#### **Engineering Mathematics**

SUBJECT NAME : Statistics and Numerical Methods

SUBJECT CODE : MA3251 QUESTION BANK

MATERIAL NAME : University Questions

REGULATION : R2017

# **Unit** – I (Testing of Hypothesis)

#### • Student's 't' test

1. A random sample of 10 boys has the following IQ's 70, 83, 88, 95, 98, 100, 101, 107, 110 and 120. Do these data support the assumption of a population mean IQ of 100 at 5% level of significance? (N/D 2017),(N/D 2018)

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2. The heights of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 inches. Is it reasonable to believe that the average height is greater than 64 inches? (A/M 2011)

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3. A sample of 10 boys had the I.Q's: 70, 120, 110, 101, 88, 83, 95, 98, 100 and 107. Test whether the population mean I.Q may be 100. (N/D 2012)

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4. The IQ's of 10 girls are respectively 120, 110, 70, 88, 101, 100, 83, 98, 95, 107. Test whether these two proportions are same. (M/J 2016)

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5. The height of six randomly chosen sailors are (in inches): 63, 65, 68, 69, 71 and 72. Those of 10 randomly chosen soldiers are 61, 62, 65, 66, 69, 69, 70, 71, 72 and 73.

Discuss, the height that these data thrown on the suggestion that sailors are on the average taller than soldiers ( $t_{0.05}(14) = 1.76$ ). (N/D 2014)

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#### F – test

1. The nicotine content in milligram of 2 samples of tobacco were found to be as follows:

Sample A						
Sample B	27	30	28	31	22	36

(A/M 2018)

Can it be said that these sample were from normal population with the same mean.

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Two independent samples of sizes 9 and 7 from a normal population had the following values of the variables. (M/J 2014)

Do the estimates of population variance differ significantly at 5% level of significance?

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3. Time taken by workers in performing a job are given below:

Test whether there is any significant difference between the variances of time distribution.  $(N/D\ 2013)$ 

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 Test whether there is any significant difference between the variances of the populations from which the following samples are taken: (N/D 2012), (N/D 2017)

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5. Test if the difference in the means is significant for the following data: (N/D 2010)

Sample I:								
Sample II:	40	48	92	85	70	76	68	22

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6. Two random samples gave the following results:

			Sum of squares of
Sample	Size	Sample mean	deviation from the
			mean
1	10	15	90
2	12	14	108

Test whether the samples have come from the same normal population. (M/J 2012)

# • Chi-Square test (Goodness of fit)

- Theory predicts that the proportion of beans in four groups A, B, C, D should be 9:3:3:1.
   In an experiment among 1600 beans, the numbers in the four groups were 882, 313, 287 and 118. Does the experiment support the theory? (M/J 2012),(M/J 2016)
- 2. Five coins are tossed 320 times. The number of heads observed is given below:

Number of Heads	0	1	2	3	4	5
Frequency	15	45	85	95	60	20

Examine whether the coin is unbiased. Use 5% level of significance. (A/M 2018)

3. 4 coins were tossed 160 times and the following results were obtained:

No. of heads: 0 1 2 3 4 Observed frequencies: 17 52 54 31 6

Under the assumption that the coins are unbiased, find the expected frequencies of getting 0, 1, 2, 3, 4 heads and test the goodness of fit. (A/M 2011)

4. The following data gives the number of aircraft accidents that occurred during the various days of a week. Find whether the accidents are uniformly distributed over the week. (N/D 2010)

Days:	Sun	Mon	Tue	Wed	Thu	Fri	Sat
No. of accidents:	14	16	8	12	11	9	14

5. The demand for a particular spare part in a factory was found to vary from day-to-day. In a sample study the following information was obtained.

Days: Mon Tues Wed Thurs Fri Sat No. of spare parts demanded: 1124 1125 1110 1120 1126 1115 Test the hypothesis that the number of parts demanded does not depend on the day of the week. (  $\chi^2_{0.05}(5) = 11.07$  ) (N/D 2014)

# • Chi-Square test (Independence of attributes)

1. Mechanical engineers testing a new arc welding technique, classified welds both with respect to appearance and an X-ray inspection. (A/M 2018)

		Appearance					
		Bad	Normal	Good			
X-ray	Bad	20	7	3			
	Normal	13	51	16			
	Good	7	12	21			

Test for independence using 0.05 level of significance.

2. Using the data given in the following table to test at 1% level of significance whether a person's ability in Mathematics is independent of his/her interest in Statistics.

