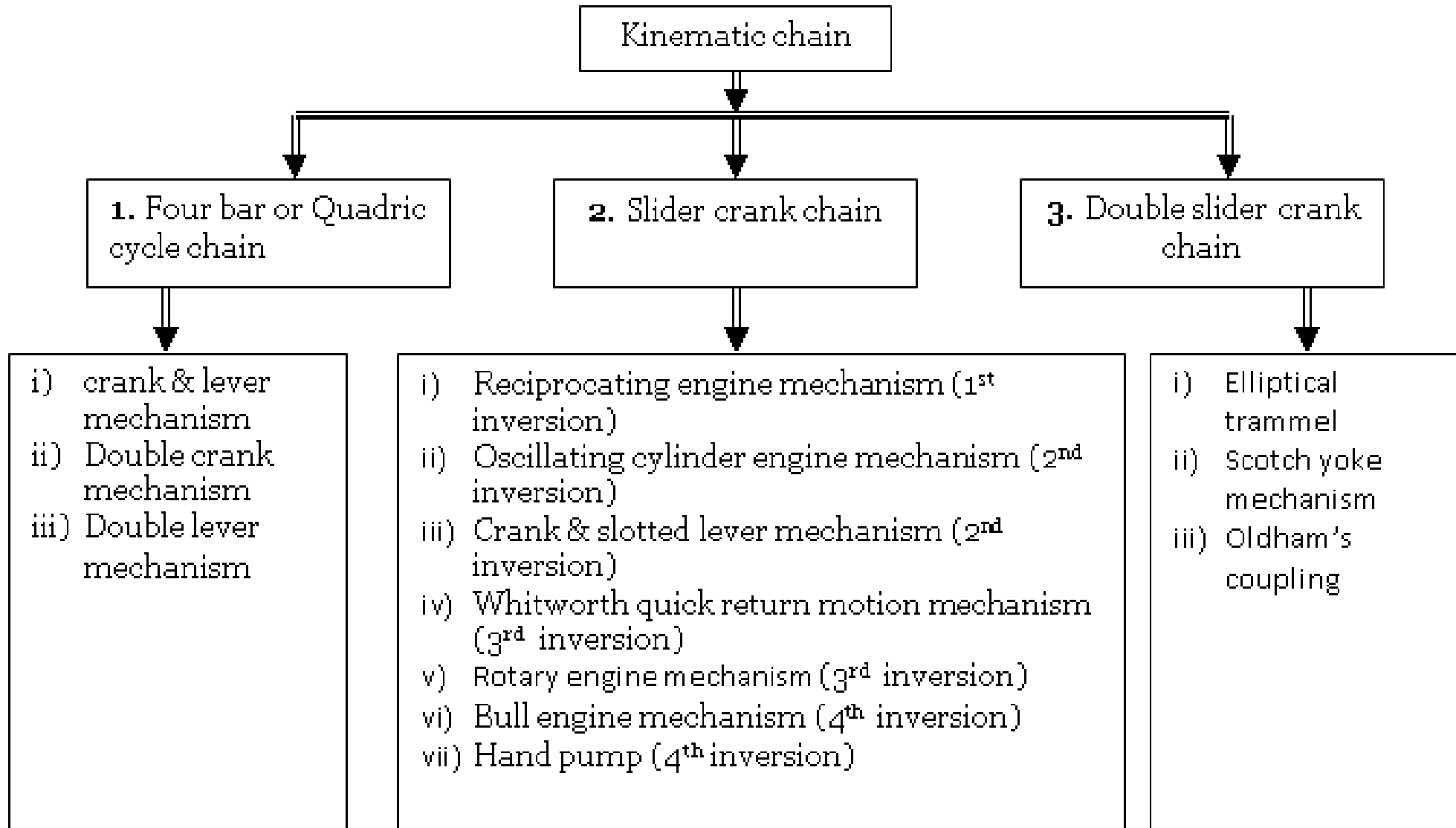
Sl. No.	Particulars	Mechanism	Machine
1	Definition	It is a constrained kinematic chain, with one link fixed, which is used to transmit or transform motion	A machine is a mechanism or group of mechanisms used to perform useful work
2	Purpose	To transmit or transform motion	To transmit energy or to do useful work
3	Dependency	No mechanism is necessarily a machine	A machine is a series or train of mechanism
4	Relationship	It is a working model of any machine	It is a practical development of any mechanism
5	Examples	Clock, mini-drafter etc	Steam engine, shaper etc

Inversion:

- The **exchange of fixedness** of an element with its mating element in a kinematic chain is called inversion.
- In the fig. any one of the links may be **arbitrary selected as the fixed link**, and each arrangement is an inversion of the others.
- Note: Relative motions between the various links is not changed in any manner through the process of inversion, but their absolute motions may be changed drastically



KINEMATIC CHAINS AND INVERSIONS



GRASHOF'S LAW

For a **four bar mechanism**, the **sum of the shortest & the longest link** lengths should not be greater than the **sum of the remaining two link lengths** if there is to be continuous relative motion between the two links

$$s + l < p + q$$

where:

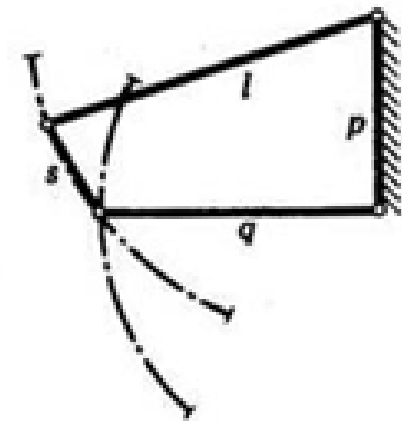
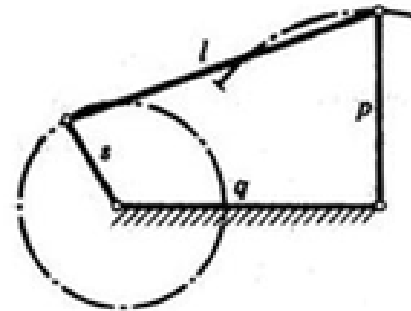
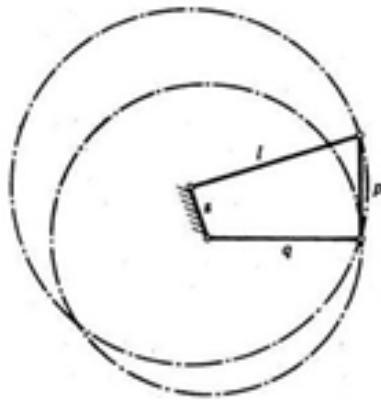
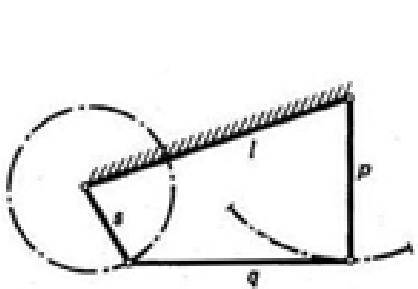
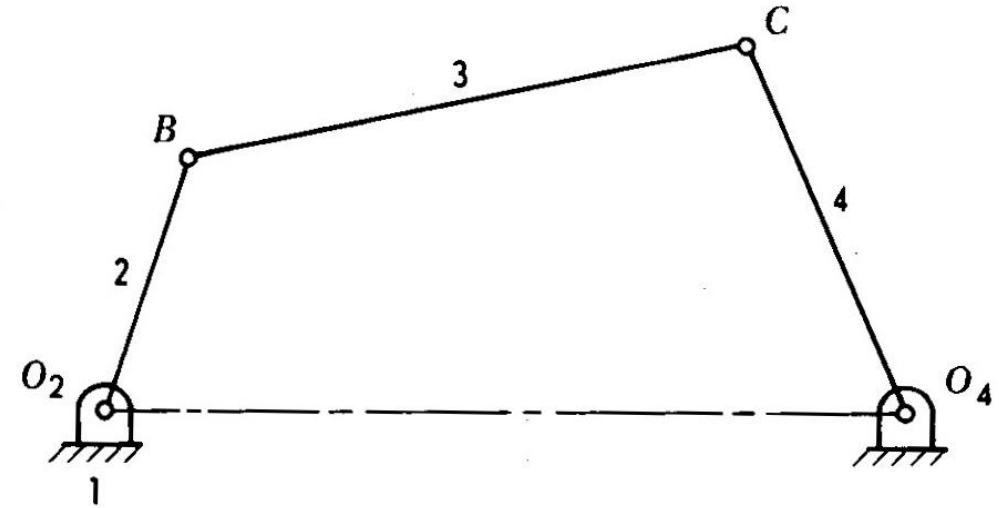
s = shortest link length,

l = longest,

p & q = intermediate length links

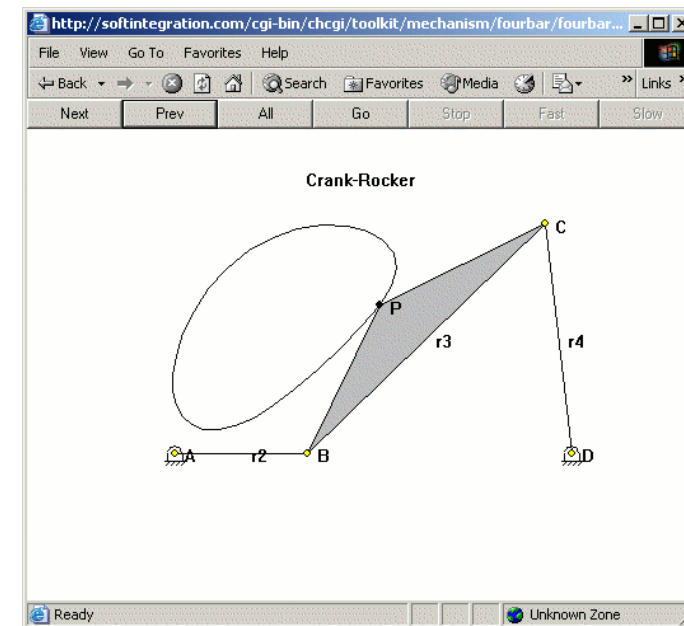
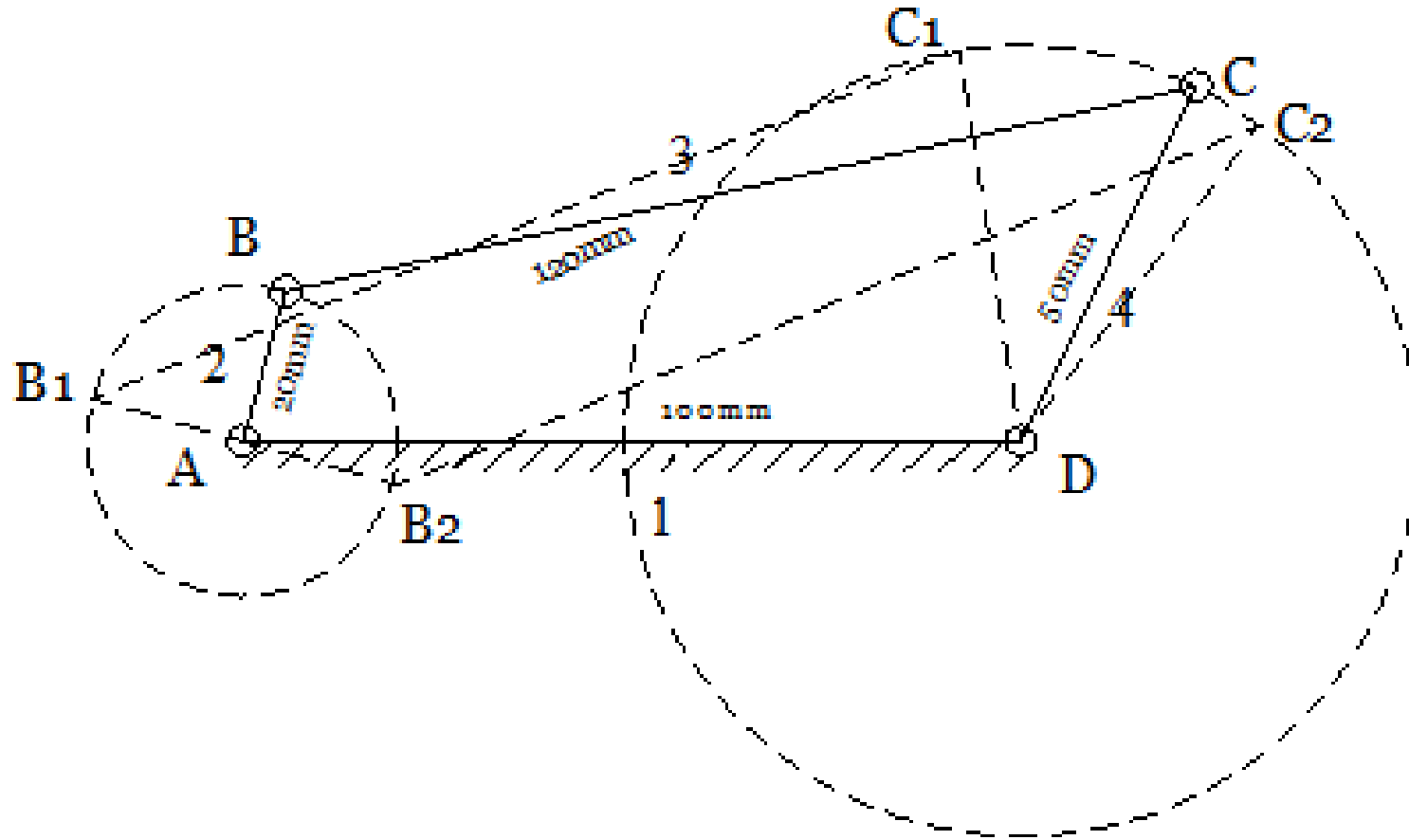
1. Four bar or Quadric cycle chain

- (link 1) frame
- (link 2) crank
- (link 3) coupler
- (link 4) rocker

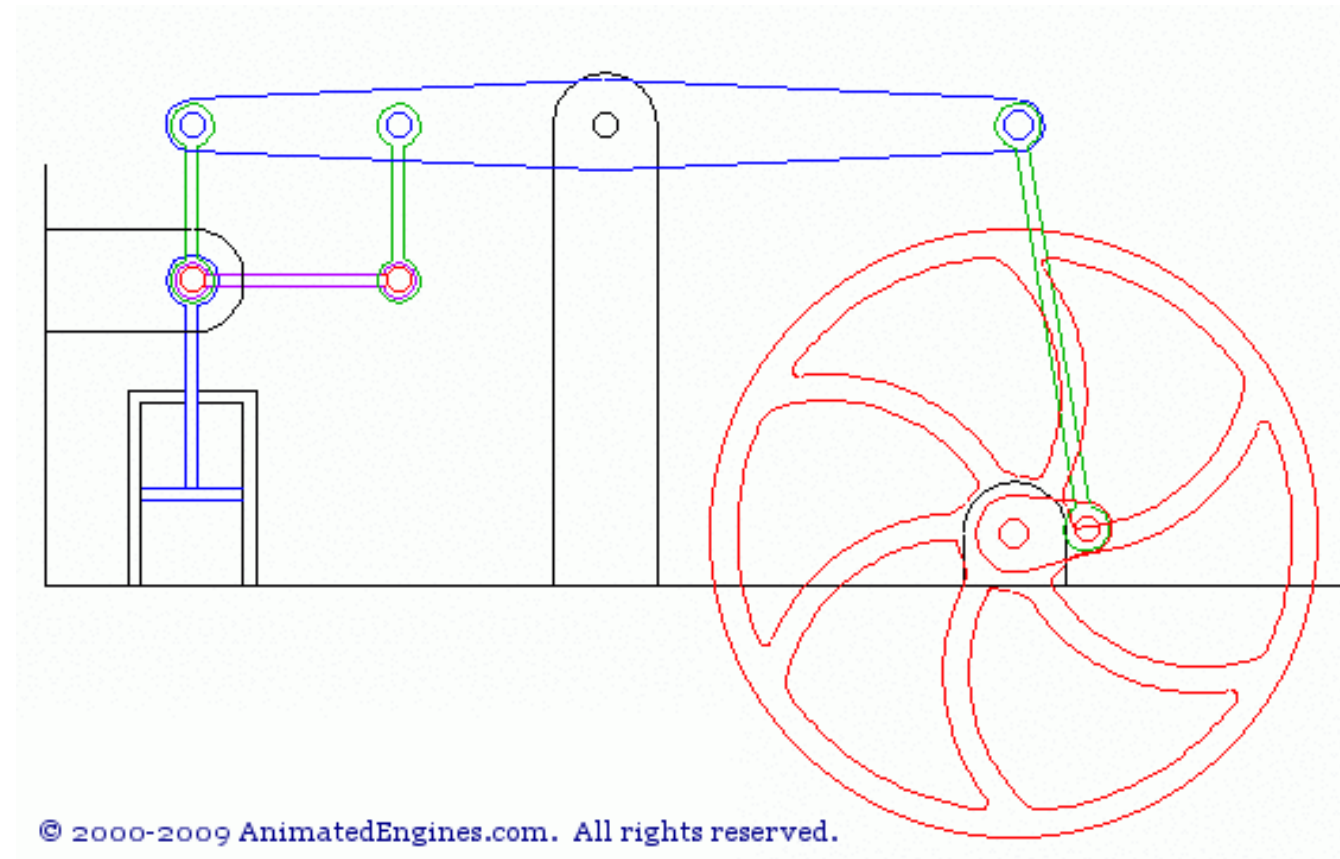
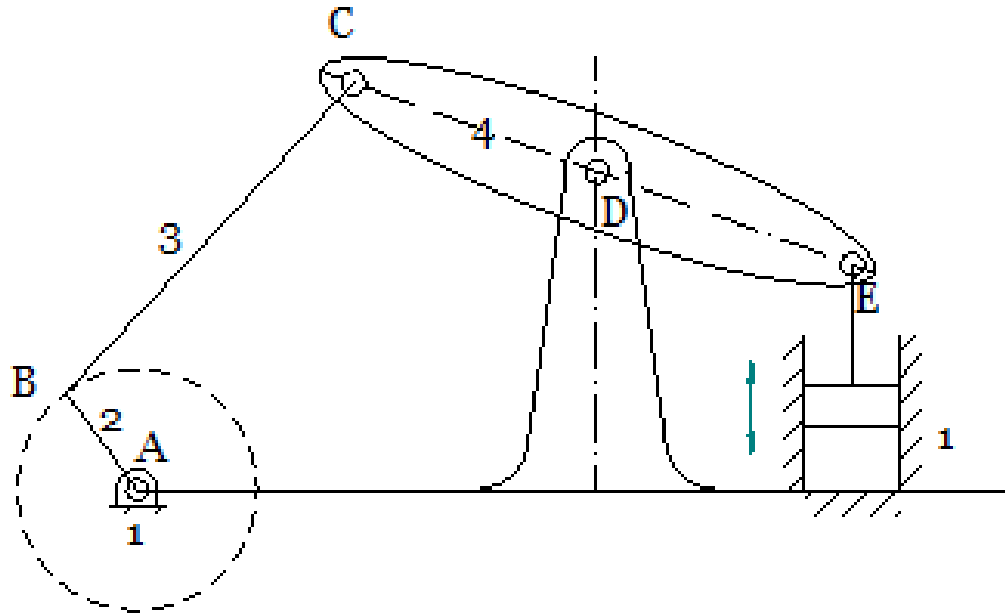


I & II inversions

Crank & lever mechanism (rotary & oscillatory motion)



