ELECTRICAL AND ELECTRONICS ENGINEERING

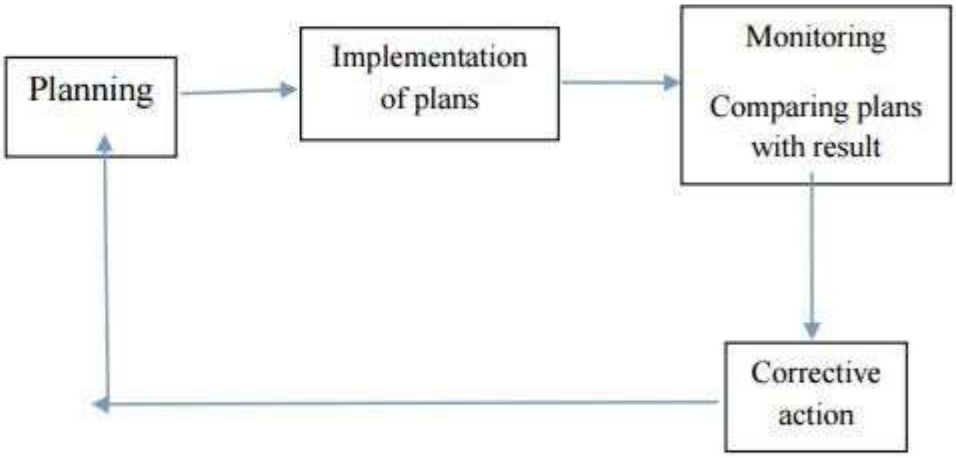
EE8501 POWER SYSTEM ANALYSIS

UNIT – I POWER SYSTEM

Need for system planning and operational studies - Power scenario in India - Power system components – Representation - Single line diagram - per unit quantities - p.u. impedance diagram - p.u. reactance diagram - Network graph, Bus incidence matrix, Primitive parameters, Bus admittance matrix from primitive parameters - Representation of off nominal transformer - Formation of bus admittance matrix of large power network.

Need for system planning and operational studies

Planning and operation of power system operational planning covers the whole period ranging from the incremental stage of system development. The system operation engineers at various points like area, space, regional & national load dispatch of power. Power system planning and operational analysis covers the maintenance of generation, transmission and distribution facilities.



Steps:

- Planning of power system
- Implementation of the plans
- Monitoring system

- Compare plans with the results
- If no undesirable deviation occurs, then directly go to planning of system
- If undesirable deviation occurs then take corrective action and then go to planning of the system

Planning and operation of power system

Planning and operation of power system the following analysis are very important

(a). Load flow analysis

(b). Short circuit analysis

(c). Transient analysis

Load flow analysis

Electrical power system operate - Steady state mode.

Basic calculation required to determine the characteristics of this state is called as Load flow.

Power flow studies - To determine the voltage current active and reactive power flows in given power system.

A number of operating condition can be analyzed including contingencies. That operating conditions are

(a). Loss of generator

(b).Loss of a transmission line

(c).Loss of transformer (or) Load

(d). Equipment over load (or) unacceptable voltage levels

The result of the power flow analysis are starting point for the stability analysis and power factor improvement.

Load flow study is done during the planning of a new system or the extension of an existing one.

Short circuit studies

To determine the magnitude of the current flowing throughout the power system at various time intervals after fault.

The objective of short circuit analysis - To determine the current and voltages at different location of the system corresponding to different types of faults.

(a). Three phase to ground fault

(b). Line to ground fault

(c). Line to line fault

(d). Double line to ground fault

(e). Open conductor fault

Transient stability analysis

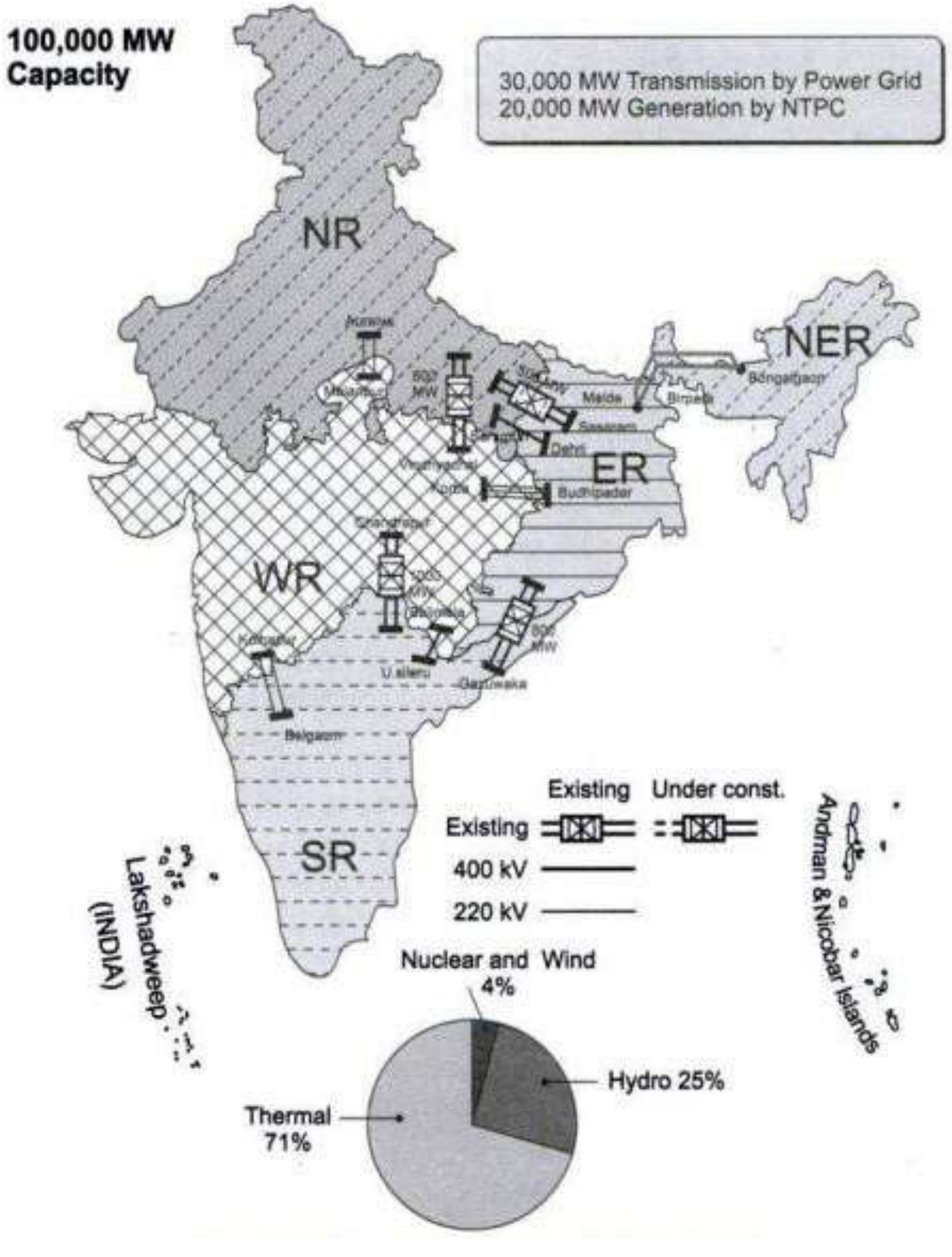
The ability of the power system consisting of two (or) more generators to continue to operate after change occur on the system is a measure of the stability.

In power system the stability depends on the power flow pattern generator characteristics system loading level and the line parameters.

Power scenario in India

	Power grid	SEBs	Total
HVDC	1,632	1,504	3,136
800 kV	550	400	950
400 kV	32,500	13,000	45,500
220/132 kV	9,000	206,000	215,000

Figures in Circuit kms


100,000 MW Capacity
30,000 MW Transmission by Power Grid
20,000 MW Generation by NTPC
NR
NER
Bongaigaon
Birpara
Malda
Sasaram
Dehri
ER
Budhipadar
500 MW
Korba
WR
1000 MW
U.sileru
Gazuwaka
Kolhapur
Belgaum
SR
Lakshadweep (INDIA)
Andman & Nicobar Islands
Existing
Under const.
Existing
400 kV
220 kV
Nuclear and Wind 4%
Hydro 25%
Thermal 71%

The first demonstration of electric light in Calcutta (now Kolkata) was conducted on 24 July 1879 by P.W. Fleury & Co. On 7 January 1897, Kilburn & Co secured the Calcutta electric lighting license as agents of the Indian Electric Co, which was registered in London on 15 January 1897. A month later, the company was renamed the Calcutta Electric Supply Corporation. The control of the company was transferred from London to Calcutta only in 1970. The introduction of electricity in Calcutta was a success, and power was next introduced in Bombay (now Mumbai). The first electric lighting demonstration in Mumbai was in 1882 at Crawford Market and the Bombay Electric Supply & Tramways Company (BEST) set up a generating station in 1905 to provide electricity for the tramway.

The first hydroelectric installation in India was installed near a tea estate at Sidrapong for the Darjeeling Municipality in 1897. The first electric street light in Asia was lit on 5 August 1905 in Bangalore. The first electric train in the country ran on the Harbour Line between Bombay's Victoria Terminus and Kurla on 3 February 1925. On 18 August 2015, Cochin International Airport became the world's first fully solar powered airport with the inauguration of a dedicated solar plant.

India began using grid management on a regional basis in the 1960s. Individual State grids were interconnected to form 5 regional grids covering mainland India, the Northern, Eastern, Western, North Eastern and Southern Grids. These regional links were established to enable transmission of surplus electricity between states in each region. In the 1990s, the Indian government began planning for a national grid. Regional grids were initially interconnected by asynchronous high-voltage direct current (HVDC) back-to-back links facilitating the limited exchange of regulated power. The links were subsequently upgraded to high capacity synchronous links.

The first interconnection of regional grids was established in October 1991 when the North Eastern and Eastern grids were interconnected. The Western Grid was interconnected with these grids in March 2003. The Northern grid was also interconnected in August 2006, forming a Central Grid that was synchronously connected and operating at one frequency. The sole remaining regional grid, the Southern Grid, was synchronously interconnected to the Central Grid on 31 December 2013 with the commissioning of the 765 kV Raichur-Solapur transmission line, establishing the National Grid.

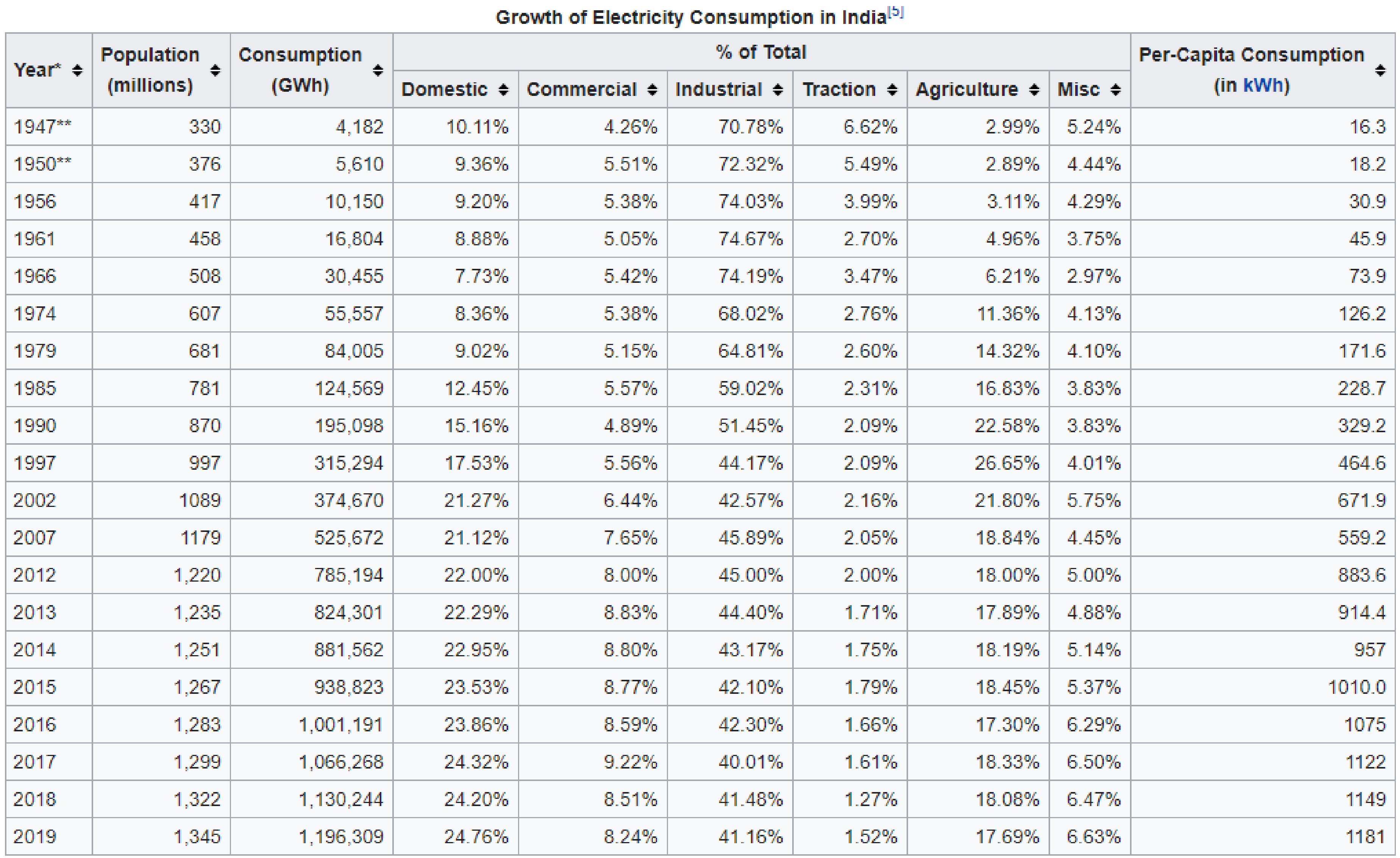
By the end of the calendar year 2015, despite poor hydroelectricity generation, India had become a power surplus nation with huge power generation capacity idling for want of demand.

The calendar year 2016 started with steep falls in the international price of energy commodities such as coal, diesel oil, naphtha, bunker fuel, and liquefied natural gas (LNG), which are used in electricity generation in India. As a result of the global glut in petroleum products, these fuels became cheap enough to compete with pit head coal-based power generators. Coal prices have also fallen. Low demand for coal has led to coal stocks building up at power stations as well as coal mines. New installations of renewable energy in India surpassed installations of fossil fuel for the first time in 2016-17.

On March 29, 2017, the Central Electricity Authority (CEA) stated that for the first time India has become a net exporter of electricity. India exported 5,798 GWh to neighbouring countries, against a total import of 5,585 GWh.

Growth of Installed Capacity in India[5]

Installed Capacity as on	Thermal (MW)				Nuclear (MW)	Renewable (MW)			Total (MW)	% Growth (on yearly basis)
	Coal	Gas	Diesel	Sub-Total Thermal		Hydro	Other Renewable	Sub-Total Renewable		
31-Dec-1947	756	-	98	854	-	508	-	508	1,362	-
31-Dec-1950	1,004	-	149	1,153	-	560	-	560	1,713	8.59%
31-Mar-1956	1,597	-	228	1,825	-	1,061	-	1,061	2,886	13.04%
31-Mar-1961	2,436	-	300	2,736	-	1,917	-	1,917	4,653	12.25%
31-Mar-1966	4,417	137	352	4,903	-	4,124	-	4,124	9,027	18.80%
31-Mar-1974	8,652	165	241	9,058	640	6,966	-	6,966	16,664	10.58%
31-Mar-1979	14,875	168	164	15,207	640	10,833	-	10,833	26,680	12.02%
31-Mar-1985	26,311	542	177	27,030	1,095	14,460	-	14,460	42,585	9.94%
31-Mar-1990	41,236	2,343	165	43,764	1,565	18,307	-	18,307	63,636	9.89%
31-Mar-1997	54,154	6,562	294	61,010	2,225	21,658	902	22,560	85,795	4.94%
31-Mar-2002	62,131	11,163	1,135	74,429	2,720	26,269	1,628	27,897	105,046	4.49%
31-Mar-2007	71,121	13,692	1,202	86,015	3,900	34,654	7,760	42,414	132,329	5.19%
31-Mar-2012	112,022	18,381	1,200	131,603	4,780	38,990	24,503	63,493	199,877	9.00%
31-Mar-2017	192,163	25,329	838	218,330	6,780	44,478	57,260	101,138	326,841	10.31%
31-Mar-2018	197,171	24,897	838	222,906	6,780	45,293	69,022	114,315	344,002	5.25%
31-Mar-2019[3]	200,704	24,937	637	226,279	6,780	45,399	77,641	123,040	356,100	3.52%
31-Mar-2020[39]	205,135	24,955	510	230,600	6,780	45,699	87,028	132,427	370,106	3.93%



Growth of Electricity Consumption in India[5]

Year*	Population (millions)	Consumption (GWh)	% of Total						Per-Capita Consumption (in kWh)
			Domestic	Commercial	Industrial	Traction	Agriculture	Misc	
1947**	330	4,182	10.11%	4.26%	70.78%	6.62%	2.99%	5.24%	16.3
1950**	376	5,610	9.36%	5.51%	72.32%	5.49%	2.89%	4.44%	18.2
1956	417	10,150	9.20%	5.38%	74.03%	3.99%	3.11%	4.29%	30.9
1961	458	16,804	8.88%	5.05%	74.67%	2.70%	4.96%	3.75%	45.9
1966	508	30,455	7.73%	5.42%	74.19%	3.47%	6.21%	2.97%	73.9
1974	607	55,557	8.36%	5.38%	68.02%	2.76%	11.36%	4.13%	126.2
1979	681	84,005	9.02%	5.15%	64.81%	2.60%	14.32%	4.10%	171.6
1985	781	124,569	12.45%	5.57%	59.02%	2.31%	16.83%	3.83%	228.7
1990	870	195,098	15.16%	4.89%	51.45%	2.09%	22.58%	3.83%	329.2
1997	997	315,294	17.53%	5.56%	44.17%	2.09%	26.65%	4.01%	464.6
2002	1089	374,670	21.27%	6.44%	42.57%	2.16%	21.80%	5.75%	671.9
2007	1179	525,672	21.12%	7.65%	45.89%	2.05%	18.84%	4.45%	559.2
2012	1,220	785,194	22.00%	8.00%	45.00%	2.00%	18.00%	5.00%	883.6
2013	1,235	824,301	22.29%	8.83%	44.40%	1.71%	17.89%	4.88%	914.4
2014	1,251	881,562	22.95%	8.80%	43.17%	1.75%	18.19%	5.14%	957
2015	1,267	938,823	23.53%	8.77%	42.10%	1.79%	18.45%	5.37%	1010.0
2016	1,283	1,001,191	23.86%	8.59%	42.30%	1.66%	17.30%	6.29%	1075
2017	1,299	1,066,268	24.32%	9.22%	40.01%	1.61%	18.33%	6.50%	1122
2018	1,322	1,130,244	24.20%	8.51%	41.48%	1.27%	18.08%	6.47%	1149
2019	1,345	1,196,309	24.76%	8.24%	41.16%	1.52%	17.69%	6.63%	1181

Power System Components

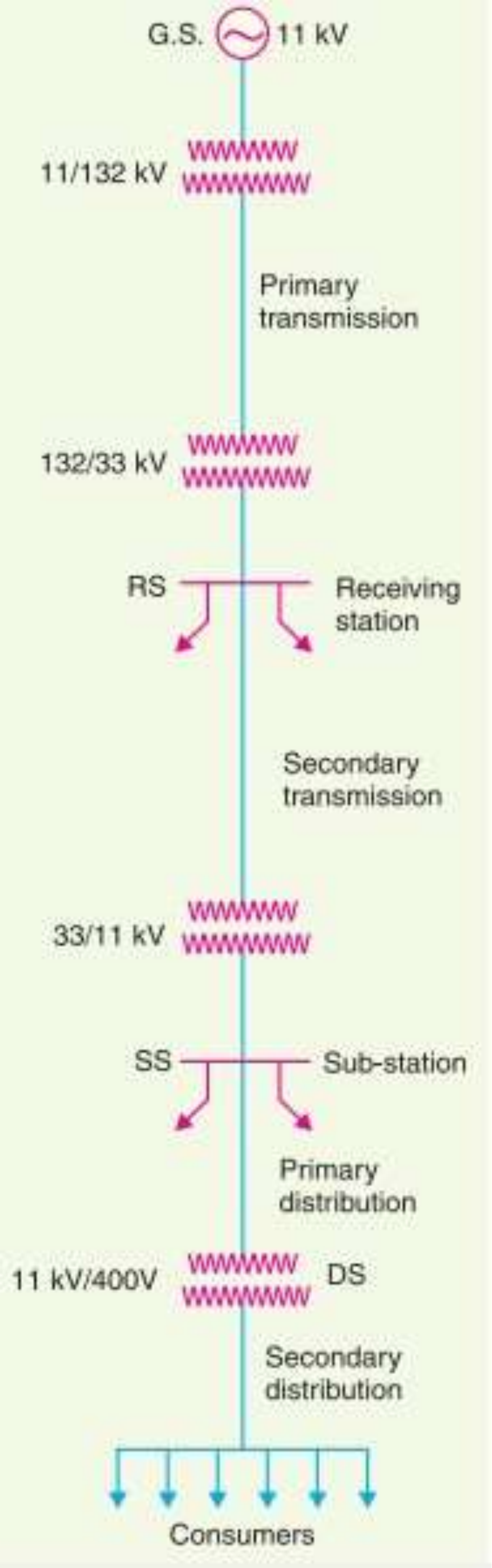
Generators - Convert mechanical energy in to electrical energy.

Transformers - Transfer Power or energy from one circuit to another circuit without change in frequency.

Transmission Lines - Transfer power from one place another place.

Control Equipment- Used for protection purpose.

Major components of a power system are- synchronous generators, synchronising equipment, circuit breakers, isolators, earthing switches, bus-bars, transformers, transmission lines, current transformers, potential transformers, relay and protection equipment, lightning arresters, station transformer, motors for driving auxiliaries in power station.



Representation & Single line diagram

In power engineering, a single-line diagram (SLD), also sometimes called one-line diagram, is a simplified notation for representing a three-phase power system.

The one-line diagram has its largest application in power flow studies. Electrical elements such as circuit breakers, transformers, capacitors, bus bars, and conductors are shown by standardized schematic symbols. Instead of representing each of three phases with a separate line or terminal, only one conductor is represented.

It is a form of block diagram graphically depicting the paths for power flow between entities of the system. Elements on the diagram do not represent the physical size or location of the electrical equipment, but it is a common convention to organize the diagram with the same left-to-right, top-to-bottom sequence as the switchgear or other apparatus represented. A one-line diagram can also be used to show a high level view of conduit runs for a PLC control system.

Balanced systems

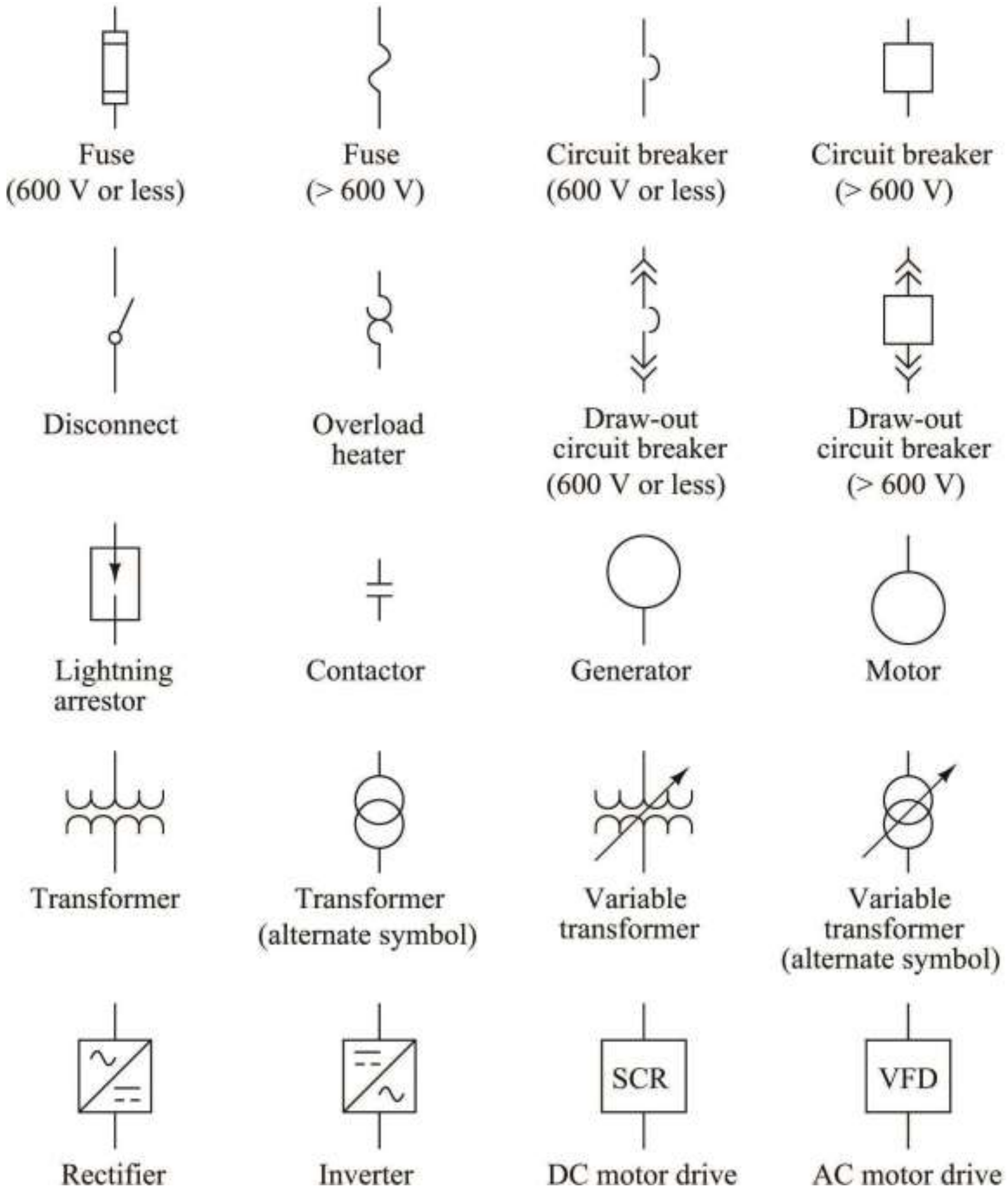
The theory of three-phase power systems tells us that as long as the loads on each of the three phases are balanced, we can consider each phase separately. In power engineering, this assumption is often useful, and to consider all three phases requires more effort with very little potential advantage. An important and frequent exception is an asymmetric fault on only one or two phases of the system.