(N/D 2017),(N/D 2018)

MY		Ability in Mathematics			
<i>Y</i>	n, K	Low	Average	High	
Interest in Statistics	Low	63	42	15	
	Average	58	61	31	
	High	14	47	29	

3. Out of 8000 graduates in a town 800 are females, out of 1600 graduate employees 120 are females. Use  $\chi^2$  to determine if any distinction is made in appointment on the basic of sex. Value of  $\chi^2$  at 5% level for one degree of freedom is 3.84. (A/M 2010)

4. An automobile company gives you the following information about age groups and the liking for particular model of car which it plans to introduce. On the basic of this data can it be concluded that the model appeal is independent of the age group.

$$(\chi^2_{0.05}(3) = 7.815)$$
 (A/M2010)

Persons who:	Below 20	20 – 39	40 – 59	60 and above
Liked the car:	140	80	40	20
Disliked the car:	60	50	30	80

5. Test of the fidelity and selectivity of 190 radio receivers produced the results shown in the following table:

	Fidelity			
Selectivity	Low	Average	High	
Low	6	12	32	
Average	33	61	18	
High	13	15	0	

Use the 0.01 level of significance to test whether there is a relationship between fidelity and selectivity. (A/M2011)

# Large Sample (n > 30)Single Mean & Difference of Means:

- 1. A sample of 900 members has a mean 3.4 cm and standard deviation 2.61 cm. Is the sample from a large population of mean 3.25 cms and standard deviation of 2.61cms? (Test at 5% level of significance. The value of z at 5% level is  $|z_{\alpha}| < 1.96$ ).(A/M 2010)
- 2. The means of two large samples of 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same populations of standard deviation 2.5 inches? (M/J 2012),(N/D 2018)

3. A mathematics test was given to 50 girls and 75 boys. The girls made an average grade of 76 with a SD of 6, while boys made an average grade of 82 with a SD of 2. Test whether there is any significant difference between the performance of boys and girls.

(N/D 2012),(M/J 2016)

- 4. A random sample of 100 bulbs from a company P shows a mean life 1300 hours and standard deviation of 82 hours. Another random sample of 100 bulbs from company Q showed a mean life 1248 hours and standard deviation of 93 hours. Are the bulbs of company P superior to bulbs of company Q at 5% level of significance? (N/D 2017)
- 5. The sales manager of a large company conducted a sample survey in two places A and B taking 200 samples in each case. The results were the following table. Test whether the average sales is the same in the 2 areas at 5% level. (N/D 2013)

Place A Place B
Average Sales Rs. 2,000 Rs. 1,700
S.D Rs. 200 Rs. 450

6. Examine whether the difference in the variability in yields is significant at 5% level of significance, for the following. (N/D 2010)

	Set of 40 plots	Set of 60 plots	
Mean yield per plot	1256	1243	
S.D. per plot	34	28	

### **Single Proportion & Difference of Proportions:**

- 7. 20 people were attacked by a disease and only 18 survived. Will you reject the hypothesis that the survival rate, if attacked by this disease is 85% is favor of the hypothesis that is more at 5% level? (N/D 2013)
- 8. A Manufacturer of light bulbs claims that an average of 2% of the bulbs manufactured by him are defective. A random sample of 400 bulbs contained 13 defective bulbs. On the basis of the sample, can you support the manufacturer's claim at 5% level of significance?

  (M/J 2014)

- 9. 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test whether these two proportions are same. (M/J 2016)
- 10. In a random sample of 100 men taken from village A, 60 were found to be consuming alcohol. In another sample of 200 men taken from village B, 100 were found to be consuming alcohol. Do the two villages differ significantly in respect of the proportion of men who consume alcohol?

  (M/J 2014)
- 11. Before an increase in excise duty on tea, 800 persons out of a sample of 1000 persons were found to be tea drinkers. After an increase in duty, 800 people were tea drinkers in a sample of 1200 people. Using standard error of proportion, state whether there is a significant decrease in the consumption of tea after the increase in excise duty. ( $z_{\alpha}$  at 5% level 1.645, 1% level 2.33). (A/M 2010)
- 12. A machine puts out 16 imperfect articles in a sample of 500. After it was overhauled, it puts out 3 imperfect articles in a sample of 100. Has the machine improved in its performance?

  (N/D 2010)(N/D2012)
- 13. Before an increase in excise duty on tea, 900 persons out of a sample of 1100 persons were found to be tea drinkers. After an increase in excise duty, 900 person were tea drinkers in a sample of 1300. Using standard error of proportion, state whether there is a significant decrease in the consumption of tea after the increase in excise duty?

(N/D 2013)

14. In a random sample of 1000 people from city A, 400 are found to be consumers of wheat. In a sample of 800 from city B, 400 are found to be consumers of wheat. Does this data give a significant difference between the two cities as far as the proportion of wheat consumers is concerned?