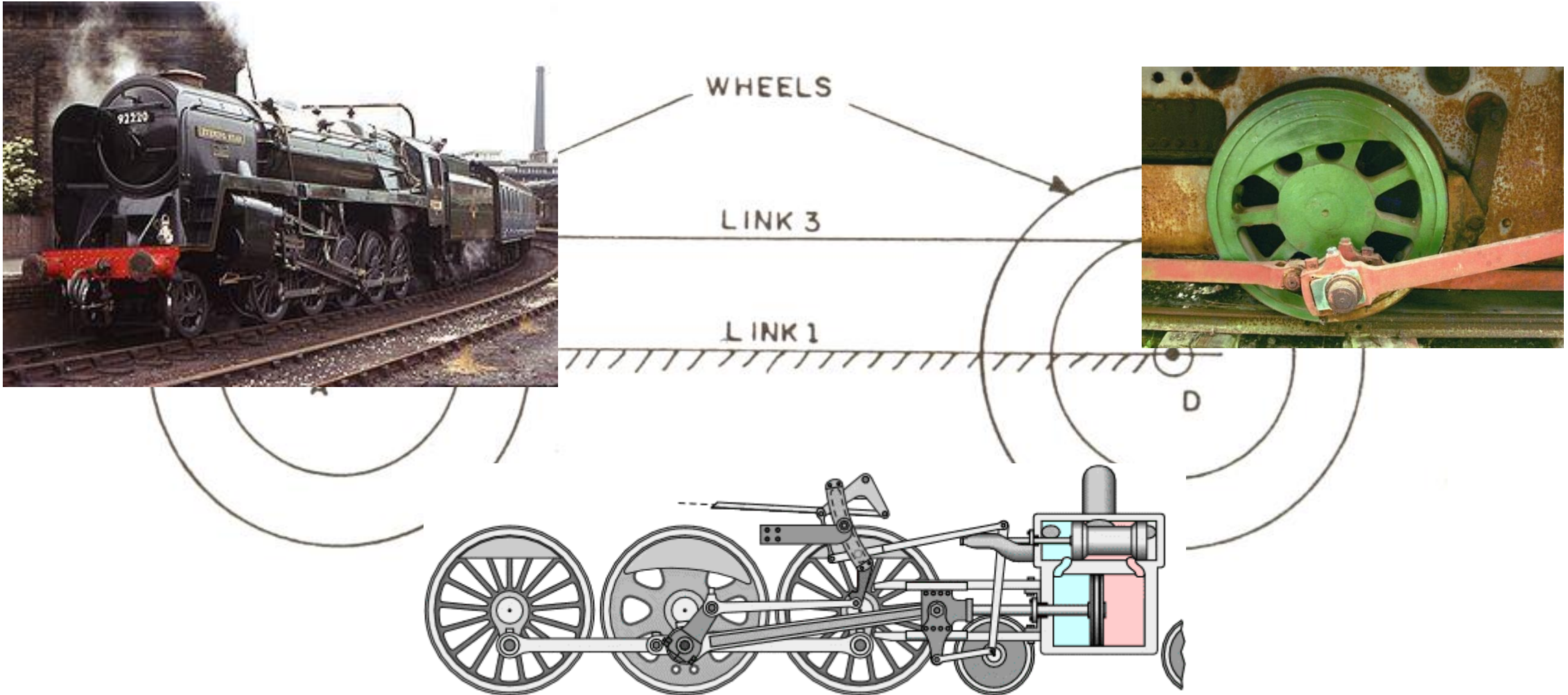
Beam engine:



III inversions

Double crank mechanism (complete rotation of the crank & follower)

1. Coupling rod of locomotive



IV inversions

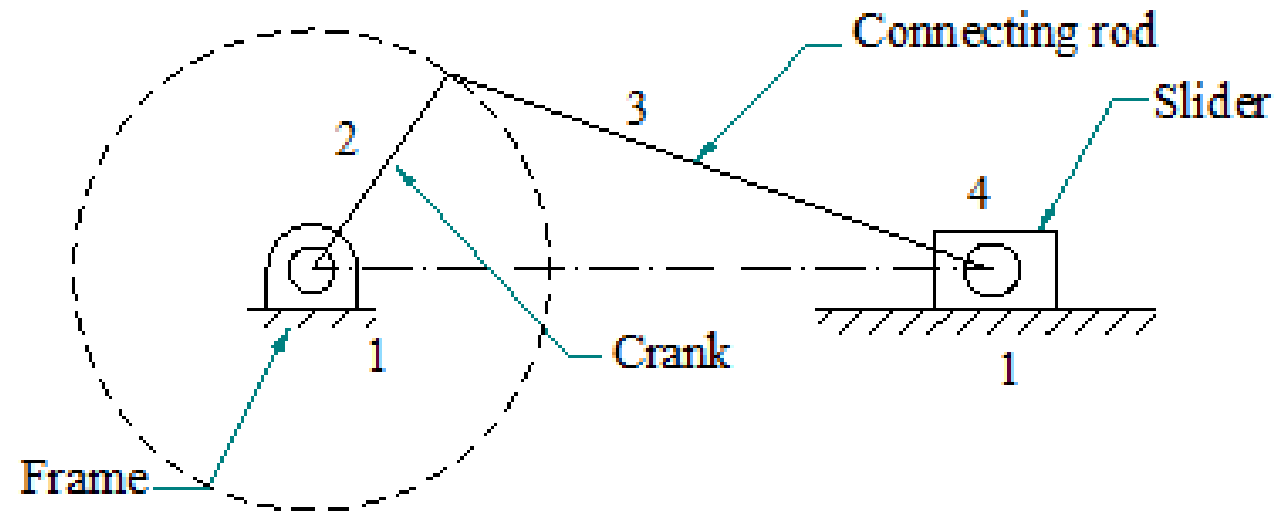
Double lever mechanism (oscillatory motion)

1. Watt's straight line mechanism



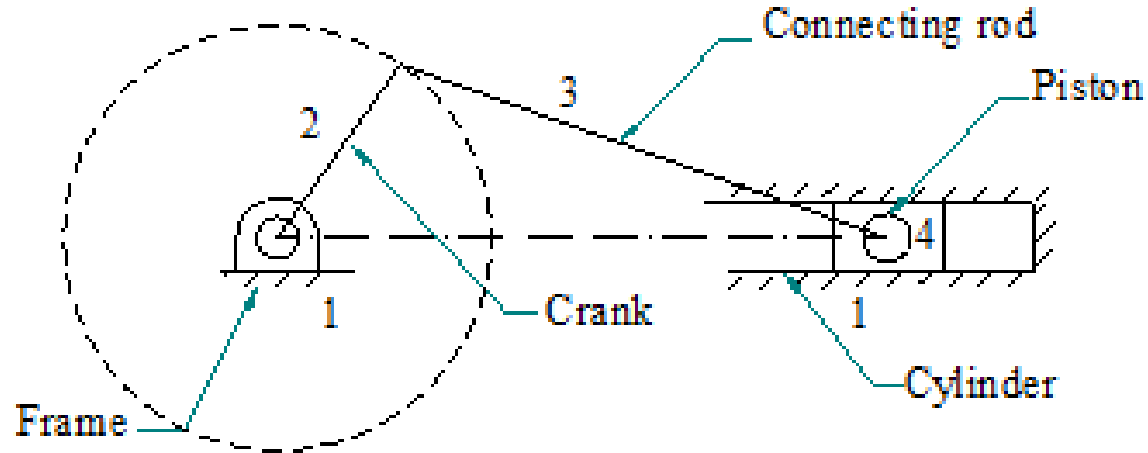
WORKING

2. Slider crank chain

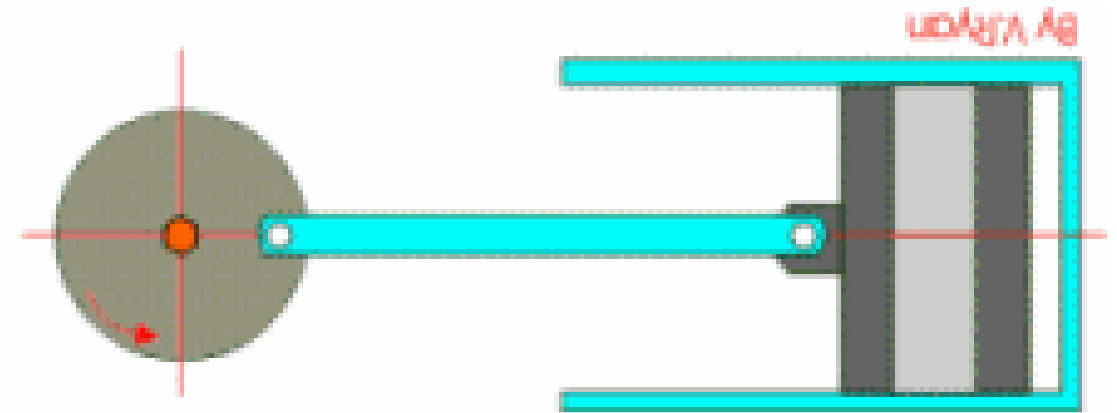


Reciprocating engine mechanism (1st inversion)

When link 1 is fixed, link 2 is made as crank and link 4 is made as slider, then first inversion of single slider crank is obtained

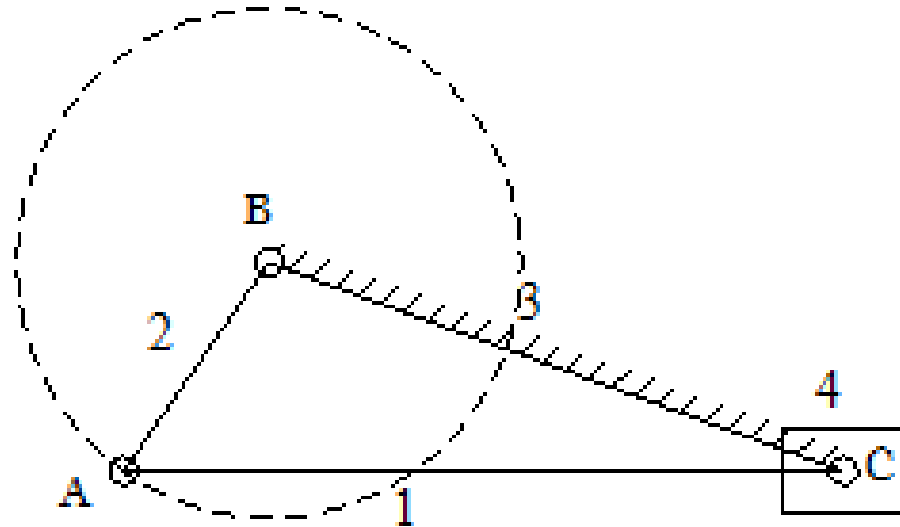


Example: steam engine, compressors, pumps, I.C. engines etc

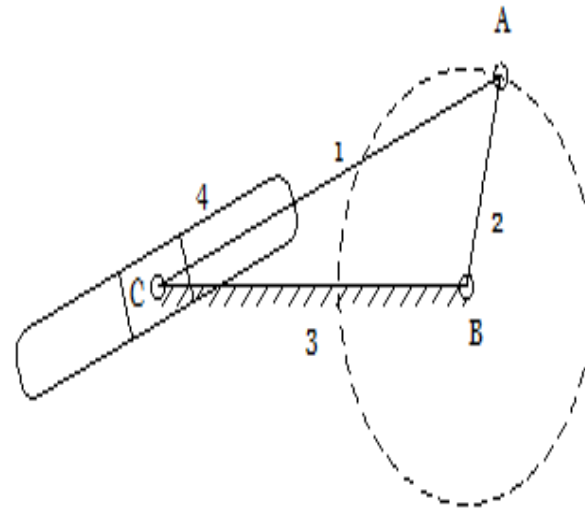
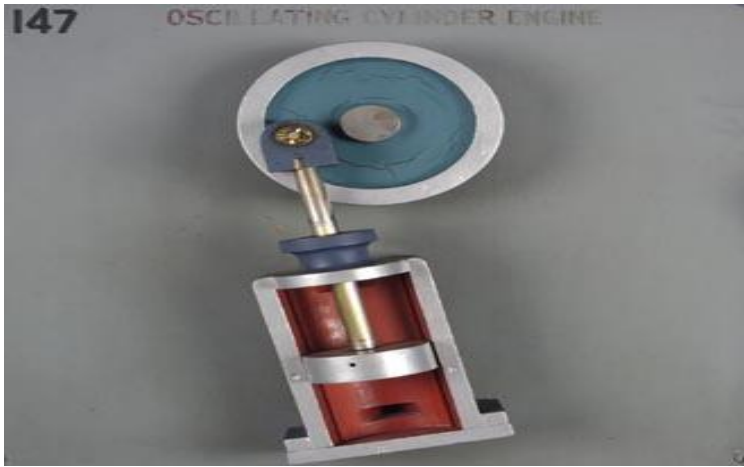


Second inversion:

- The second inversion is obtained by fixing the link 3 (connecting rod).
- Link 2 acts as a crank and is rotating about the point B.
- Link 4 oscillates.

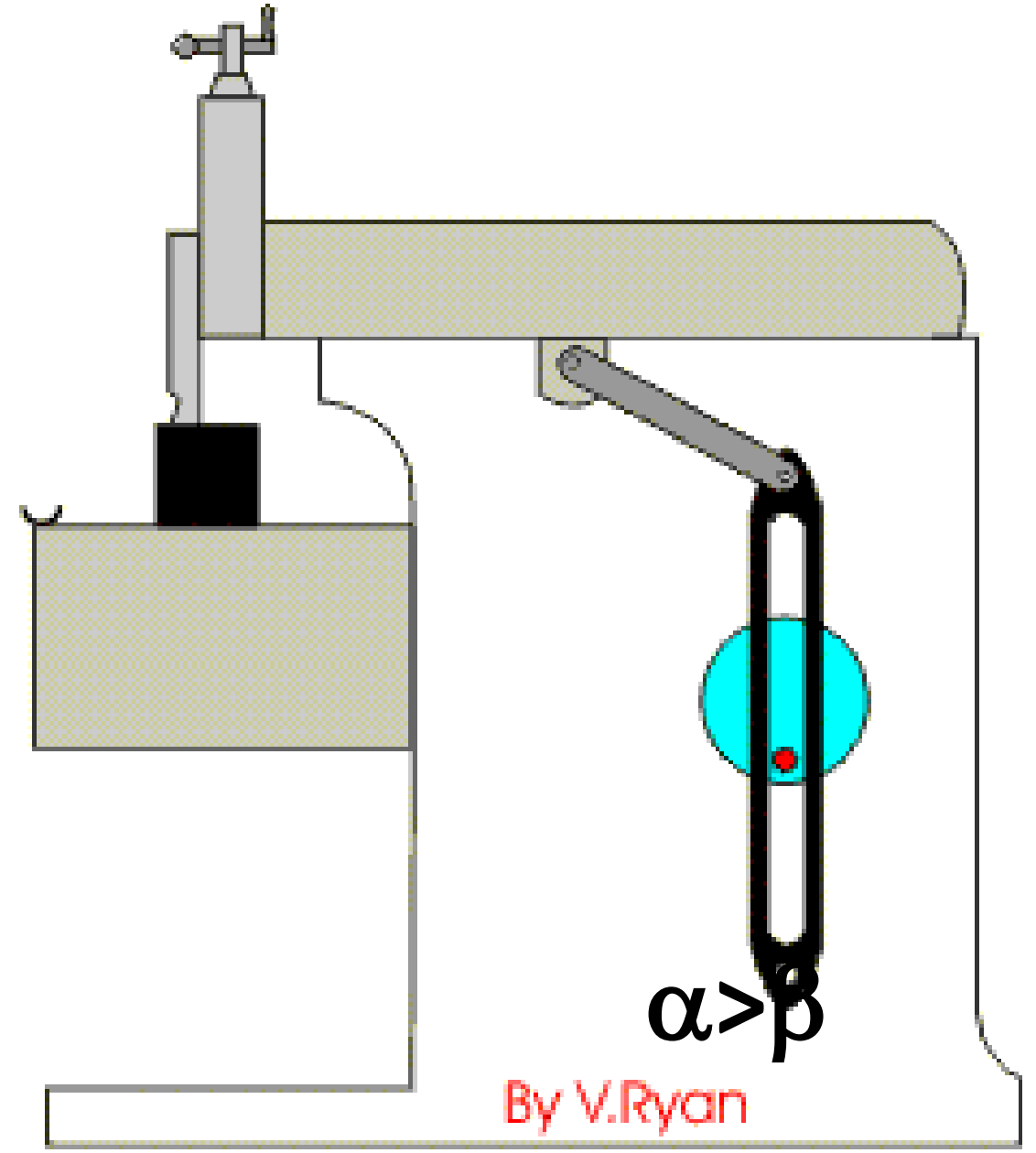
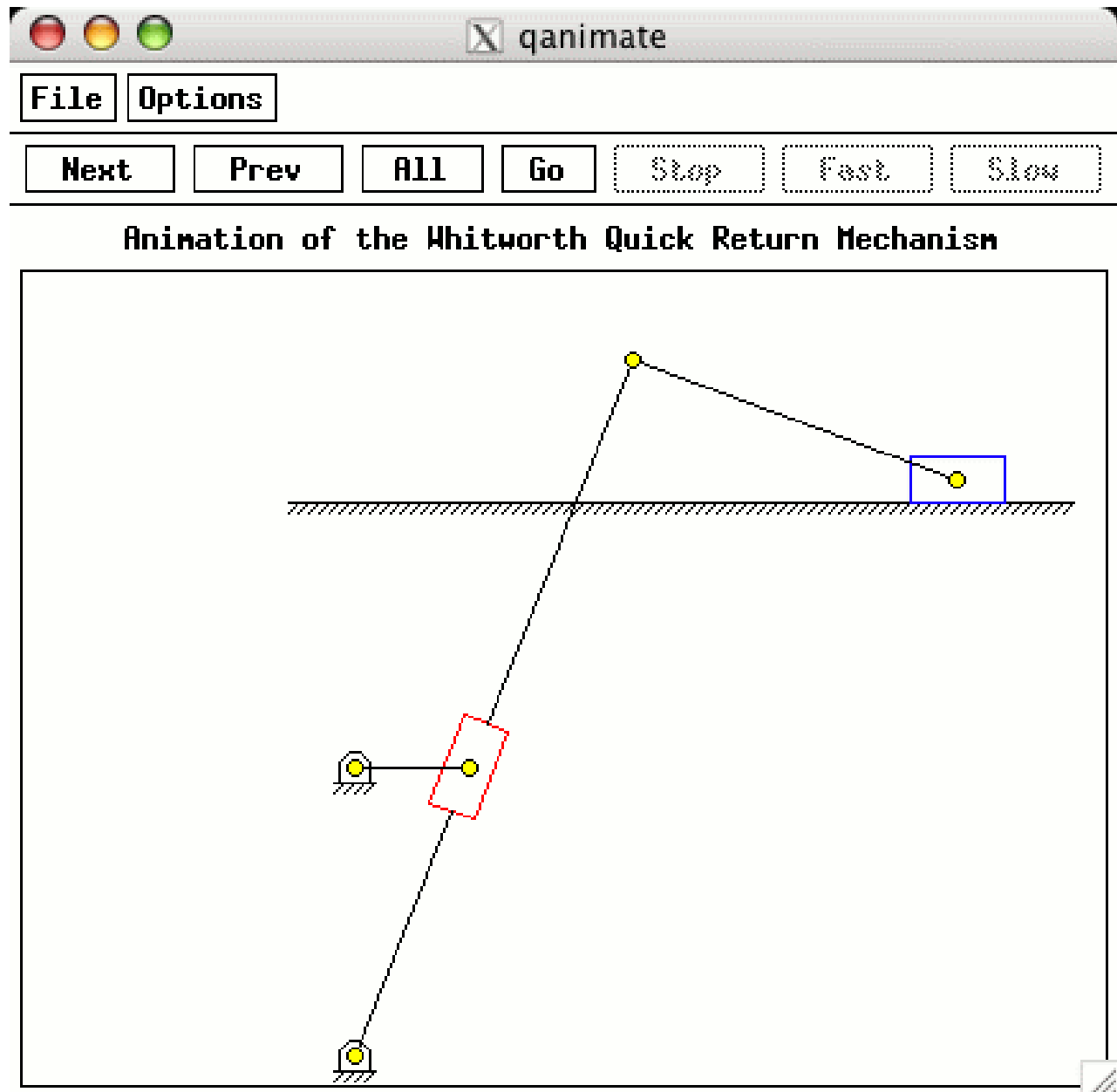


Oscillating cylinder engine mechanism (2nd inversion)



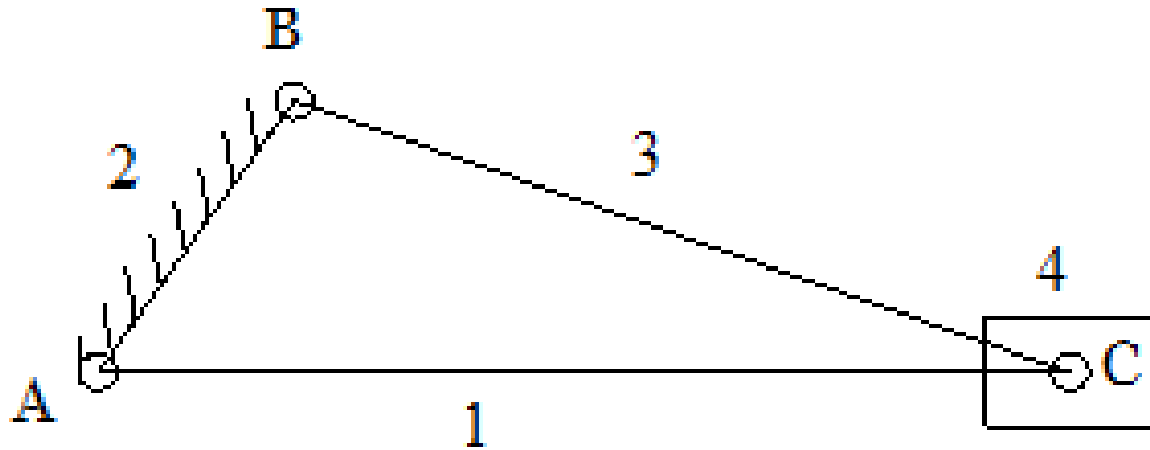
WORKING

- It is used to convert reciprocating motion into rotary motion.
- In this mechanism, the link 3 forming the turning pair is fixed which corresponds to connecting rod of a reciprocating steam engine mechanism.
- When the crank (link 2) rotates, the piston attached to the piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at C.