A one-line diagram is usually used along with other notational simplifications, such as the per-unit system. A secondary advantage to using a one-line diagram is that the simpler diagram leaves more space for non-electrical, such as economic, information to be included.

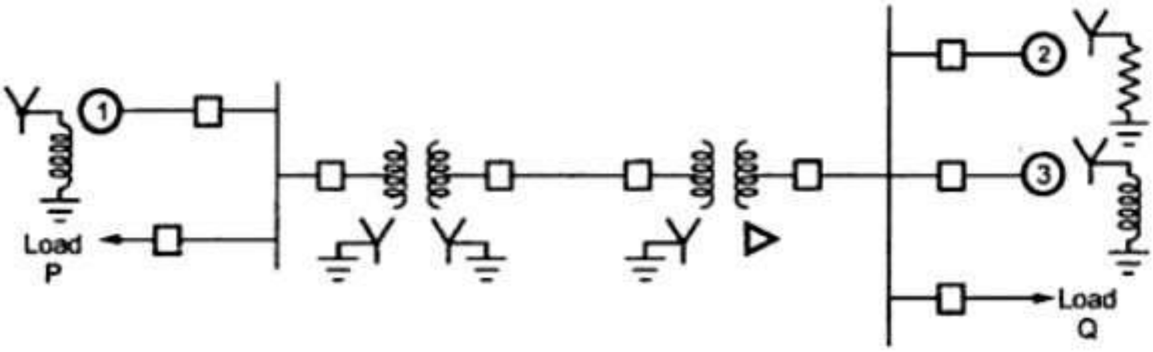
Unbalanced systems

When using the method of symmetrical components, separate one-line diagrams are made for each of the positive, negative and zero-sequence systems. This simplifies the analysis of unbalanced conditions of a poly phase system. Items that have different impedances for the different phase sequences are identified on the diagrams. For example, in general a generator will have different positive and negative sequence impedance, and certain transformer winding connections block zero-sequence currents. The unbalanced system can be resolved into three single line diagrams for each sequence, and interconnected to show how the unbalanced components add in each part of the system.

Symbols of Power System Components



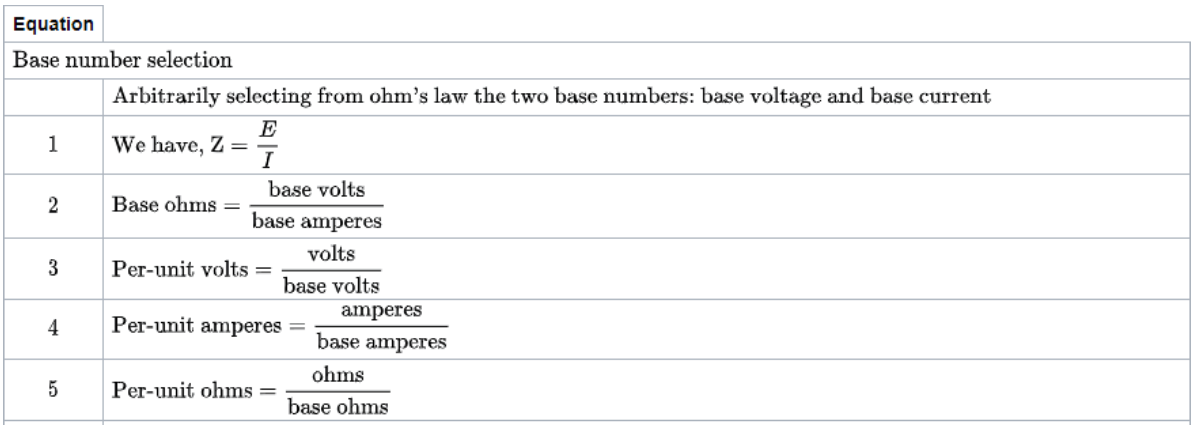
Single Line diagram of an Electrical system



- One line diagram of a very simple power system.
- Two generators one grounded through a reactor and one through a resister connected to a bus and through a step up transformer to a transmission lines.
- Another generator grounded a reactor is connected a bus and through a transformer to the opposite end of the transmission line.
- A load is connected to each bus.
- On the diagram information about the loads the ratings of the generators and transformers and reactance of different components of the circuit is often given.
- It is important to know the location of points where a system is connected to ground to calculate the amount of current flowing when an unsymmetrical fault involving ground occur.

Per-unit system

A per-unit system provides units for power, voltage, current, impedance, and admittance.

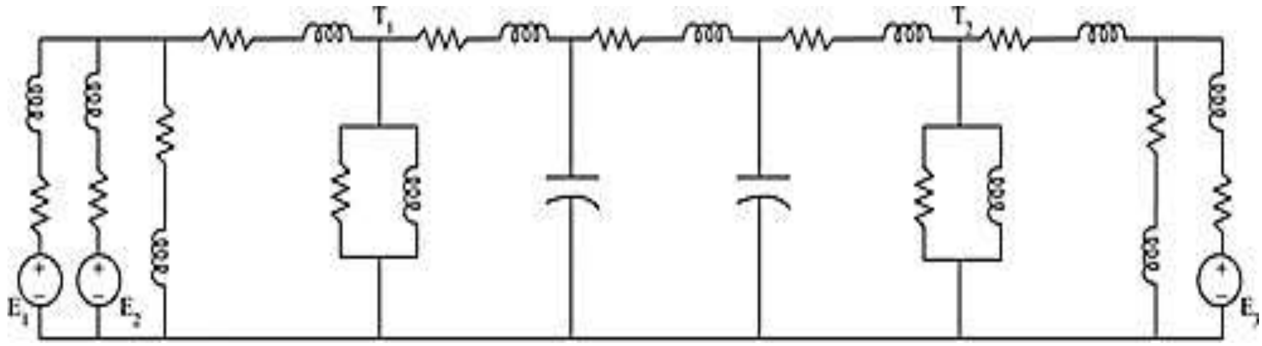


Equation	
Base number selection	
	Arbitrarily selecting from ohm's law the two base numbers: base voltage and base current
1	We have, $Z = \frac{E}{I}$
2	$\text{Base ohms} = \frac{\text{base volts}}{\text{base amperes}}$
3	$\text{Per-unit volts} = \frac{\text{volts}}{\text{base volts}}$
4	$\text{Per-unit amperes} = \frac{\text{amperes}}{\text{base amperes}}$
5	$\text{Per-unit ohms} = \frac{\text{ohms}}{\text{base ohms}}$

With the exception of impedance and admittance, any two units are independent and can be selected as base values; power and voltage are typically chosen. All quantities are specified as multiples of selected base values. For example, the base power might be the rated power of a transformer, or perhaps an arbitrarily selected power which makes power quantities in the system more convenient. The base voltage might be the nominal voltage of a bus. Different types of quantities are labelled with the same symbol (pu); it should be clear whether the quantity is a voltage, current, or other unit of measurement.

P.U Impedance Diagram

The impedance diagram on single-phase basis for use under balanced conditions can be easily drawn from the SLD. The following assumptions are made in obtaining the impedance diagrams.



Assumptions:

- The single phase transformer equivalents are shown as ideals with impedance on appropriate side (LV/HV),
- The magnetizing reactance of transformers are negligible,
- The generators are represented as constant voltage sources with series resistance or reactance,
- The transmission lines are approximated by their equivalent -Models,
- The loads are assumed to be passive and are represented by a series branch of resistance or reactance.
- Since the balanced conditions are assumed, the neutral grounding impedance do not appear in the impedance diagram.

P.U Reactance Diagram

- With some more additional and simplifying assumptions, the impedance diagram can be simplified further to obtain the corresponding reactance diagram. The following are the assumptions made.

Additional assumptions:

- The resistance is often omitted during the fault analysis. This causes a very negligible error since, resistances are negligible
- Loads are Omitted
- Transmission line capacitances are ineffective &
- Magnetizing currents of transformers are neglected.

Per Phase and Per Unit Representation

During the power system analysis, it is a usual practice to represent current, voltage, impedance, power, etc., of an electric power system in per unit or percentage of the base or reference value of the respective quantities. The numerical per unit (pu) value of any quantity is its ratio to a chosen base value of the same dimension. Thus a pu value is a normalized quantity with respect to the chosen base value.

Per Unit value of a given quantity is the ratio of the actual value in any given unit to the base value in the same unit. The percent value is 100 times the pu value. Both the pu and percentage methods are simpler than the use of actual values. Further, the main advantage in using the pu system of computations is that the result that comes out of the sum, product, quotient, etc. of two or more pu values is expressed in per unit itself.

Per unit value.

The per unit value of any quantity is defined as the ratio of the actual value of the any quantity to the base value of the same quantity as a decimal.

Advantages of per unit system

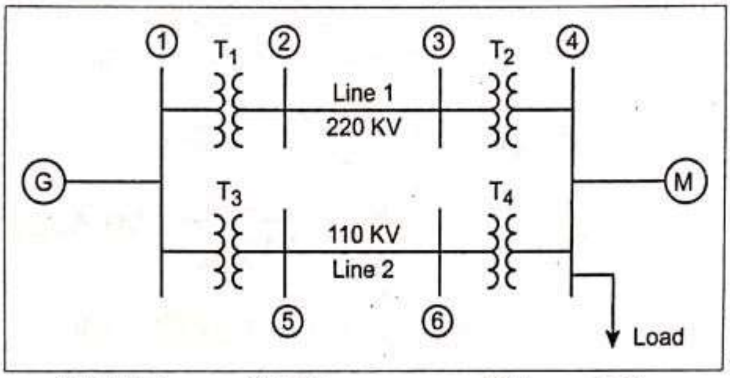
- Per unit data representation yields valuable relative magnitude information.
- Circuit analysis of systems containing transformers of various transformation ratios is greatly simplified.
- The p.u systems are ideal for the computerized analysis and simulation of complex power system problems.
- Manufacturers usually specify the impedance values of equivalent in per unit of the equipment rating. If the any data is not available, it is easier to assume its per unit value than its numerical value.
- The ohmic values of impedances are refereed to secondary is different from the value as referee to primary. However, if base values are selected properly, the p.u impedance is the same on the two sides of the transformer.
- The circuit laws are valid in p.u systems, and the power and voltages equations are simplified since the factors of √3 and 3 are eliminated.

In an electrical power system, the parameters of interest include the current, voltage, complex power (VA), impedance and the phase angle. Of these, the phase angle is dimensionless and the other four quantities can be described by knowing any two of them. Thus clearly, an arbitrary choice of any two base values will evidently fix the other base values.

Normally the nominal voltage of lines and equipment is known along with the complex power rating in MVA. Hence, in practice, the base values are chosen for complex power (MVA) and line voltage (KV). The chosen base MVA is the same for all the parts of the system. However, the base voltage is chosen with reference to a particular section of the system and the other base voltages (with reference to the other sections of the systems, these sections caused by the presence of the transformers) are then related to the chosen one by the turns-ratio of the connecting transformer.

P.U Example

19 **Example 2.7** *The one line diagram of a three phase power system is shown in Fig. Select a common base of 100 MVA and 20 KV on the generator side. Draw an impedance diagram with all impedances including the load impedance marked in p.u.*



G	:	*85 MVA,*	*20 KV,*	$X'' = 16\%$
T_1	:	*60 MVA,*	*20/220 KV,*	$X = 10\%$
T_2	:	*50 MVA,*	*220/11 KV,*	$X = 5\%$
T_3	:	*50 MVA,*	*20/110 KV,*	$X = 7\%$
T_4	:	*40 MVA,*	*110/11 KV,*	$X = 9\%$
M	:	*65 MVA,*	*10.5 KV,*	$X'' = 17\%$

The three phase load at bus 4 absorbs 62 MVA, 0.8 power factor lagging at 10.5 KV. Line 1 and Line 2 have reactances of 45 Ω and 60 Ω respectively.

☺ ***Solution :***

$$\text{MVA}_{b\ \text{new}} = 100$$

$$\text{KV}_{b\ \text{new}} = 20\ \text{KV on generator side}$$

Generator G_1 :

$$Z_{\text{p.u. new}} = Z_{\text{p.u. given}} \times \left[\frac{\text{KV}_{b\ \text{given}}}{\text{KV}_{b\ \text{new}}}\right]^2 \left[\frac{\text{MVA}_{b\ \text{new}}}{\text{MVA}_{b\ \text{given}}}\right]$$

$$= j0.16 \times \left[\frac{20}{20}\right]^2 \times \frac{100}{85} = j0.188 \quad \text{p.u.}$$

Transformer T_1 referred to primary (LV side) :

$$\text{KV}_{b\ \text{new}} = 20\ \text{KV}$$

$$Z_{\text{p.u. new}} = Z_{\text{p.u. given}} \times \left[\frac{\text{KV}_{b\ \text{given}}}{\text{KV}_{b\ \text{new}}}\right]^2 \left[\frac{\text{MVA}_{b\ \text{new}}}{\text{MVA}_{b\ \text{given}}}\right]$$

$$= j0.1 \times \left[\frac{20}{20}\right]^2 \left[\frac{100}{60}\right] = j0.167 \quad \text{p.u.}$$

Transformer T_3 referred to primary (LV side) :

$$KV_{b\ new} = 20\ KV$$

$$Z_{p.u.\ new} = Z_{p.u.\ given} \times \left[\frac{KV_{b\ given}}{KV_{b\ new}}\right]^2 \times \left[\frac{MVA_{b\ new}}{MVA_{b\ given}}\right]$$

$$= j0.07 \times \left[\frac{20}{20}\right]^2 \times \left[\frac{100}{50}\right] = j0.14\ \text{p.u.}$$

Line 1 : Transformer T_1 secondary side change occurs, so calculate $KV_{b\ new}$ as

$$KV_{b\ new} = KV_{b\ old} \times \left[\frac{\text{H.T side rating of } T_1}{\text{L.T side rating of } T_1}\right]$$

$$KV_{b\ new} = 20 \times \frac{220}{20} = 220\ KV$$

$$Z_{p.u.\ new} = \frac{Z_{actual}}{Z_{base}} = \frac{Z_{actual}}{KV_{b\ new}^2} \times MVA_{b\ new}$$

$$= \frac{j45}{220^2} \times 100 = j0.093\ \text{p.u.}$$

Transformer T_2 referred to primary side :

$$KV_{b\ new} = 220\ KV$$

$$Z_{p.u.\ new} = Z_{p.u.\ given} \times \left[\frac{KV_{b\ given}}{KV_{b\ new}}\right]^2 \times \left[\frac{MVA_{b\ new}}{MVA_{b\ given}}\right]$$

$$= j0.05 \times \left[\frac{220}{220}\right]^2 \times \left[\frac{100}{50}\right] = j0.1\ \text{p.u.}$$

Line 2 : Transformer T_3 secondary side change occurs, so calculate $KV_{b\ new}$ as

$$KV_{b\ new} = 20 \times \frac{110}{20} = 110\ KV$$

$$Z_{p.u.\ new} = \frac{Z_{actual}}{KV_b^2} \times MVA_b = \frac{j60}{110^2} \times 100 = j0.496\ \text{p.u.}$$

Transformer T_4 referred to primary :

$$KV_{b\ new} = 110\ KV$$

$$Z_{p.u.\ new} = Z_{p.u.\ given} \times \left[\frac{KV_{b\ given}}{KV_{b\ new}}\right]^2 \times \left[\frac{MVA_{b\ new}}{MVA_{b\ given}}\right]$$

$$= j0.09 \times \left[\frac{110}{110}\right]^2 \times \left[\frac{100}{40}\right] = j0.225\ \text{p.u.}$$

Motor M : Transformer T_4 secondary side change occurs, so calculate $KV_{b\text{ new}}$ as

$$KV_{b\text{ new}} = 110 \times \frac{11}{110} = 11 \text{ KV}$$

$$Z_{\text{p.u. new}} = Z_{\text{p.u. given}} \times \left[\frac{KV_{b\text{ given}}}{KV_{b\text{ new}}}\right]^2 \times \left[\frac{MVA_{b\text{ new}}}{MVA_{b\text{ given}}}\right]$$

$$= j0.17 \times \left[\frac{10.5}{11}\right]^2 \times \left[\frac{100}{65}\right] = j0.238 \text{ p.u.}$$

Load at bus 4 :

Load apparent power at 0.8 power factor lagging is given by

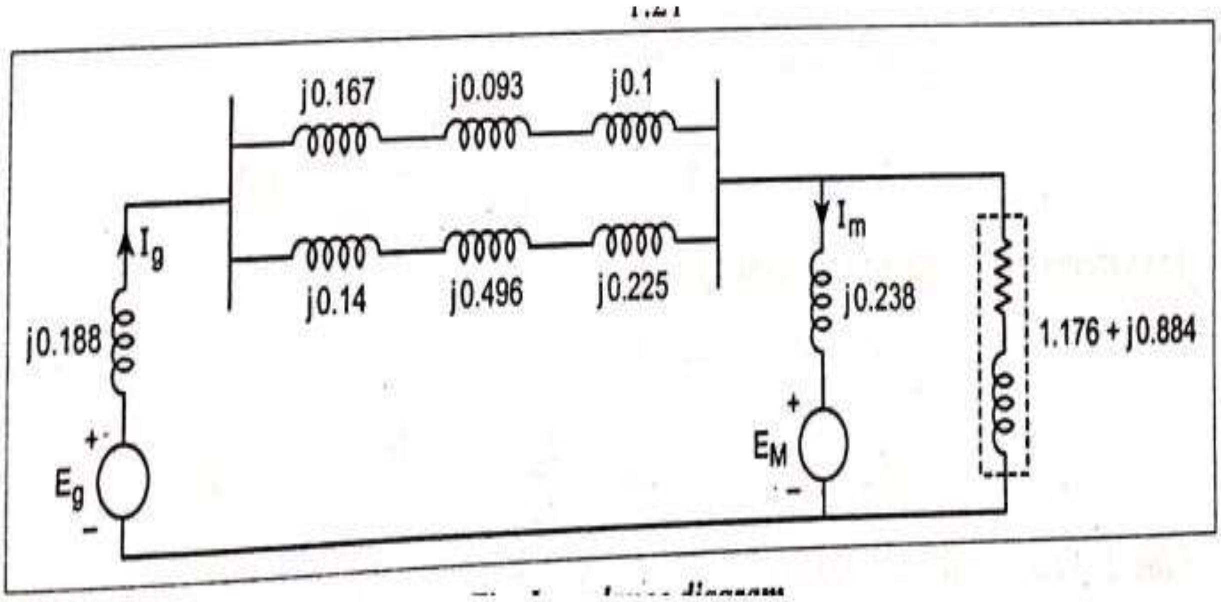
$$S_{L\,(3\phi)} = 62\,\angle\cos^{-1}(0.8) = 62\,\angle 36.87°$$

$$\text{Actual load impedance } Z_L = \frac{V_{LL}^2}{S^{*}_{L\,(3\phi)}} = \frac{10.5^2}{62\,\angle{-36.87°}} = 1.4226 + j1.07\,\Omega$$

$$\text{Base impedance } Z_b = \frac{KV_b{}^2}{MVA_b} = \frac{11^2}{100} = 1.21\,\Omega$$

$$Z_{L\text{ p.u.}} = \frac{Z_{\text{actual}}}{Z_{\text{base}}}$$

$$= \frac{1.4226 + j1.07}{1.21} = 1.176 + j0.884 \text{ p.u.}$$



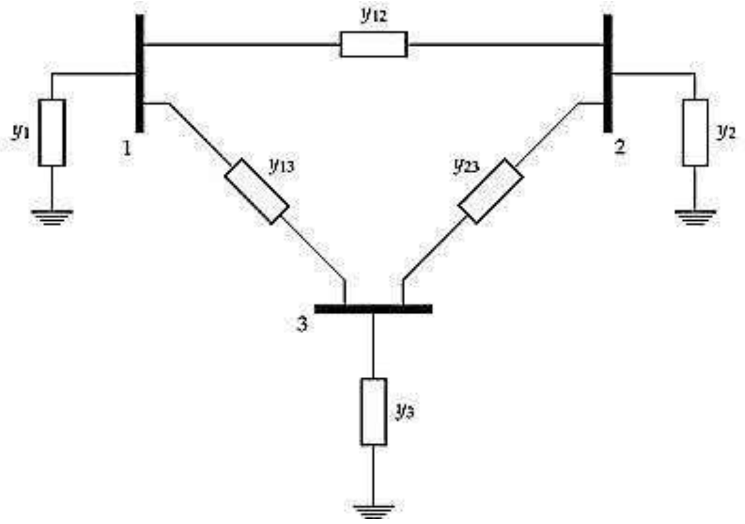
Bus Incidence matrix

Incidence matrix is that matrix which represents the graph such that with the help of that matrix we can draw a graph. This matrix can be denoted as [AC] As in every matrix, there are also rows and columns in incidence matrix [AC].

The rows of the matrix [AC] represent the number of nodes and the column of the matrix [AC] represent the number of branches in the given graph.

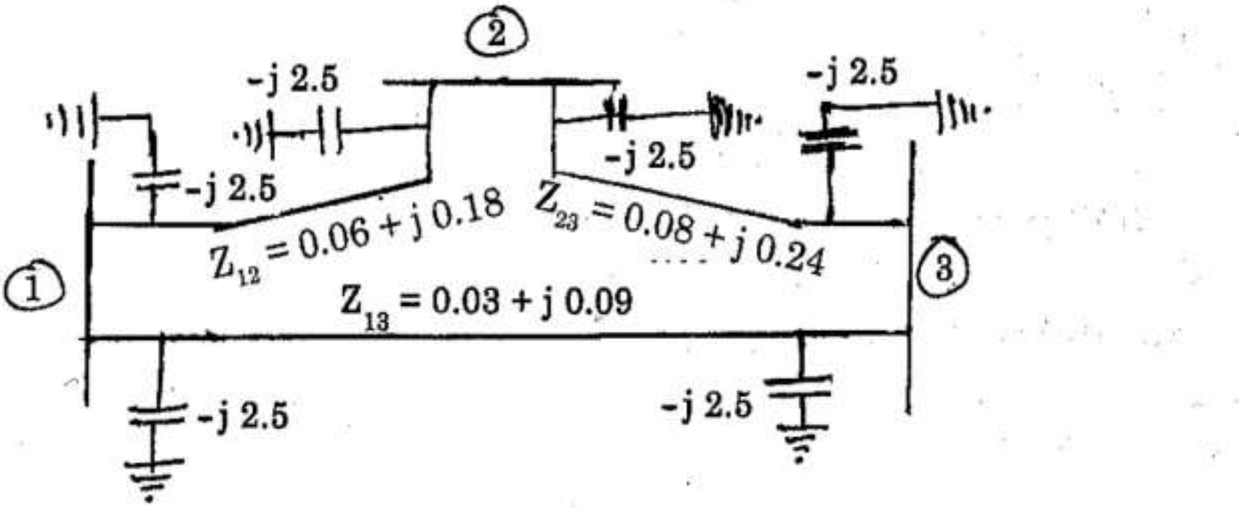
If there are ‘n’ number of rows in a given incidence matrix, that means in a graph there are ‘n’ number of nodes. Similarly, if there are ‘m’ number of columns in that given incidence matrix, that means in that graph there are ‘m’ number of branches.