(A/M 2011)

## **Unit** – **II** (**Design** of **Experiments**)

# • Completely Randomized Design (One – Way Classification)

1. The following are the number of mistakes made in 5 successive days by 4 technicians working for a photographic laboratory test at a level of significance  $\alpha = 0.01$ . Test whether the difference among the four sample means can be attributed to chance. (A/M 2011)

Technician						
I	II	III	IV			
6	14	10	9			
14	9	12	12			
10	12	7	8			
8	10	15	10			
11	14	11	11			

# • Randomized Block Design (Two - Way Classification)

A chemist wishes to test the effect of four chemical agents on the strength of a particular type of cloth. Because there might be variability from one bolt to another, the chemist decides to use a randomized block design, with the bolts of cloth consider as blocks, she selects five bolts and applies all four chemicals in random order to each bolt. The resulting tensile strengths follows:

 (A/M 2018)

		Bolt					
		1	2	3	4	5	
Chemical	1	73	68	74	71	67	
	2	73	67	75	72	70	
	3	75	68	78	73	68	
	4	73	71	75	75	69	

Does the tensile strength depend on chemical? Test at  $\alpha = 0.10$ .

2. Three varieties of coal were analysed by 4 chemists and the ash content is tabulated here. Perform an anatysis of variance. (M/J 2016)

	Chemists					
		Α	В	С	D	
Coal	I	8	5	5	7	
	II	7	6	4	4	
	III	3	6	5	4	

3. Analyse the following RBD and find your conclusion.

(N/D 2013)

**Treatments** 

4. A set of data involving four "four tropical feed stuffs A, B, C, D" tried on 20 chicks is given below. All the twenty chicks are treated alike in all respects except the feeding treatments and each feeding treatment is given to 5 chicks. Analyze the data. Weight gain of baby chicks fed on different feeding materials composed of tropical feed stuffs.

(A/M 2010)

		1_				Total
Α	55	49	42	21	52	219
В	61	112	30	89	63	355
С	42	97	81	95	92	407
D	169	137	169	85	154	714
		G = 1695				

5. Four verities A, B, C, D of a fertilizer are tested in a RBD with 4 replications. The plot yields in pounds are as follows:

A12 D20 C16 B10 D18 A14 B11 C14 B12 C15 D19 A13 C16 B11 A15 D20 Analyse the experimental yield.

(M/J 2012),(M/J 2014)

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6. The result of an RBD experiment on 3 blocks with 4 treatments A, B, C, D are tabulated here. Carry out an analysis of variance. (M/J 2016)

	Treatment effects						
I	A36	D35	C21	B36			
II	D32	B29	A28	C31			
III	B28	C29	D29	A26			

7. Carry out ANOVA (Analysis of variance) for the following. (N/D 2010)

8. The following data represent the number of units of production per day turned out by 5 different workers using 4 different types of machines. (A/M 2011), (M/J 2013)

- (i) Test whether the mean production is the same for the different machine types.
- (ii) Test whether the 5 men differ with mean productivity.
- 9. The sales of 4 salesmen in 3 seasons are tabulated here. Carry out an analysis of variance. (N/D 2012)

#### Salesmen

Seasons A B C D

36 21

35

36

Summer

Winter 28 29 31 32

Monsoon 26 28 29 29

# Latin Square (Three – Way Classification)

 A Latin square design was used to compare the bond strengths of gold semi conductor lead wires bounded to the lead terminal by 5 different methods, A, B, C, D and E. The bonds were made by 5 different operators and the devices were encapsulated using 5 different plastics. With the following results, expressed as pounds of force required to break the bond. (A/M 2018)

		Operator							
		1	2	3	4	5			
	1	A 3	B 2.4	C 1.9	D 2.2	E 1.7			
Plastics	2	B 2.1	C 2.7	D 2.3	E 2.5	A 3.1			
	3	C 2.1	D 2.6	E 2.5	A 2.9	B 2.1			
	4	D 2.0	E 2.5	D 3.2	B 2.5	C 2.2			
	5	E 2.1	A 3.6	B 2.4	C 2.4	D 2.1			

Analyse these results and test with 0.01 level of significance.

2. A variable trial was conducted on wheat with 4 varieties in a Latin Square design. The plan of the experiment and per plot yield are given below:

Analyse the data.

(M/J 2012)

3. A farmer wishes to test the effect of 4 fertilizers A, B, C, D on the yield of wheat. The fertilizers are used in a LSD and the result are tabulated here. Perform an analysis of variance. (N/D 2012)

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4. Analyse the following of Latin square experiment.

(M/J 2013)

5. The following is a Latin square of a design when 4 varieties of seed are being tested. Set up the analysis of variance table and state your conclusion. You can carry out the suitable charge of origin and scale.

(N/D 2013)

6. Analyse the variance in the Latin square of yields (in kgs) of paddy where P,Q, R, S denote the different methods of cultivation: (M/J 2014)

Examine whether different method of cultivation have significantly different yields.

7. In a Latin square experiment given below are the yields in quintals per acre on the paddy crop carried out for testing the effect of five fertilizers A, B, C, D, E. Analyze the data for variations.