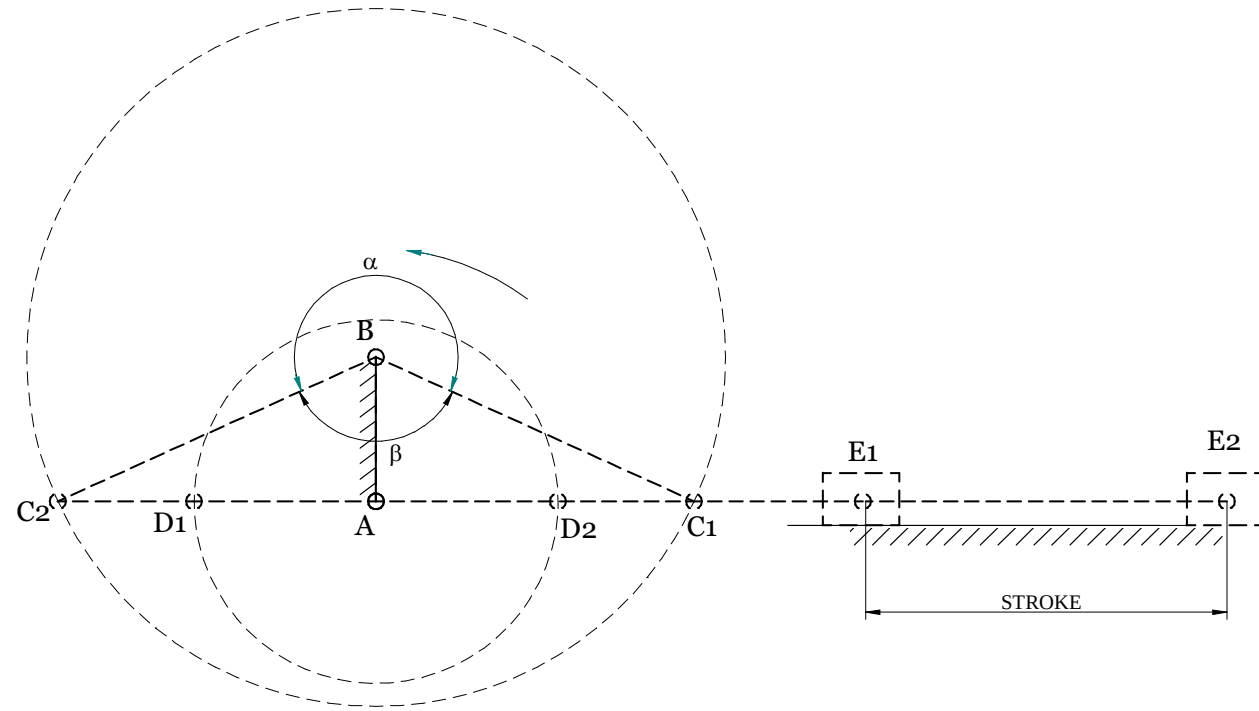
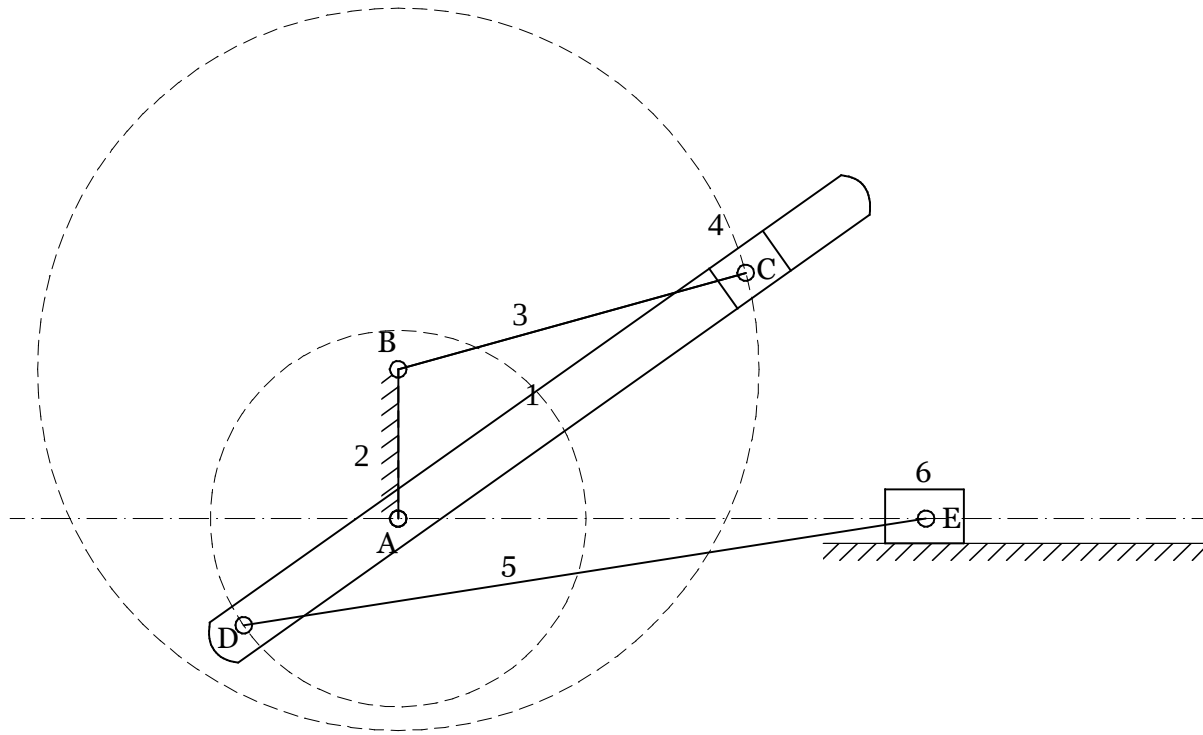
THE SHAPING MACHINE

Third inversion



- By fixing the link 2 (crank) third inversion is obtained.
- Link 3 along with slider at its end C, becomes a crank.
- Hence link 3 along with slider (link 4) rotates about B.
- By doing so, the link 1 rotates about A along with the slider (link 4) which reciprocates on link 1.

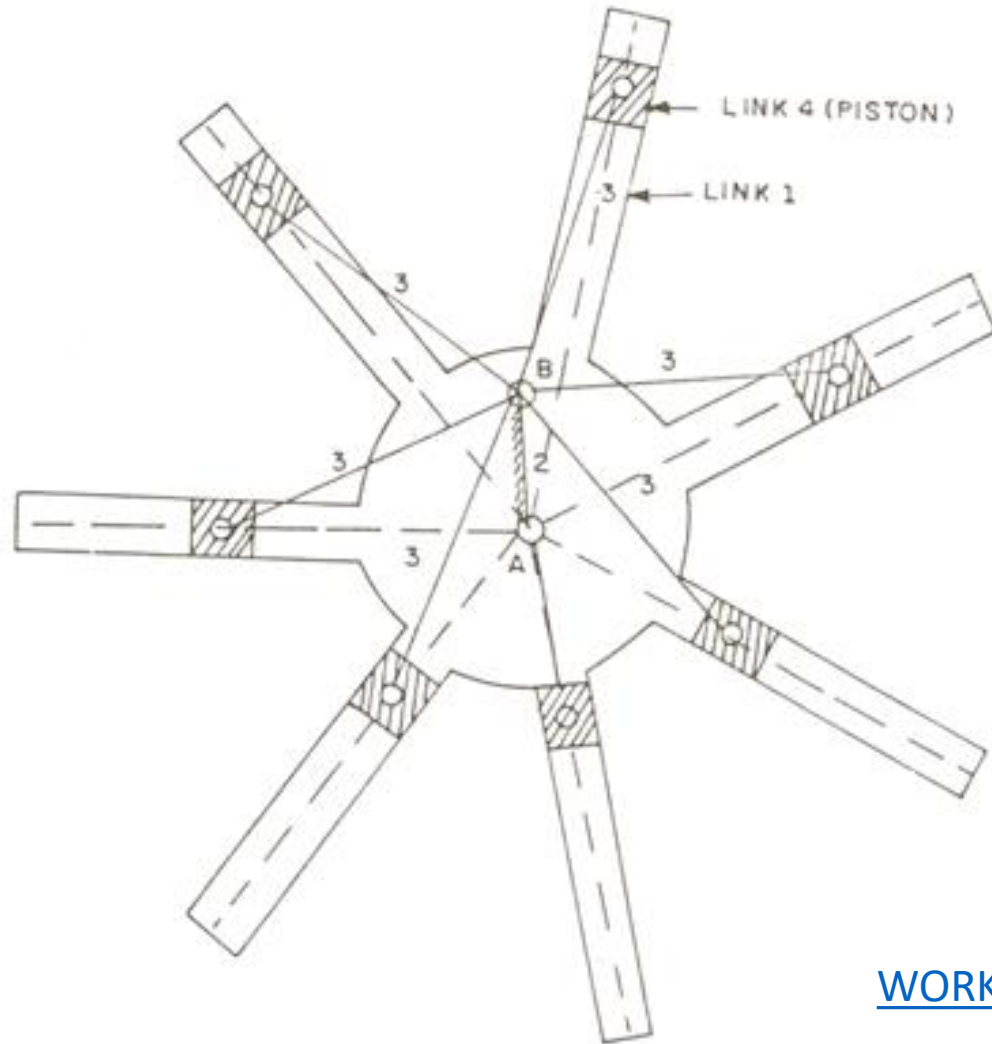
Whitworth quick return motion mechanism (3rd inversion)



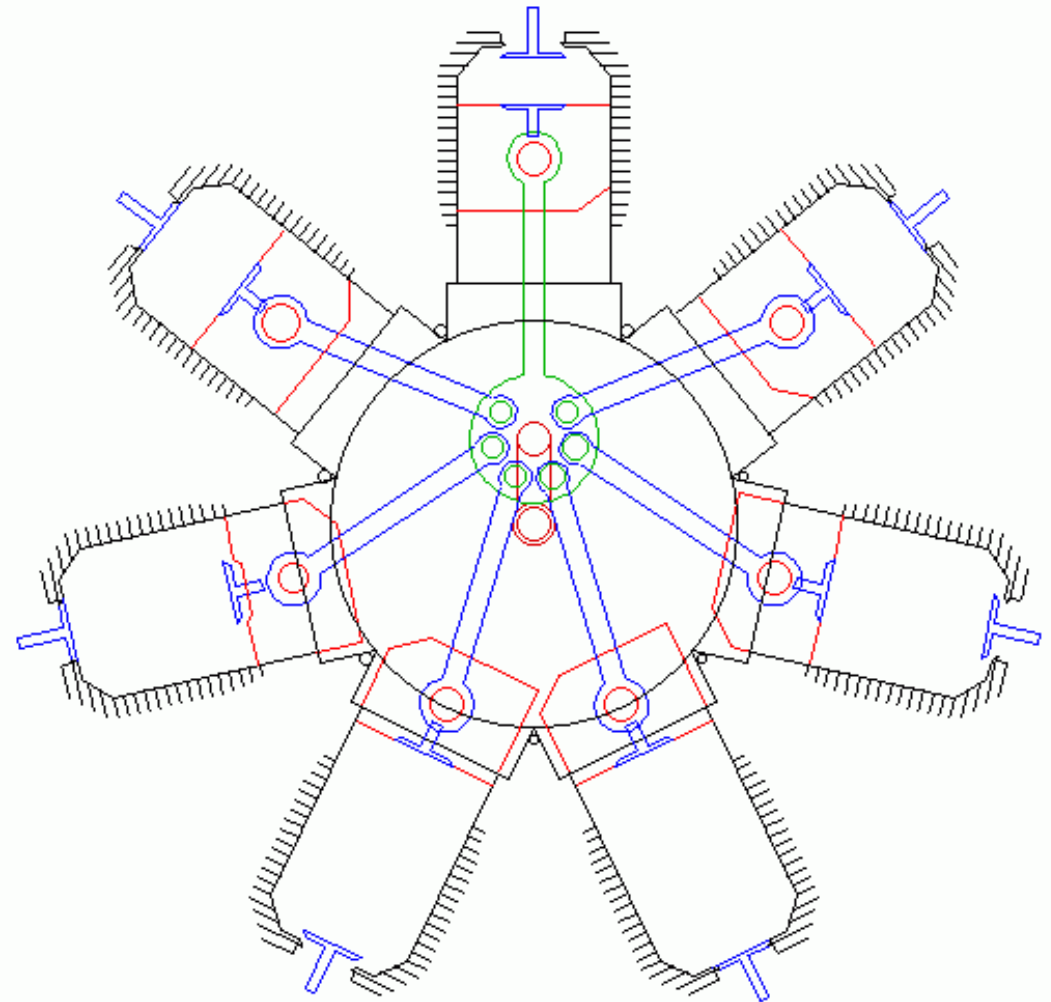
$$\alpha > \beta$$

WORKING

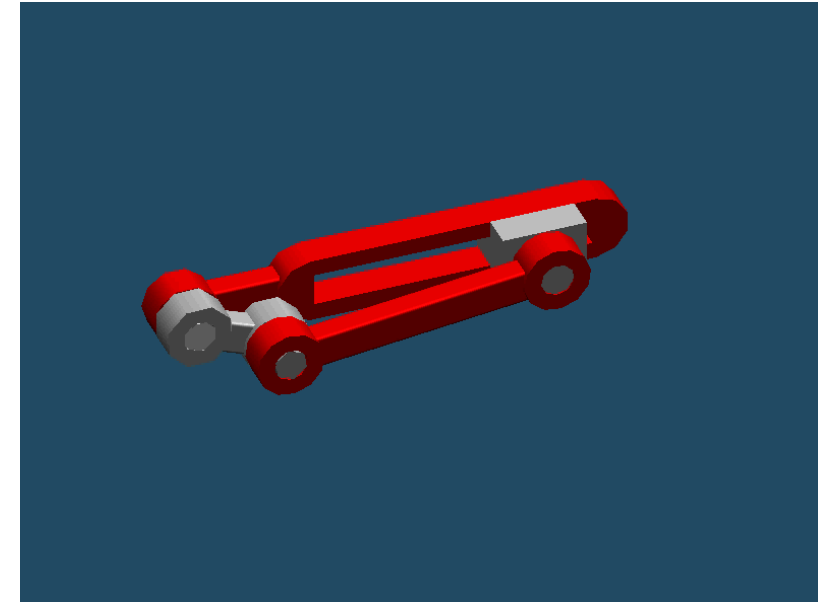
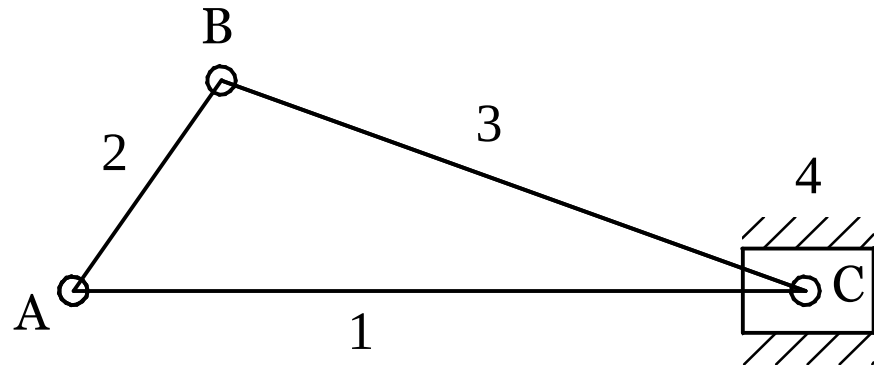
Rotary engine or Gnome engine mechanism (3rd inversion)



- It is a rotary cylinder V-type internal combustion engine used as an aero-engine, which now has been replaced by gas turbine.

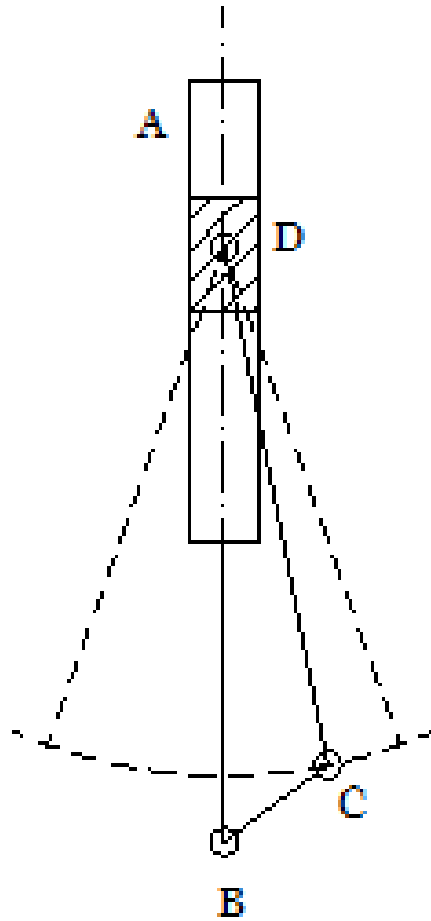


Fourth inversion

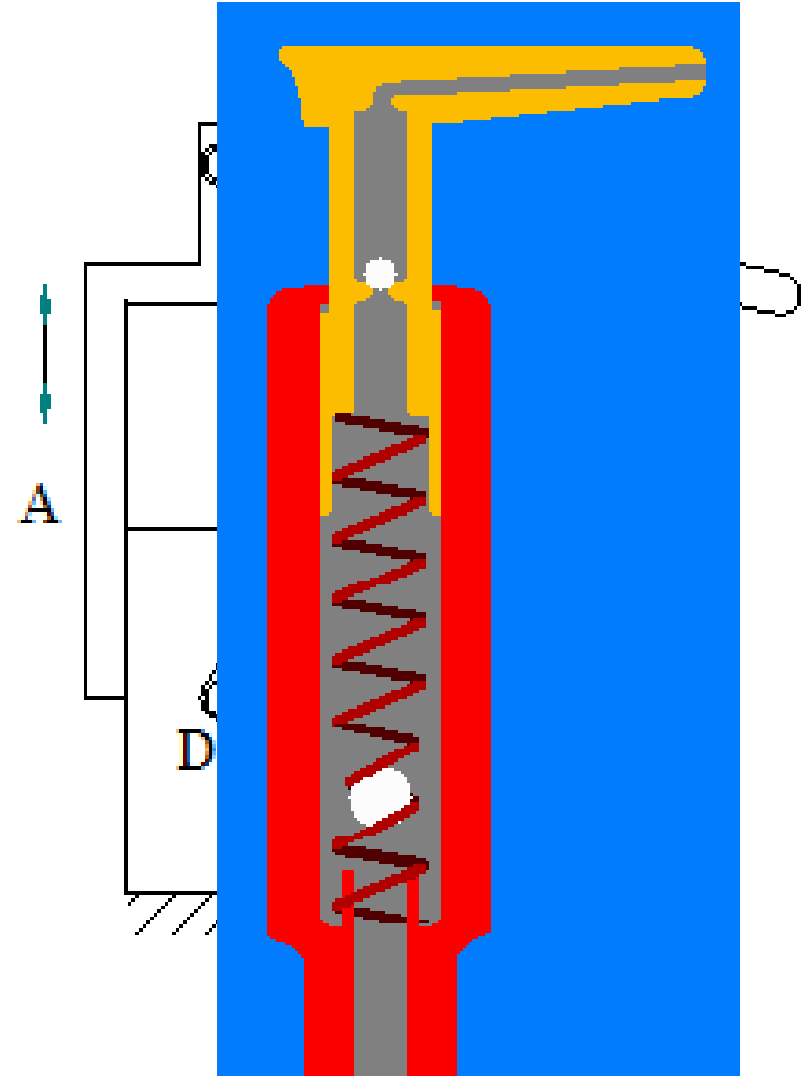


- By fixing the link 4 (sliding pair or cylinder) fourth inversion is obtained.
- Link 3 can oscillate about the fixed point C on link 4.
- This makes end B of link 2 to oscillate about C and end A reciprocates along the axis of the fixed link 4.