Bus Admittance Matrix

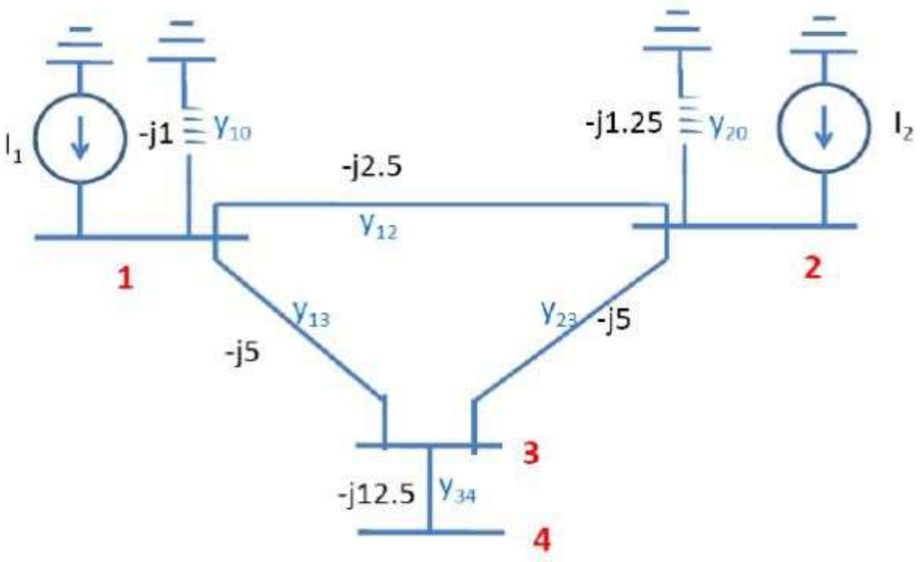


$$\begin{bmatrix} I_1 \\ I_2 \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{1n} \\ Y_{21} & Y_{22} & Y_{2n} \\ Y_{n1} & Y_{n2} & Y_{n1} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_n \end{bmatrix}$$

Determine the bus admittance matrix for the given power system.



Example



$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} -j8.5 & j2.5 & j5 & 0 \\ j2.5 & -j8.75 & j5 & 0 \\ j5.0 & j5.0 & -j22.5 & j12.5 \\ 0 & 0 & j12.5 & -j12.5 \end{bmatrix}$$

ELECTRICAL AND ELECTRONICS ENGINEERING

EE8501 POWER SYSTEM ANALYSIS

UNIT – II POWER FLOW ANALYSIS

Bus classification - Formulation of Power Flow problem in polar coordinates - Power flow solution using Gauss Seidel method - Handling of Voltage controlled buses - Power Flow Solution by Newton Raphson method.

Power flow study or Load flow study

The study of various methods of solution to power system network is referred to as load flow study.

The solution provides the voltages at various buses, power flowing in various lines and line losses.

Information’s that are obtained from a load flow study

The information obtained from a load flow study is magnitude and phase angle of voltages, real and reactive power flowing in each line and the line losses.

The load flow solution also gives the initial conditions of the system when the transient behaviour of the system is to be studied.

Need for load flow study

The load flow study of a power system is essential to decide the best operation of existing system and for planning the future expansion of the system. It is also essential for designing a new power system.

Quantities associated with each bus in a system

Each bus in a power system is associated with four quantities and they are

- Real power (P),
- Reactive power (Q),
- Magnitude of voltage (V),
- Phase angle of voltage (δ).

Work involved (or) to be performed by a load flow study

(i) Representation of the system by a single line diagram

(ii) Determining the impedance diagram using the information in single line diagram

(iii) Formulation of network equation

(iv) Solution of network equations

Iterative methods to solve load flow problems

The load flow equations are non linear algebraic equations and so explicit solution as not possible. The solution of non linear equations can be obtained only by iterative numerical techniques.

Mainly used for solution of load flow study

The Gauss seidal method, Newton Raphson method and Fast decouple methods.

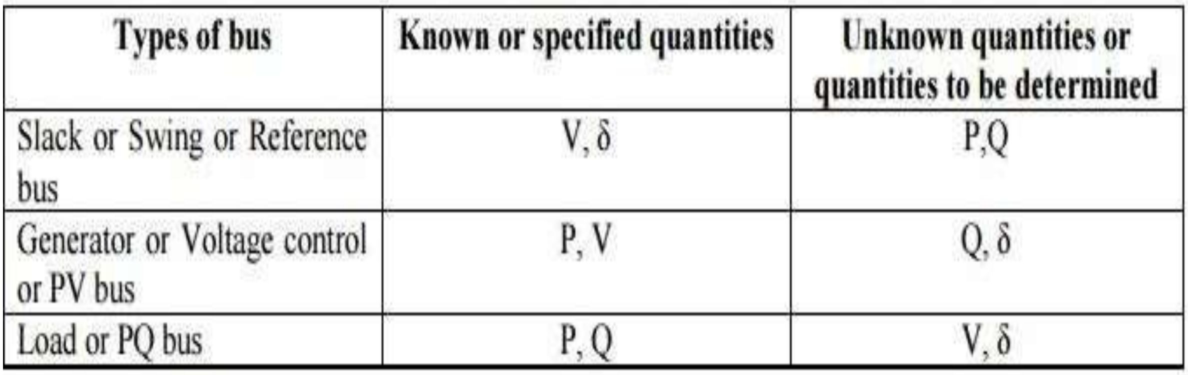
Flat voltage start

In iterative method of load flow solution, the initial voltages of all buses except slack bus assumed as 1+j0 p.u. This is referred to as flat voltage start.

Classification of Buses

The meeting point of various components in a power system is called a bus. The bus is a conductor made of copper or aluminum having negligible resistance .

At some of the buses power is being injected into the network, whereas at other buses it is being tapped by the system loads.



Types of bus	Known or specified quantities	Unknown quantities or quantities to be determined
Slack or Swing or Reference bus	V, δ	P,Q
Generator or Voltage control or PV bus	P, V	Q, δ
Load or PQ bus	P, Q	V, δ

Bus admittance matrix

The matrix consisting of the self and mutual admittance of the network of the power system is called bus admittance matrix (Y bus).

Methods available for forming bus admittance matrix

Direct inspection method.

Singular transformation method. (Primitive Network)

Need for slack bus (Swing Bus)

The slack bus is needed to account for transmission line losses. In a power system the total power generated will be equal to sum of power consumed by loads and losses.

In a power system only the generated power and load power are specified for buses. The slack bus is assumed to generate the power required for losses. Since the losses are unknown the real and reactive power are not specified for slack bus.

Effect of acceleration factor in load flow study

Acceleration factor is used in Gauss Seidal method of load flow solution to increase the rate of convergence. Best value of A.F=1.6

Generator buses are treated as load bus

If the reactive power constraint of a generator bus violates the specified limits then the generator is treated as load bus.

Gauss-Seidel Method

Step1: Assume all bus voltage be 1+ j0 except slack bus. The voltage of the slack bus is a constant voltage and it is not modified at any iteration

Step 2: Assume a suitable value for specified change in bus voltage which is used to compare the actual change in bus voltage between K th and (K+1) th iteration

Step 3: Set iteration count K = 0 and the corresponding voltages are V10, V20, V30, …… Vn0 except slack bus

Step 4: Set bus count P = 1

Step 5: Check for slack bus. It is a slack bus then goes to step 12 otherwise go to next step

Step 6: Check for generator bus. If it is a generator bus go to next step. Otherwise go to step 9

Step 7: Set │VPK│= │VP│ specified and phase of │VPK│ as the K th iteration value if the bus P is a generator bus where │VP│ specified is the specified magnitude of voltage for bus P. Calculate reactive power rating.

$$Q_P{}^{K+1}{}_{Cal} = (-1)\ \text{Imag}\ [(V_P{}^{K})^{A} (\sum_{q=1}^{P-1} Y_{pq} Vq^{k+1} + \sum_{q=P}^{n} Y_{pq} V_q{}^{K}$$

Step 8: If calculated reactive power is within the specified limits then consider the bus as generator bus and then set

QP = QP K+1 Cal for this iteration go to step 10

Step 9 : If the calculated reactive power violates the specified limit for reactive power then treat this bus as load bus

If QP K+1 Cal < QP min then QP = QP min

QPK+1Cal > QP max then QP = QP max

Step10: For generator bus the magnitude of voltage does not change and so for all iterations the magnitude of bus voltage is the specified value. The phase of the bus voltage can be calculated using

$$V_P{}^{K+1}\ \bar{t}emp = 1 / Y_{PP}\ [(P_P - jQ_P / \bar{V}_P{}^{K\,*}) - \sum Y_{pq} V_q{}^{K+1} - \sum Y_{pq} V_q{}^{K}]$$

Step 11: For load bus the (k+1)th iteration value of load bus P voltage VPK+1 can be calculated using $V_P{}^{K+1}$ temp = 1 / Y_{PP} [(P_P –jQ_P / $V_P{}^{K\,*}$) - $\sum Y_{pq} V_q{}^{K+1}$ - $\sum Y_{pq} V_q{}^{K}$]

Step 12: An acceleration factor α can be used for faster convergence. If acceleration factor is specified then modify the (K+1)th iteration value of bus P using $V_{Pacc}{}^{K+1} = V_P{}^{K} + \alpha\ (V_P{}^{K+1} - V_P{}^{K})$ then

Set $V_P{}^{K+1} = V_{Pacc}{}^{K+1}$

Step 13: Calculate the change in bus-P voltage using the relation $\Delta V_P{}^{K+1} = V_P{}^{K+1}$ $-V_P{}^{K}$

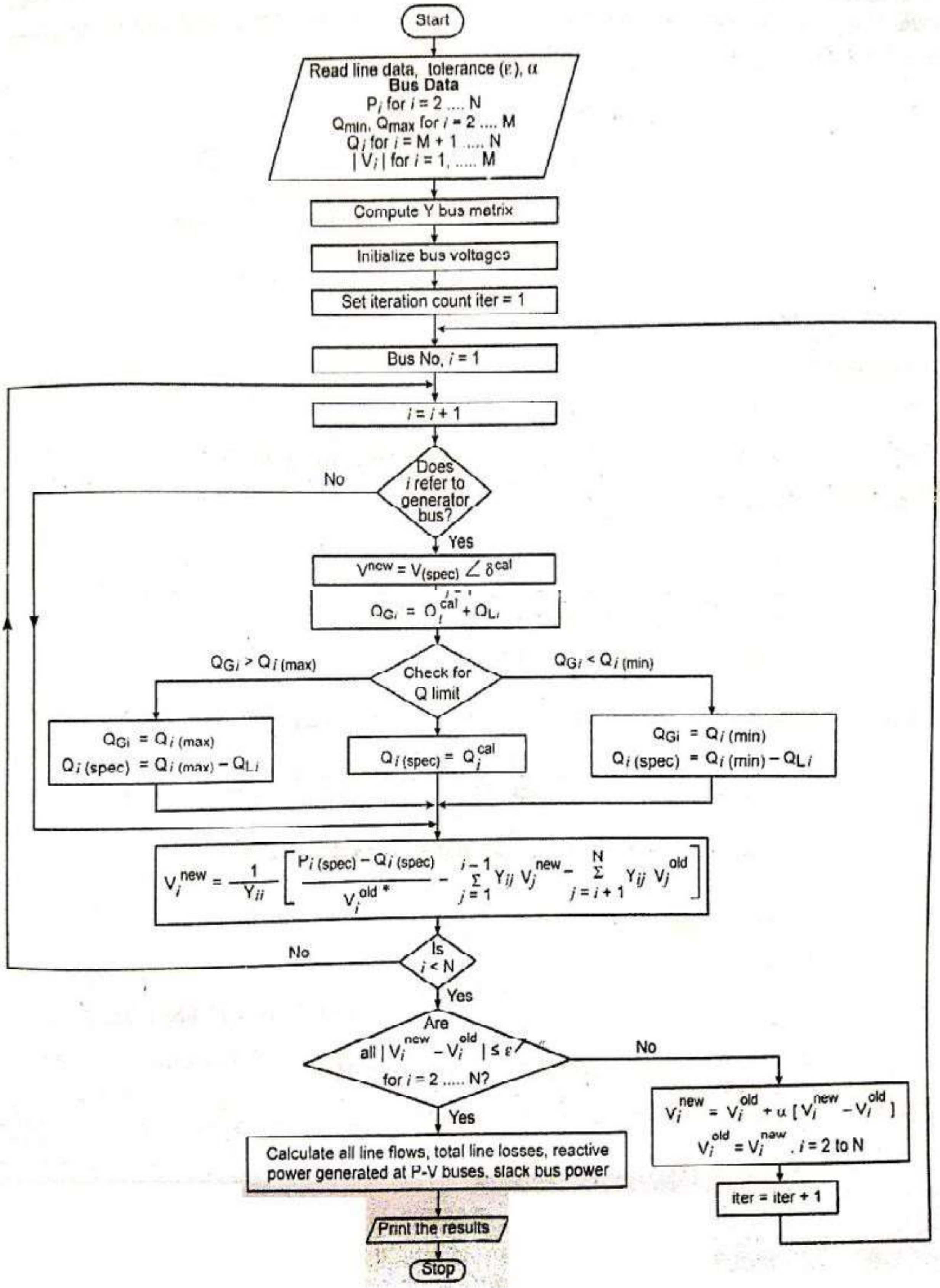
Step 14: Repeat step 5 to 12 until all the bus voltages have been calculated. For this increment the bus count by 1 go to step 5 until the bus count is n

Step 15: Find the largest of the absolute value of the change in voltage

$|\Delta V_1{}^{K+1}|, |\Delta V_2{}^{K+1}|, |\Delta V_3{}^{K+1}|, \ldots\ldots\ldots\ldots |\Delta V_n{}^{K+1}|$

Let this largest value be the $|\Delta V_{max}|$. Check this largest change $|\Delta V_{max}|$ is less than pre specified tolerance. If $|\Delta V_{max}|$ is less go to next step. Otherwise increment the iteration count and go to step 4

Step 16: Calculate the line flows and slack bus power by using the bus voltages.



Advantages and Disadvantages of Gauss-Seidel method

Advantages:

Calculations are simple and so the programming task is lessees.

The memory requirement is less.

Useful for small systems;

Disadvantages:

Requires large no. of iterations to reach converge.

Not suitable for large systems.

Convergence time increases with size of the system

Newton-Raphson Method

Algorithm of Newton-Raphson method

Step 1: Assume a suitable solution for all buses except the slack bus. Let Vp = a+j0 for P = 2,3,……n V1 = a+j0

Step 2 : Set the convergence criterion = ε0

Step 3 : Set iteration count K= 0

Step 4 : Set bus count P = 2

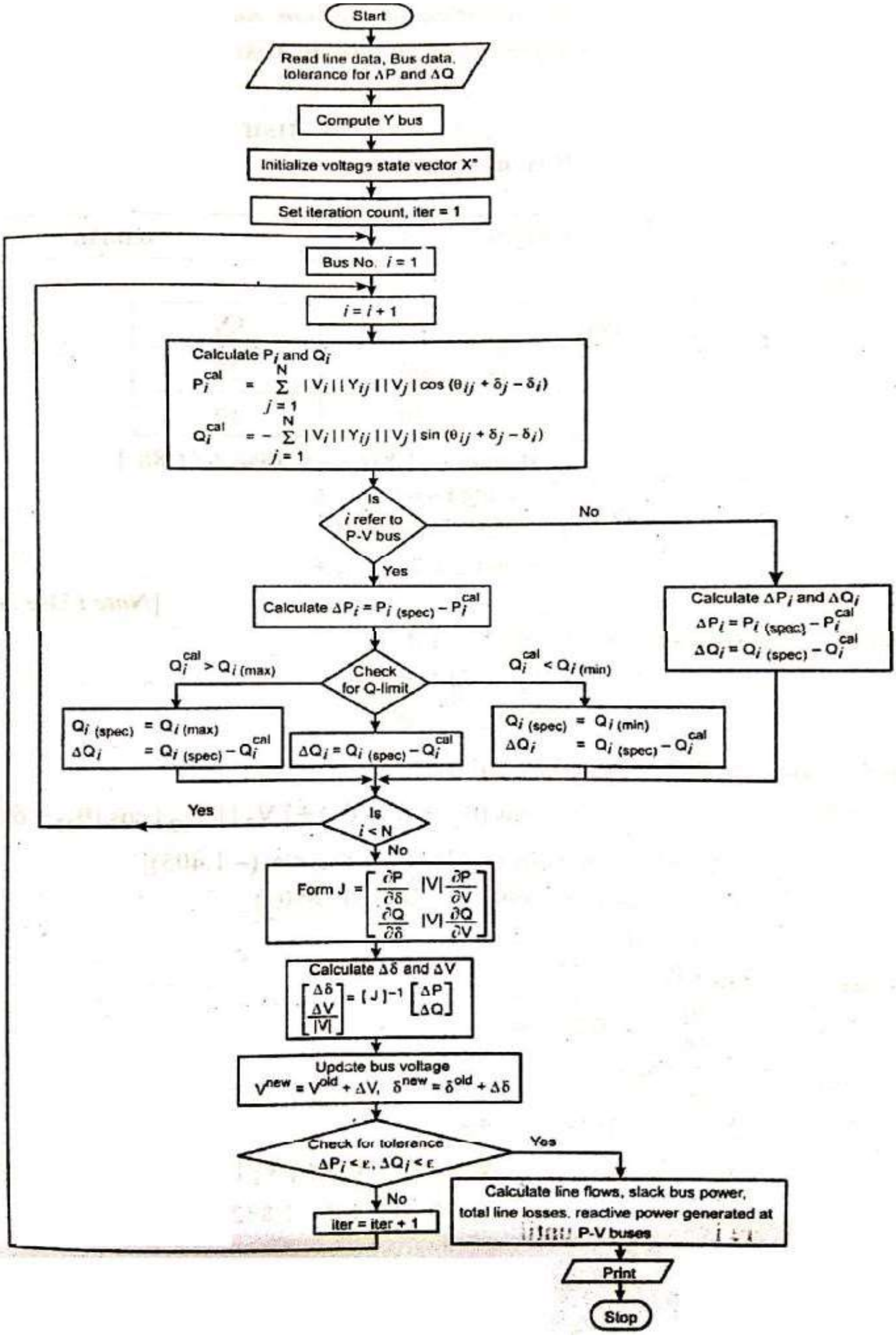
Step 5 : Calculate Pp and Qp using n

Step 6 : Evaluate ΔPPK = Pspec - PPK

Step 7 : Check if the bus is the question is a PV bus. If yes compare QPK with the limits. If it exceeds the limit fix the Q value to the corresponding limit and treat the bus as PQ for that iteration and go to next step (or) if the lower limit is not violated evaluate │ΔVP│2 = │ Vspec│2 - │VPK│2 and go to step 9

Step 8: Evaluate ΔQPK = Qspec - QPK

Step 9 : Advance bus count P = P+1 and check if all buses taken in to account if not go to step 5


Start
Read line data, Bus data, tolerance for ΔP and ΔQ
Compute Y bus
Initialize voltage state vector X°
Set iteration count, iter = 1
Bus No. $i = 1$
$i = i + 1$
Calculate P_i and Q_i
$P_i^{cal} = \sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \cos(\theta_{ij} + \delta_j - \delta_i)$
$Q_i^{cal} = -\sum_{j=1}^{N} |V_i||Y_{ij}||V_j| \sin(\theta_{ij} + \delta_j - \delta_i)$
Is i refer to P-V bus
No
Yes
Calculate $\Delta P_i = P_{i\,(spec)} - P_i^{cal}$
Calculate ΔP_i and ΔQ_i
$\Delta P_i = P_{i\,(spec)} - P_i^{cal}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$
$Q_i^{cal} > Q_{i\,(max)}$
Check for Q-limit
$Q_i^{cal} < Q_{i\,(min)}$
$Q_{i\,(spec)} = Q_{i\,(max)}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$
$Q_{i\,(spec)} = Q_{i\,(min)}$
$\Delta Q_i = Q_{i\,(spec)} - Q_i^{cal}$
Yes
Is $i < N$
No
Form $J = \begin{bmatrix} \frac{\partial P}{\partial \delta} & |V| \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \delta} & |V| \frac{\partial Q}{\partial V} \end{bmatrix}$
Calculate $\Delta\delta$ and ΔV
$\begin{bmatrix} \Delta\delta \\ \frac{\Delta V}{|V|} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}$
Update bus voltage
$V^{new} = V^{old} + \Delta V, \ \delta^{new} = \delta^{old} + \Delta\delta$
Check for tolerance $\Delta P_i < \varepsilon, \Delta Q_i < \varepsilon$
Yes
No
iter = iter + 1
Calculate line flows, slack bus power, total line losses, reactive power generated at P-V buses
Print
Stop

Step 10 : Determine the largest value of │ΔVP│2

Step 11: If ΔVP < ε go to step 16

Step 12: Evaluate the element of Jacobin matrices J1, J2, J3, J4, J5 and J6

Step 13: Calculate ΔePK and ΔfPK

Step 14: Calculate ePK+1 = ePK + ΔePK and fPK+1 = fPK + ΔfPK

Step 15 : Advance count (iteration) K=K+1 and go to step 4

Step 16: Evaluate bus and line power and print the result

Advantages and disadvantages of N.R method

Advantages:

Faster,

More reliable and

Results are accurate,

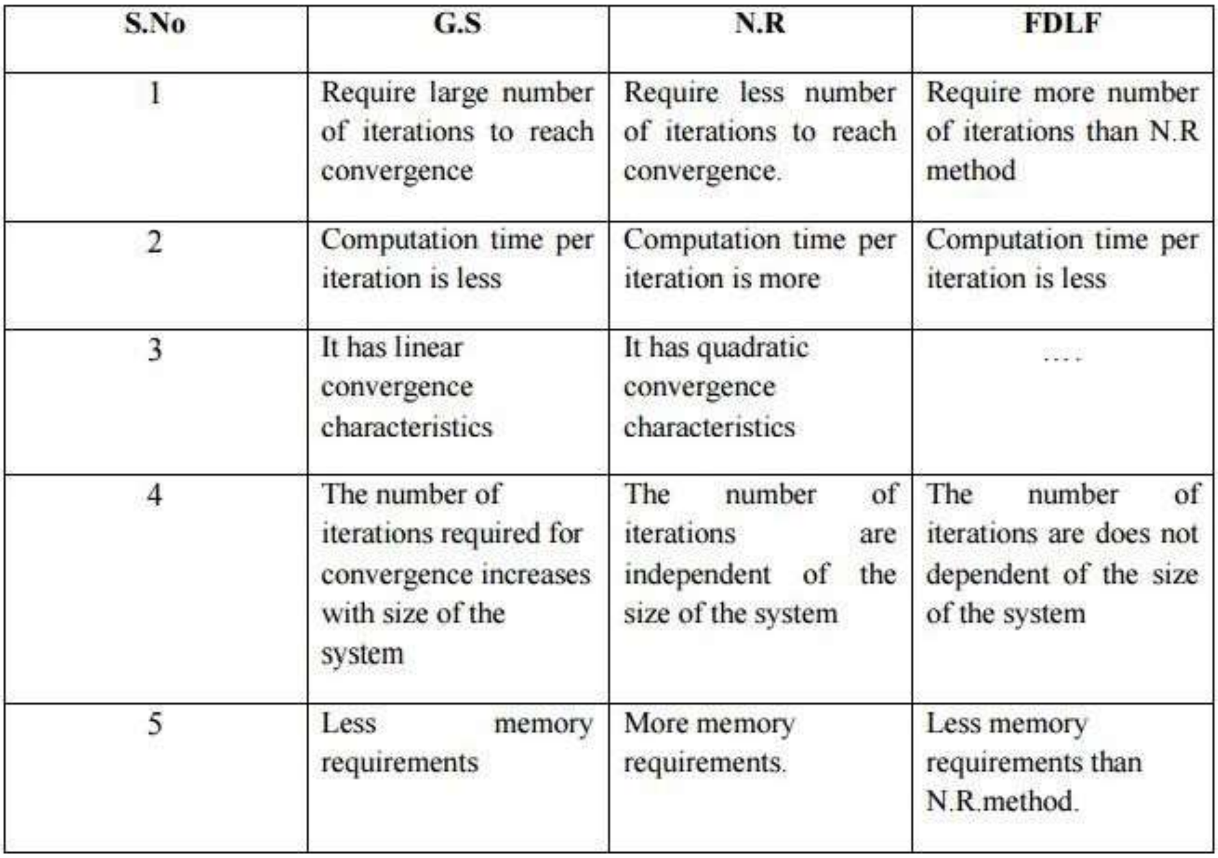
Require less number of iterations;

Disadvantages:

Program is more complex,

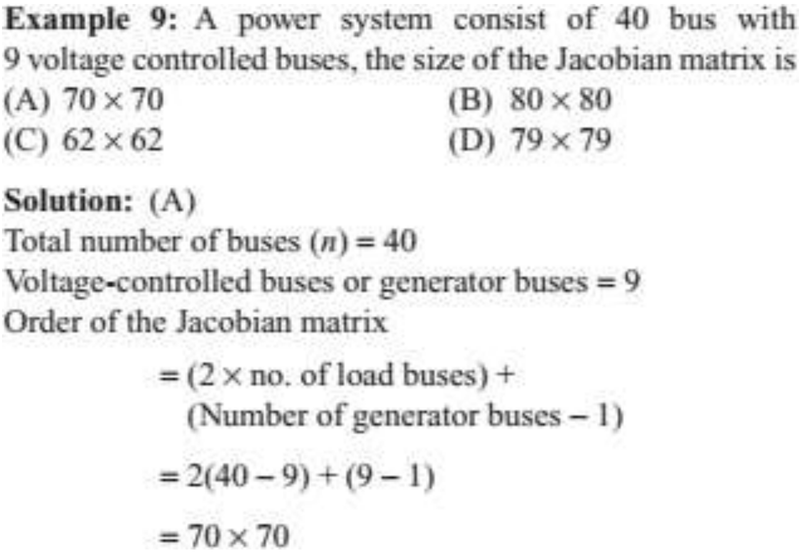
Memory is more complex.