(A/M 2011)

8. The following is a Latin square of a design when 4 varieties of seeds are being tested. Set up the analysis of variance table and state your conclusion. You may carry out suitable change of origin and scale. (M/ J 2013)

9. A company wants to produce cars for its own use. It has to select the make of the car out of the four makes A, B, C and D available in the market. For this he tries four cars of each make by assigning the cars to four drivers to run on four different routes. The efficiency of cars is measured in terms of time in hours. The layout and time consumed is as given below.

10	1	Drivers		
Routes	1	2	3	4
1	18 (C)	12 (D)	16 (A)	20 (B)
2	26 (D)	34 (A)	25 (B)	31 (C)
3	15 (B)	22 (C)	10 (D)	28 (A)
4	30 (A)	20 (B)	15 (C)	9 (D)

Analyse the experimental data and draw conclusions. (  $F_{0.05}(3,5) = 5.41$  ) (N/D 2014)

# **Unit – III (Solution of Equations and Eigenvalue Problems)**

#### Newton – Raphson method

1. Using Newton-Raphson method, solve  $x \log_{10} x = 12.34$  taking the initial value  $x_0$  as 10.

(M/J 2012)

(Or)

Solve the equation  $x \log_{10} x = 1.2$  using Newton-Raphson method. (M/J 2014)

#### Gauss – Jacobi method & Gauss – Seidel method

1. Solve, by Gauss-Jacobi method, the system of following equations correct to three decimal places 27x + 6y - z = 85; 6x + 15y + 2z = 72; x + y + 54z = 110.

(N/D 2018)

- 2. Using Gauss-Seidel method, solve the following system of linear equations x + 5y z = 10; 4x + 2y + z = 14; x + y + 8z = 20.
- 3. Apply Gauss-Seidel method to solve the equations 20x + y 2z = 17; 3x + 20y z = -18; 2x 3y + 20z = 25.
- 4. Solve the system of equations using Gauss-Seidal method x + y + z = 9; 2x 3y + 4z = 13; 3x + 4y + 5z = 40. (N/D 2012)
- 5. Find the solution, to three decimals, of the system using Gauss-Seidal method 8x + 11y 4z = 95; 7x + 52y + 13z = 104; 3x + 8y + 29z = 71. (N/D 2014)
- 6. Solve the following set of equations using Gauss-Seidal iterative procedure -10x + 2y + 2z = 4; x 10y + 2z = 18; x + y 10z = 45. (M/J 2014)
- 7. Solve the following system of equations using Gauss-Seidal iterative method 27x+6y-z=85, 6x+15y+2z=72, x+y+54z=110. (M/J 2012)

- 8. Solve the following equations by Gauss-Seidal method  $x+y+54z=110,\ 27x+6y-z=85,\ 6x+15y+2z=72$ . (A/M 2011), (N/D 2012), (N/D 2017), (A/M 2018)
- 9. Solve 5x y + z = 10; 2x + 4y = 12 and x + y + 5z = -1 using Gauss Seidel method. (A/M 2010)
- 10. Solve by Gauss-Seidel method 6x + 3y + 12z = 35; 8x 3y + 2z = 20; 4x + 11y z = 33. (N/D 2010)
- 11. Solve by Gauss-Seidal method 28x + 4y z = 32; x + 3y + 10z = 24; 2x + 17y + 4z = 35. (M/J 2013)
- 12. Solve by Gauss-Seidal method x + y + 9z = 15; x + 17y 2z = 48; 30x 2y + 3z = 75. (M/J 2016)

#### Gauss Elimination method & Gauss Jordan method

- 1. Solve the system of equations using Gauss-elimination method 5x-2y+z=4, 7x+y-5z=8, 3x+7y+4z=10. (N/D 2014)
- 2. Solve the following equations by Gauss elimination method  $x+y+z=9, \ 2x-3y+4z=13, \ 3x+4y+5z=40$ . (N/D 2012)
- 3. Solve the following system of equation by Gauss elimination method: 2x + y + z = 10; 3x + 2y + 3z = 18; x + 4y + 9z = 16. (A/M 2010), (N/D 2018)
- 4. Solve by Gauss-Elimination method 2x + 3y z = 5; 4x + 4y 3z = 3; 2x 3y + 2z = 2. (M/J 2009)
- 5. Solve by Gauss-Elimination method 3x + 4y + 5z = 18; 2x y + 8z = 13; 5x 2y + 7z = 20. (M/J 2013)
- 6. Using Gauss-Jordan method to solve 2x y + 3z = 8, -x + 2y + z = 4, 3x + y 4z = 0. (N/D 2015)