Bull engine mechanism (4th inversion)

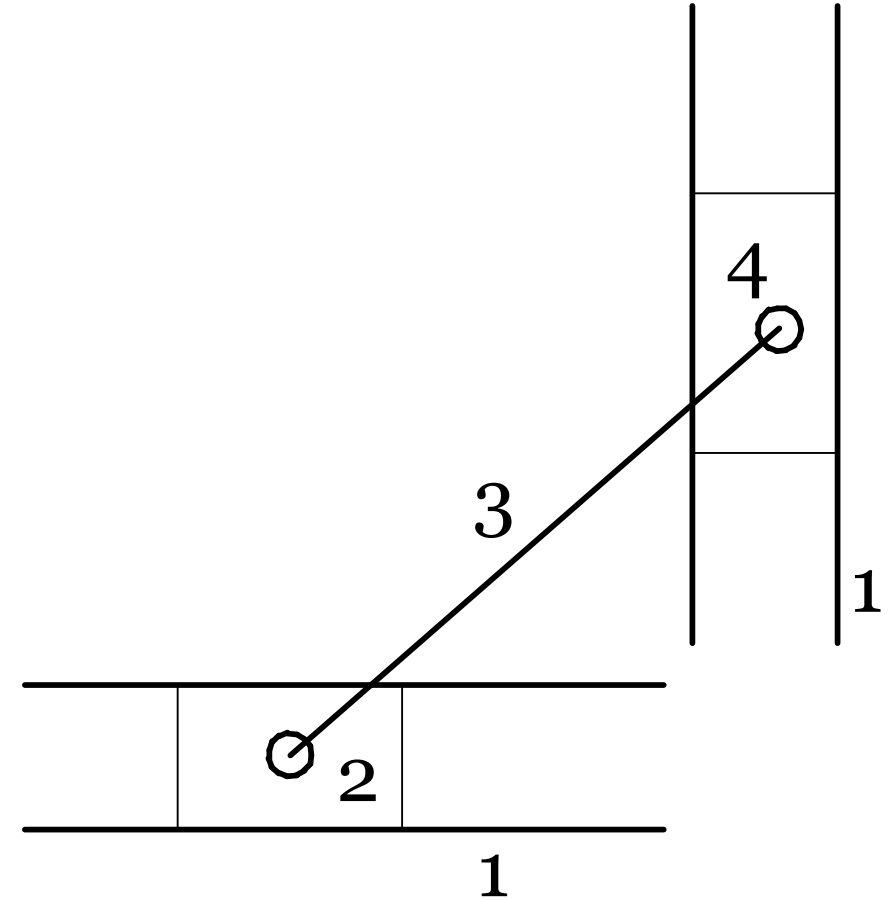


Hand pump (4th inversion)



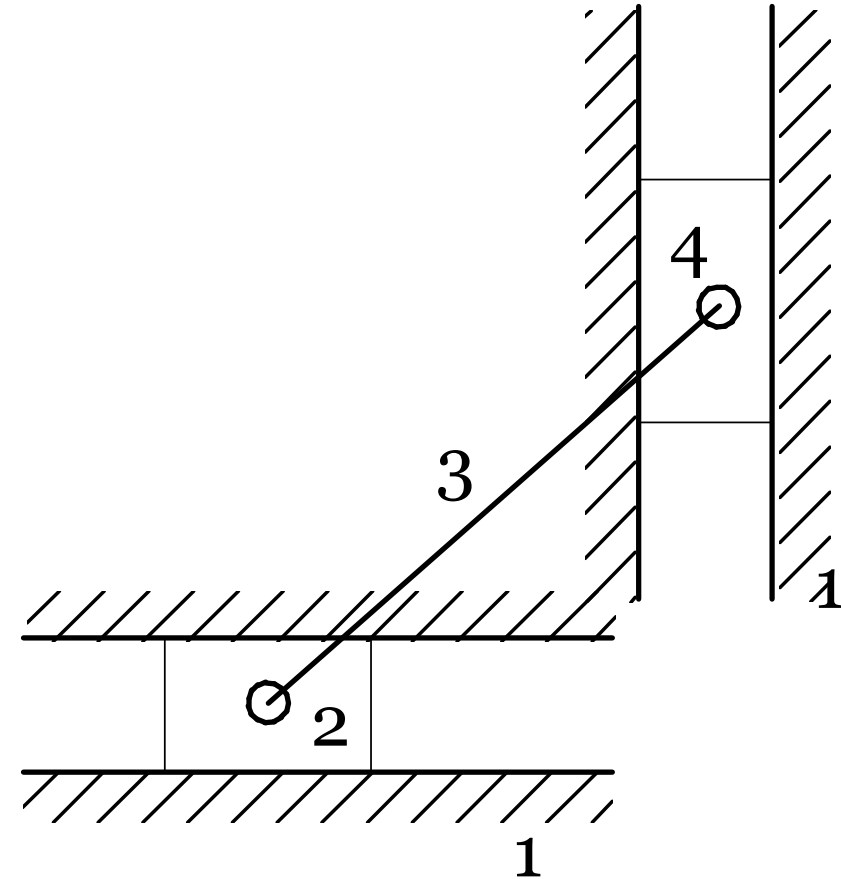
3. Double slider crank chain

- It is a four-bar kinematic chain containing two turning pairs and two sliding pairs.
- Link 1 & link 2 is sliding pair, link 2 & link 3 is turning pair, link 3 & link 4 is second turning pair, link 4 & link 1 is second sliding pair.
- Also the two pairs of the same kind are adjacent (To adjacent pairs 23 & 34 are turning pairs whereas the other two pairs 12 & 14 are sliding pairs)



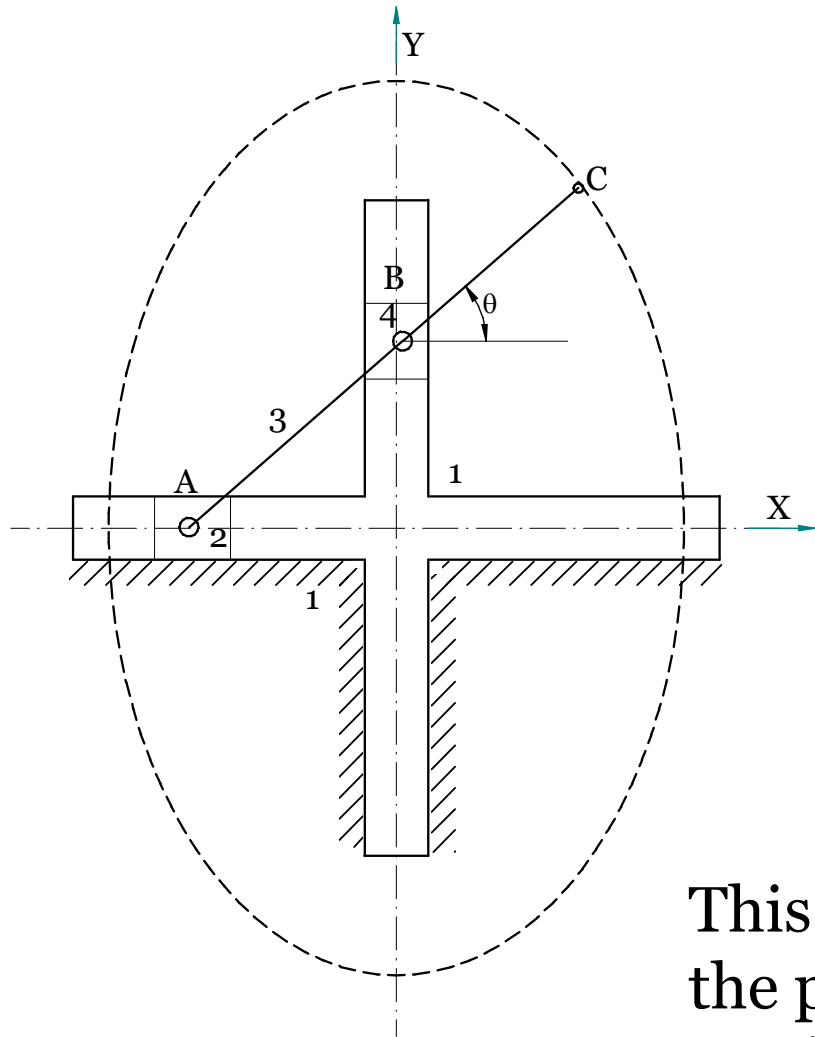
First Inversion

- When link 1 is fixed, the first inversion is obtained as shown in the fig.
- Two adjacent pairs 23 and 34 are turning pairs where as the other two pairs 12 and 14 are sliding pair.

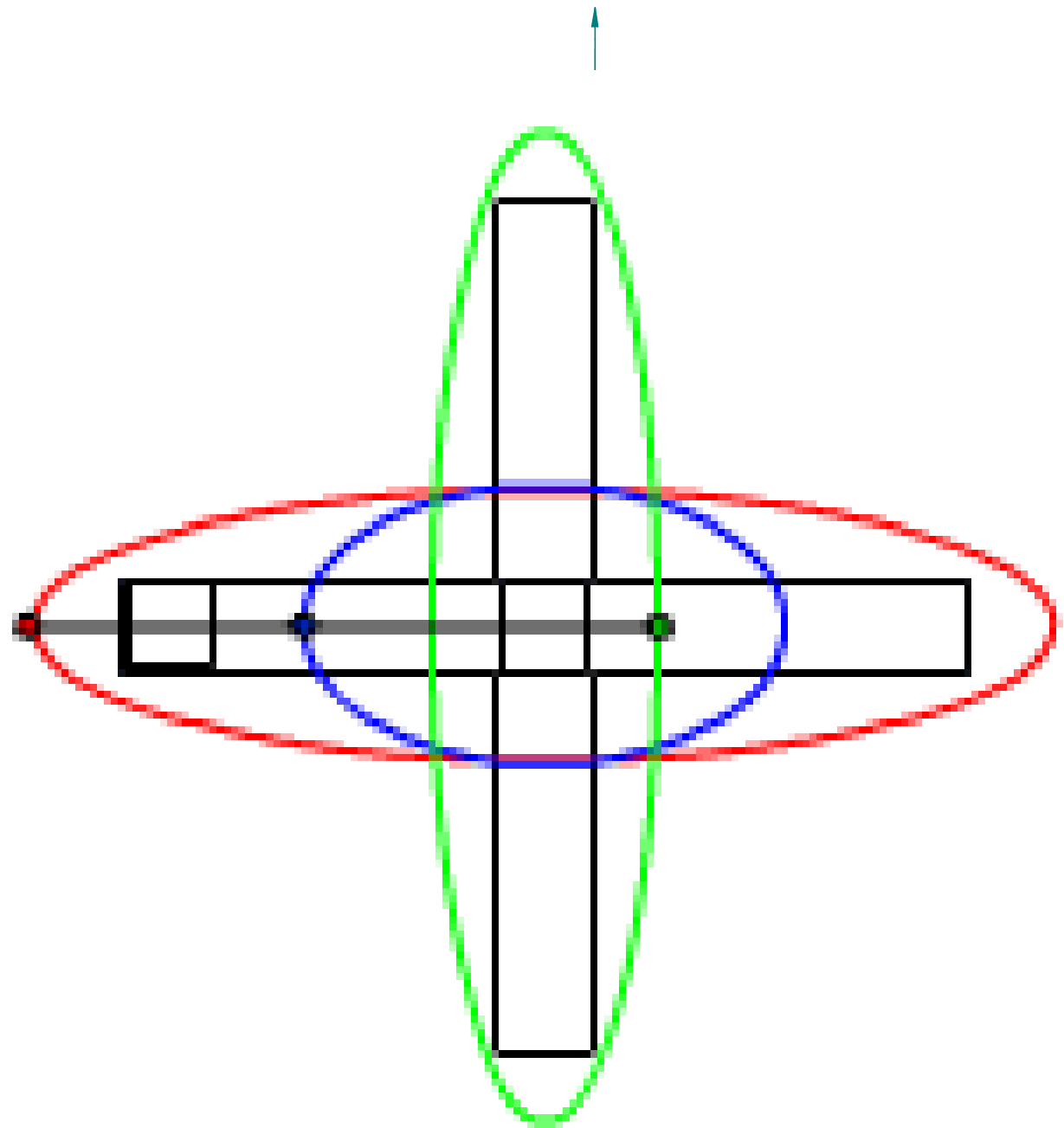


WORKING

Elliptical trammel

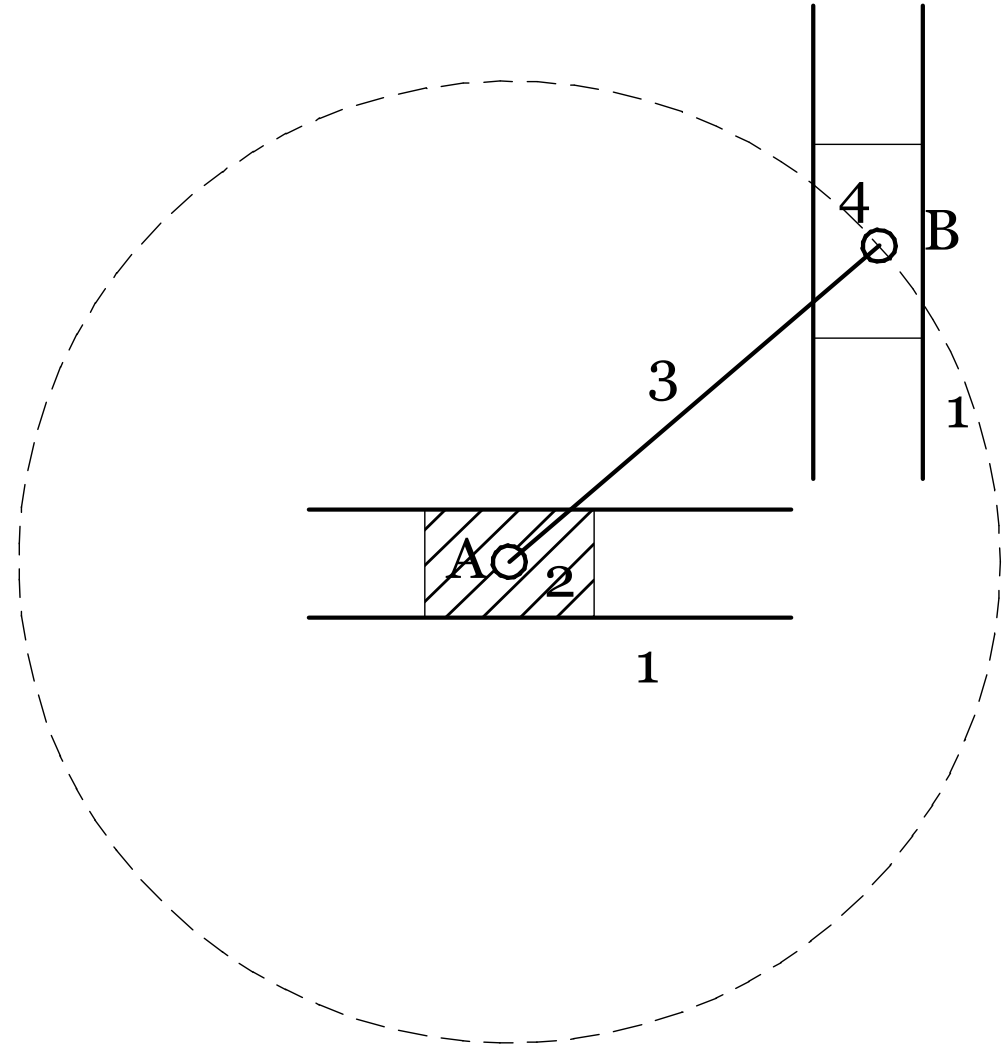


This is
the po
semi n

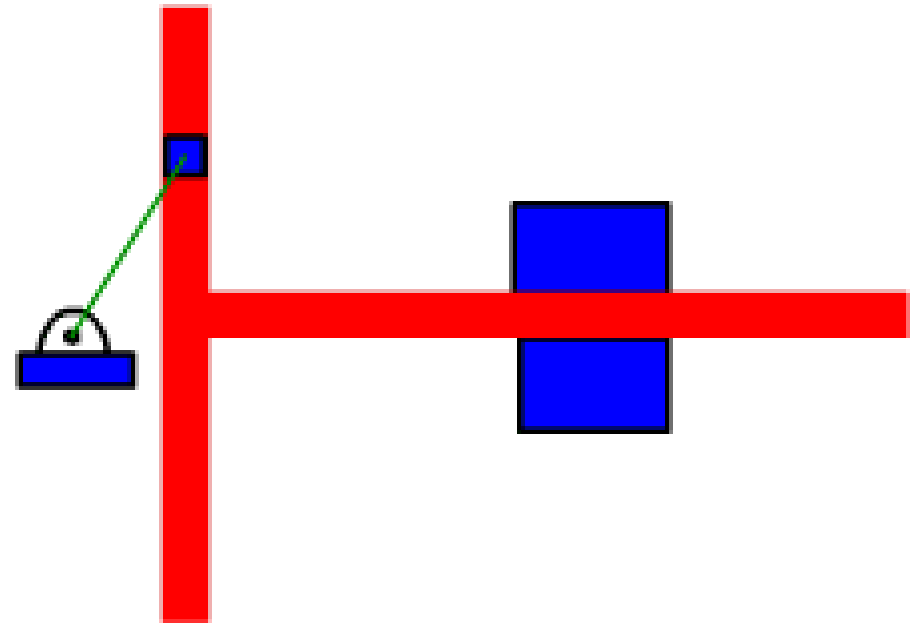
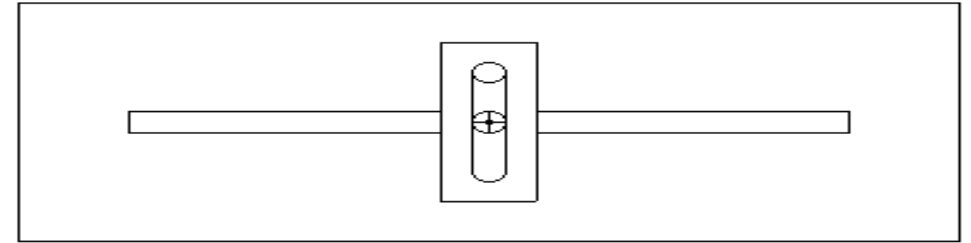
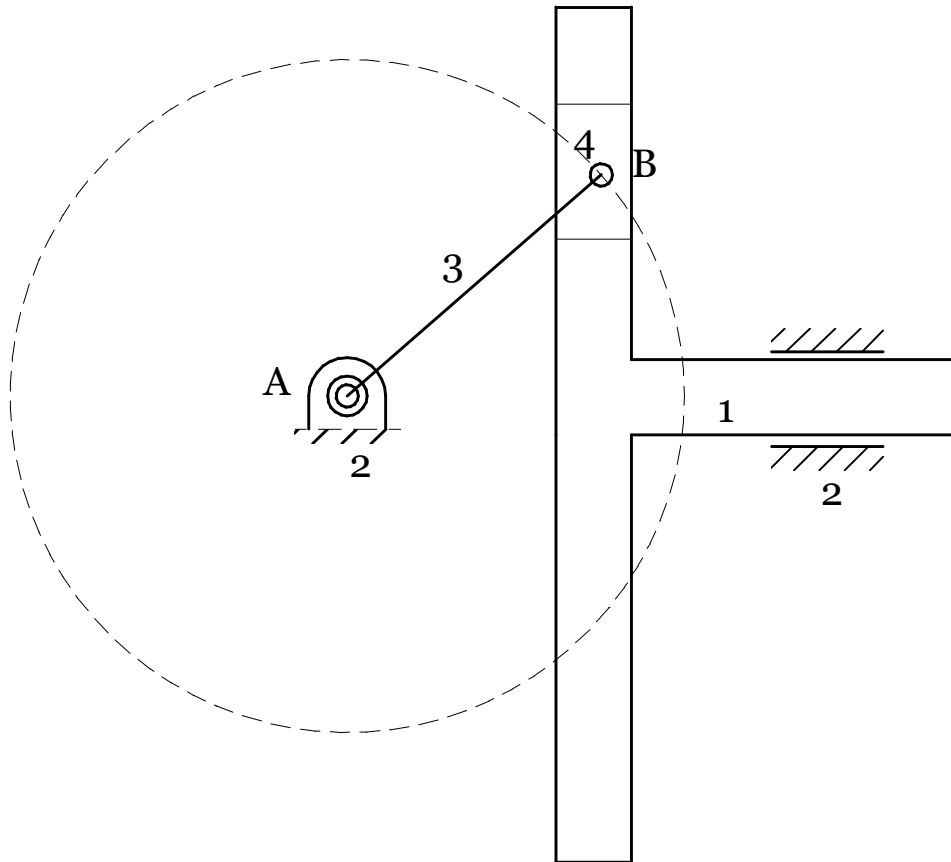


Second inversion

- When link 2 or link 4 of the double slider crank chain is fixed, the second inversion is obtained.
- Here link 2 is fixed and end B of the link 3 rotates about the about A and link 4 will reciprocate in the vertical slot.
- Hence, link 1 reciprocates in the horizontal direction.