Comparison of Gauss Seidel and Newton Raphson Methods of Load Flow Study



S.No	G.S	N.R	FDLF
1	Require large number of iterations to reach convergence	Require less number of iterations to reach convergence.	Require more number of iterations than N.R method
2	Computation time per iteration is less	Computation time per iteration is more	Computation time per iteration is less
3	It has linear convergence characteristics	It has quadratic convergence characteristics	....
4	The number of iterations required for convergence increases with size of the system	The number of iterations are independent of the size of the system	The number of iterations are does not dependent of the size of the system
5	Less memory requirements	More memory requirements.	Less memory requirements than N.R.method.

Jacobian matrix Calculation



Example 9: A power system consist of 40 bus with 9 voltage controlled buses, the size of the Jacobian matrix is

(A) 70×70 (B) 80×80
(C) 62×62 (D) 79×79

Solution: (A)

Total number of buses (n) = 40

Voltage-controlled buses or generator buses = 9

Order of the Jacobian matrix

$$= (2 \times \text{no. of load buses}) + (\text{Number of generator buses} - 1)$$

$$= 2(40 - 9) + (9 - 1)$$

$$= 70 \times 70$$

ELECTRICAL AND ELECTRONICS ENGINEERING

EE8501 POWER SYSTEM ANALYSIS

UNIT – III SYMMETRICAL FAULT ANALYSIS

Assumptions in short circuit analysis - Symmetrical short circuit analysis using Thevenin's theorem - Bus Impedance matrix building algorithm (without mutual coupling) – Symmetrical fault analysis through bus impedance matrix - Post fault bus voltages - Fault level – Current limiting reactors.

Importance Short Circuit (Or) For Fault Analysis

Fault

A fault in a circuit is any failure which interferes with the normal flow of current. The faults are associated with abnormal change in current, voltage and frequency of the power system.

Faults occur in a power system

The faults occur in a power system due to

(i). Insulation failure of equipment

(ii). Flashover of lines initiated by a lighting stroke

(iii). Due to permanent damage to conductors and towers or due to accidental faulty operations.

Various types of faults

(i) Series fault or open circuit fault

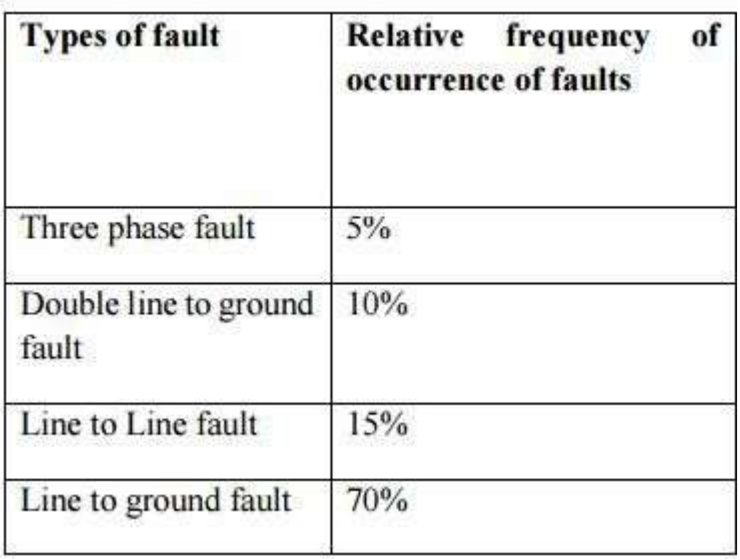
- One open conductor fault
- Two open conductor fault

(ii) Shunt fault or short circuit fault. Symmetrical fault or balanced fault

- Three phase fault
- Unsymmetrical fault or unbalanced fault
- Line to ground (L-G) fault

- Line to Line (L-L) fault
- Double line to ground (L-L-G) fault

Relative frequency of occurrence of various types of fault



Types of fault	Relative frequency of occurrence of faults
Three phase fault	5%
Double line to ground fault	10%
Line to Line fault	15%
Line to ground fault	70%

Symmetrical fault or balanced three phase fault

This type of fault is defined as the simultaneous short circuit across all the three phases. It occurs infrequently, but it is the most severe type of fault encountered. Because the network is balanced, it is solved by per phase basis using Thevenin’s theorem or bus impedance matrix or KVL, KCL laws.

Basic Assumptions in Fault Analysis of Power Systems

(i). Representing each machine by a constant voltage source behind proper reactance which may be X”, X’, or X

(ii). Pre-fault load current are neglected

(iii). Transformer taps are assumed to be nominal

(iv). Shunt elements in the transformers model that account for magnetizing current and core loss are neglected

(v). A symmetric three phase power system is conducted

(vi). Shunt capacitance and series resistance in transmission are neglected

(vii). The negative sequence impedances of alternators are assumed to be the same as their positive sequence impedance Z+ = Z-

Need for short circuit studies or fault analysis

Short circuit studies are essential in order to design or develop the protective schemes for various parts of the system .To estimate the magnitude of fault current for the proper choice of circuit breaker and protective relays.

Bolted fault or Solid fault

A Fault represents a structural network change equivalent with that caused by the addition of impedance at the place of a fault. If the fault impedance is zero, the fault is referred as bolted fault or solid fault.

Reason for transients during short circuits

The faults or short circuits are associated with sudden change in currents. Most of the components of the power system have inductive property which opposes any sudden change in currents, so the faults are associated with transients.

Doubling effect

If a symmetrical fault occurs when the voltage wave is going through zero then the maximum momentary short circuit current will be double the value of maximum symmetrical short circuit current. This effect is called doubling effect.

DC off set current

The unidirectional transient component of short circuit current is called DC off set current.

Short circuit capacity of power system or Fault level.

Short circuit capacity (SCC) or Short circuit MVA or fault level at a bus is defined as the product of the magnitude of the pre fault bus voltage and the post fault current

$$\text{SCC or Short circuit MVA} = |V_{prefault}| \times |I_f|$$

(OR)

$$\text{SCC} = \frac{1}{X_{th}} \; p.u \;\; MVA$$

Thevenin’s theorem:

(i). Fault current = Eth / (Zth+Zf)

(ii). Determine current contributed by the two generators

IG1 = If * (Z2/(Z1+Z2))

IG2 = If * (Z1 / (Z1+Z2))

(iii). Determine Post fault voltage Vif = Vi°+ΔV = V°+(-Zi2*IGi)

(iv). Determine post fault voltage line flows   Iij = (Vi –Vj) / Zij series

(v). Short circuit capacity If = $|\text{Eth}|^2$ / Xth

Fault Analysis Using Z-Bus Matrix – Algorithm

Bus impedance matrix

Bus impedance matrix is the inverse of the bus admittance matrix. The matrix consisting of driving point impedance and transfer impedances of the network is called as bus impedance matrix. Bus impedance matrix is symmetrical.

Methods available for forming bus impedance matrix

(i). Form bus admittance matrix and take the inverse to get bus impedance matrix.

(ii). Using bus building algorithm.

(iii). Using L-U factorization of Y-bus matrix.

Z_{BUS} Formulation:

Z_{BUS} Formulation is given by

By Inventing YBUS

$$J_{\mathrm{BUS}} = Y_{\mathrm{BUS}}\ V_{\mathrm{BUS}}$$

$$V_{\mathrm{BUS}} = [Y_{\mathrm{BUS}}]^{-1}\ J_{\mathrm{BUS}} = Z_{\mathrm{BUS}}\ J_{\mathrm{BUS}}$$

$$Z_{\mathrm{BUS}} = [Y_{\mathrm{BUS}}]^{-1}$$

The sparsity of YBUS may be retained by using an efficient inversion technique and nodal impedance matrix can then be calculated directly from the factorized admittance matrix.

Current Injection Method:

Above Equation can be written in the expanded form

$$\begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 + \ldots + Z_{1n}I_n \\ V_2 &= Z_{21}I_1 + Z_{22}I_2 + \ldots + Z_{2n}I_n \\ &\text{-------------------------} \\ V_n &= Z_{n1}I_1 + Z_{n2}I_1 + \ldots + Z_{nn}I_n \end{aligned}$$

It immediately follows from Eq. that

$$Z_{ij} = \left.\frac{V_i}{I_j}\right|_{\substack{I_1 = I_2 = \cdots = I_n = 0 \\ I_j \neq 0}}$$

Also Zij = Zji; (Z_{BUS} Formulation is a symmetrical matrix).

As per Eq. if a unit current is injected at bus (node) j, while the other buses are kept open circuited, the bus voltages yield the values of the jth column of Z_{BUS}. However, no organized computerizable techniques are possible for finding the bus voltages. The technique had utility in AC Network Analyzers where the bus voltages could be read by a voltmeter.

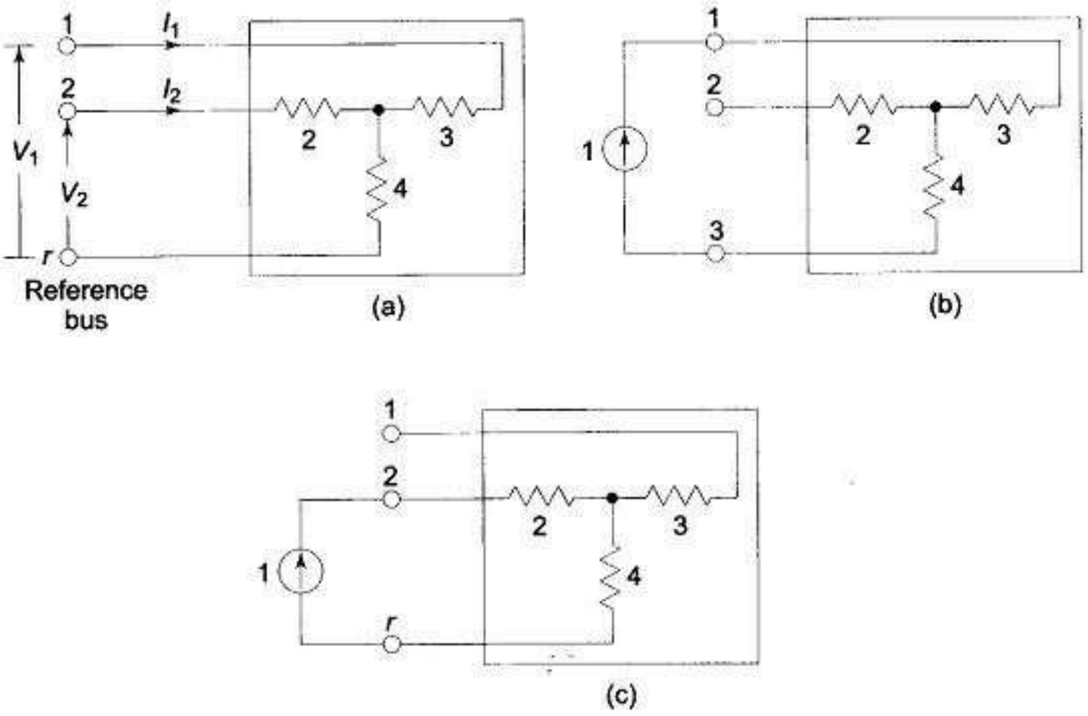
Z_{BUS} Building Algorithm:

It is a step-by-step programmable technique which proceeds branch by branch. It has the advantage that any modification of the network does not require complete rebuilding of Z_{BUS} Formulation.

Consider that ZBUS Formulation has been formulated upto a certain stage and another branch is now added. Then

$$Z_{BUS}\ (\text{old}) \xrightarrow{Z_b = \text{branch impedance}} Z_{BUS}\ (\text{new})$$

Upon adding a new branch, one of the following situations is presented.



Current Injection Method

Current Injection Method

1. Z_b is added from a new bus to the reference bus (i.e. a new branch is added and the dimension of Z_{BUS} goes up by one). This is type-I modification.
2. Z_b is added from a new bus to an old bus (i.e., a new branch is added and the dimension of Z_{BUS} goes up by one). This is type-2 modification.
3. Z_b connects an old bus to the reference branch (i.e., a new loop is formed but the dimension of Z_{BUS} does not change). This is type-3 modification.
4. Z_b connects two old buses (i.e., new loop is formed but the dimension of Z_{BUS} does not change). This is type-4 modification.
5. Z_b connects two new buses (Z_{BUS} remains unaffected in this case). This situation can be avoided by suitable numbering of buses and from now on wards will be ignored.

   Notation: i, j—old buses; r—reference bus; k—new bus.

Type-1 Modification:

Figure shows a passive (linear) n-bus network in which branch with impedance Z_b is added to the new bus k and the reference bus r. Now

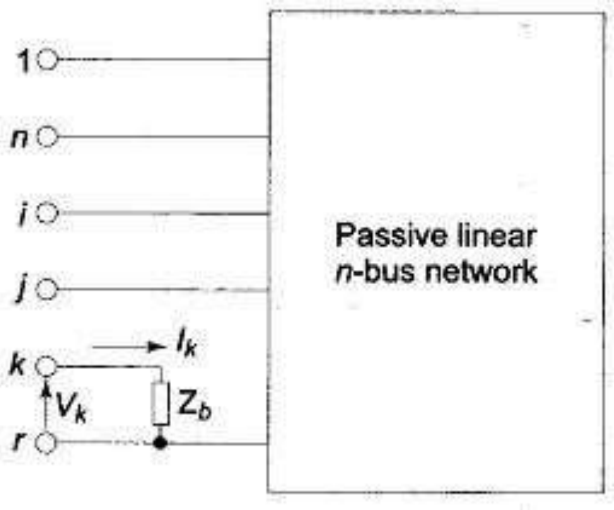
$$V_k = Z_b I_k$$

$$Z_{ki} = Z_{ik} = 0;\ \text{i} = 1, 2, \ldots, n$$

$$Z_{kk} = Z_b$$

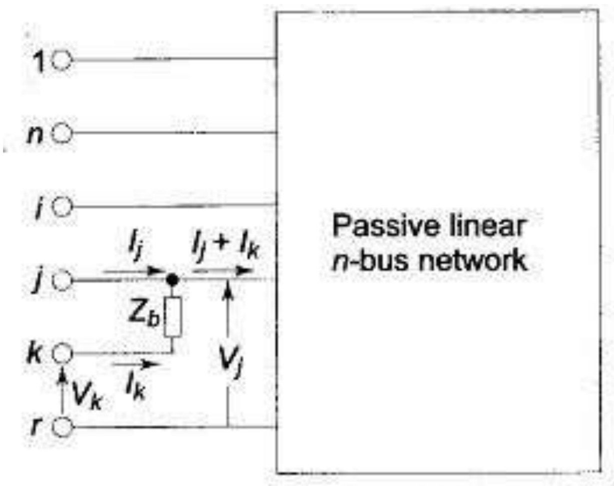
Hence

$$Z_{BUS}\ (\text{new}) = \left[\begin{array}{c|c} & 0 \\ Z_{Bus}(\text{old}) & \vdots \\ & 0 \\ \hline 0 \qquad \ldots 0 & Z_b \end{array}\right]$$



Type-2 Modification:

Z_b is added from new bus k to the old bus j as in Fig. It follows from this figure that



$$V_k = Z_b I_k + V_j$$
$$= Z_b I_k + Z_{j1} I_1 + Z_{j2} I_2 + \dots + Z_{jj}(I_j + I_k) + \dots + Z_{jn} I_n$$

Rearranging,

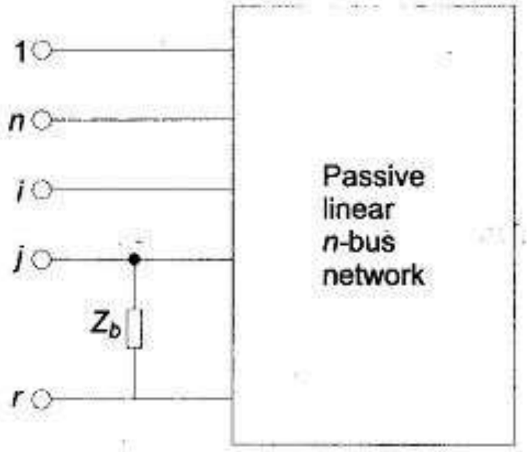
$$V_k = Z_{j1} I_1 + Z_{j2} I_2 + \dots + Z_{jj} I_j + \,.\,.\, + Z_{jn} I_n + (Z_{jj} + Z_b) I_k$$

Consequently

$$Z_{\text{BUS}}\text{ (new)} = \left[\begin{array}{c|c} & Z_{1j} \\ & Z_2 j \\ Z_{\text{BUS}}(\text{old}) & \vdots \\ & Z_{nj} \\ \hline Z_{ji} Z_{j2} \dots Z_{jn} & Z_{jj} + Z_b \end{array}\right]$$

Type–3 Modification:

Z_b connects an old bus (j) to the reference bus (r) as in Fig. This case follows from Fig. by connecting bus k to the reference bus r, i.e. by setting Vk = 0.



Thus

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ 0 \end{bmatrix} = \left[\begin{array}{c|c} & Z_{1j} \\ & Z_2j \\ Z_{BUS}(\text{old}) & \vdots \\ & Z_{nj} \\ \hline Z_{j1}Z_{j2}\ldots Z_{jn} & Z_{jj}+Z_b \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ I_k \end{bmatrix}$$

Eliminate Ik in the set of equations contained in the matrix operation,

$$0 = Z_{j1}I_1 + Z_{j2}I_2 + \ldots + Z_{jn}I_n + (Z_{jj} + Z_b)I_k$$

$$I_k = -\frac{1}{Z_{jj} - Z_b}(Z_{j1}I_1 + Z_{j2}I_2 + \ldots + Z_{jn}I_n)$$

Substituting Eq.

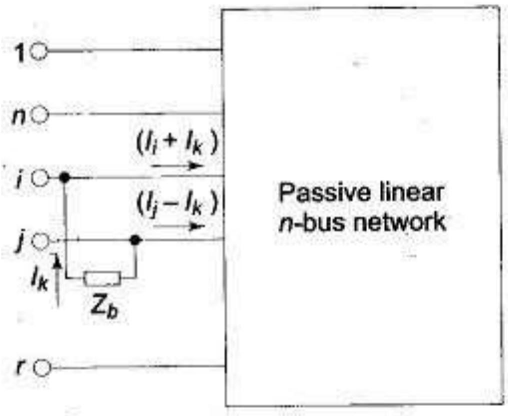
$$V_i = \left[Z_{i1} - \frac{1}{Z_{jj}+Z_b}(Z_{ij}Z_{j1})\right]I_1 + \left[Z_{i2} - \frac{1}{Z_{jj}+Z_b}(Z_{ij}Z_{j2})\right]I_2 + \ldots + \left[Z_{in} - \frac{1}{Z_{jj}+Z_b}(Z_{ij}\ Z_{jn})\right]I_n$$

Equation can be written in matrix form as

$$Z_{BUS}\ (\text{new}) = Z_{BUS}\ (\text{old} - \frac{1}{Z_{jj}+Z_b}\begin{bmatrix} Z_{1j} \\ \vdots \\ Z_{nj} \end{bmatrix}[Z_{j1}\ldots Z_{jn}]$$

Type-4 Modification:

Z_b connects two old buses as in Fig. Equations can be written as follows for all the network buses.



Similar equations follow for other buses.

$$V_i = Z_{i1}I_1 + Z_{i2}I_2 + \dots + Z_{1i}\,(I_i + I_k) + Z_{ij}\,(I_j - I_k) + \dots + Z_{in}I_n$$

The voltages of the buses i and j are, however, constrained by the equation (Fig.)

$$V_j = Z_bI_k + V_i \tag{9.44}$$

$$\begin{aligned} \text{or}\quad & Z_{j1}I_1 + Z_{j2}I_2 + \dots + Z_{ji}\,(I_i + I_k) + Z_{jj}\,(I_j - I_k) + \dots + Z_{jn}I_n \\ &= Z_bI_k + Z_{i1}I_1 + Z_{i2}I_2 + \dots + Z_{ii}\,(I_i + I_k) + Z_{ij}(I_j - I_k) + \dots + Z_{in}I_n \end{aligned}$$

Rearranging

$$\begin{aligned} 0 = {} & (Z_{i1} - Z_{j1})\,I_1 + \dots + (Z_{ii} - Z_{ji})\,I_i + (Z_{ij} - Z_{jj})\,I_j \\ & + \dots + (Z_{in} - Z_{jn})\,I_n + (Z_b + Z_{ii} + Z_{jj} - Z_{ij} - Z_{ji})\,I_k \end{aligned}$$

Collecting equations similar to Eq. we can write

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \\ - \\ 0 \end{bmatrix} = \left[\begin{array}{c|c} & (Z_{1i} - Z_{1j}) \\ Z_{\text{BUS}} & | \\ & (Z_{ni} - Z_{nj}) \\ \hline (Z_{i1} - Z_{j1})\dots(Z_{in} - Z_{jn}) & Z_b + Z_{ii} + Z_{jj} - 2Z_{ij} \end{array}\right] \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \\ \hline I_j \end{bmatrix}$$

Eliminating Ik in Eq. on lines similar to what was done in Type-2 modification, it follows that

$$Z_{\text{BUS}}\text{ (new)} = Z_{\text{BUS}}\text{ (old)} - \frac{1}{Z_b + Z_{ii} + Z_{jj} - 2Z_{ij}} \begin{bmatrix} Z_{1i} & - & Z_{1j} \\ & \vdots & \\ Z_{ni} & - & Z_{nj} \end{bmatrix}$$

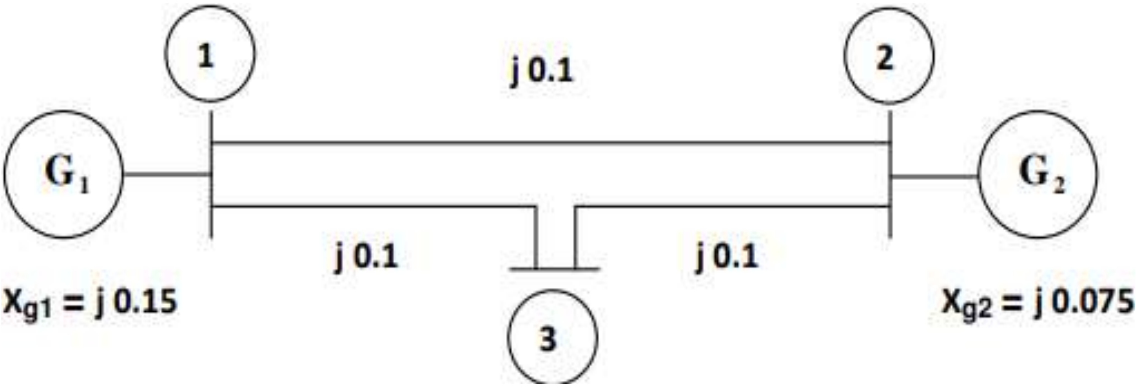
$$[Z_{i1} - Z_{j1}] \ \dots \ (Z_{in} - Z_{jn})] \qquad (9.47)$$

With the use of four relationships Eqs bus impedance matrix can be built by a step-by-step procedure

When the network undergoes changes, the modification procedures can be employed to revise the bus impedance matrix of the network. The opening of a line (Zij) is equivalent to adding a branch in parallel to it with impedance — Zij.

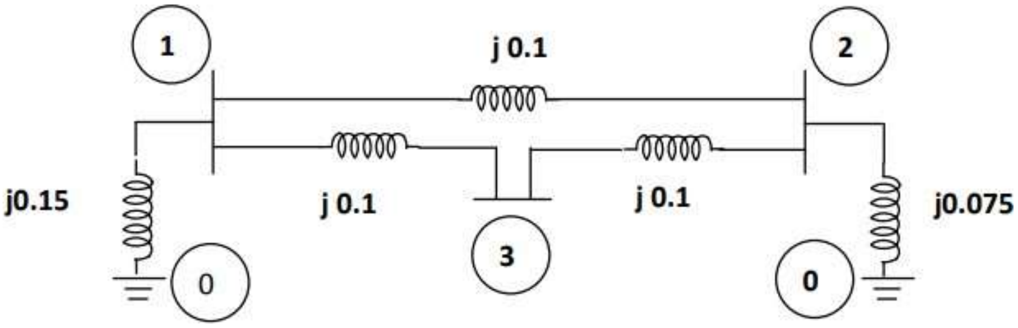
Example

Consider the power system shown in Fig. The values marked are p.u. impedances. The p.u. reactance’s of the generator 1 and 2 are 0.15 and 0.075 respectively. Compute the bus impedance matrix of the generator – transmission network.