- 7. Using Gauss-Jordan, solve the following system 10x + y + z = 12; 2x + 10y + z = 13; x + y + 5z = 7. (N/D 2010)
- 8. Solve the following equations using Gauss-elimination method 5x y = 9, -x + 5y z = 4, -y + 5z = -6.
- 9. Solve the following system of equation by Gauss-Jordan 2x + y + 4z = 12; 8x 3y + 2z = 20; 4x + 11y z = 33. (A/M 2018)
- 10. Solve the following equations using Gauss-elimination method 2x + y + 4z = 12, 8x 3y + 2z = 20, 4x + 11y z = 33. (M/J 2016)
- 11. Solve the following system of equations by Gauss-Elimination method 2y-3z=-5; x+4y-7z+t=-8; 2x-y-t=-4; x+y+z=6; (A/M 2018)
- 12. Solve the system of equations by Gauss-Elimination method  $x_1+x_2+x_3+x_4=2$ ;  $2x_1-x_2+2x_3-x_4=-5;\ 3x_1+2x_2+3x_3+4x_4=7;\ x_1-2x_2-3x_3+2x_4=5.$  (N/D 2013)
- 13. Solve the system of equations by Gauss-Jordan method x + y + z + w = 1; 2x y + 2z w = -5; 3x + 2y + 3z + 4w = 7; x 2y 3z + 2w = 5. (M/J 2013)
  - Matrix inversion by Gauss Jordan method
- 1. Using Gauss Jordan method, find the inverse  $A = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$ . (A/M 2018)
- 2. Using Gauss Jordon method, find the inverse of  $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ .

  (A/M 2010), (M/J 2012)
- 3. If  $A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ , find  $A^{-1}$  by Gauss-Jordan method. (N/D 2012)

- 4. By Gauss-Jordan method, find the inverse of  $A=\begin{pmatrix}4&1&2\\2&3&-1\\1&-2&2\end{pmatrix}$ . (A/M 2011), (M/J 2016), (N/D 2017)
- 5. Find the inverse of the matrix  $\begin{pmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{pmatrix}$  by Gauss-Jordan method. (N/D 2013)
- 6. Find the inverse of the matrix  $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & 4 \end{pmatrix}$  using Gauss-Jordan method.
- 7. Find the inverse of  $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 3 & -1 & -4 \end{pmatrix}$  using Gauss-Jordon method. (N/D 2010)
- 8. By Gauss Jordan elimination method, find the inverse of the matrix  $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 0 & -1 \\ 2 & -1 & 2 \end{pmatrix}$ . (M/J 2014)

# • Eigen values of a matrix by Power method

- 1. Using the power method, find the largest eigenvalue and the corresponding eigenvector for the matrix  $A = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$ . Let  $X_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ . (A/M2018)
- 2. Find the dominant eigenvalue of  $\begin{pmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{pmatrix}$  by power method. (N/D 2012)
- 3. Find all the eigenvalue of  $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$  using power method. Using  $x_1 = \begin{pmatrix} 1,0,0 \end{pmatrix}^T$  as initial vector. (N/D 2010)

- 4. Find the numerically largest eigenvalue of  $A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$  and the corresponding eigenvector. (A/M 2013),(N/D 2017),(M/J 2016)
- 5. Find the dominant eigenvalue and its eigenvector of the matrix by power method

$$A = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}. \tag{N/D 2014}$$

- 6. Using power method, find all the eigenvalues of  $A = \begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$ . (M/J 2013)
- 7. Determine the largest eigenvalue and the corresponding eigenvector of the matrix

$$A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \text{ with the initial vector } X^{(0)} = [1,1,1]^T.$$
 (A/M 2010)

- 8. Find the numerically largest eigenvalue of  $A = \begin{pmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{pmatrix}$  by power method.

  (M/J 2012), (M/J 2014)
- 9. Find the largest eigenvalue of the matrix  $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & -1 & 0 \end{pmatrix}$  by power method. Also find its corresponding eigenvector. (N/D 2013)

# **Unit – IV** (Interpolation, Numerical Differential & Integration)

# • Lagrange's & Newton's divided difference interpolation

1. Using Lagrange's interpolation formula, find the polynomial f(x) from the following data:

(N/D 2017)

2. Using Lagrange's interpolation formula, find y(10) from the following table. (A/M 2011)

3. Using Lagrange's interpolation, find the value of f(3), from the following table: (M/J 2012)

$$x: 0 1 2 5$$
  
 $f(x): 2 3 12 147$ 

4. Using Lagrange's method, find the value of f(3) from the following table: (N/D 2013)

$$x:$$
 0 1 2 3  $f(x):$  2 3 12 147

5. Use Lagrange's formula to fit a polynomial to the following data hence find y(x=1). (N/D 2010)

6. Find polynomial f(x) by using Lagrange's formula and hence find  $f(\mathbf{4})$  for (M/J 2014)

x:	1	3	5	7
f(x):	24	120	336	720

7. Given the table of values

x:	50	52	54	56
$\sqrt[3]{x}$ :	3.684	3.732	3.779	3.825

Use Lagrange's formula to find  $\sqrt[3]{53}$  .