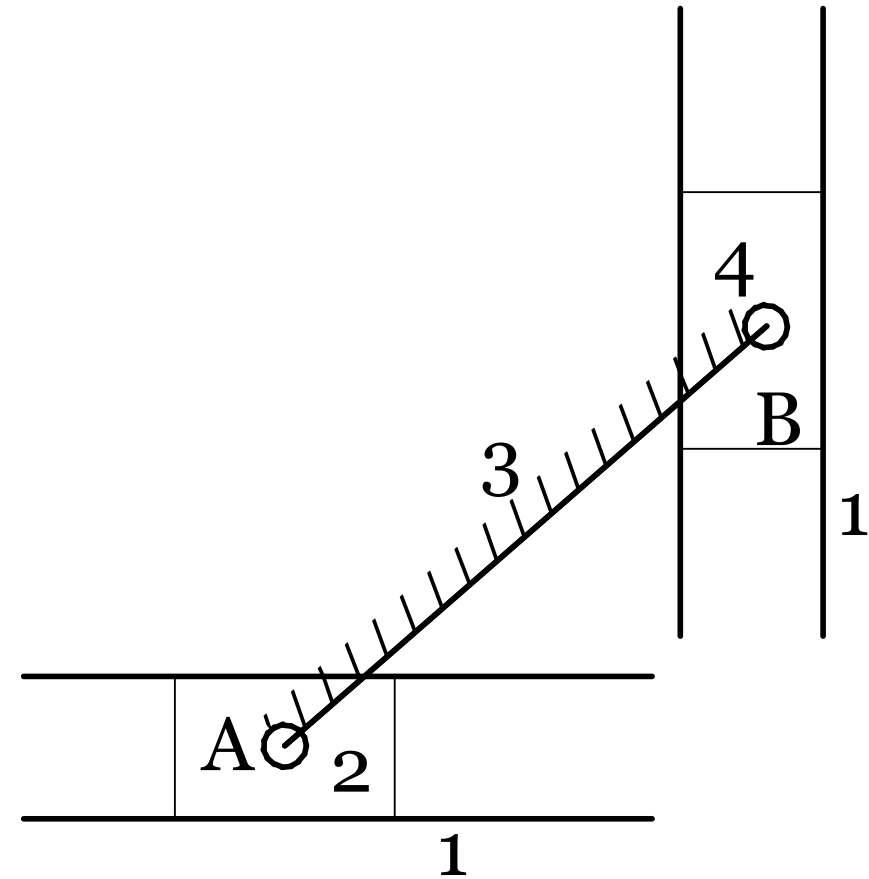
Scotch yoke mechanism



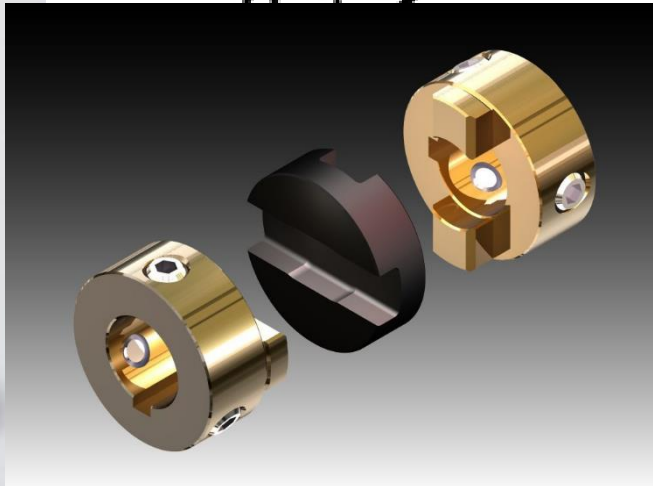
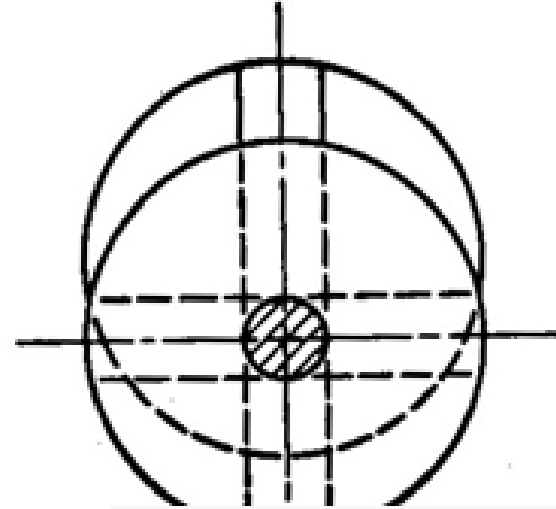
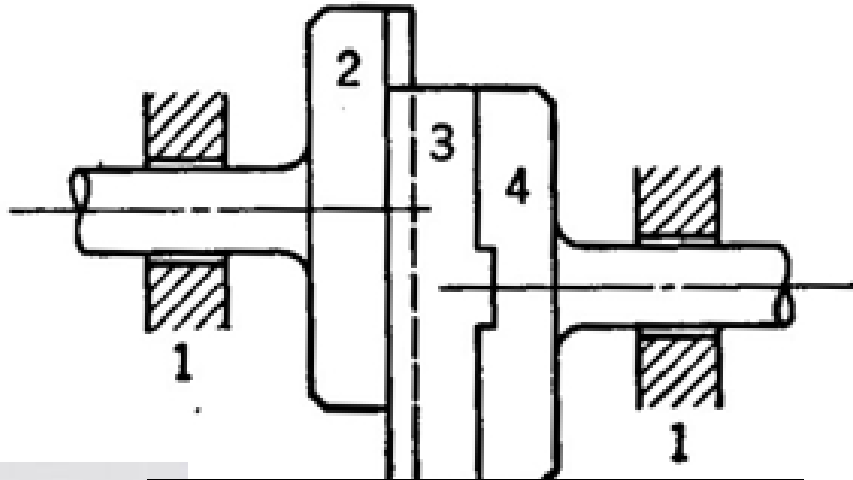
WORKING

Third inversion:

- When the link 3, of the double slider crank chain is fixed & link 1 is free to move, the third inversion is obtained.
- In this case each of the slider blocks (i.e. link 2 & link 4) can turn about the pins A & B. if one slide block (say link 2) is turned through an definite angle, the frame (i.e. link 1) and other block (i.e. link 4) must turn through the same angle.



Oldhaum's coupling



shafts when the c



Degrees of freedom OR Mobility of Mechanism

- It is defined as the **number of independent relative motions**, both translational & rotational, a pair can have.
- An unconstrained rigid body moving in space can describe the following independent motions.
 - **Translation motion** along three mutually perpendicular **axes x, y & z**
 - **Rotation motion** about these axes.
- Thus a rigid body possesses **6 degrees of freedom**.
- The connection of a link with another imposes certain constraints on their relative motion.
- The number of restraints can never be zero (joint is disconnected) or six (joint becomes solid).

$$\text{Degrees of freedom} = 6 - \text{number of restraints}$$

GRUBLER'S CRITERION

- The Grubler's mobility equation for a planar mechanism is

$$F = 3(l - 1) - 2j_1 - j_2$$

Where,

F = mobility or number of degrees of freedom

l = number of links including frame

j_1 = joints with single (one) degree of freedom (lower pairs or binary joints)

j_2 = joints with two degrees of freedom (higher pairs)

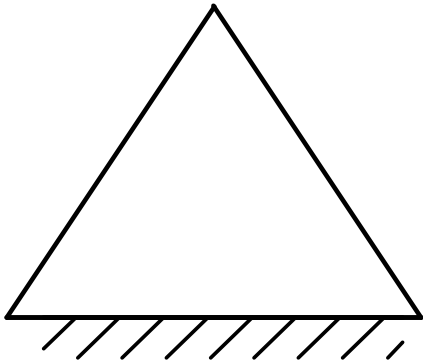
F > 0, results a mechanism with F degrees of freedom

F = 0, results in a statically determinate structure

F < 0, results in a statically indeterminate structure

A joint connecting l links at a single point must be counted as (l-1) joints.

Examples: (mechanisms with lower pairs)



$$F = 3(l - 1) - 2j_1 - j_2$$

$$l=3$$

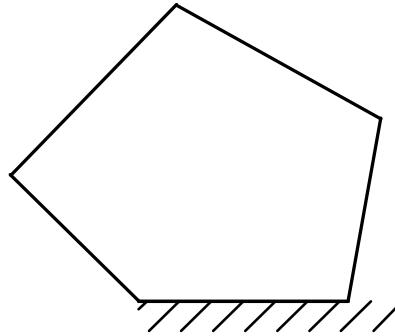
$$j_1=3$$

$$j_2=0$$

$$F = 3(3 - 1) - 2 \times 3 - 0$$

$$F=0$$

Therefore statically determinate structure



$$F = 3(l - 1) - 2j_1 - j_2$$

$$l=5$$

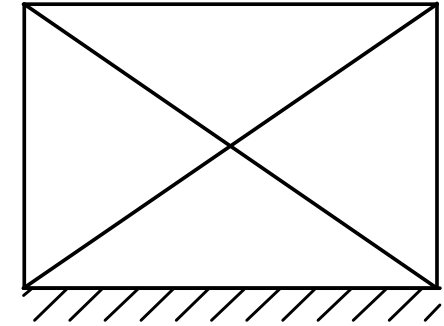
$$j_1=5$$

$$j_2=0$$

$$F = 3(5 - 1) - 2 \times 5 - 0$$

$$F=2$$

Therefore mechanism with 2 degrees of freedom



$$F = 3(l - 1) - 2j_1 - j_2$$

$$l=6$$

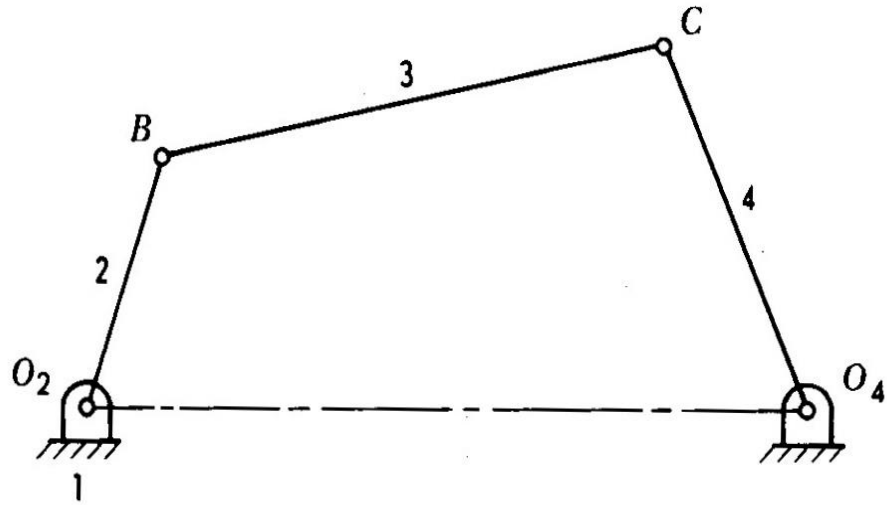
$$j_1=8$$

$$j_2=0$$

$$F = 3(6 - 1) - 2 \times 8 - 0$$

$$F=-1$$

Therefore statically indeterminate structure



$$F = 3(l - 1) - 2j_1 - j_2$$

$$l=4$$

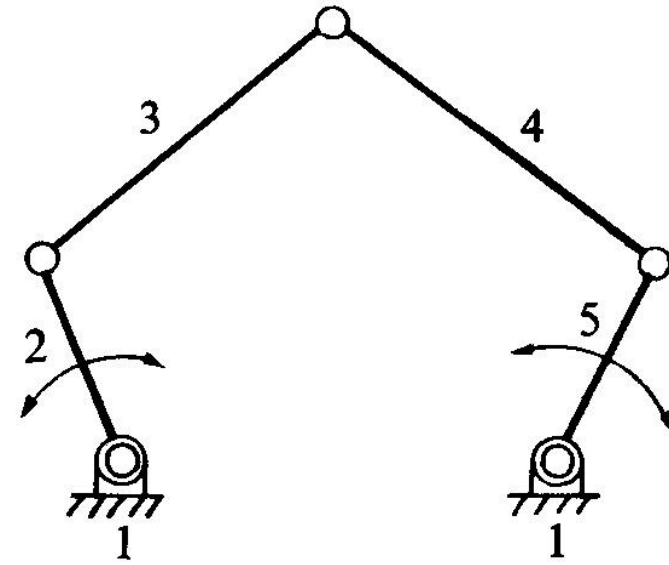
$$j_1=4$$

$$j_2=0$$

$$F = 3(4 - 1) - 2 \times 4 - 0$$

$$F=1$$

Therefore mechanism with 1 degrees of freedom I.e., one input to any one link will result in definite motion of all the links.



$$F = 3(l - 1) - 2j_1 - j_2$$

$$l=5$$

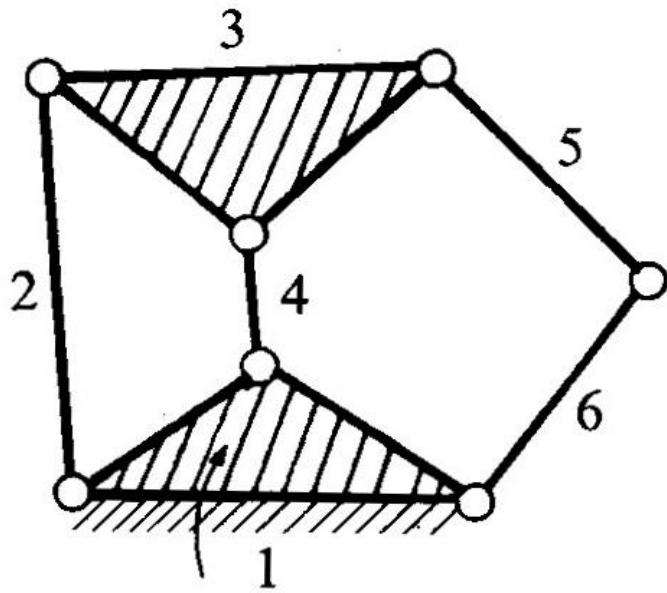
$$j_1=5$$

$$j_2=0$$

$$F = 3(5 - 1) - 2 \times 5 - 0$$

$$F=2$$

Therefore mechanism with 2 degrees of freedom I.e., two inputs to any two links are required to yield definite motions in all the links.



$$F = 3(l - 1) - 2j_1 - j_2$$

$$l=6$$

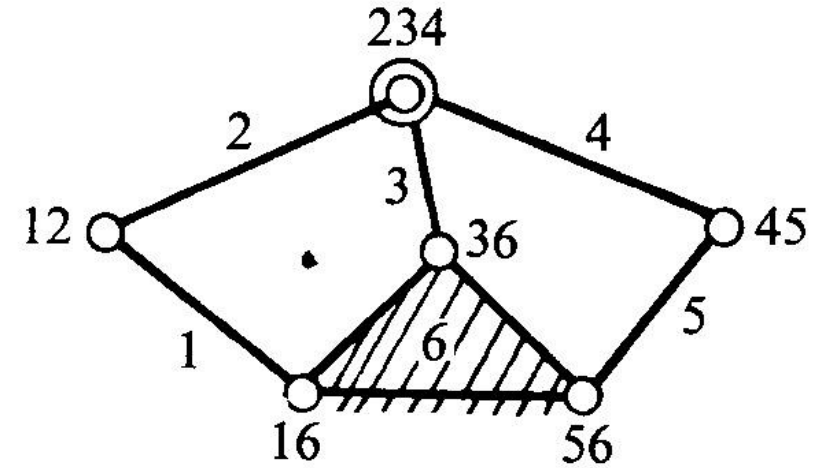
$$j_1=7$$

$$j_2=0$$

$$F = 3(6 - 1) - 2 \times 7 - 0$$

$$F=1$$

Therefore mechanism with 1 degrees of freedom i.e., one input to any one link will result in definite motion of all the links.