Solution

The ground bus is numbered as 0 and it is taken as reference bus. The p.u. impedance diagram is shown in Fig.



When element $0-1$ is included

$$Z_{bus} = j \begin{array}{cc} & 1 \\ 1 & \left[0.15\right] \end{array} \; ; \text{ When element } 0-2 \text{ is included } \; Z_{bus} = j \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 0.15 & 0 \\ 0 & 0.075 \end{bmatrix} \end{array}$$

Element $1-2$ is added; it is a link between buses 1 and 2. With bus ℓ

Eliminating the ℓ^{th} bus

$$Z_{bus} = j \begin{array}{c|cc|c} & 1 & 2 & \ell \\ \hline 1 & 0.15 & 0 & 0.15 \\ 2 & 0 & 0.075 & -0.075 \\ \hline \ell & 0.15 & -0.075 & 0.325 \end{array} \; ; \qquad Z_{bus} = j \begin{array}{c|cc|} & 1 & 2 \\ \hline 1 & 0.08077 & 0.034615 \\ 2 & 0.034615 & 0.05769 \end{array}$$

Add element $1-3$. It is a branch from bus 1 and it creates bus 3.

$$Z_{bus} = j \begin{array}{c|ccc|} & 1 & 2 & 3 \\ \hline 1 & 0.08077 & 0.034615 & 0.08077 \\ 2 & 0.034615 & 0.05769 & 0.034615 \\ 3 & 0.08077 & 0.034615 & 0.18077 \end{array}$$

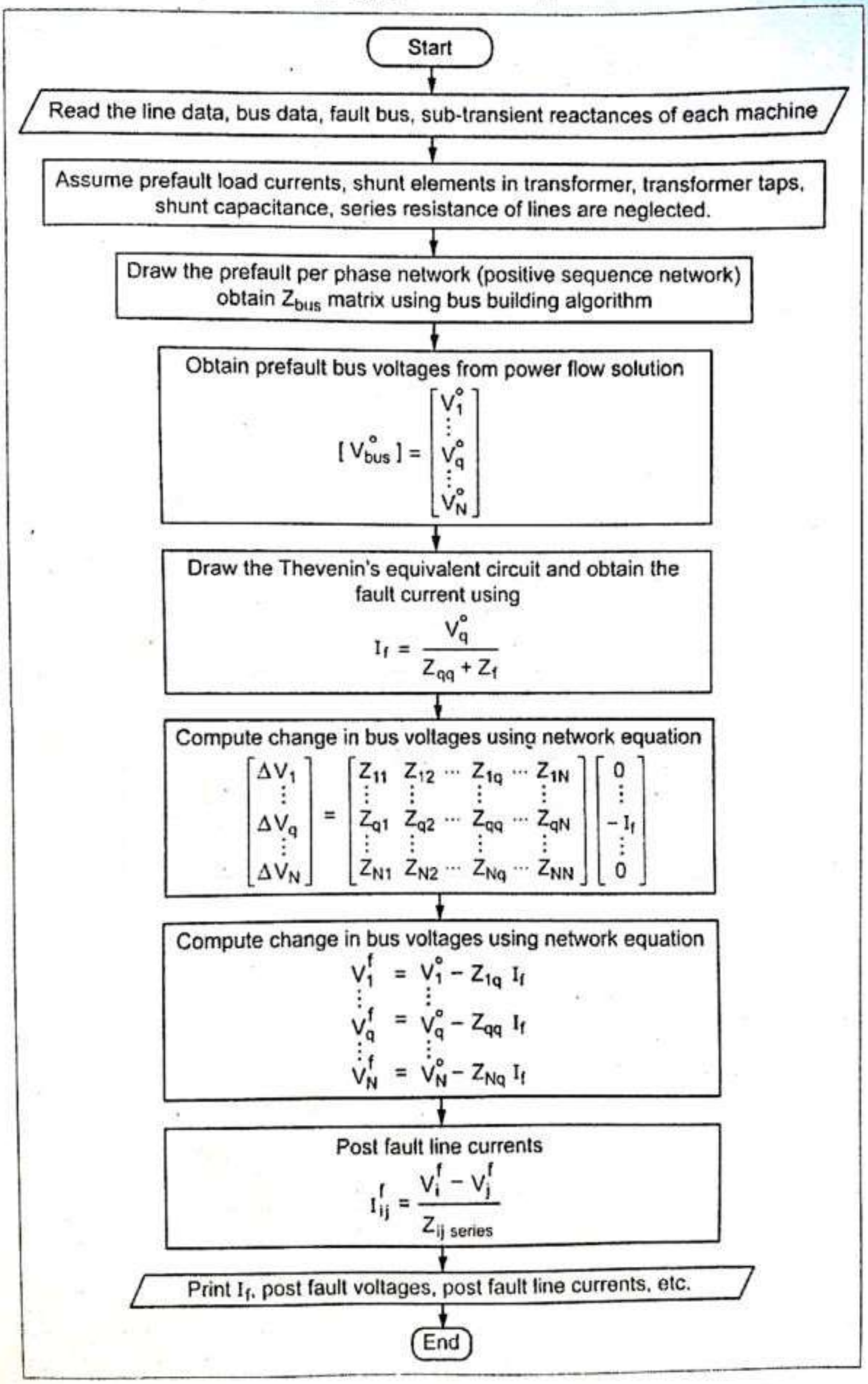
Finally add element $2-3$. It is a link between buses 2 and 3. With bus ℓ

$$Z_{bus} = j \begin{array}{c|ccc|c|} & 1 & 2 & 3 & \ell \\ \hline 1 & 0.08077 & 0.034615 & 0.08077 & -0.046155 \\ 2 & 0.034615 & 0.05769 & 0.034615 & 0.023075 \\ 3 & 0.08077 & 0.034615 & 0.18077 & -0.146155 \\ \hline \ell & -0.046155 & 0.023075 & -0.146155 & 0.26923 \end{array}$$

Eliminating the ℓ^{th} bus, final bus impedance matrix is obtained as

$$Z_{bus} = j \begin{array}{c|ccc|} & 1 & 2 & 3 \\ \hline 1 & 0.07286 & 0.03857 & 0.05571 \\ 2 & 0.03857 & 0.05571 & 0.04714 \\ 3 & 0.05571 & 0.04714 & 0.10143 \end{array}$$

Symmetrical Fault Analysis using Z_{bus} (Flow chart)



Start

Read the line data, bus data, fault bus, sub-transient reactances of each machine

Assume prefault load currents, shunt elements in transformer, transformer taps, shunt capacitance, series resistance of lines are neglected.

Draw the prefault per phase network (positive sequence network) obtain Z_{bus} matrix using bus building algorithm

Obtain prefault bus voltages from power flow solution

$$[V^{\circ}_{bus}] = \begin{bmatrix} V^{\circ}_1 \\ \vdots \\ V^{\circ}_q \\ \vdots \\ V^{\circ}_N \end{bmatrix}$$

Draw the Thevenin's equivalent circuit and obtain the fault current using

$$I_f = \frac{V^{\circ}_q}{Z_{qq} + Z_f}$$

Compute change in bus voltages using network equation

$$\begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_q \\ \vdots \\ \Delta V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1q} & \cdots & Z_{1N} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{q1} & Z_{q2} & \cdots & Z_{qq} & \cdots & Z_{qN} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{Nq} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ -I_f \\ \vdots \\ 0 \end{bmatrix}$$

Compute change in bus voltages using network equation

$$\begin{aligned} V^f_1 &= V^{\circ}_1 - Z_{1q}\, I_f \\ &\vdots \\ V^f_q &= V^{\circ}_q - Z_{qq}\, I_f \\ &\vdots \\ V^f_N &= V^{\circ}_N - Z_{Nq}\, I_f \end{aligned}$$

Post fault line currents

$$I^f_{ij} = \frac{V^f_i - V^f_j}{Z_{ij\ series}}$$

Print I_f, post fault voltages, post fault line currents, etc.

End

Current Limiting Reactor

The current limiting reactor is an inductive coil having a large inductive reactances in comparison to their resistance and is used for limiting short circuit currents during fault conditions. Current-voltage reactors also reduced the voltage disturbances on the rest of the system. It is installed in feeders and ties, in generators leads, and between bus sections, for reducing the magnitude of short circuit currents and the effect of the respective voltage disturbance.

Current reactor allows free interchange of power under normal condition, but when the fault occurs the disturbance is restricted by the current reactor to the faulty section. As the resistance of the system is very small as compared to their reactance. Hence, the efficiency of the system is not much affected.

Main Function of Current Limiting Reactor

The main purpose of the current limiting reactor is that its reactance should not decrease when a large short current flows through its windings. When the fault current exceeds about three times rated full-load current then large cross section iron cored reactor is used for limiting the fault current. Because of the large cross-section area, the iron cored reactor becomes very costly and heavy. Therefore, the air cored reactor is usually used to limit the short circuit or fault current.

The iron-cored reactor produces hysteresis and eddy current loss due to which more power is consumed as compared to air cored reactor. Normally, in an air cored reactor, the total losses are of the order of 5% of KVA rating of the reactor.

Functions of Current Limiting Reactor

- Current limiting reactor reduces the flow of short circuit current so as to protect the appliances from mechanical stress and overheating.
- Current reactor reduced the magnitude of voltage disturbances which is caused by short circuits.
- It limits the fault current to flow into the healthy feeders or parts of the system, thereby avoiding the fault from spreading. This increase the chances of continuity of supply.

Drawbacks of current limiting reactor

The main drawbacks of the current limiting reactors are as follows

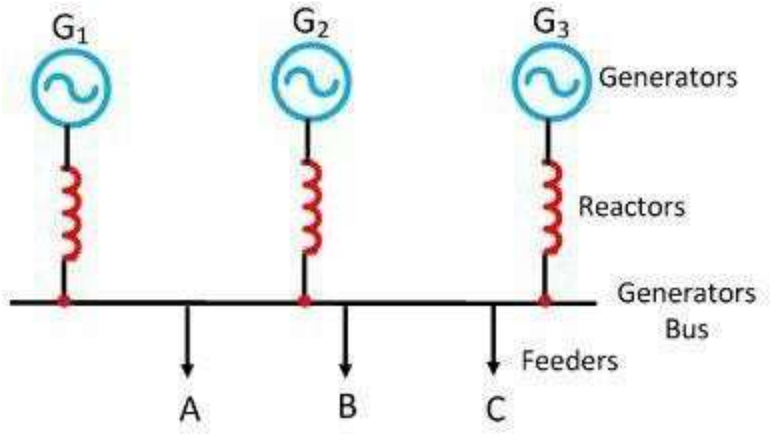
- When the reactor is installed on the network, the total percentage reactance of the circuit increases.
- It decreases the power factor and thus the regulation becomes poorer.

Location of Reactors

Reactors are located at different location in a power system for reducing the short circuit current. These reactors may be connected in series with the generators, feeders or in bus-bars as explained below.

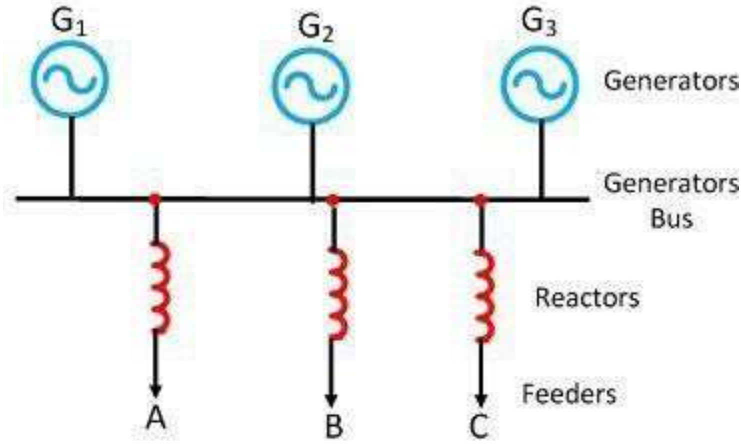
Generators Reactors

Generator reactors are inserted between the generator and the generator bus. Such reactors protect the machines individually. In power station generator, reactors are installed along with the generators. The magnitude of reactors is approximately about 0.05 per unit. The main disadvantages of such type of reactors are that if the fault occurs on one feeder, then the whole of the system will be adversely affected by it.



Feeders Reactors

Reactors, which is connected in series with the feeder is called feeders reactor. When the fault occurs on any one feeder, then the voltage drops occur only in its reactors and the bus bar is not affected much. Hence the machines continue to supply the load. The other advantage is that the fault occurs on a feeder will not affect the others feeders, and thus the effects of fault are localized.



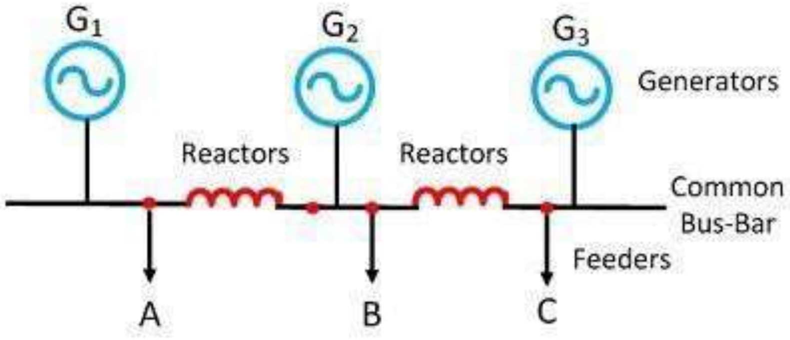
The disadvantage of such type of reactors is that it does not provide any protection to the generators against short circuit faults occurs across the bus bars. Also, there is a constant voltage drop and constant power loss in reactors during normal operating conditions.

Bus-Bar Reactor

When the reactors are inserted in the bus bar, then it is called bus-bar reactors. The constant voltage drop and constant power loss in reactors may be avoided by inserting the reactors in the bus bars. The bus bar reactor for ring system and the tie system are explained below.

Bus-Bar Reactors (Ring System)

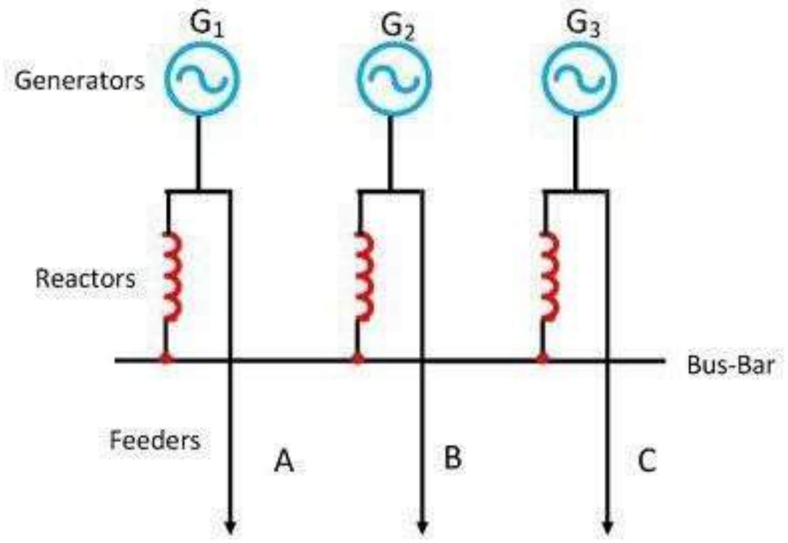
Bus-bar reactors are used to tie together the separate bus sections. In this system sections are made of generators and feeders and these sections are connected to each other to a common bus bar. In such type of system normally one feeder is fed from one generator. In normal operating conditions a small amount of power flows through the reactors. Therefore voltage drop and the power loss in the reactor is low. The bus bar reactor, therefore, made with high ohmic resistance so that there is not much voltage drop across it.



When the fault occurs on any one feeders, only one generator feeds the fault while the current of the other generator is limited because of the presence of the bus-bar reactors. The heavy current and voltage disturbances caused by a short circuit on a bus section are reduced and restricted to that faulty section only. The only drawback of such type of reactor is that it does not protect the generators connected to the faulty sections.

Bus-bar Reactors (Tie-Bus System)

This is the modification of the above system. In tie-bus system, the generator is connected to the common bus-bar through the reactors, and the feeder is fed from generator side.



The operation of the system is similar to the ring system, but it has got additional advantages. In this system, if the number of sections is increased, the fault current will not exceed a certain value, which is fixed by the size of the individual reactors.

Example 3.5

Fig. 3.31 shows four identical alternators in parallel. Each machine is rated for 25 MVA, 11 kV and has a subtransient reactance of 16 % on its rating. Compute the short circuit MVA when a three phase fault occurs at one of the outgoing feeders.

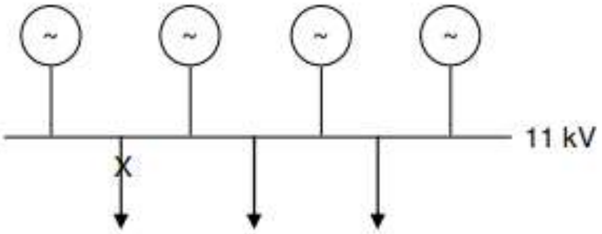


Fig. 3.31 Four alternators – Example 3.5

Solution

Fault is simulated by closing the switch shown in the p.u. reactance diagram shown in Fig. 3.32 (a). Its Thevenin's equivalent is shown in Fig. 3.32 (b).

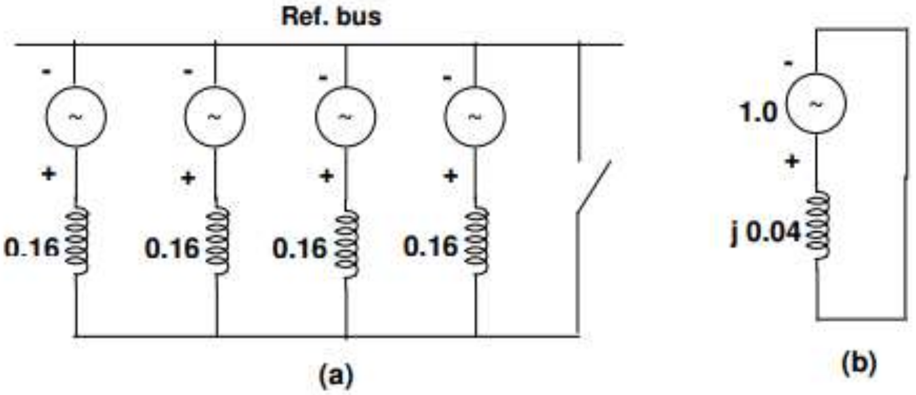


Fig. 3.32 Reactance diagram and its Thevenin's equivalent circuit

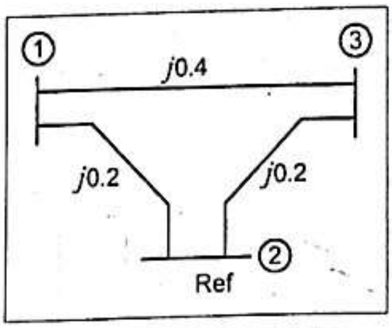
Fault current $|I_F| = \dfrac{1}{0.04} = 25$ **p.u.**

Short circuit MVA = prefault voltage in p.u. x fault current in p.u. x Base MVA

= 1.0 x 25 x 25

= 625

Example 4.2 ***For the system shown in Fig., form the bus impedance matrix using building algorithm.***



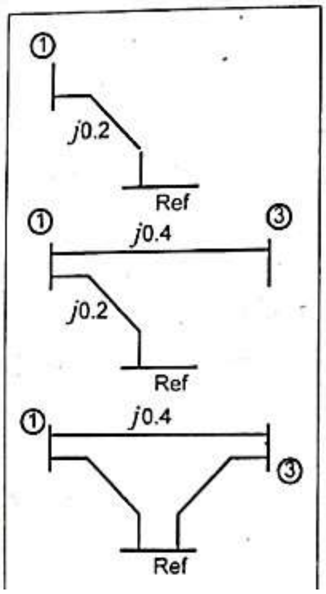
☺ ***Solution :***

Step 1 : Add an element between reference and node (1).

$$Z_{bus} = \begin{array}{c|c} & 1 \\ \hline 1 & [j0.2] \end{array}$$

Step 2 : Add an element between existing node (1) and the new node (3).

$$Z_{bus} = \begin{array}{c|cc} & 1 & 3 \\ \hline 1 & j0.2 & j0.2 \\ 3 & j0.2 & j0.6 \end{array}$$



Step 3 : Add an element between existing node (3) and the reference node.

$$Z_{bus} = \begin{array}{c|ccc} & 1 & 3 & 0 \\ \hline 1 & j0.2 & j0.2 & j0.2 \\ 3 & j0.2 & j0.6 & j0.6 \\ 0 & j0.2 & j0.6 & j0.8 \end{array}$$

Using Kron's reduction method,

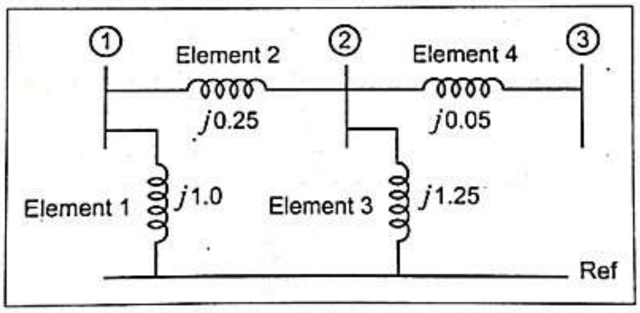
$$Z_{11} = Z_{11} - \frac{Z_{13}\,Z_{31}}{Z_{33}} = j0.2 - \frac{j0.2 \times j0.2}{j0.8} = j0.15$$

$$Z_{12} = Z_{21} = Z_{12} - \frac{Z_{13}\,Z_{32}}{Z_{33}} = j0.2 - \frac{j0.2 \times j0.6}{j0.8} = j0.05$$

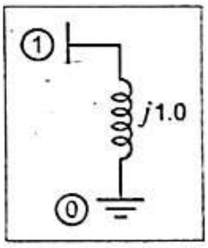
$$Z_{22} = Z_{22} - \frac{Z_{23}\,Z_{32}}{Z_{33}} = j0.6 - \frac{j0.6 \times j0.6}{j0.8} = j0.15$$

$$Z_{bus} = \begin{bmatrix} j0.15 & j0.05 \\ j0.05 & j0.15 \end{bmatrix}$$

Example 4.3 ***Determine Z_{bus} using bus building algorithm by adding the lines as per increasing element number. The reactance diagram of the system is shown in Fig.***

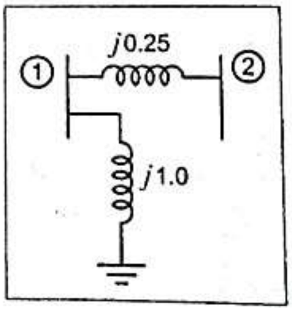


☺ ***Solution :*** ***Step 1 :*** Add an element between ref and node (1).



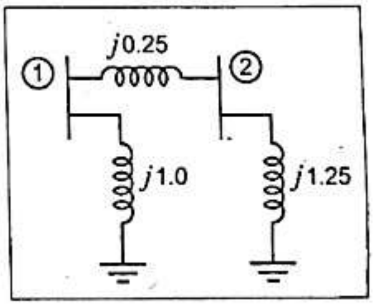
$$Z_{bus} = \begin{array}{cc} & 1 \\ 1 & [j1.0] \end{array}$$

Step 2 : Add an element between the existing node (1) and new node (2).