(N/D 2014)

8. From the following values, find f(x) and hence find f(6) by Newton's divided difference formula. (N/D 2017)

x:	1	2	7	8
f(x):	1	5	5	4

9. Use Newton divided difference formula to calculate f(3), f'(3) and f''(3) from the following table: (A/M 2010)

10. Using Newton's divided difference formula, find the values of f(2), f(8) and f(15) given the following table. (A/M 2011), (M/J 2013), (N/D 2013)

$$x:$$
 4 5 7 10 11 13  $f(x):$  48 100 294 900 1210 2028

- 11. Given the set of tabulated points (1,-3), (3,9), (4,30) and (6,132) obtain the value of y when x=2 using Newton's divided difference formula. (N/D 2014)
- 12. Find the cubic polynomial y(x) for

(N/D 2012)

$$x:$$
 -1 0 2 3  $y(x):$  -8 3 1 12

Newton's forward and backward difference interpolation

1. Interpolate y(12), if

(N/J 2016)

$\boldsymbol{x}$				25		
y(x)	35	33	29	27	22	14

2. Using Newton's forward interpolation formula, find the polynomial f(x) satisfying the following data. Hence evaluate f(x) at x = 5. (M/J 2012)

$$x:$$
 4 6 8 10  $f(x):$  1 3 8 16

3. Construct Newton's forward interpolation polynomial for the following data:

x:	1	2	3	4	5
f(x):	1	-1	1	-1	1

and hence find f(3.5), f'(3.5).

(M/J 2014)

4. Find y(1976) from the following

(N/D 2010)

<i>x</i> :	1941	1951	1961	1971	1981	1991
<i>y</i> :	20	24	29	36	46	51

5. From the following table of half-yearly premium for policies maturing at different ages, estimate the premium for policies maturing at age 46 and 63. (A/M 2011)

Age x: 45 50 55 60 65

Premium y: 114.84 96.16 83.32 74.48 68.48

6. Find y(22), given that (N/D 2012)

x: 20 25 30 35 40 45

y(x): 354 332 291 260 231 204

# • Approximation of derivatives using interpolation

1. Find the first derivative of f(x) at x = 0.4 from the following table: (A/M 2018)

 	· · · · · · · · · · · · · · · · · · ·	,		, , ,
X	0.1	0.2	0.3	0.4
f(x)	1.10517	1.22140	1.34986	1.49182

2. Find the first two derivatives of  $x^{1/3}$  at x = 56, given the table below: (A/M 2018)

X	50	51	52	53	54	55	56
$y = x^{1/3}$	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

3. Compute f'(0) and f''(4) from the following data:

(N/D 2012)

$$x:$$
 0 1 2 3 4  $f(x):$  1 2.718 7.381 20.086 54.598

4. From the following table of values of x and y, obtain  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  for x = 1.2. (A/M2010)

I	x:	1.0	1.2	1.4	1.6	1.8	2.0	2.2
ĺ	<i>y</i> :	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

5. Find the first and second derivatives of f(x) at x = 1.5 if

(N/D 2014)

$$x:$$
 1.5 2.0 2.5 3.0 3.5 4.0  $f(x):$  3.375 7.000 13.625 24.000 38.875 59.000

6. The population of a certain town is given below. Find the rate of growth of the population in 1931, 1941, 1961 and 1971. (M/J 2013)

Year 
$$x$$
: 1931 1941 1951 1961 1971 Population in thousands  $y$ : 40.62 60.80 79.95 103.56 132.65

7. Find the value of cos(1.74), using suitable formula from the given data. (N/D 2017)

<i>x</i> :	1.7	1.74	1.78	1.82	1.86
$\sin x$ :	0.9916	0.9857	0.9781	0.9691	0.9584

8. Find y'(1), if

(M/J 2016)

$$x:$$
 -1 0 2 3  $y(x):$  -8 3 1 12

9. Find y'(1), if

(N/D 2012)

$$x:$$
 0 2 3 4 7 9  $y(x):$  4 26 58 112 466 922

# • Numerical integration using Trapezoidal & Simpson's Rule

1. Evaluate  $\int_{-1}^{1} \frac{dx}{1+x^2}$  by Trapezoidal rule, dividing the range into eight equal parts.

(M/J 2016)

- 2. Evaluate  $\int_{0}^{2} e^{x} dx$  by using Trapezoidal rule taking 6 sub intervals. (A/M 2018)
- 3. Evaluate  $\int_0^1 \frac{dx}{1+x}$  by using Simpson's one-third rule and hence deduce the value of  $\log_e 2$ . (M/J 2014)
- 4. Evaluate  $\int_{0}^{\pi/2} \sin x \ dx$ , by trapezoidal and Simpson's  $\left(\frac{1}{3}\right)$  rules by dividing the range into 10 equal parts. Verify your answer with integration. (N/D 2012),(M/J 2013)
- 5. Taking  $h = \frac{\pi}{5}$ , evaluate  $\int_{0}^{2\pi} \sin x \, dx$  by Simpson's 1/3 rule. Verify the answer with integration. (N/D 2010)
- 6. Evaluate  $\int_{0}^{2} \frac{dx}{x^2 + x + 1}$  to three decimals, dividing the range of integration into 8 equal parts using Simpson's rule. (M/J 2012)
- 7. Evaluate  $\int_0^1 \frac{dx}{1+x^2}$  by Simpson's (1/3) rule, dividing the range into four equal parts.