$$F = 3(l - 1) - 2j_1 - j_2$$

$$l=6$$

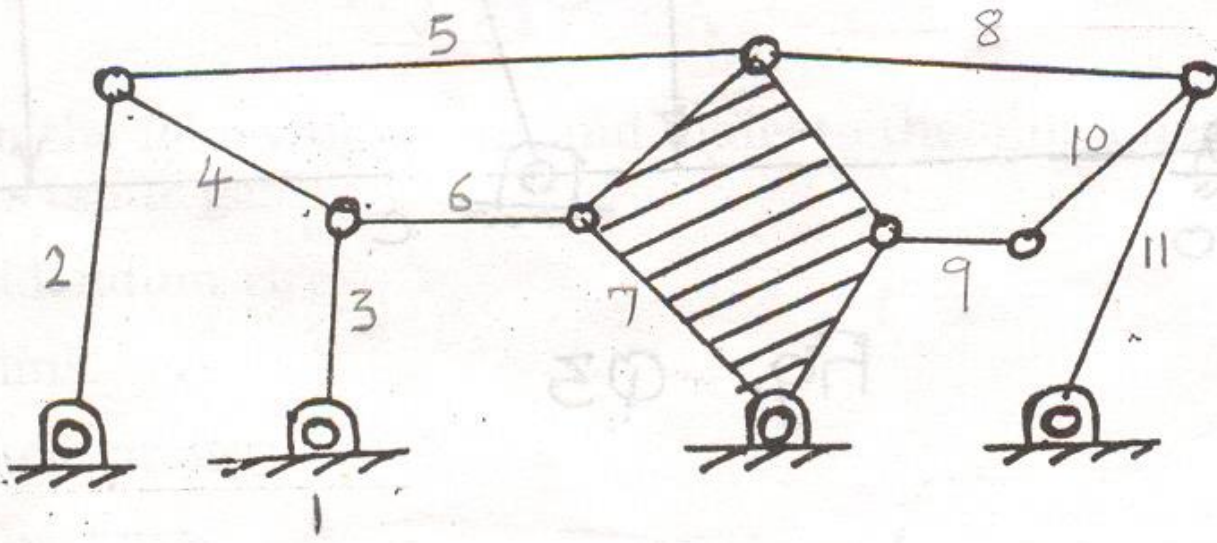
$$j_1=7$$

$$j_2=0$$

$$F = 3(6 - 1) - 2 \times 7 - 0$$

$$F=1$$

Therefore mechanism with 1 degrees of freedom i.e., one input to any one link will result in definite motion of all the links.



$$F = 3 (l - 1) - 2j_1 - j_2$$

$$l=11$$

$$j_1=15$$

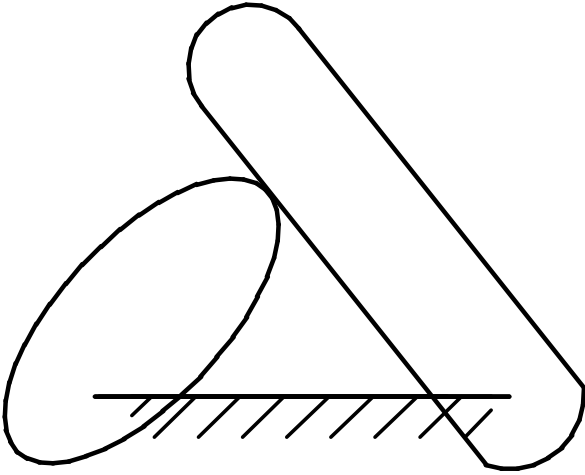
$$j_2=0$$

$$F = 3 (11 - 1) - 2 \times 15 - 0$$

$$F=0$$

Therefore statically determinate structure

Example : (mechanisms with higher pairs)



$$F = 3(l - 1) - 2j_1 - j_2$$

$$l=3$$

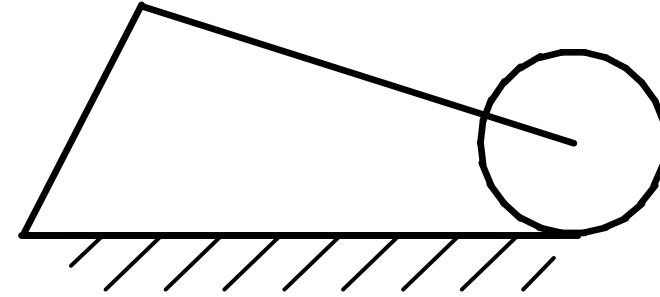
$$j_1=2$$

$j_2=1$ (there exist a rolling & sliding between 2 & 3)

$$F = 3(3 - 1) - 2 \times 2 - 1$$

$$F=1$$

Therefore mechanism is of 1 degree freedom system



$$F = 3(l - 1) - 2j_1 - j_2$$

$$l=4$$

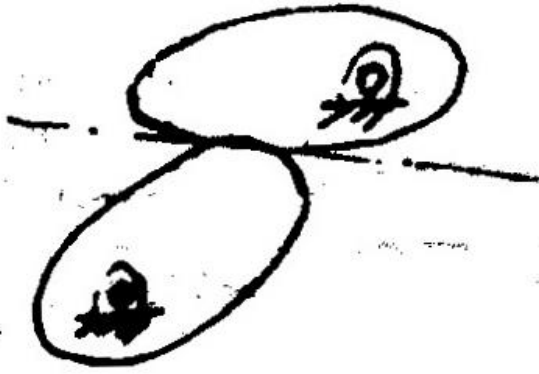
$$j_1=3$$

$j_2=1$ (there exist a rolling & sliding between 4 & 1)

$$F = 3(4 - 1) - 2 \times 3 - 1$$

$$F= 2$$

Therefore mechanism is of 2 degree freedom system



$$F = 3 (l - 1) - 2j_1 - j_2$$

$$l=3$$

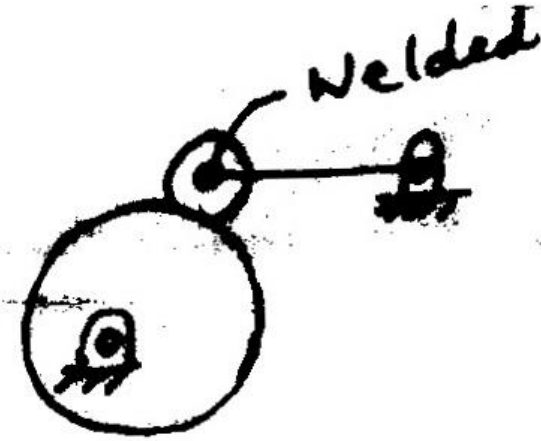
$$j_1=2$$

$j_2=1$ (there exist a rolling & sliding between 2 & 3)

$$F = 3 (3 - 1) - 2 \times 2 - 1$$

$$F=1$$

Therefore mechanism is of 1 degree freedom system



$$F = 3 (l - 1) - 2j_1 - j_2$$

$$l=3$$

$$j_1=2$$

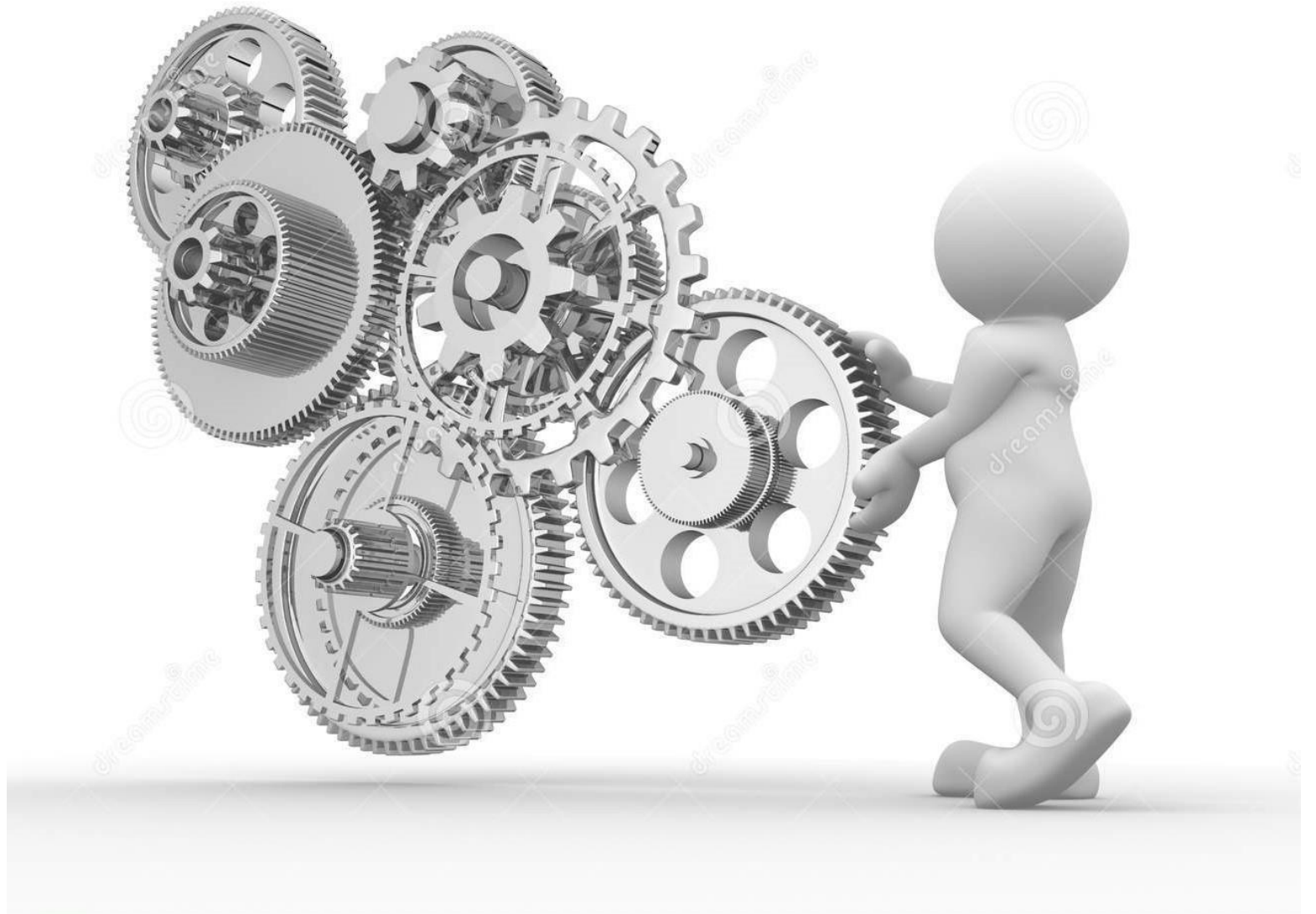
$j_2=1$ (there exist a rolling & sliding between 2 & 3)

$$F = 3 (3 - 1) - 2 \times 2 - 1$$

$$F=1$$

Therefore mechanism is of 1 degree freedom system

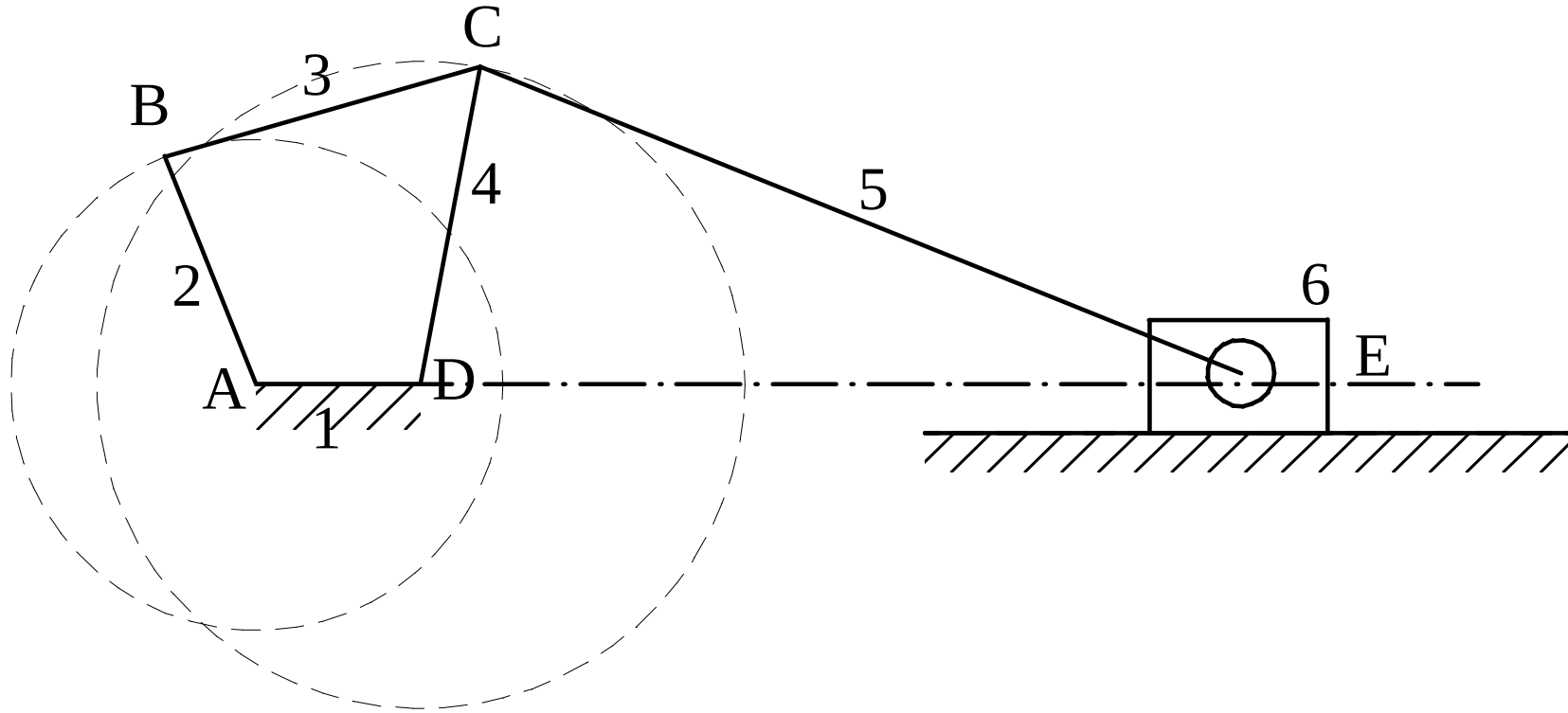
MECHANISMS



Quick – Return mechanisms

- Repetitive operations.
- Mechanism is under load, called working stroke.
- Return stroke.
- Time ratio.
- To produce quick return, the time ratio must be greater than unity and as large as possible.
- Quick return mechanisms are used on machine tools to give a slow cutting stroke and a quick return stroke for a constant angular velocity of the driving crank.
- The most commonly used types of quick return mechanisms are
 - Drag link mechanism
 - Whitworth mechanism
 - Crank & Slotted lever Mechanism

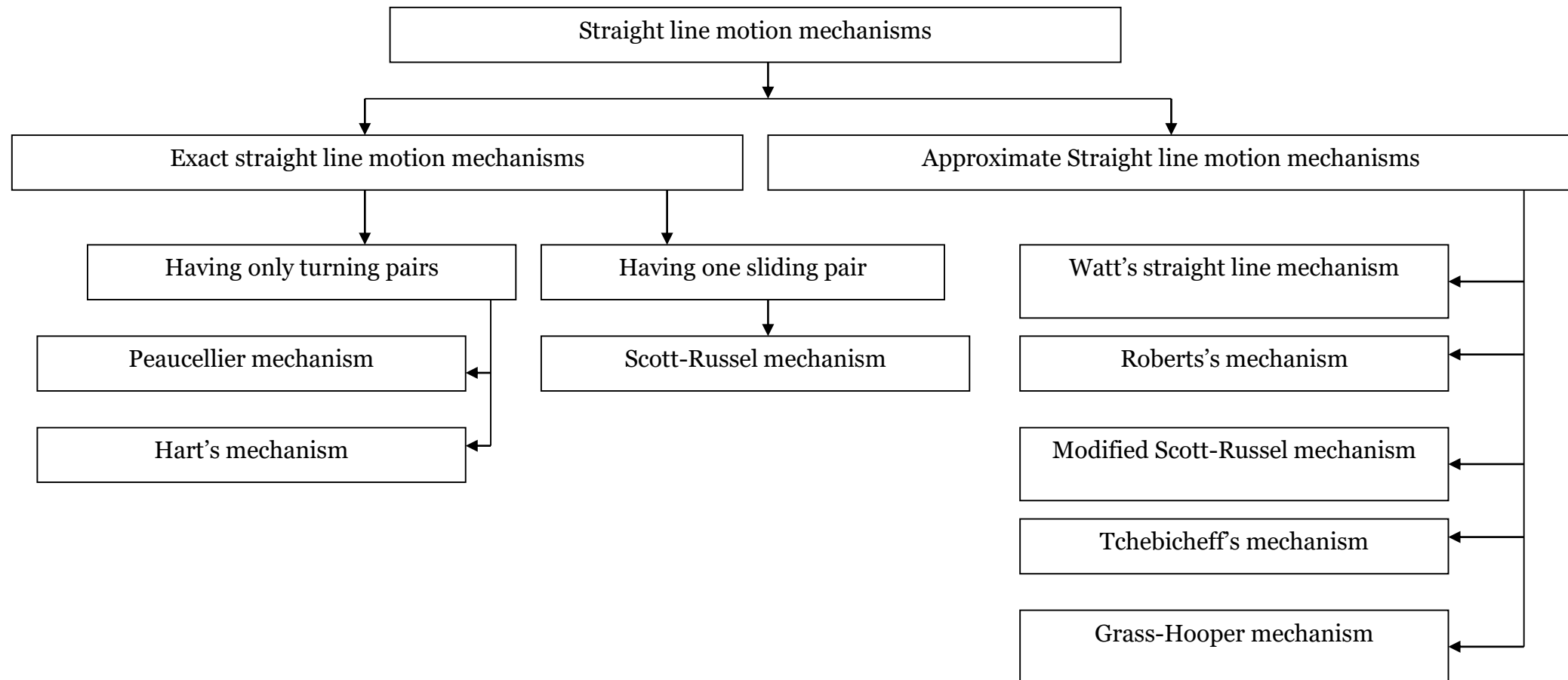
Drag link quick return mechanism (complete rotation of the crank & follower)



WORKING

Straight line motion mechanisms

- Used to produce straight motions.
- Mechanisms may produce exactly straight line motion or approximate straight line motion.
- Either only turning pairs are connected or one sliding pair is used.



Condition for exact straight line motion mechanisms using only turning pairs

- **Principle:**

- Let O be the centre of a circle of diameter AD.
- AB is any chord.
- The triangle inscribed on a semi circle (i.e. $\triangle ABD$) will be right angled triangle.
- The chord produced up to the point C.
- From C, draw a line CE perpendicular to the diameter AD produced.
- Then locus of point C will be straight line, perpendicular to the diameter AD, provided the product of $AB \times AC$ is a constant.