$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cc} 1 & \quad 2 \end{array} \\ \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.0+j0.25 \end{bmatrix} \end{array} = \begin{bmatrix} j1.0 & j1.0 \\ j1.0 & j1.25 \end{bmatrix}$$

Step 3 : Add an element between existing node (2) and ref.



$$Z_{bus} = \begin{array}{c|ccc} & (1) & (2) & (0) \\ \hline (1) & j1.0 & j1.0 & j1.0 \\ (2) & j1.0 & j1.25 & j1.25 \\ (0) & j1.0 & j1.25 & j1.25+j1.25 \end{array} \begin{bmatrix} j1.0 & j1.0 & j1.0 \\ j1.0 & j1.25 & j1.25 \\ j1.0 & j1.25 & j2.5 \end{bmatrix}$$

Fictitious node can be eliminated using

$$Z_{ij}^{new} = Z_{ij}^{old} - \frac{Z_{i\,(n+1)}\,Z_{(n+1)\,j}}{Z_{(n+1)\,(n+1)}}$$

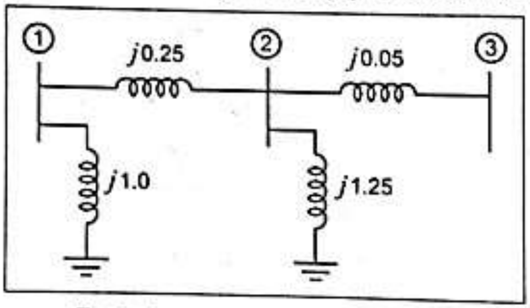
$$Z_{11}^{new} = Z_{11}^{old} - \frac{Z_{13}\,Z_{31}}{Z_{33}} = j1.0 - \frac{j1.0 \times j1.0}{j2.5} = j0.6$$

$$Z_{12}^{new} = Z_{21}^{new} = Z_{12}^{old} - \frac{Z_{13}\,Z_{32}}{Z_{33}} = j1.0 - \frac{j1.0 \times j1.25}{j2.5} = j0.5$$

$$Z_{22}^{new} = Z_{22}^{old} - \frac{Z_{23}\,Z_{32}}{Z_{33}} = j1.25 - \frac{j1.25 \times j1.25}{j2.5} = j0.625$$

$$Z_{bus} = \begin{array}{c} \\ (1) \\ (2) \end{array}\begin{array}{c} \begin{array}{cc} (1) & \quad (2) \end{array} \\ \begin{bmatrix} j0.6 & j0.5 \\ j0.5 & j0.625 \end{bmatrix} \end{array}$$

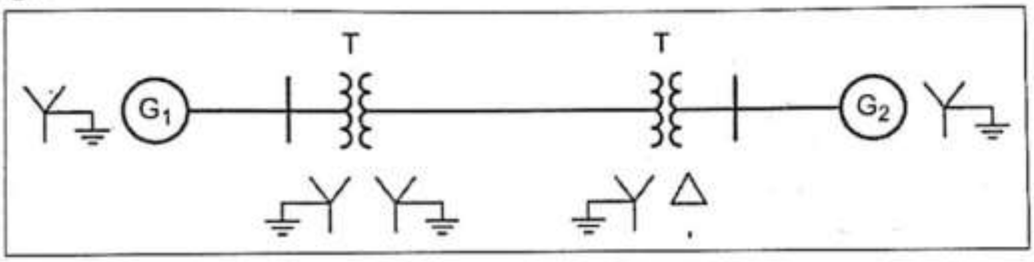
Step 4 : Add an element between existing node (2) and new node (3).



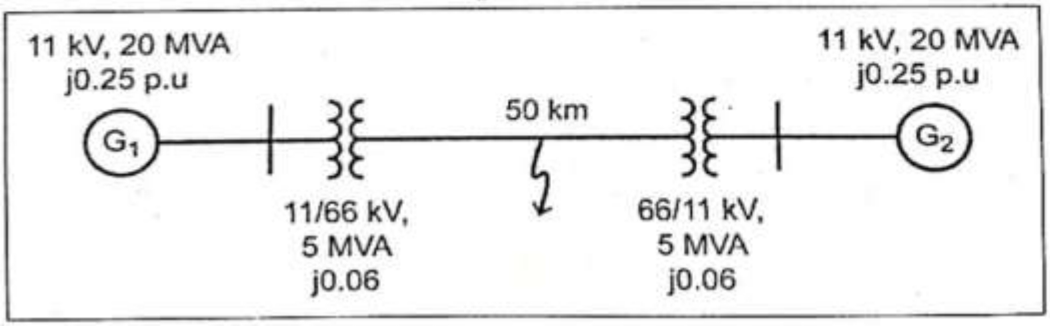
$$Z_{bus} = \begin{bmatrix} j0.6 & j0.5 & j0.5 \\ j0.5 & j0.625 & j0.625 \\ j0.5 & j0.625 & j0.625 + j0.05 \end{bmatrix}$$

$$Z_{bus}^{new} = \begin{bmatrix} j0.6 & j0.5 & j0.5 \\ j0.5 & j0.625 & j0.625 \\ j0.5 & j0.625 & j0.675 \end{bmatrix}$$

(a) Generator G_1 and G_2 are identical and rated 11 kV, 20 MVA and have a transient reactance of 0.25 p.u at own MVA base. The transformers T_1 and T_2 are also identical and are rated 11/66 kV, 5 MVA and have a reactance of 0.06 p.u to their own MVA base. A 50 km long transmission line is connected between the two generators. Calculate three phase fault current, when fault occurs at middle of the line as shown in Fig.3.



Solution



$$\text{Base MVA} = 20\text{ MVA}$$

$$\text{Generator 1, } Z_{\text{p.u.new}} = j0.25 \times \left[\frac{11}{11}\right]^2 \times \left[\frac{20}{20}\right] = j0.25 \text{ p.u}$$

$$\text{Transformer 1, } Z_{\text{p.u.new}} = j0.06 \times \left[\frac{11}{11}\right]^2 \times \left[\frac{20}{5}\right] = j0.24 \text{ p.u}$$

$$\text{Assume transmission line reactance} = j1 \text{ p.u}$$

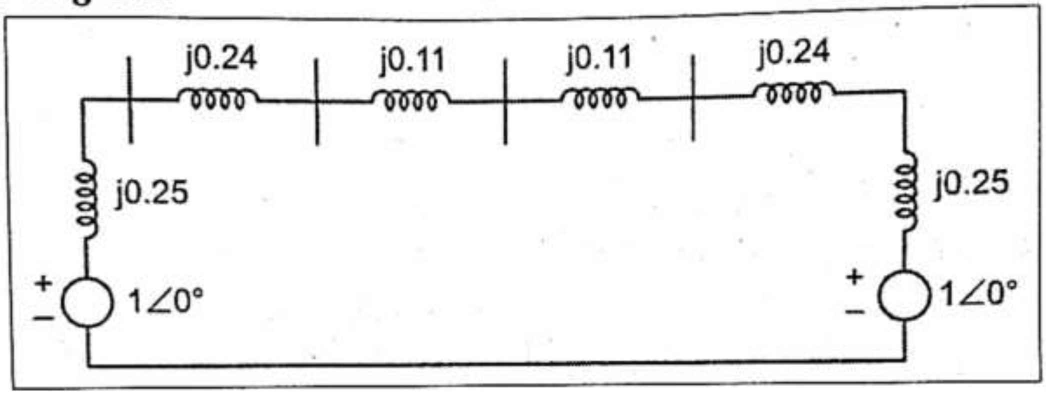
$$\text{Transmission line, } Z_{\text{p.u.new}} = (j1 \times 50) \times \frac{20}{66^2} = j0.22 \text{ p.u}$$

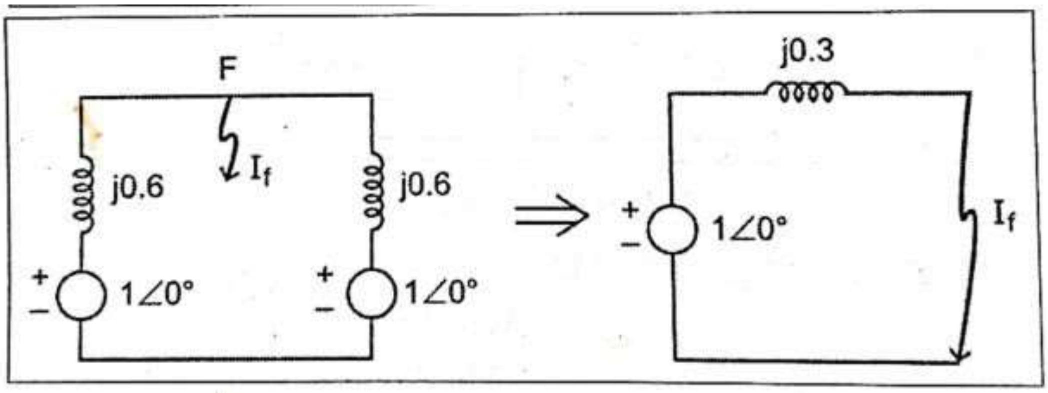
$$\text{Fault at middle, } Z_{\text{p.u.new (middle)}} = \frac{j0.22}{2} = j0.11 \text{ p.u}$$

$$\text{Transformer 2, } Z_{\text{p.u.new}} = j0.06 \times \left(\frac{66}{66}\right)^2 \times \left(\frac{20}{5}\right) = j0.24 \text{ p.u}$$

$$\text{Generator 2, } Z_{\text{p.u.new}} = j0.25 \times \left(\frac{11}{11}\right)^2 \times \frac{20}{20} = j0.25 \text{ p.u}$$

Reactance diagram:





Faultcurrent I_1 - $\frac{Erb}{XTh}$- $\frac{1 \angle 0°}{J0.3}$ -j3.333

1_1 - 3.333 L - 90°

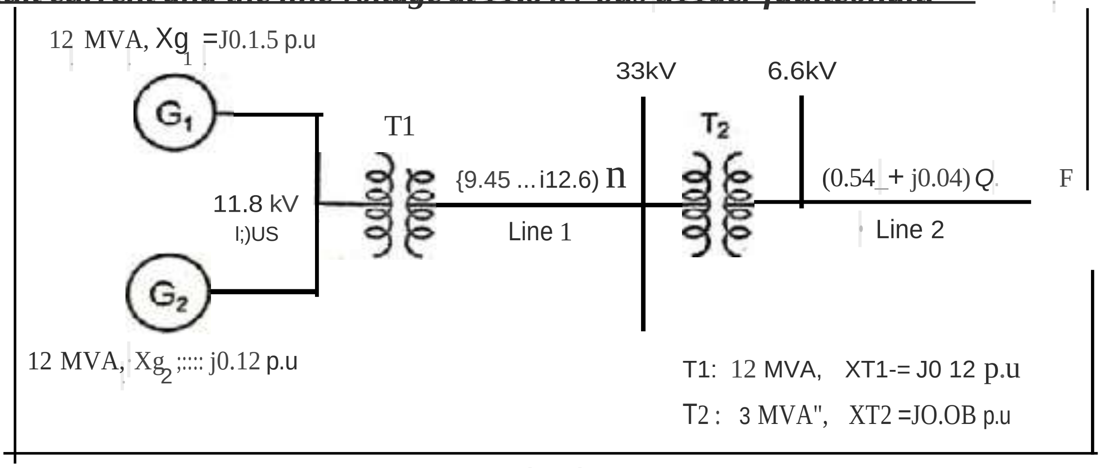
Actual fault current in amperes - I I/ Ix *lbase*

Basecurrent *Ibase* - -fJx $\frac{20 \times 10^3}{6\,6}$ = 174.95 A

ActuaI fault current - 3.333 x I74.95

- S83.12 A

(a) For the ra£!ial network sltow11 it,,.Fig.-3, 3(> jault o.ccurs_at point F. D termine the fault current and the line voltage at 11.8 kV bus u11der faultconditi -



Stei 1: Draw re tance dia

Base MVA = 12 MYA

Bas(f KV l-1.8 K..V

Generator 1 : KV $_b$ new = 11.8 KV

Zn ew = $_0$Diwn X $\left[\frac{KV_{,,\ given}}{KV_{b\ new}}\right]^2$ x $\left[\frac{MVAb\ n}{MV'A_{b\ pvcn}}\right]$

'015 (ll.8)2,,-(i2)--015

_.==J' • x 11.9 x 12 =*1* .. P

Generato,: 2 . KV,bn w == i1.8 KV .

Znew = J0.l2x $\left(\frac{11.8}{lfB}\right)^2$ >< $\left(\frac{12}{12}\right)$ = *10..12* p.u.

Transformer 1 ! rilary si<l:e KV$_h$ new = U.8KV

znew = jO.U x $\left(\frac{1-1.8}{11.8}\right)$ x $\left(\frac{12}{12}\right)$ = jO.12 p.u.

Transmission line 1 : $\text{KV}_{b\ \text{new}} = 11.8 \times \frac{33}{11.8} = 33 \text{ KV}$

$$Z = \frac{Z_{\text{actual}}}{Z_{\text{base}}} \doteq \frac{Z_{\text{actual}}}{\text{KV}_b^2} \times \text{MVA}_b$$

$$= \frac{9.45 + j12.6}{33^2} \times 11.8$$

$$= 0.1024 + j0.136 \text{ p.u.}$$

Transformer 2 : $\text{KV}_{b\ \text{new}} = 33 \text{ KV (primary)}$

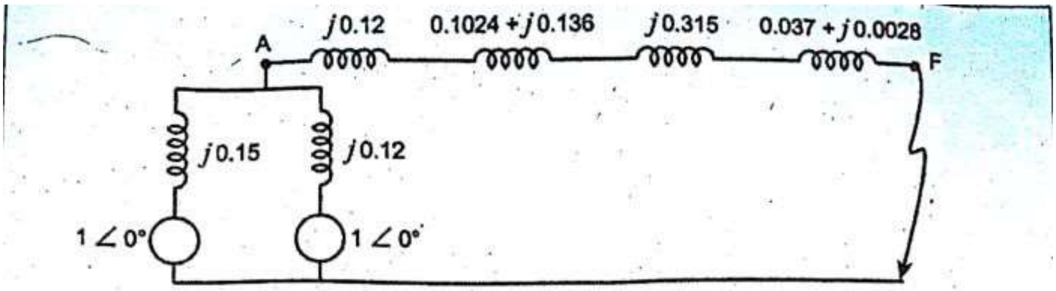
$$\text{MVA}_{b\ \text{given}} = 3 \text{ MVA}$$

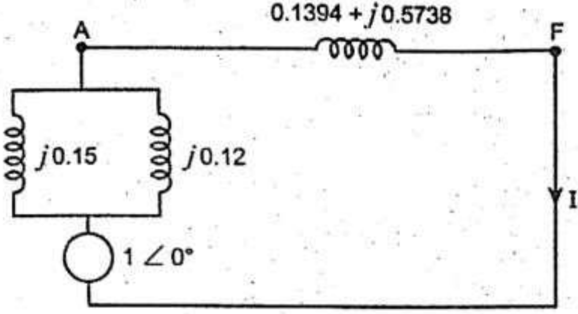
$$Z_{\text{new}} = j0.08 \times \left(\frac{33}{33}\right)^2 \times \left(\frac{11.8}{3}\right) = j0.315 \text{ p.u.}$$

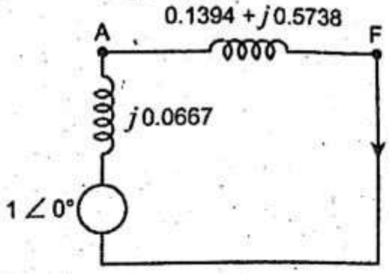
Transmission line 2 : $\text{KV}_{b\ \text{new}} = 33 \times \frac{6.6}{33} = 6.6 \text{ KV (sec of Transformer 2)}$

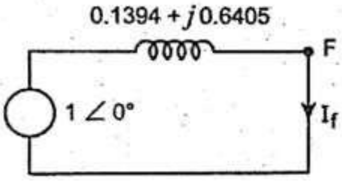
$$Z_{\text{new}} = \frac{Z_{\text{actual}}}{\text{KV}_b^2} \times \text{MVA}_b = \frac{(0.54 + j0.04)}{6.6^2} \times 3$$

$$= 0.037 + j0.0028 \text{ p.u.}$$









$$I_f = \frac{E_{Th}}{Z_{Th}} = \frac{1 \angle 0^\circ}{0.1394 + j0.6405} = 0.324 - j1.49$$

$$= 1.525 \angle -77.72^\circ \text{ p.u.}$$

$$\text{Base current } I_B = \frac{MVA_b}{\sqrt{3} \times KV_b} = \frac{12 \times 10^3}{\sqrt{3} \times 6.6} = 1049.7 \text{ Amp}$$

[KV_b for cable because fault point at F]

$$I_f = 1.525 \angle -77.72^\circ \text{ p.u.} \times I_B$$

$$= 1.525 \angle -77.72^\circ \times 1049.7 = 1600.8 \text{ Amp}$$

Voltage at 11.8 KV bus :

$$Z_{AF} = 0.1394 + j0.5738$$

$$= 0.59 \angle 76.34^\circ$$

$$\text{Voltage at 11.8 KV bus} = Z_{AF\ p.u.} \times I_{F\ p.u.}$$

$$= 0.59 \times 1.525 = 0.8998 \text{ p.u}$$

$$= 0.8998 \times 11.8 = 10.617 \text{ KV}$$

ELECTRICAL AND ELECTRONICS ENGINEERING

EE8501 POWER SYSTEM ANALYSIS

UNIT – IV UNSYMMETRICAL FAULT ANALYSIS

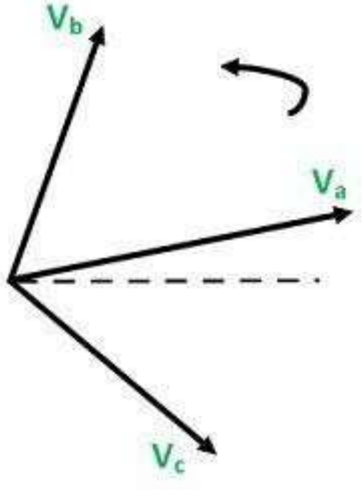
Symmetrical components - Sequence impedances - Sequence networks - Analysis of unsymmetrical faults at generator terminals: LG, LL and LLG - unsymmetrical fault occurring at any point in a power system - computation of post fault currents in symmetrical component and phasor domains

Symmetrical Components

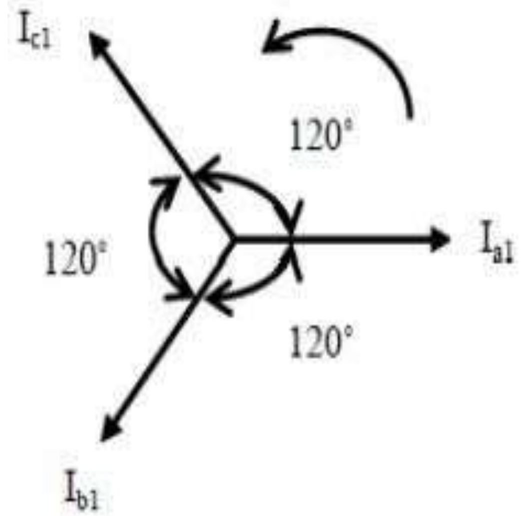
When the system is unbalanced the voltages, currents and the phase impedances are in general unequal. Such a system can be solved by a symmetrical per phase technique, known as the method of symmetrical components. This method is also called a three-component method. The method of symmetrical components simplified the problems of the unbalanced three-phase system. It is used for any number of phases but mainly used for the three-phase system.

The unbalanced three phase system is solved regarding symmetrical components, and then it can be transferred back to the actual circuit. The balanced set of components can be given as a positive sequence component, negative sequence component, and zero phase sequence component.

Consider an unbalanced voltage phasor system shown in the figure below. Suppose that the phasors are represented by V_a, V_b and V_c and their phase sequence is V_a, $V_{b,}$ and V_c. The phase sequence of the positive component is V_a, V_b and V_c and the phase sequence of negative components is V_a, V_c, and V_b.



In positive phase sequence component, the set of three phasors are equal in magnitude, spaced 120° apart from each other and having the same phase sequence as the original unbalanced phasors. The positive sequence component of the unbalanced three phase system is shown below.

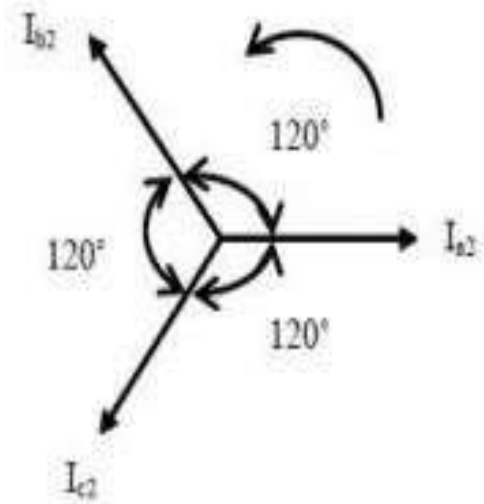


$$I_{a1} = I_{a1}\angle 0^{\circ}$$

$$I_{b1} = I_{a1}\angle 240^{\circ} = I_{a1}\angle -120^{\circ}$$

$$I_{c1} = I_{a1}\angle 120^{\circ}$$

In negative phase sequence component, the set of the three phasors are equal in magnitude, spaced 120° apart from each other and having the phase sequence opposite to that of the original phasors. The negative phase sequence is shown in the figure below

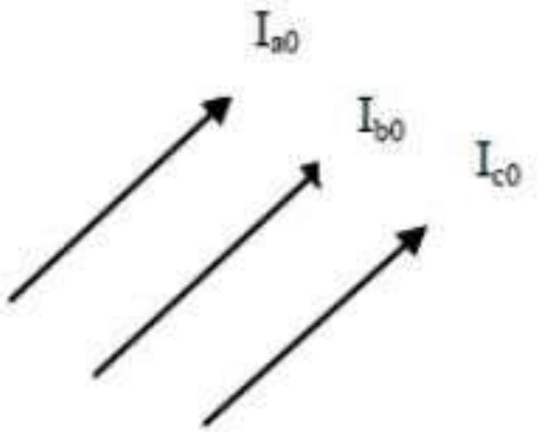


$$I_{a2} = I_{a2}\angle 0^{\circ}$$

$$I_{b2} = I_{a2}\angle 120^{\circ\circ}$$

$$I_{c1} = I_{a2}\angle 240^{\circ} = I_{a2}\angle -120^{\circ}$$

In zero phase sequence components, the set of three phasors is equal in magnitude to zero phase displacement from each other. The zero phase sequence component is shown in the figure below.



$$I_{a0} = I_{b0} = I_{c0}$$

The three phase balanced system is a special case of a general three-phase system in which zero and negative sequence components are zero.

Sequence operator

In unbalanced problem, to find the relationship between phase voltages and phase currents, we use sequence operator „a‟.

$$a = 1\angle 120^\circ == -0.5+j0.86$$

$$a^2 = 1\angle 240^\circ = -0.5 - j0.866$$

$$1 + a + a^2 = 0$$

Unbalanced currents from symmetrical currents

Let, Ia, Ib, Ic be the unbalanced phase currents

Let, Ia0, Ia1, Ia2 be the symmetrical components of phase a

$$\begin{bmatrix} I_a \\ I_b \\ I_b \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

Determination of symmetrical currents from unbalanced currents.

Let, Ia, Ib, Ic be the unbalanced phase currents

Let, Ia0, Ia1, Ia2 be the symmetrical components of phase a

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_b \end{bmatrix}$$

Sequence Impedances Sequence Networks

The sequence impedances are the impedances offered by the power system components or elements to +ve, -ve and zero sequence current.

The single phase equivalent circuit of power system consisting of impedances to current of any one sequence only is called sequence network.

The phase voltage across a certain load are given as

$$V_a = (176 - j132)\ Volts$$
$$V_b = (-128 - j96)\ Volts$$
$$V_c = (-160 + j100)\ Volts$$

Compute positive, negative and zero sequence component of voltage

Solution:

$$V_{a1} = \frac{1}{3}\left(V_a + \beta V_b + \beta^2 V_c\right)$$

$$V_{a2} = \frac{1}{3}\left(V_a + \beta^2 V_b + \beta V_c\right)$$

$$V_{a0} = \frac{1}{3}\left(V_a + V_b + V_c\right)$$

$$V_{a1} = \frac{1}{3}\left\{176 - j132 + 1\underline{\lfloor 120^\circ} \times \left(-128 - j96\right) + 1\underline{\lfloor 240^\circ}\left(-160 \ \ + j100\right)\right\}$$
$$V_{a1} = (163.24 - j35.10)\ \ \text{Volts}$$

$$V_{a2} = \frac{1}{3}\left\{176 - j132 + 1\underline{\lfloor 240^\circ}\left(-128 - j96\right) + 1\underline{\lfloor 120^\circ}\left(-160 + j100\right)\right\}$$
$$V_{a2} = (50.1 - j53.9)\ \ \text{Volts}$$

$$V_{a0} = \frac{1}{3}\left(176 - j132 - 128 - j96 - 160 + j100\right) \text{ Volts}$$

A balanced delta connected load is connected to a three phase system and supplied to it is a current of 15 amps. If the fuse is one of the lines melts, compute the symmetrical components of line currents.