  (M/J 2016)
- 8. Evaluate  $\int_{0}^{6} \frac{1}{1+x} dx$  using Trapezoidal rule. Verify the answer with direct integration. (N/D 2010)
- 9. The table below gives the velocity V of a moving particle at time t seconds. Find the distance covered by the particle in 12 seconds and also the acceleration at t=2 seconds, using Simpson's rule. (A/M 2011)

*t*: 0 2 4 6 8 10 12 *V*: 4 6 16 34 60 94 136

10. The velocity v of a particle at a distance s form a point on its path is given as follows:

s in meter: 0 10 20 30 40 50 60 v m/sec: 47 58 64 65 61 52 38

Estimate the time taken to travel 60 meters by using Trapezoidal rule and Simpson's rule. (M/J 2014)

11. The velocities of a car (running on a straight road) at intervals of 2 minutes are given below. (N/D 2014)

Time in minutes:		2	4	6	8	10	12
Velocities in km/hr:	0	22	30	27	18	7	0

Apply Simpson's rule to find the distance covered by the car.

12. A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is in the table below. Using trapezoidal rule and Simpson's 1/3 rule, find the velocity of the rocket at t=80 sec. (A/M 2010)

t(sec):	0	10	20	30	40	50	60	70	80
$t(cm/\sec)$ :	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	40.67

- Double integrals by Trapezoidal and Simpsons's rules
- 1. Using Trapezoidal rule, evaluate  $\int_{1}^{2} \int_{1}^{2} \frac{dxdy}{x^2 + y^2}$  numerically with h = 0.2 along x direction and k = 0.25 along y -direction. (M/J 2012)
- 2. Evaluate  $\int_{0}^{1} \int_{0}^{1} \frac{1}{1+x+y} dx dy$  by trapezoidal rule. (N/D 2014)
- 3. Evaluate  $\int_{1}^{1.21.4} \int_{1}^{4.21.4} \frac{dxdy}{x+y}$  by trapezoidal formula by taking h=k=0.1.

(A/M 2010),(N/D 2017)

4. Using Trapezoidal rule, evaluate 
$$\int_{1}^{2} \int_{1}^{2} \frac{dxdy}{x+y}$$
 with  $h = k = 0.5$ . (M/J 2016)

- 5. Evaluate  $\int_{0}^{2} \int_{0}^{1} 4xy \, dxdy$  using Simpson's rule by taking  $h = \frac{1}{4}$  and  $k = \frac{1}{2}$ . (N/D 2012)
- 6. Evaluate  $\int_0^{1/2} \int_0^{1/2} \frac{\sin(xy)}{1+xy} dxdy$  using Simpson's rule with  $h=k=\frac{1}{4}$ . (M/J 2012),(M/J 2014)
- 7. Evaluate  $\int_{1}^{1.22.4} \frac{1}{xy} dxdy$  using Simpon's one-third rule. (M/J 2013)
- 8. Using Simpson's 1/3 rule, to evaluate  $\int_{0}^{1} \int_{0}^{1} \frac{dxdy}{1+xy}$  with  $\Delta x = \Delta y = 0.25$ . (A/M2018)

# Unit - V (Numerical Solution of Ordinary Differential Eqns.)

# Taylor's series, Euler's and Modified Euler's method

1. Using Taylor series method, find the value of y at x=0.1, if y satisfies the equation  $\frac{dy}{dx} = x^2 - y$  given that y=1 when x=0, correct to 3 decimal places.

(N/D 2017),(N/D2018)

- 2. Using Taylor's series method, find y at x = 0.1 if  $\frac{dy}{dx} = x^2y 1$ , y(0) = 1.
- 3. Apply Taylor series method to find and approximate value of y when x = 0.1, 0.2 given that  $\frac{dy}{dx} = x + y$ , y(0) = 1. (M/J 2014)
- 4. Given  $y' = x^2 y$ , y(0) = 1, y(0.1) = 0.9052, y(0.2) = 0.8213, find y(0.3) using Taylor's series method. (N/D 2013)