- **Proof:**

- The $\triangle$ s AEC & ABD are similar as $\angle DAB = \angle EAC$ (common angle) and $\angle ABD = \angle AEC = 90^\circ$

- Hence,

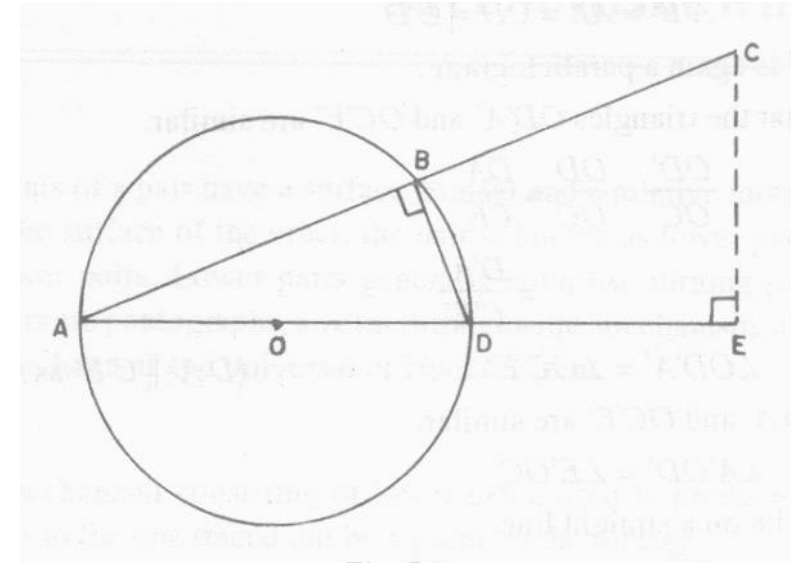
- Or
$$\frac{AD}{AC} = \frac{AB}{AE}$$

- $$AB \times AC = AD \times AE$$

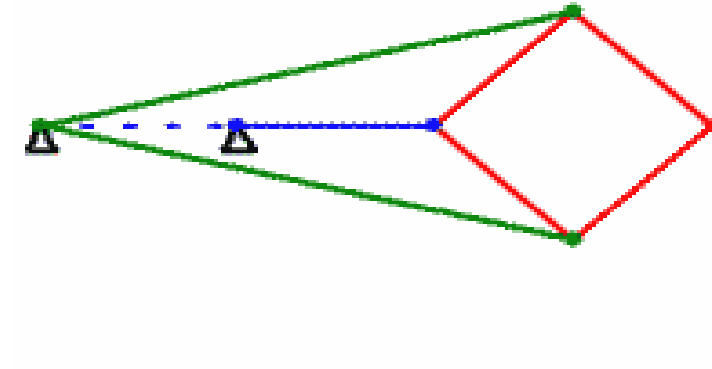
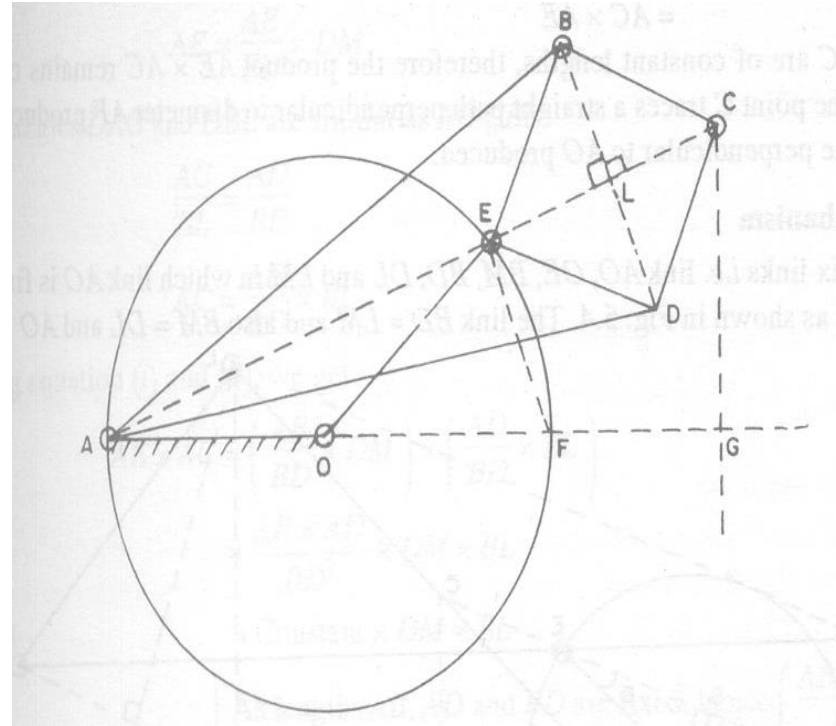
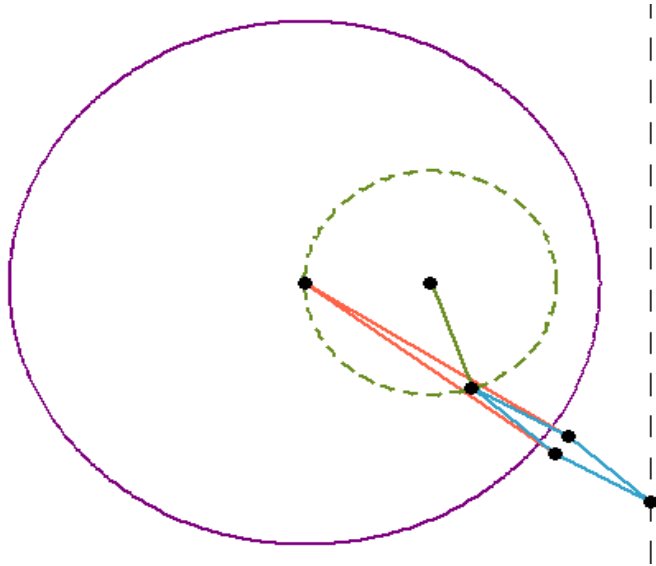
But AD is the diameter of the circle and hence it is constant.

If AE also constant then $AB \times AC$ will be a constant when the perpendicular from the point C always coincides with point E.

Hence the projection of C should always be at E.



PEAUCELLIER MECHANISM



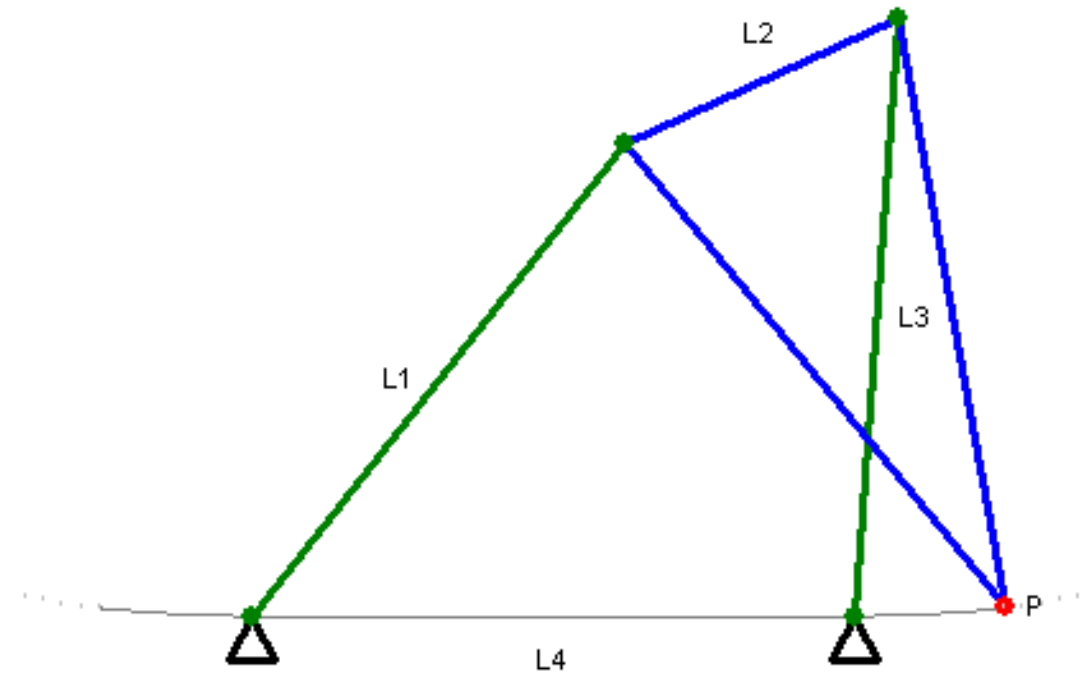
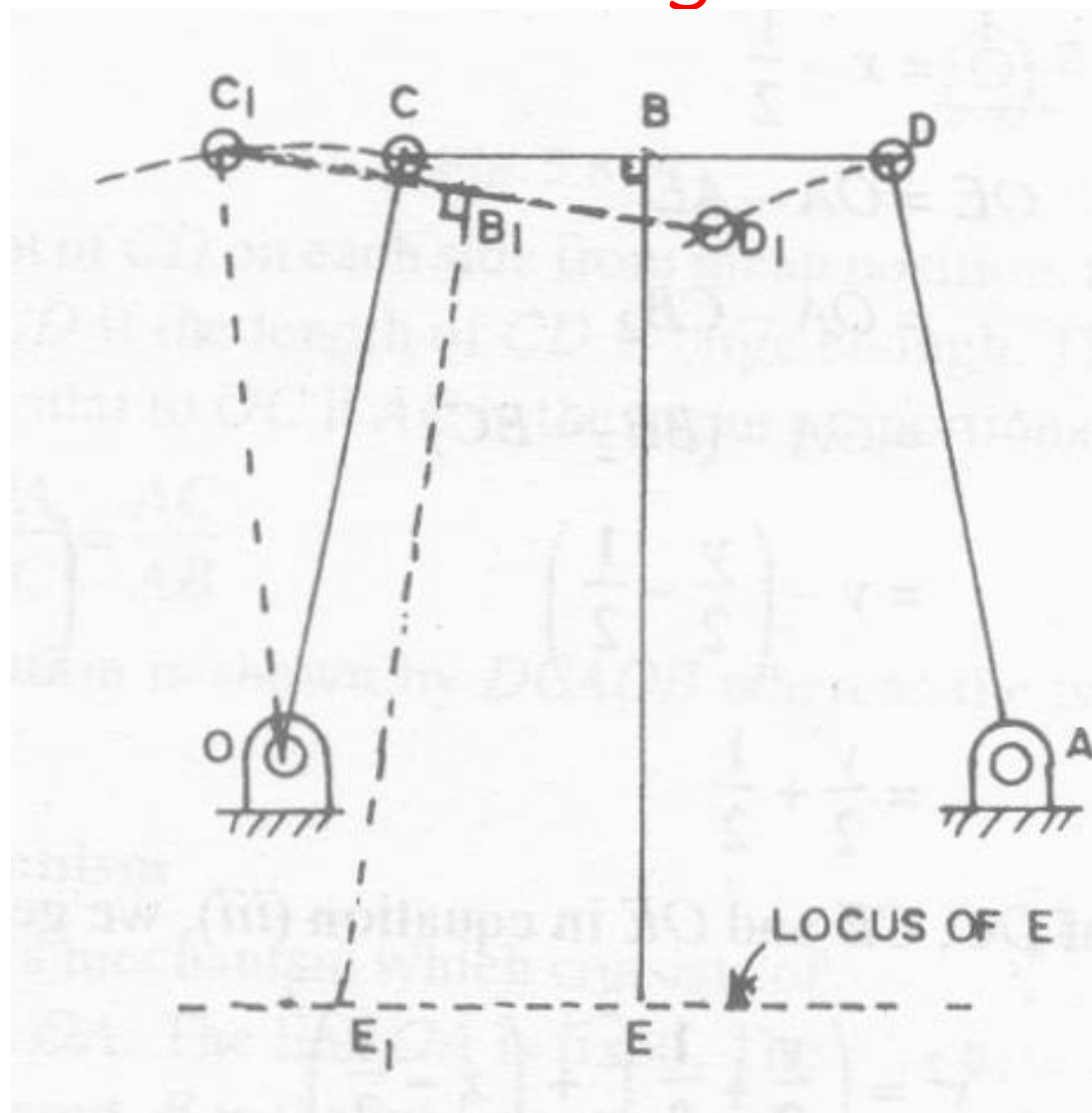
•WORKING

- It consists of eight links i.e. links AO, OE, AB, AD, EB, BC, CD and DE in which link AO is fixed and the link OE is rotating about point O, as shown in the fig.
- Links $BC = CD = DE = EB$, thus form a rhombus and link $AB = AD$.
- All the links are connected to pin joints.
- The pin at E is constrained to move along the circumference of a circle of diameter AF by means of link OE, thus $OA = OE$.
- As the link OE moves around O, the point C moves in the straight line perpendicular to AO produced.

- Since BCDE is a rhombus, the diagonals EC & BD will bisect each other at right angles. Hence $\angle ELB = \angle CLB = 90^\circ$.
- Also the $\angle AEF$ will be right angle for all positions of E, as it is angle subtended by a diameter of the circle on the circumference of the circle.
- In $\triangle AEB$ & $\triangle ADE$, $AB = AD$, $BE = ED$ & AE is common. Therefore these two triangles are similar.
- Hence $\angle EAB = \angle EAD$. Therefore point E lies on the bisector of $\angle DAB$(1)
- In $\triangle ACB$ & $\triangle ACD$, $AB = AD$, $BC = CD$ & AC is common. Therefore these two triangles are similar.
- Hence $\angle CAB = \angle CAD$. Therefore point C lies on the bisector of $\angle DAB$(2)
- From (1) & (2), it is clear that AEC is a straight line.
- Now in right angled triangle ALB,
- $AB^2 = AL^2 + LB^2$ (3)
- Now in right angled triangle CLB,
- $BC^2 = CL^2 + LB^2$ (4)
- Subtracting equation (4) from (3), we get

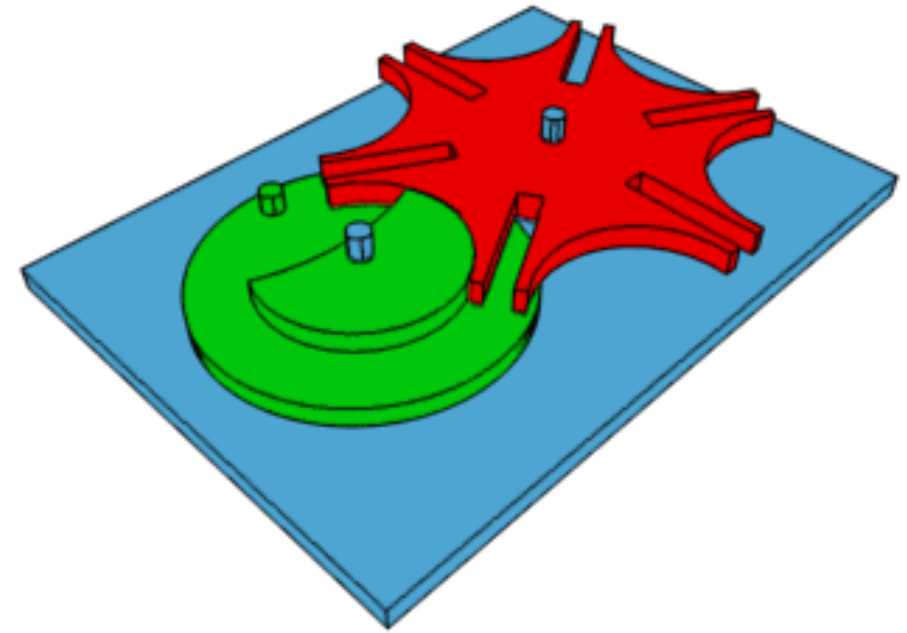
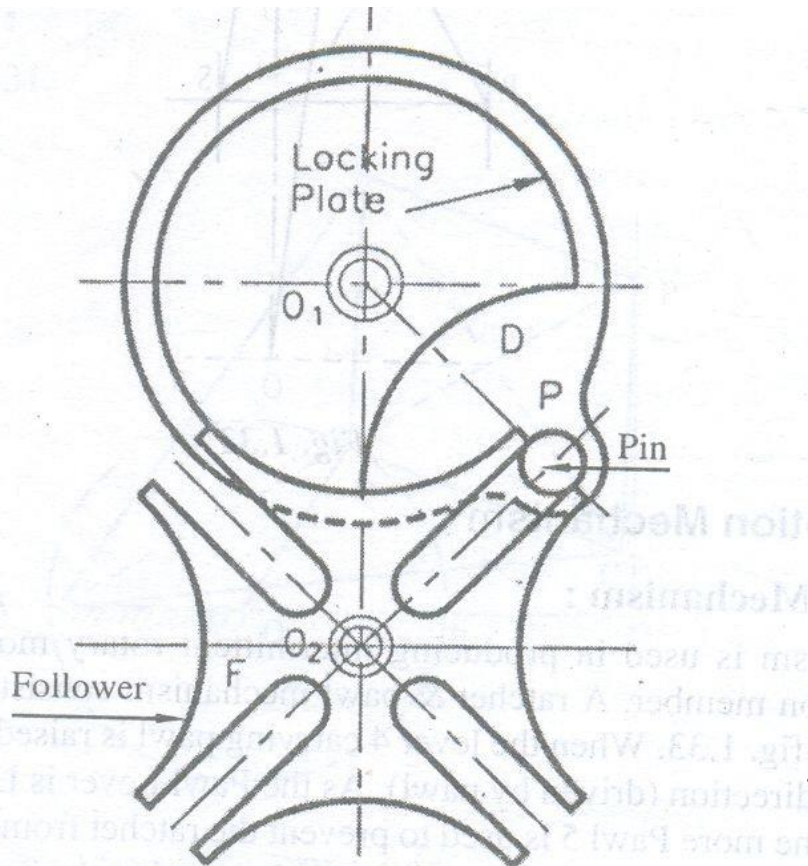
$$\begin{aligned}
 AB^2 - BC^2 &= AL^2 - CL^2 \\
 &= (AL + CL)(AL - CL) \\
 &= AC \times (AL - EL) \\
 &= AC \times AE
 \end{aligned}$$
- But AB & BC are of constant lengths; therefore the product $AE \times AC$ remains constant.
- Hence point C traces a straight path perpendicular to the diameter AF produced.
- Hence point C moves in a straight line perpendicular to AO produced.

Robert's Straight Line Mechanism



INTERMITTENT MOTION MECHANISM

Geneva mechanism



D- Driving Wheel

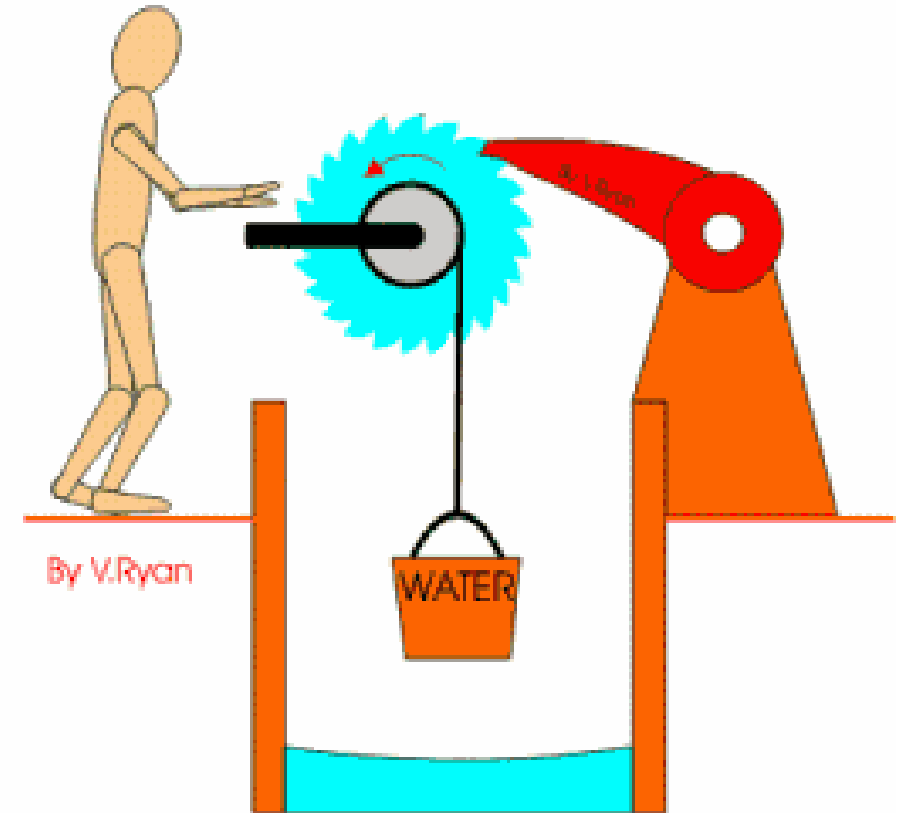
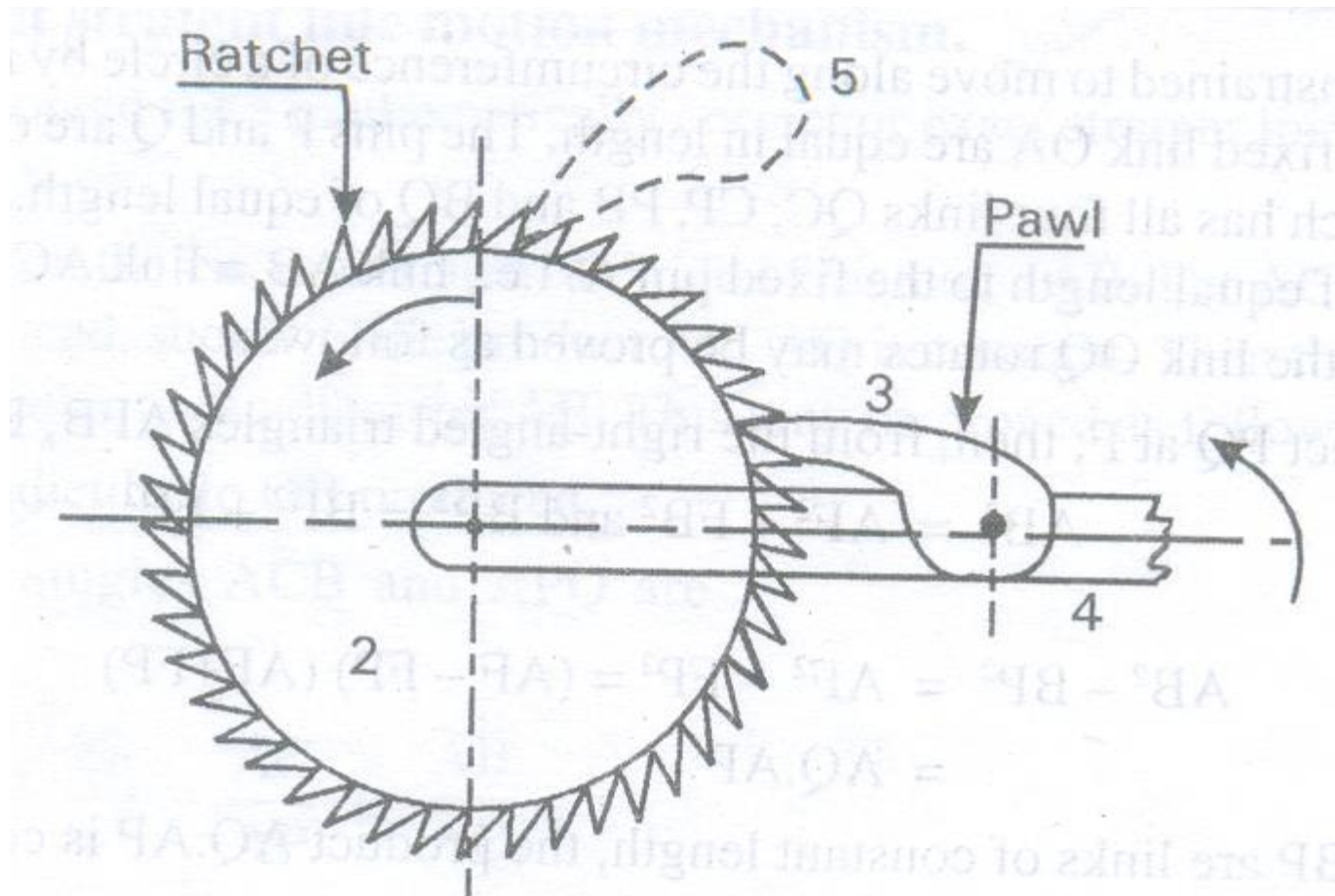
P- Pin