Solution:

$$I_a = -I_c, \quad I_b = 0$$

$$I_a = 15\angle 0^\circ; \quad I_c = 15\angle 180^\circ = -15$$

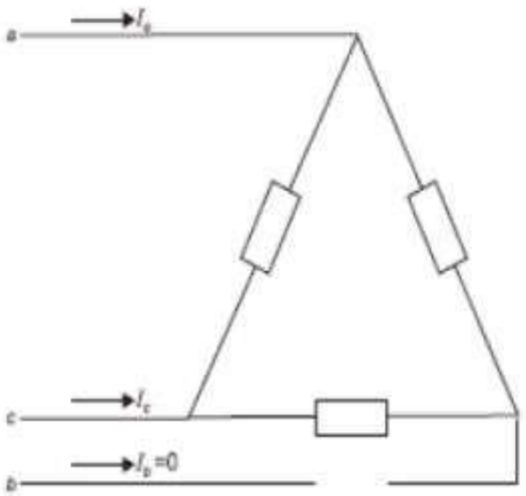
$$\therefore \quad I_{a1} = \frac{1}{3}\left(I_a + \beta I_c + \beta^2 I_b\right)$$

$$= (7.5 + j4.33) \quad \text{Amp.}$$

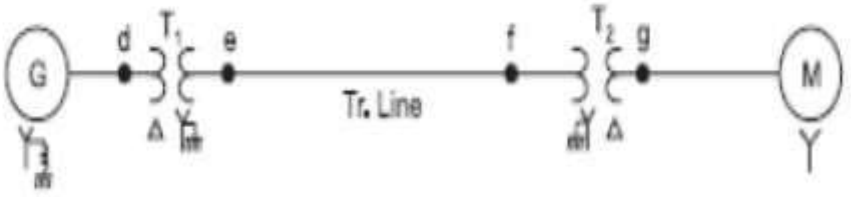
$$I_{a2} = \frac{1}{3}\left(I_a + \beta^2 I_c + \beta I_b\right)$$

$$= (7.5 + j4.33) \quad \text{Amp.}$$

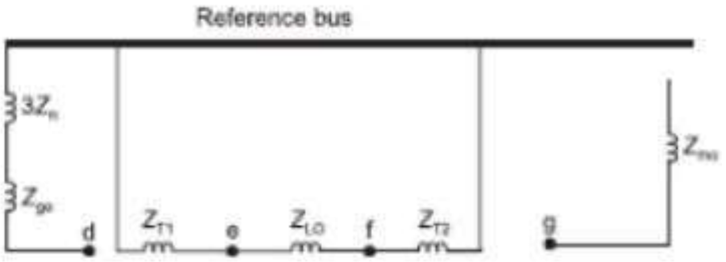
$$I_{a0} = \frac{1}{3}\left(I_a + I_b + I_c\right) = 0.0$$



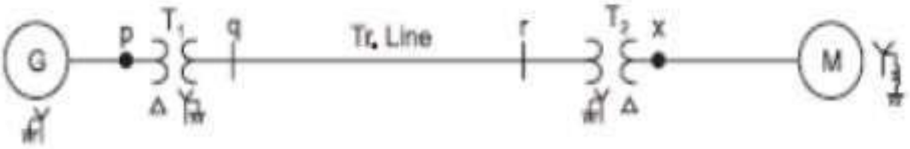
Draw zero sequence network of the power system as shown in fig.



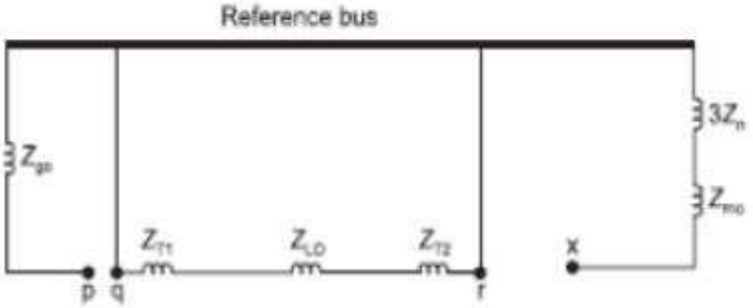
Solution:



Draw zero sequence network of the power system as shown in fig.

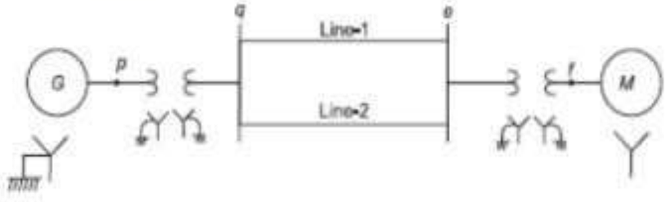


Solution:

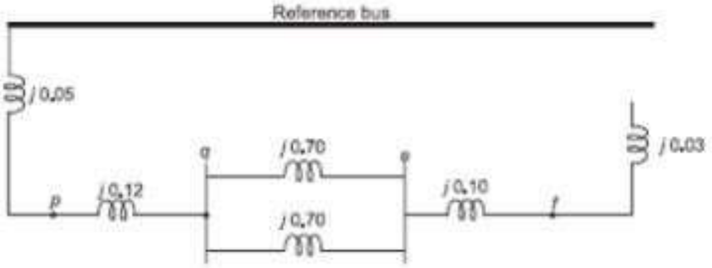


Draw zero sequence network of the power system as shown in fig. Data are given below.

G: $x_{g0} = 0.05$ pu
M: $x_{m0} = 0.03$ pu
T₁: $x_{T1} = 0.12$ pu
T₂: $x_{T2} = 0.10$ pu
Line-1: $x_{L10} = 0.70$ pu
Line-2: $x_{L20} = 0.70$ pu

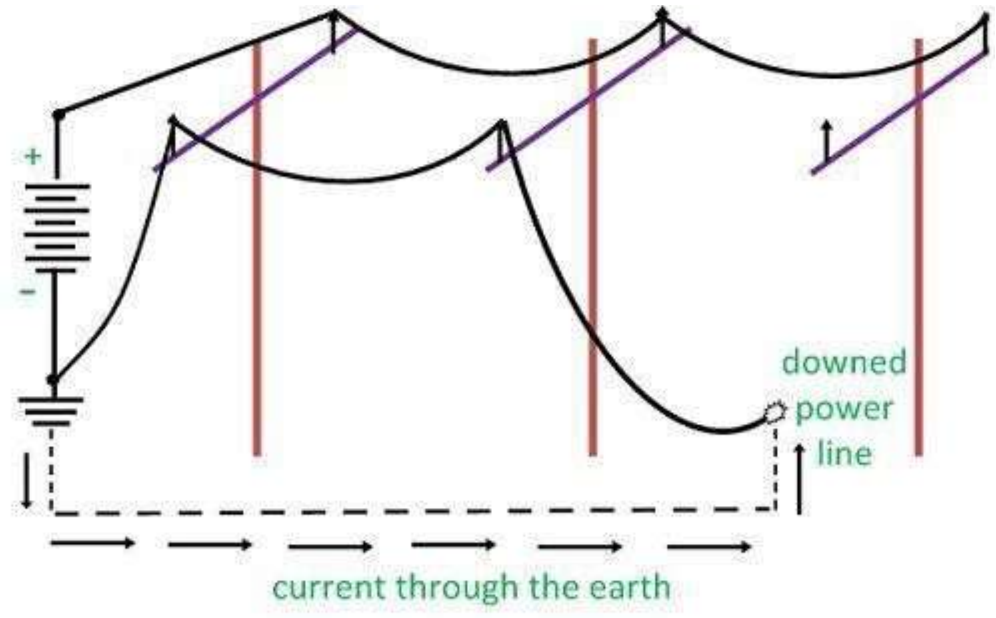


Solution:

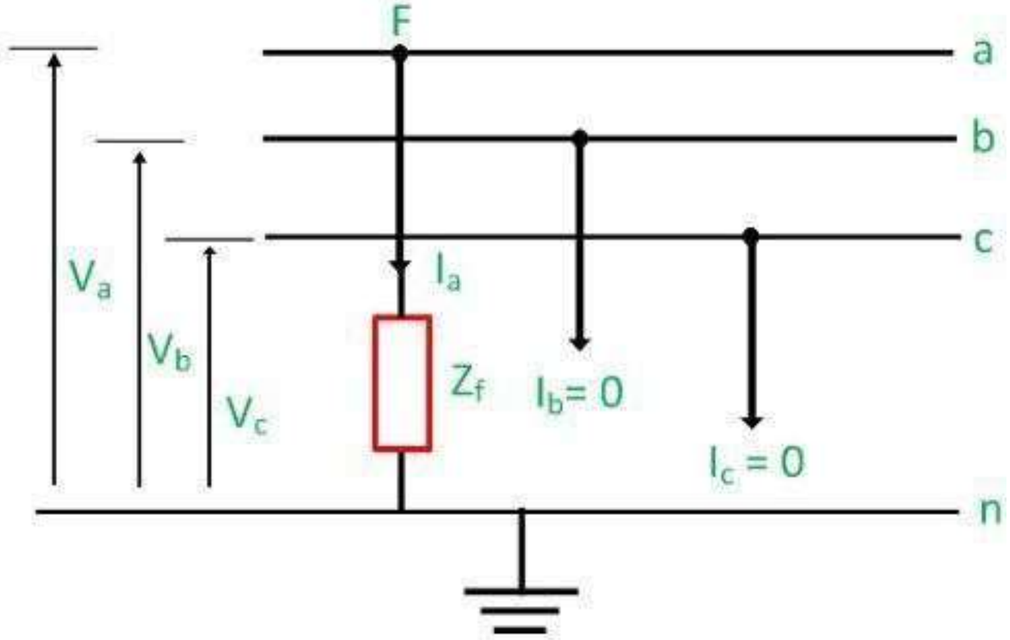


Single Line-to-Ground Fault

Generally, a single line-to-ground fault on a transmission line occurs when one conductor drops to the ground or comes in contact with the neutral conductor. Such types of failures may occur in power system due to many reasons like high-speed wind, falling off a tree, lightning, etc.



Circuit diagram of single line-to-ground fault



Suppose the phase a is connected to ground at the fault point F as shown in a figure below. Ia, Ib and Ic are the current and Va, Vb and Vc are the voltage across the three phase line a, b and c respectively. The fault impedance of the line is Zf.

Since only phase a is connected to ground at the fault, phase b and c are open circuited and carries no current; i.e fault current is Ia and Ib = 0, Ic = 0. The voltage at the fault point F is Va = Zf Ia.

The symmetrical component of the fault current in phase “a” at the fault point can be written as

$$I_{a0} = \frac{1}{3}(I_a + I_b + I_c) = \frac{1}{3}(I_a + 0 + 0) = \frac{1}{3}I_a$$

$$I_{a1} = \frac{1}{3}(I_a + \alpha I_b + \alpha^2 I_c) = \frac{1}{3}(I_a + 0 + 0) = \frac{1}{3}I_a$$

$$I_{a2} = \frac{1}{3}(I_a + \alpha^2 I_b + \alpha I_c) = \frac{1}{3}(I_a + 0 + 0) = \frac{1}{3}I_a$$

$$I_{a0} = I_{a1} = I_{a2} = \frac{1}{3}I_a$$

This relation can also be found by matrix method as follows:-

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{I_a}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$I_{a0} = I_{a1} = I_{a2} = \frac{1}{3} I_a$$

In the case of a single line-to-ground fault, the sequence currents are equal. The sequence voltage at the fault point is determined by the equations:-

$$V_{a0} = E_{a0} - Z_{a0} I_{a0}$$

$$V_{a1} = E_{a1} - Z_{a1} I_{a1}$$

$$V_{a2} = E_{a2} - Z_{a2} I_{a2}$$

Where, Ea0, Ea1, and Ea2 are the sequence voltages of phase a, and Za0, Za1 and Za2 are the sequence impedances to the flow of currents Ia0, Ia1, and Ia2 respectively. For a balanced system

$$E_{a0} = 0, \qquad E_{a2} = 0, \qquad E_{a1} = V_f$$

We know that

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$Z_f I_a = -Z_{ao} I_a + V_f - Z_{a1} I_{a1} - Z_{a2} I_{a2}$$

On substituting the Ia0 = Ia1 = Ia2 = Ia in above equation we get,

$$Z_f I_a = V_f - \frac{I_a}{3}(Z_{a0} + Z_{a1} + Z_{a2})$$

$$Z_f I_a + \frac{I_a}{3}(Z_{a0} + Z_{a1} + Z_{a2}) = V_f$$

$$I_a \left[Z_f + \frac{1}{3}(Z_{a0} + Z_{a1} + Z_{a2})\right] = V_f$$

$$I_a = \frac{V_f}{Z_f + \frac{1}{3}(Z_{a0} + Z_{a1} + Z_{a2})}$$

The sequence current is given by equation,

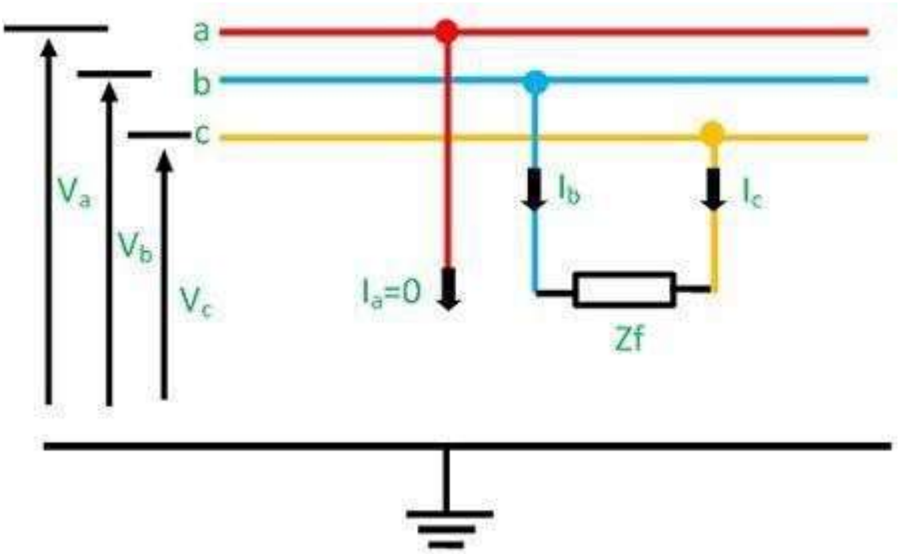
$$3I_{a0} = 3I_{a1} = 3I_{a2} = \frac{V_f}{Z_f + \frac{1}{3}(Z_{a0} + Z_{a1} + Z_{a2})}$$

$$I_{ao} = I_{a1} = I_{a2} = \frac{V_f}{3 \times [Z_f + \frac{1}{3}(Z_{a0} + Z_{a1} + Z_{a2})]}$$

$$I_{ao} = I_{a1} = I_{a2} = \frac{V_f}{[3Z_f + (Z_{a0} + Z_{a1} + Z_{a2})]}$$

Line-to-Line Fault

A line to line fault or unsymmetrical fault occurs when two conductors are short circuited. In the figure shown below shows a three phase system with a line-to-line fault phases b and c. The fault impedance is assumed to be Zf. The LL fault is placed between lines b and c so that the fault be symmetrical with respect to the reference phase a which is un-faulted.



The symmetrical components of a fault current in phase „a‟ at the fault point can be divided into three component. The zero sequence component of current at phase a is

$$I_{a0} = \frac{1}{3}\,(I_a + I_b + I_c)$$

$$I_{a0} = \frac{1}{3}(0 + I_b - I_b) \ldots\ldots\ldots\ldots\ldots equ(1)$$

In the equation(1) Ib = -Ic. Positive sequence component of phase a is expressed as

$$I_{a1} = \frac{1}{3}(I_a + \alpha I_b + \alpha^2 I_c)$$

$$I_{a1} = \frac{1}{3}(0 + \alpha I_b - \alpha^2 I_b)$$

$$I_{a1} = \frac{1}{3}(\alpha - \alpha^2)I_b \ldots\ldots\ldots\ldots\ldots equ(2)$$

and the negative sequence component of phase a is given by the equation,

$$I_{a2} = \frac{1}{3}(I_a + \alpha^2 I_b + \alpha I_c)$$

$$I_{a2} = \frac{1}{3}(0 + \alpha^2 I_b - \alpha I_b)$$

$$I_{a2} = \frac{1}{3}(\alpha - \alpha^2) \ldots\ldots\ldots\ldots\ldots .equ(3)$$

The sequence current can also be found by matrix method

$$\begin{bmatrix} I_{ao} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\begin{bmatrix} I_{ao} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ I_c \end{bmatrix}$$

Therefore, we get

$$I_{a0} = 0 \quad and \quad I_{a1} = -I_{a2}$$

Expressing Va, Vb and Vc regarding voltages at the fault point are found by the relations given by

$$(V_{ao} + \alpha^2 V_{a1} + \alpha V_{a2}) - (V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2}) = Z_f(I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2})$$
$$\ldots\ldots\ldots .equ(5)$$

Combination of equation (1), (4) and (5) gives

$$(\alpha^2 - \alpha)V_{a1} - (\alpha^2 - \alpha)V_{a2} = Z_f(\alpha^2 - \alpha)I_{a1}$$

$$V_{a1} - V_{a2} = Z_f I_{a1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots .equ(6)$$

The sequence current of voltage at the fault point are determined by the relations shown below

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ V_{a1} \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{a0} & 0 & 0 \\ 0 & Z_{a1} & 0 \\ 0 & 0 & Z_{a2} \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$V_{a0} = -Z_{a0} I_{a0} \ldots \ldots \ldots . equ(7)$$

$$V_{a1} = -V_f - Z_{a1} I_{a1} \ldots \ldots equ(8)$$

$$V_{a2} = -Z_{a2} I_{a2} \ldots \ldots \ldots equ(9)$$

From equation (8) and (9) we get

$$V_{a1} - V_{a2} = V_f - Z_{a1} I_{a1} + Z_{a2} I_{a2} \ldots \ldots \ldots \ldots \ldots . . equ(10)$$

Combination of equation (4), (10) and (9) gives

$$Z_f I_{a1} = V_f - Z_{a1} I_{a1} + Z_{a2} I_{a2}$$

$$\left(Z_{a1} + Z_{a2} + Z_f\right) I_{a1} = V_f$$

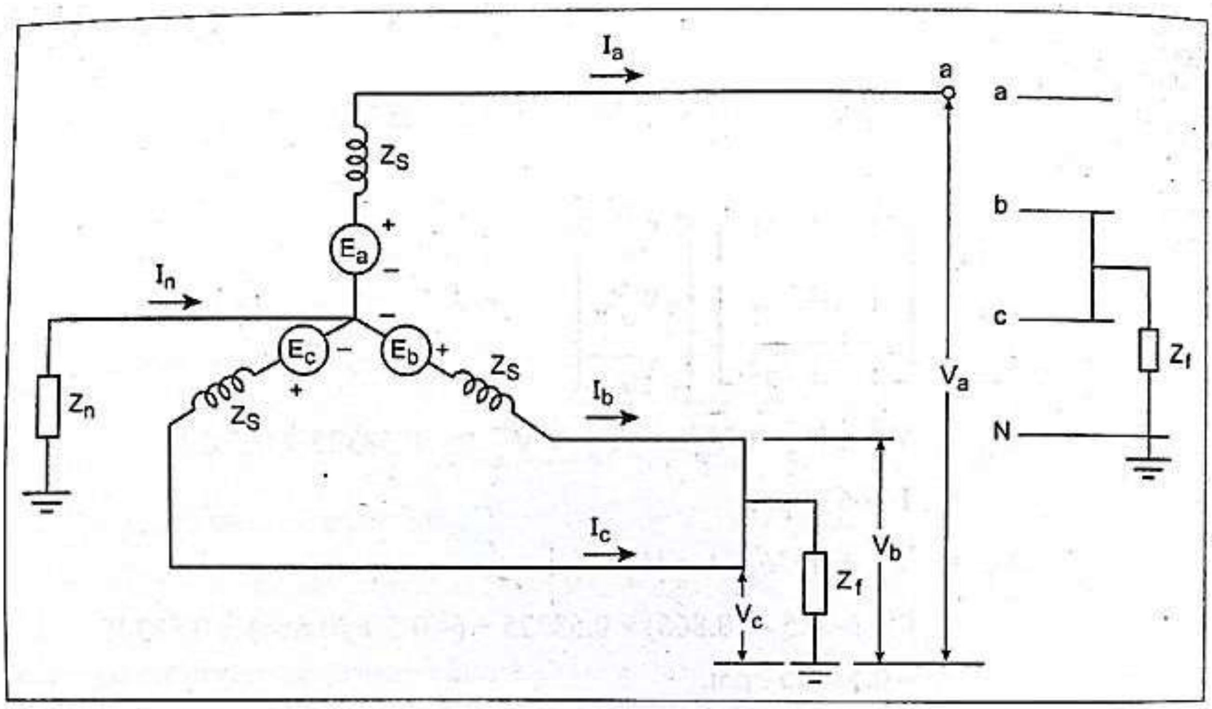
$$I_{a1} = \frac{V_f}{Z_{a1} + Z_{a2} + Z_f} \ldots \ldots . . equ(11)$$

The fault current is given by the equation

$$I_f = \frac{(\alpha^2 - \alpha) V_f}{Z_{a1} + Z_{a2} + Z_{a3}} \ldots \ldots \ldots \ldots \ldots equ(12)$$

From equation (1) it is clear that the line-to-line fault the zero sequence component of current Ia0 is equal to zero. Equation (4) shows that the positive-sequence component of current is opposite in phase to the negative-sequence component of current.

Double Line to Ground Fault (L-L-G Fault)



$$I_a = 0$$

$$I_b + I_c = I_f$$

$$V_b = V_c = Z_f I_f = Z_f(I_b + I_c)$$

The symmetrical components of voltages are

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Substitute $V_b = V_c$ in equation (10.34), we get

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix}$$

$$V_a^0 = \frac{1}{3}(V_a + V_b + V_b) = \frac{1}{3}(V_a + 2\,V_b)$$

$$V_a^+ = \frac{1}{3}(V_a + a\,V_b + a^2\,V_b)$$

$$= \frac{1}{3}[\,V_a + V_b\,(a + a^2)\,] \qquad [1 + a + a^2 = 0\,;\; a + a^2 = -1]$$

$$= \frac{1}{3}[\,V_a - V_b\,]$$

$$V_a^- = \frac{1}{3}[\,V_a + a^2\,V_b + a\,V_b\,]$$

$$= \frac{1}{3}[\,V_a + V_b\,(a^2 + a)\,] = \frac{1}{3}[\,V_a - V_b\,]$$

$$\boxed{V_a^+ = V_a^-} \qquad \text{... (10.35)}$$

The phase currents are given by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$$I_a = I_a^0 + I_a^+ + I_a^-$$

$$I_b = I_a^0 + a^2\,I_a^+ + a\,I_a^-$$

$$I_c = I_a^0 + a\,I_a^+ + a^2\,I_a^-$$

$$I_f = I_b + I_c = I_a^0 + a^2\,I_a^+ + a\,I_a^- + I_a^0 + a\,I_a^+ + a^2\,I_a^-$$

$$= 2\,I_a^0 + I_a^+\,(a^2 + a) + I_a^-\,(a + a^2)$$

$$= 2\,I_a^0 + I_a^+\,(-1) + I_a^-\,(-1)$$

$$= 2\,I_a^0 - I_a^+ - I_a^- = 2\,I_a^0 - (I_a^+ + I_a^-)$$

From the condition, $\quad I_a = I_a^0 + I_a^- + I_a^- = 0$

$$(I_a^+ + I_a^-) = -I_a^0$$

Substituting (10.37) in (10.36), we get

$$I_b + I_c = 2\,I_a^0 + I_a^0 = 3\,I_a^0$$

From the condition, $\quad V_b = Z_f(I_b + I_c)$

$$V_b = 3\,Z_f\,I_a^0$$

The phase voltages are given by

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix}$$

$$\begin{aligned} V_a &= V_a^0 + V_a^+ + V_a^- \\ V_b &= V_a^0 + a^2 V_a^+ + a\,V_a^- \\ &= V_a^0 + a^2 V_a^+ + a\,V_a^+ \\ &= V_a^0 + V_a^+\ (a^2 + a) \\ V_b &= V_a^0 - V_a^+ \qquad [\therefore V_b = 3\,Z_f I_a^0] \end{aligned}$$

Equate (10.38) and (10.39),

$$V_a^0 - V_a^+ = 3\,Z_f I_a^0$$

The symmetrical component voltages are given by

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{KK}^0 & 0 & 0 \\ 0 & Z_{KK}^+ & 0 \\ 0 & 0 & Z_{KK}^- \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

The symmetrical component voltages are given by

$$\begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_{KK}^0 & 0 & 0 \\ 0 & Z_{KK}^+ & 0 \\ 0 & 0 & Z_{KK}^- \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix}$$

$$\left.\begin{aligned} V_a^0 &= -Z_{KK}^0 \; I_a^0 \\ V_a^+ &= E_a - Z_{KK}^+ \; I_a^+ \\ V_a^- &= -Z_{KK}^- \; I_a^- \end{aligned}\right\}$$