- 5. Using Taylor method, compute y(0.2) and y(0.4) correct to 4 decimal places given  $\frac{dy}{dx} = 1 2xy \text{ and } y(0) = 0, \text{ by taking } h = 0.2. \tag{A/M2011}$
- 6. Solve  $y' = y^2 + x$ , y(0) = 1 using Taylor series method for y(0.1) and y(0.2).
- 7. By Taylor series method find y(0.1), y(0.2) and y(0.3) if  $\frac{dy}{dx} = x y^2, \ y(0) = 1.$ (N/D 2012)
- 8. Given  $\frac{dy}{dx} = 1 + y^2$ , where x = 0, find y(0.2), y(0.4) and y(0.6), using Taylor series method. (A/M 2010)
- 9. Solve by Taylor's method to find an approximate value of y at x=0.2 for the differential equation  $\frac{dy}{dx}=2y+3e^x$ , y(0)=0. Compare the numerical solution with the exact solution. Use first three non-zero terms in the series. (N/D 2014)
- 10. Find the value of y at x = 0.1 given that  $\frac{dy}{dx} = x^2 y$ , y(0) = 1 by modified Euler's method. (A/M2018)
- 11. Using modified Euler method, find y(0.2), y(0.1) given  $\frac{dy}{dx} = x^2 + y^2$ , y(0) = 1.

  (A/M 2011)
- 12. Use Euler's method, with h = 0.1 to find the solution of  $y' = x^2 + y^2$  with y(0) = 0 in  $0 \le x \le 5$ . (N/D 2010)
- 13. Solve by Euler's method, the equation  $\frac{dy}{dx} = x + y$ , y(0) = 0, chose h = 0.2 and compute y(0.4) and y(0.6). (N/D 2013)
- 14. Consider the initial value problem  $\frac{dy}{dx} = y x^2 + 1$ , y(0) = 0.5. Compute y(0.2) by Euler's method and modified Euler method. (M/J 2012),(N/D 2014)

- 15. By Modified Euler's method, find y(0.1), y(0.2) and y(0.3) if  $\frac{dy}{dx} = x + y$ , y(0) = 1.

  (N/D 2012)
- 16. Solve  $(1+x)\frac{dy}{dx} = -y^2$ , y(0) = 1 by modified Euler method by choosing h = 0.1, find y(0.1) and . (N/D 2016)

# • Runge – Kutta method for 1<sup>st</sup> order equations

- 2. Find y(0.8) given that  $y' = y x^2$ , y(0.6) = 1.7379 by using R-K method of order 4, taking h = 0.1. (M/J 2012)
- 3. Using 4<sup>th</sup> order Runge-Kutta method, solve  $\frac{dy}{dx} = \frac{y^2 x^2}{y^2 + x^2}$ , y(0) = 1 for x = 0.2 and x = 0.4 with h = 0.2. (A/M 2010), (A/M 2011), (M/J 2013)
- 4. Use R.K Method fourth order to find the y(0.2) if  $\frac{dy}{dx} = x + y^2$ , y(0) = 1, h = 0.1. (N/D 2010),(N/D 2017)
- 5. Solve  $\frac{dy}{dx} = xy + y^2$ , y(1) = 1, for y(1.1), using fourth order Runge-Kutta method. (N/D 2014)

# • Milne's predictor - corrector methods for 1<sup>st</sup> order eqn.

- 1. Given  $\frac{dy}{dx} = x^3 + y$ , y(0) = 2, y(0.2) = 2.073, y(0.4) = 2.452, y(0.6) = 3.023, compute y(0.8) by Milne's method. (N/D 2017)
- 2. If  $\frac{dy}{dx} = \frac{y^2 x^2}{y^2 + x^2}$ , y(0) = 1, find y(0.2), y(0.4) and y(0.6) by Runge-Kutta method of fourth order and hence find y(0.8) by Milne's method. (N/D 2012), (M/J 2016)

- 3. Compute y(0.4) and y(0.5), given that  $y' = y \frac{2x}{y}$ , y(0) = 1, y(0.1) = 1.0954, y(0.2) = 1.1832, y(0.3) = 1.2649 using Milne's predictor-corrector method. (N/D 2014)
- 4. If  $\frac{dy}{dx} = x^2 + y^2$ , y(0) = 1 find y(0.1), y(0.2) and y(0.3) by Taylor series method. Hence find y(0.4) by Milne's Predictor-Corrector method. (M/J 2016)
- 5. Using Milne's predictor and corrector method find y(4.4) given  $5xy' + y^2 2 = 0$  given y(4) = 1, y(4.1) = 1.0049, y(4.2) = 1.0097, y(4.3) = 1.0143.

  (M/J 2012),(M/J 2014),(A/M 2018)
- 6. Using Milne's method, obtain the solution of  $\frac{dy}{dx} = x y^2$  at x = 0.8 given y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795, y(0.6) = 0.176. (N/D 2010)
- 7. Using Milne's predictor-corrector method, find y(0.4), given that  $y' = \frac{\left(1 + x^2\right)y^2}{2}$ , y(0) = 1, y(0.1) = 1.06, y(0.2) = 1.12, y(0.3) = 1.21. (N/D 2013)

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