F- Follower

[WORKING](#)

Used in preventing overriding of main springs in clocks and watches, feeding of film in projectors and indexing working table on machine tool

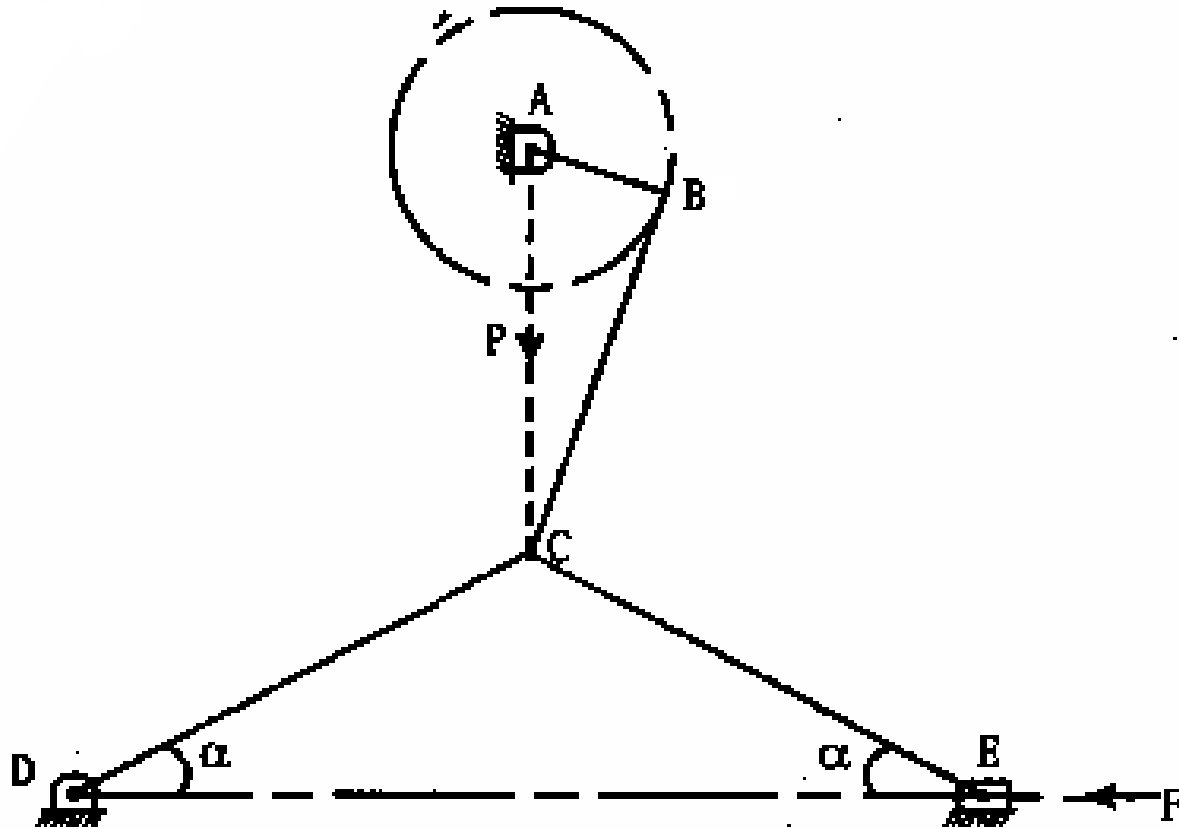
RATCHET & PAWL MECHANISM



- 2-Ratchet wheel
- 3- Pawl
- 4- Lever
- 5- Pawl

Used in Feed mechanisms, lifting jacks, clocks, watches & counting devices

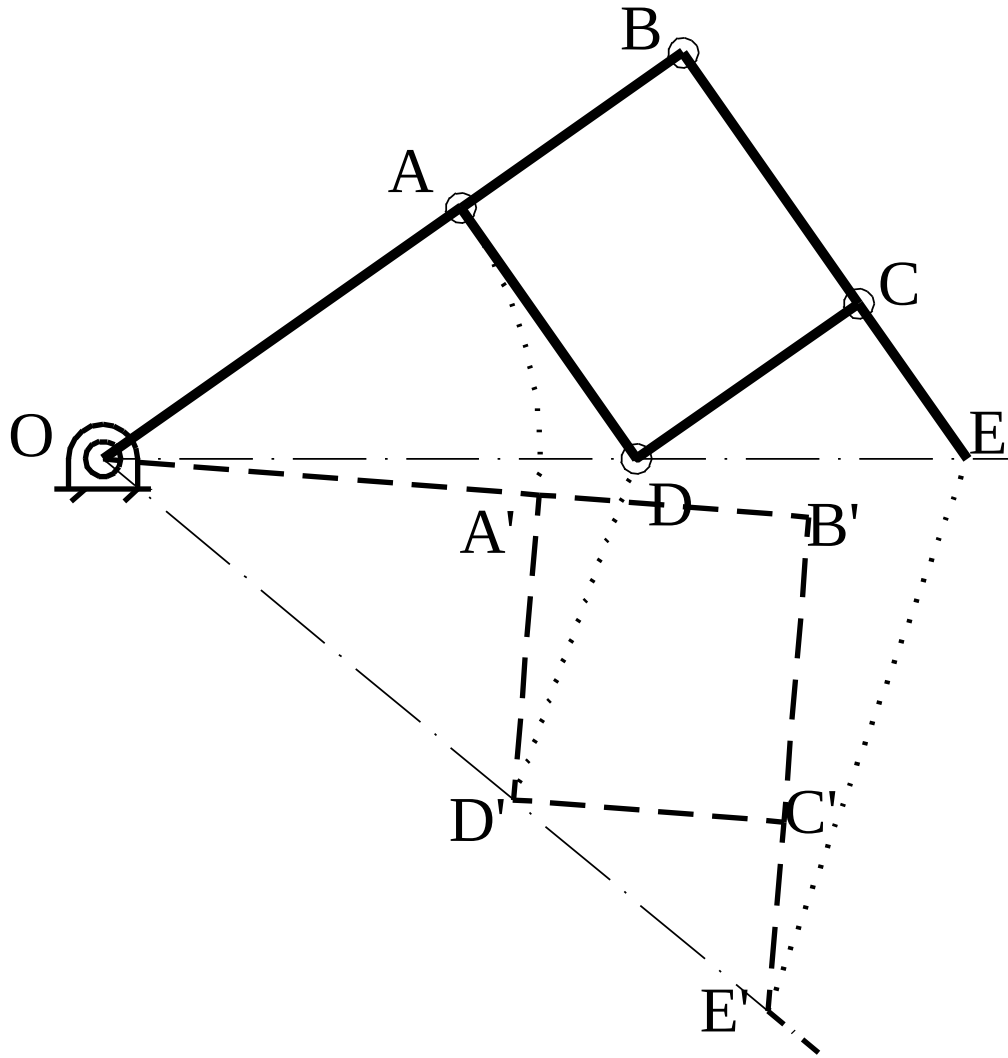
Toggle Mechanism



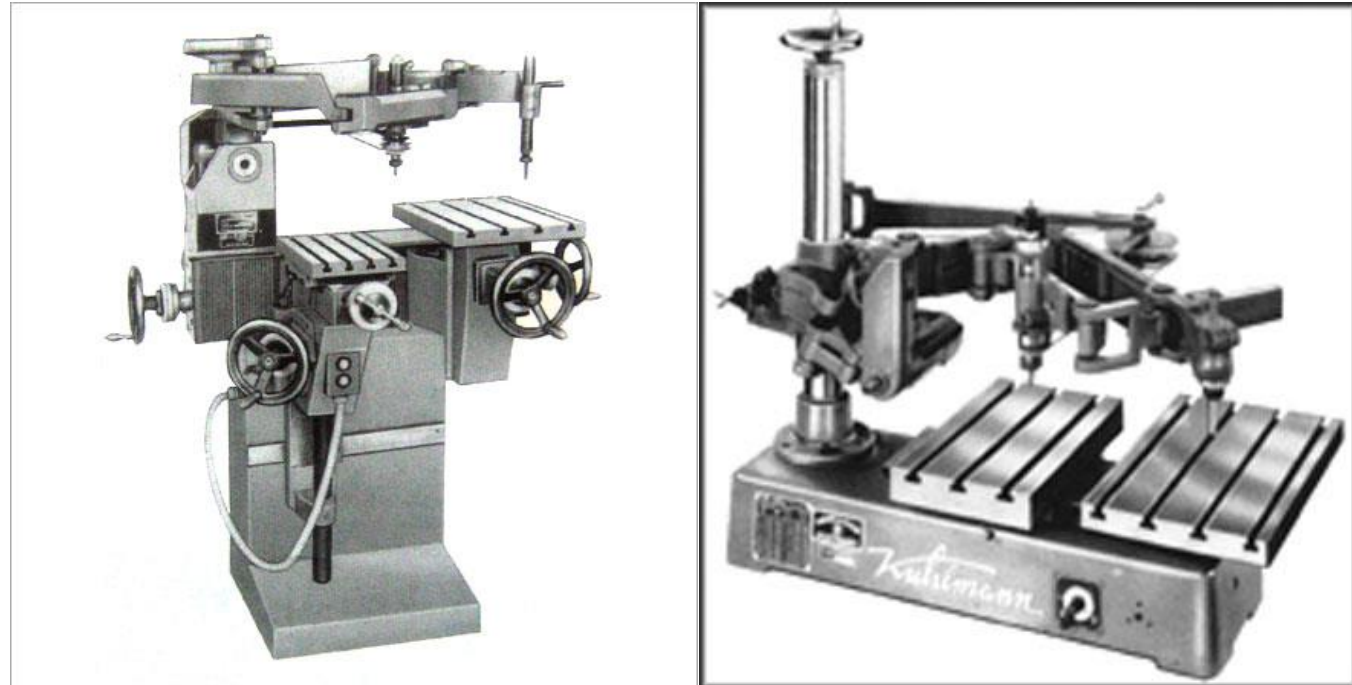
WORKING

Used in toggle clamps, riveting machines, punch presses, stone crushers, etc.

PANTOGRAPH



WORKING



Uses of Pantograph

- Pantograph is used as geometrical instrument
- Pantograph is used to guide the cutting tools
- Used as an indicator rig to reproduce the displacement of cross head of reciprocating engine

Condition for correct steering in motor cars

$$AC = EF = EG - FG \dots (1)$$

$$\text{In } \triangle CFG, \tan \theta = CF/FG$$

$$FG = CF / \tan \theta = CF \cot \theta \dots (2)$$

$$\text{In } \triangle AEG, \tan \phi = AE/EG$$

$$EG = AE / \tan \phi = AE \cot \phi \dots (3)$$

Substituting (2) & (3) in (1), we get

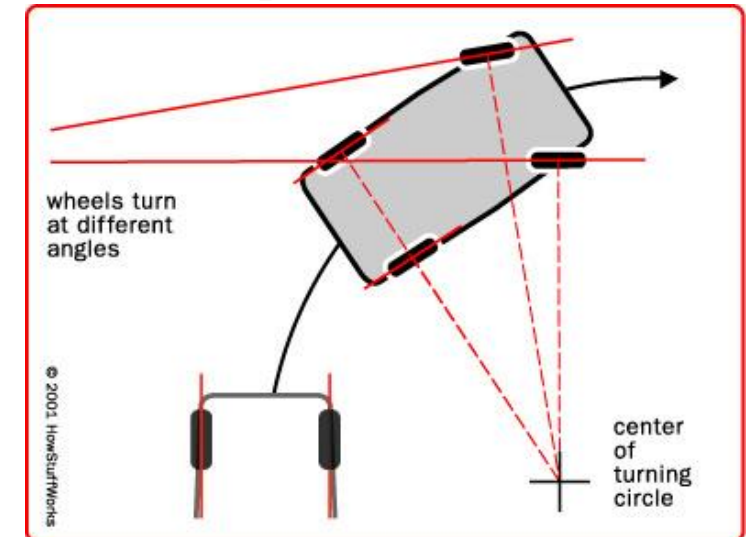
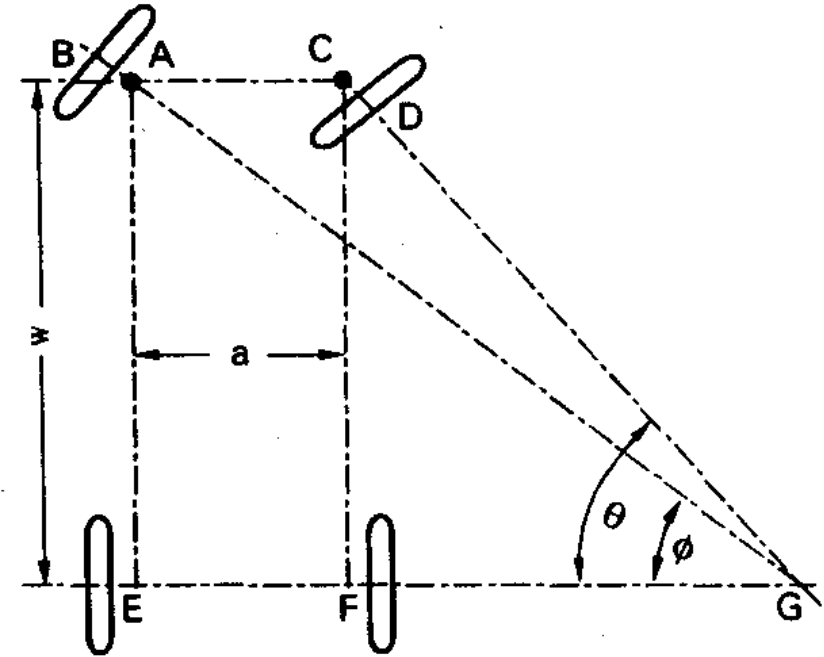
$$AC = AE \cot \phi - CF \cot \theta$$

$$= AE (\cot \phi - \cot \theta)$$

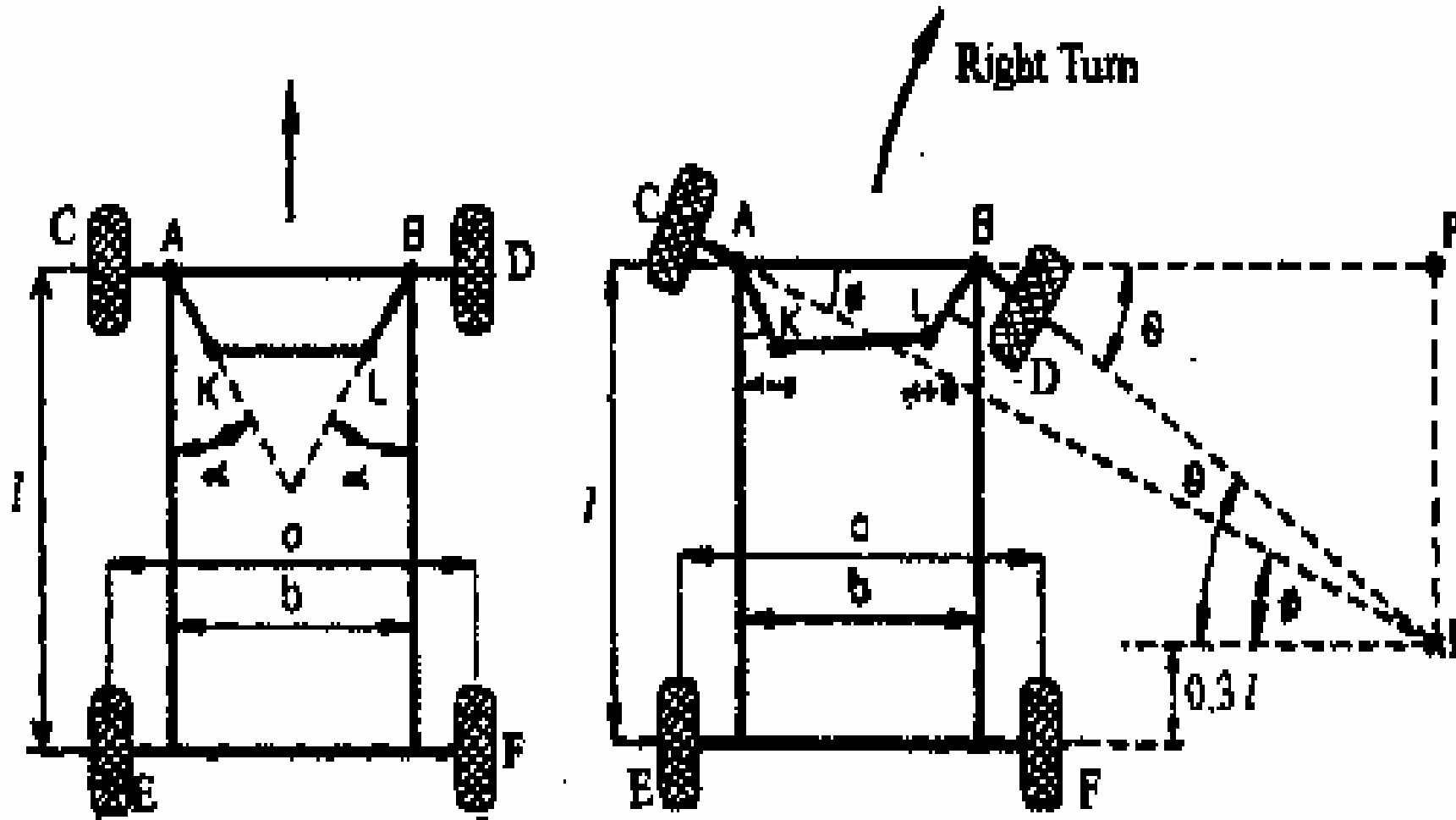
$$\cot \phi - \cot \theta = AC/AE = a/w$$

$$\cot \phi - \cot \theta = AC/AE = a/w$$

Condition for correct steering



Ackermann steering gear mechanism



THANK YOU