Substitute V_a^0, V_a^+ in equation (10.40), we get

$$-Z_{KK}^0 \; I_a^0 - [E_a - Z_{KK}^+ \; I_a^+] = 3\,Z_f I_a^0$$

$$-[E_a - Z_{KK}^+ \; I_a^+] = [Z_{KK}^0 + 3\,Z_f]\, I_a^0$$

$$I_a^0 = \frac{-[E_a - Z_{KK}^+ \; I_a^+]}{Z_{KK}^0 + 3\,Z_f}$$

From equation (10.35), $V_a^+ = V_a^-$

$$E_a - Z_{KK}^+ \; I_a^+ = -Z_{KK}^- \; I_a^-$$

$$I_a^- = \frac{-(E_a - Z_{KK}^+ \; I_a^+)}{Z_{KK}^-}$$

From equation (10.37), $-I_a^0 = I_a^+ + I_a^-$

$$I_a^+ = -[I_a^- + I_a^0] = -I_a^- - I_a^0$$

$$= \left[\frac{E_a - Z_{KK}^+ \; I_a^+}{Z_{KK}^-}\right] + \left[\frac{E_a - Z_{KK}^+ \; I_a^+}{Z_{KK}^0 + 3\,Z_f}\right]$$

$$= I_a^+ \times \left[1 + \frac{Z_{KK}^+}{Z_{KK}^-} + \frac{Z_{KK}^+}{Z_{KK}^0 + 3\,Z_f}\right]$$

$$= \frac{E_a}{Z_{KK}^-} + \frac{E_a}{Z_{KK}^0 + 3\,Z_f}$$

$$I_a^+ [Z_{KK}^- (Z_{KK}^0 + 3\,Z_f) + Z_{KK}^+ (Z_{KK}^0 + 3\,Z_f) + Z_{KK}^+ \; Z_{KK}^-] = E_a [Z_{KK}^0 + 3\,Z_f + Z_{KK}^-]$$

$$I_a^+ [\,Z_{KK}^+ (Z_{KK}^0 + 3\,Z_f + Z_{KK}^-) + Z_{KK}^- \; (Z_{KK}^0 + 3\,Z_f) = E_a (Z_{KK}^0 + 3\,Z_f + Z_{KK}^-)$$

$$I_a^+ = \frac{E_a (Z_{KK}^0 + 3\,Z_f + Z_{KK}^-)}{(Z_{KK}^0 + 3\,Z_f + Z_{KK}^-)\left[Z_{KK}^+ + \dfrac{Z_{KK}^- \; (Z_{KK}^0 + 3\,Z_f)}{(Z_{KK}^0 + 3\,Z_f + Z_{KK}^-)}\right]}$$

$$\boxed{I_a^+ = \frac{E_a}{Z_{KK}^+ + \dfrac{Z_{KK}^- \; (Z_{KK}^0 + 3\,Z_f)}{Z_{KK}^0 + 3\,Z_f + Z_{KK}^-}}} \qquad \text{... (10.}$$

The fault current, $I_f = 3\,I_a^0$

Substituting from equation (10.42), we get

$$I_f = -3 \times \left[\frac{E_a - Z_{KK}^+ \; I_a^+}{Z_{KK}^0 + 3\,Z_f}\right]$$

Substituting I_a^+ from equation (10.44), we get

$$I_f = \frac{-3}{Z_{KK}^0 + 3\,Z_f}\left[E_a - \frac{Z_{KK}^+ \; E_a}{Z_{KK}^+ + \dfrac{Z_{KK}^- \; (Z_{KK}^0 + 3\,Z_f)}{Z_{KK}^0 + Z_{KK}^- + 3\,Z_f}}\right]$$

$$I_f = \frac{-3}{Z_{KK}^0 + 3\,Z_f}\left[\frac{E_a \times Z_{KK}^- \; (Z_{KK}^0 + 3\,Z_f)}{Z_{KK}^+ \times Z_{KK}^0 + 3\,Z_f\,Z_{KK}^+ + Z_{KK}^+\,Z_{KK}^- + Z_{KK}^- \; Z_{KK}^0 + 3\,Z_f\,Z_{KK}^-}\right] \qquad \text{(10.45}$$

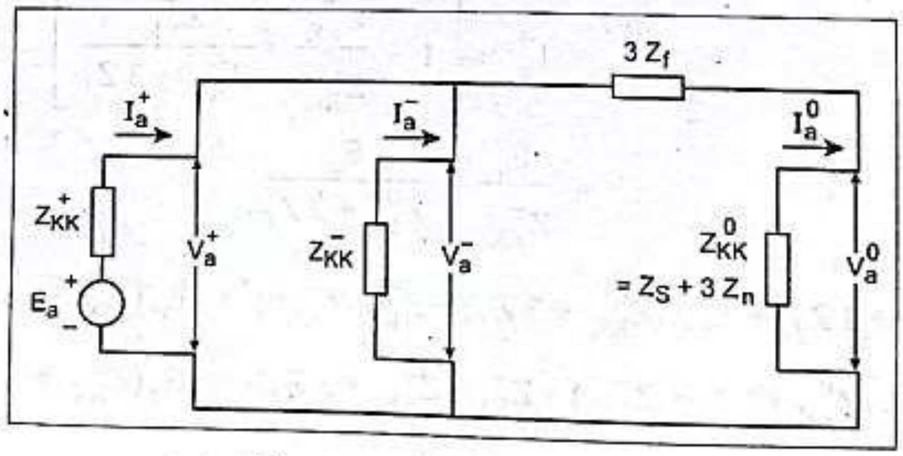
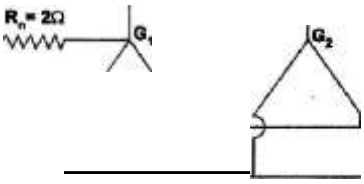


Fig. 10.10

Q39. Two 11 kV, 20 MVA, tflree phase star connected generators operate In paral,el as shown in figure. The positive, negative and zero sequence reactanc;e of each being respec;tively J 0.18,J 0.15, j 0.10 p.u. The s'8r point of oneof the generator Is isolated and that Qf the other Is earthed hrough a 2.0 ohm resistor. A single line to ground fault occurs aJ the terminals of one of the generators.
Estlrriat&: (I) fault current (11) current.In grounded resl,tor and(Iii) Voltage across grounding resistor.



Figure

Ans: **(Model Paper-2, Q18(b) | Aprll/May-17, (R13), Q14(b) | Aprll/May-11, (R08), Q14(b))**

Given that,

Two 3-4> star connected generators.

Voltage= 11 kV

Rating = 20 MVA

Positive sequence reactance. X_1 =JO.I8 p.u

Negative sequence reactapce,X_2 =$j0.15$ p.u

Zero seque ce reactance, X_0 =JO.I p.u

Neutral resistance, Rn= 2 n

Required to determine, for a single line to ground fault,

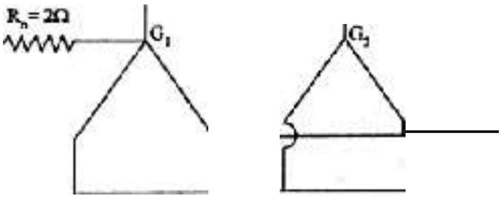
(i) faul(current, 1_1=? ,

(ii) Current in grounded resistor, In::: ?

(iii) Voltage across grounding resistor, Vn =?

The given circuit is shown in figure below.

If



Figure

Let,

Base MVA = 20 MVA

Base kV= 11 kV

E_0=(I +jO) p.n

Per unit value of neutral resistance is given by,

$$Rn(p.u) = Rn \times \frac{BaseMVA}{(Base\ kV)2} = 2 \times \frac{x...1Q}{(l\ I)2} = 0.3305\ p.u$$

Since the two identical generators operate in parallel, we have,

Eqmva ent pos1b ve sequence reactance, $X_{1eq} = \frac{Xi}{2} = \frac{jO.I8}{2} = j0.09$

Equsva ent negati ve sequence reactance, $X_{ieq} = \frac{X2}{2} = \frac{j0.15}{2} = j0.075$

The zero sequence reactance of generator G_2 is neglected, since its star point is isolated.

∴ Equivalent z.ero sequence impedance, Z0eq =jO.I + 3Rr:

=JO.I + 3(0.3305)

= 0.9915 +J0.1

For a single line to ground fault, we have,

$$I_{a1} = I_{a2} = I_{a0} \text{ and } I_f = 3I_{a1}$$

(i) Fault Current

The fault current is given by,

$$I_f = 3I_{a1} = 3 \times \frac{E_a}{X_{1eq} + X_{2eq} + Z_{0eq}}$$

$$= 3 \times \frac{(1 + j0)}{j0.09 + j0.075 + 0.9915 + j0.1}$$

$$= 3 \times \frac{1}{0.9915 + j0.265}$$

$$= 3\ (0.9413 - j0.2515)$$

$$= 2.8239 - j0.754$$

$$= 2.922\ \angle -14.94^\circ \text{ p.u}$$

Now,

$$\text{Base current, } I_b = \frac{\text{Base MVA}}{\sqrt{3} \times \text{Base kV}}$$

$$= \frac{20 \times 10^6}{\sqrt{3} \times 11 \times 10^3}$$

$$= 1049.72 \text{ A}$$

∴ Actual value of fault current, $|I_f|$ = p.u fault current × I_b

$$= 2.922 \times 1049.72$$

$$= 3067.28 \text{ A}$$

$$= 3.067 \text{ kA}$$

(ii) Current in Grounded Resistor

The current in grounded resistor is equal to the fault current.

∴ $I_n = I_f = 3.067$ kA

(iii) Voltage Across Grounding Resistor

The voltage across grounding resistor is given by,

$$V_n = \text{p.u fault current } (I_f) \times R_{n(\text{p.u})}$$

$$= 2.922 \angle{-14.94°} \times 0.3305$$

$$= 0.9657 \angle{-14.94°} \text{ p.u}$$

∴ Actual value of voltage across grounding resistor is,

$$V_n = 0.9657 \times \frac{11}{\sqrt{3}}$$

$$= 0.9657 \times 6.3508$$

$$= 6.1329 \text{ kV}$$

ELECTRICAL AND ELECTRONICS ENGINEERING

EE8501 POWER SYSTEM ANALYSIS

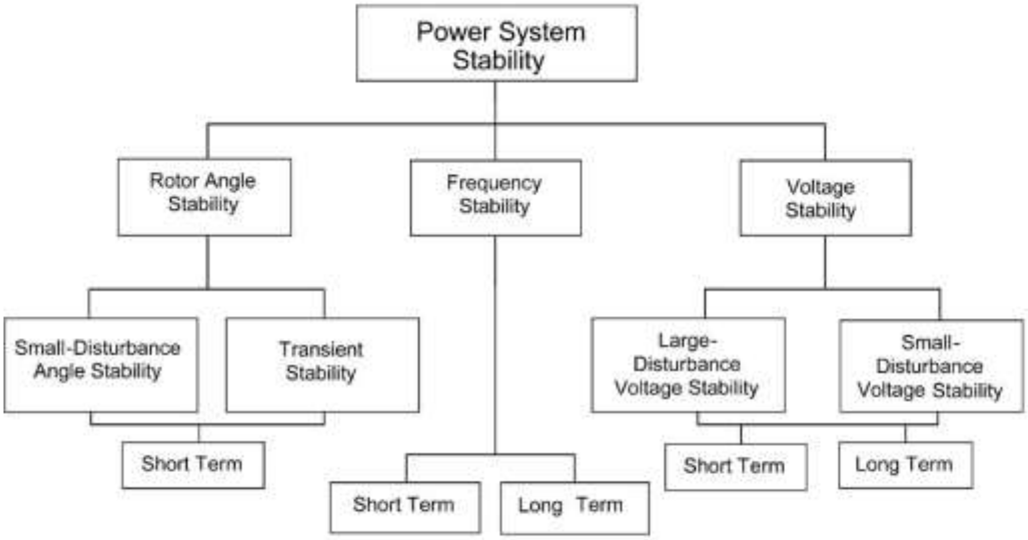
UNIT – V STABILITY ANALYSIS

Classification of power system stability – Rotor angle stability - Swing equation – Swing curve - Power-Angle equation - Equal area criterion - Critical clearing angle and time - Classical step-by-step solution of the swing equation – modified Euler method.

Basic Concepts and Definitions of Power System Stability

"Power system stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most of the system variables bounded so that practically the entire system remains intact". The disturbances mentioned in the definition could be faults, load changes, generator outages, line outages, voltage collapse or some combination of these. Power system stability can be broadly classified into rotor angle, voltage and frequency stability. Each of these three stabilities can be further classified into large disturbance or small disturbance, short term or long term. The classification is depicted in Fig.

Classification of power system stability



Rotor angle stability

"It is the ability of the system to remain in synchronism when subjected to a disturbance". The rotor angle of a generator depends on the balance between the electromagnetic torque due to the generator electrical power output and mechanical torque due to the input mechanical power through a prime mover. Remaining in synchronism means that all the generators electromagnetic torque is exactly equal to the mechanical torque in the opposite direction. If in a generator the balance between electromagnetic and mechanical torque is disturbed, due to disturbances in the system, then this will lead to oscillations in the rotor angle. Rotor angle stability is further classified into small disturbance angle stability and large disturbance angle stability.

Small-disturbance or small-signal angle stability

"It is the ability of the system to remain in synchronism when subjected to small disturbances". If a disturbance is small enough so that the nonlinear power system can be approximated by a linear system, then the study of rotor angle stability of that particular system is called as small-disturbance angle stability analysis. Small disturbances can be small load changes like switching on or off of small loads, line tripping, small generators tripping etc. Due to small disturbances there can be two types of instability: non-oscillatory instability and oscillatory instability. In non-oscillatory instability the rotor angle of a generator keeps on increasing due to a small disturbance and in case of oscillatory instability the rotor angle oscillates with increasing magnitude.

Large-disturbance or transient angle stability

"It is the ability of the system to remain in synchronism when subjected to large disturbances". Large disturbances can be faults, switching on or off of large loads, large generators tripping etc. When a power system is subjected to large disturbance, it will lead to large excursions of generator rotor angles. Since there are large rotor angle changes the power system cannot be approximated by a linear representation like in the case of small-disturbance stability. The time domain of interest in case of large-disturbance as well as small-disturbance angle stability is anywhere between 0.1- 10 s. Due to this reason small and large-disturbance angle stability are considered to be short term phenomenon. It has to be noted here that though in some literature "dynamic stability" is used in place of transient stability, according to IEEE task force committee report, only transient stability has to be used.

Voltage stability

"It is the ability of the system to maintain steady state voltages at all the system buses when subjected to a disturbance. If the disturbance is large then it is called as large-disturbance voltage stability and if the disturbance is small it is called as small-disturbance voltage stability". Unlike angle stability, voltage stability can also be a long term phenomenon. In case voltage fluctuations occur due to fast acting devices like induction motors, power electronic drive, HVDC etc then the time frame for understanding the stability is in the range of 10-20 s and hence can be treated as short term phenomenon. On the other hand if voltage variations are due to slow change in load, over loading of lines, generators hitting reactive power limits, tap changing transformers etc then time frame for voltage stability can stretch from 1 minute to several minutes.

The main difference between voltage stability and angle stability is that voltage stability depends on the balance of reactive power demand and generation in the system where as the angle stability mainly depends on the balance between real power generation and demand.

Frequency stability

"It refers to the ability of a power system to maintain steady frequency following a severe disturbance between generation and load". It depends on the ability to restore equilibrium between system generation and load, with minimum loss of load. Frequency instability may lead to sustained frequency swings leading to tripping of generating units or loads. During frequency excursions, the characteristic times of the processes and devices that are activated will range from fraction of seconds like under frequency control to several minutes, corresponding to the response of devices such as prime mover and hence frequency stability may be a short-term phenomenon or a long-term phenomenon.

Though, stability is classified into rotor angle, voltage and frequency stability they need not be independent isolated events. A voltage collapse at a bus can lead to large excursions in rotor angle and frequency. Similarly, large frequency deviations can lead to large changes in voltage magnitude.

Each component of the power system i.e. prime mover, generator rotor, generator stator, transformers, transmission lines, load, controlling devices and protection systems should be mathematically represented to assess the rotor angle, voltage and frequency stability through appropriate analysis tools. In fact entire power system can be represented by a set of Differential Algebraic Equations

(DAE) through which system stability can be analyzed. In the next few Chapters we will be concentrating on power system components modeling for stability analysis.

Rotor Angle Stability:

Rotor angle stability is the ability of the interconnected synchronous machines running in the power system to remain in the state of synchronism. Two synchronous generators running parallel and delivering active power to the load depends on the rotor angle of the generator (load sharing between alternators depends on the rotor angle).

During normal operation of the generator, rotor magnetic field and stator magnetic field rotates with the same speed, however there will be an angular separation between the rotor magnetic field and stator magnetic field which depends on the electrical torque (power) output of the generator.

An increase in the prime mover speed (turbine speed) will result in the advancement of the rotor angle to a new postion realtive to the rotating magnetic field of the stator. On the other hand reduction in the mechanical torque will result in the fall back of the rotor angle relative to the stator field.

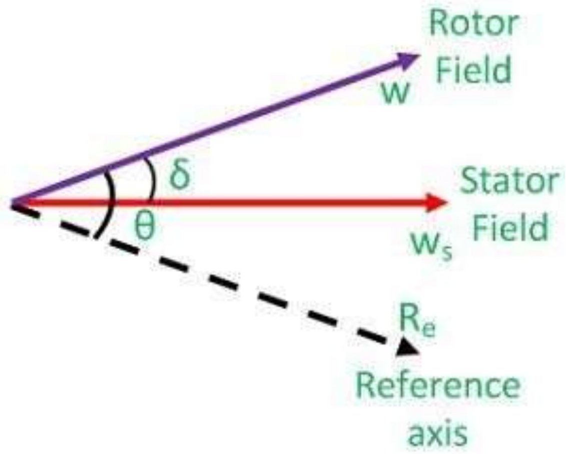
In equilibrium condition there will be equilibrium between the input mechanical torque and output electrical torque of each machine (generator) in the power system and speed of the machines will remain same. If the equilibrium is upset which results in the acceleration or deceleration of rotors of the machines.

If one of the inter connected generator moves faster temporarily with respect to the other machine. rotor angle of the machine will advance with respect to slow machine. This results in the load deliver by faster generator increases and load delivered by slow machine decreases. This tends to reduce the speed difference between the two generators and also the angular separation between the slow generator and fast generator.

Beyond certain point the increase in the angular separation will result in decrease of power transfer by the fast machine. This increases the angular separation further and also may lead to instability and synchronous generators fall out of synchroism.

Swing equation

The transient stability of the system can be determined by the help of the swing equation. Let θ be the angular position of the rotor at any instant t. θ is continuously changing with time, and it is convenient to measure it with respect to the reference axis shown in the figure below. The angular position of the rotor is given by the equation



$$\theta = w_s t + \delta \ldots\ldots\ldots\ldots\ldots equ(1)$$

Where,

θ – angle between rotor field and a reference axis

ws – synchronous speed

δ – angular displacement

Differentiation of equation (1) gives

$$\frac{d\theta}{dt} = w_s + \frac{d\delta}{dt} \ldots\ldots\ldots\ldots\ldots.. equ(2)$$

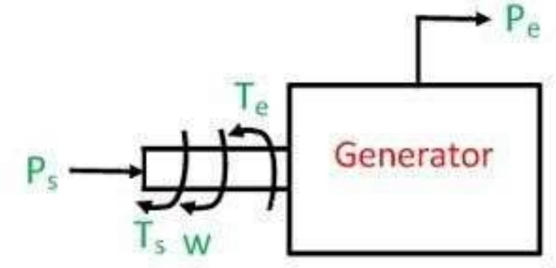
Differentiation of equation (2) gives

$$\frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2} \ldots\ldots\ldots\ldots\ldots\ldots equ(3)$$

Angular acceleration of rotor

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2}\ elect.rad/s^2$$

Power flow in the synchronous generator is shown in the diagram below. If the damping is neglected the accelerating torques, Ta in a synchronous generator is equal to the difference of mechanical input shaft and the electromagnetic output torque, i.e.,



$$T_a = T_s - T_e \ldots\ldots\ldots\ldots\ldots\ldots equ(5)$$

Where,

Ta – accelerating torque

Ts – shaft torque

Te – electromagnetic torque

Angular momentum of the rotor is expressed by the equation

$$M = Jw \ldots\ldots\ldots\ldots\ldots\ldots equ(6)$$

Where,

w- the synchronous speed of the rotor

J – moment of inertia of the rotor

M – angular momentum of the rotor

Multiplying both the sides of equation (5) by w we get

$$wT_a = wT_s - wT_e$$

$$P_a = P_s - P_e$$

Where,

Ps – mechanical power input

Pe – electrical power output

Pa – accelerating power

But,

$$J\frac{d^2\delta}{dt^2} = T_a$$

$$J\frac{d^2\theta}{dt^2} = T_a$$

$$wJ\frac{d^2\delta}{dt^2} = wT_a$$

$$M\frac{d^2\delta}{dt^2} = P_a = P_s - P_e \ldots\ldots\ldots\ldots\ldots\ldots\ldots equ(7)$$

Equation (7) gives the relation between the accelerating power and angular acceleration. It is called the swing equation. Swing equation describes the rotor dynamics of the synchronous machines and it helps in stabilizing the system.

Swing Curve

The above equation describes the behaviour of the rotor dynamics and hence is known as the swing equation. The angle δ is the angle of the internal EMF of the generator and it dictates the amount of power that can be transferred. This angle is therefore called the load angle.

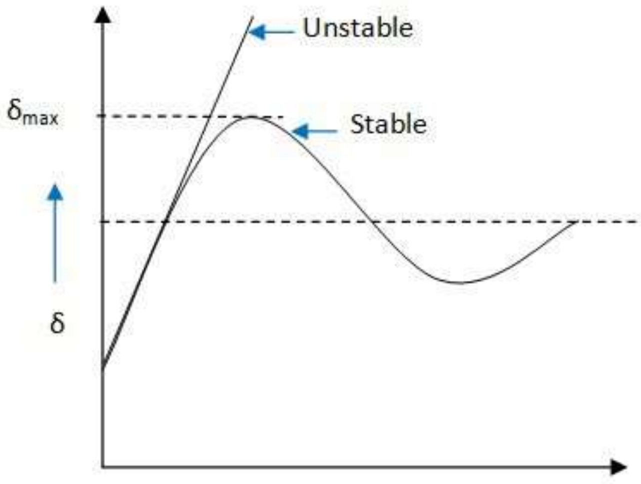
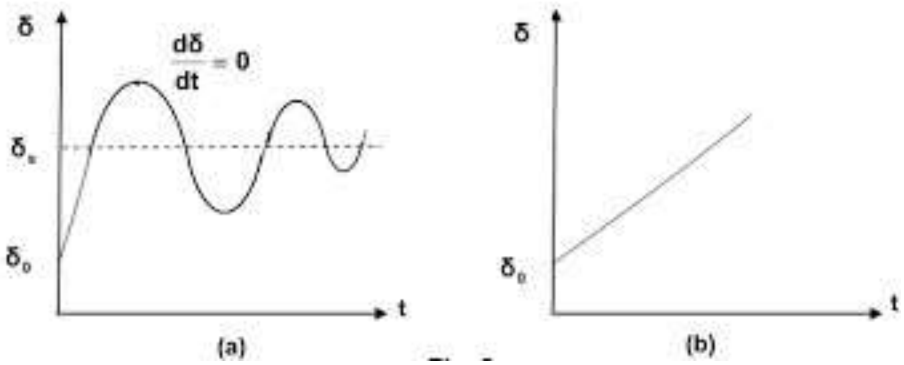


Figure 1

- Stable system; $\frac{d\delta}{dt} = 0$ at some time.
- Unstable system, $\frac{d\delta}{dt}$>0 for long time.

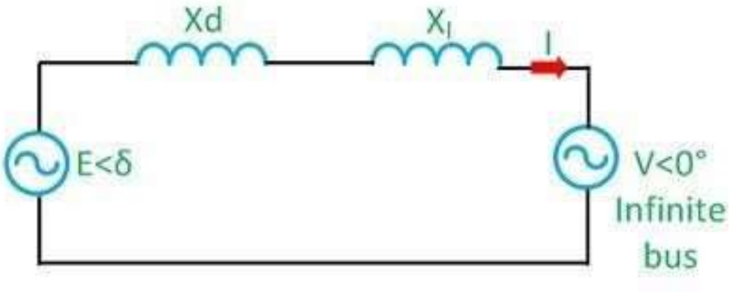


(a)

(b)

Power-Angle Equation

Consider a synchronous machine connected to an infinite bus through a transmission line of reactance X_l shown in a figure below. Let us assume that the resistance and capacitance are neglected.



Equivalent diagram of synchronous machine connected to an infinite bus through a transmission line of series reactance Xl is shown below:

Let,

V = V<0º – voltage of infinite bus

E = E<δ – voltage behind the direct axis synchronous reactance of the machine.

Xd = synchronous / transient resistance of the machine

The complex power delivered by the generator to the system is

S = VI

$$S = V\left[\frac{E < \delta - V < 0^0}{j(X_d + X_l)}\right]$$

Let,

$$X_d + X_l = X$$

$$S = V\left[\frac{E < \delta}{X < 90^0} + j\frac{V}{X}\right]$$

$$S = \frac{EV}{X} < (90^0 - \delta) - j\frac{V^2}{X}$$

$$S = V\left[\frac{EV}{X} sin\delta + j\frac{EV}{X} cos\delta - j\frac{V^2}{X}\right]$$

$$P_e + jQ_e = \frac{EV}{X} sin\delta + j\left(\frac{EV}{X} cos\delta - \frac{V^2}{X}\right)$$

Active power transferred to the system

$$P_e = \frac{EV}{X} sin\delta$$

The reactive power transferred to the system

$$Q_e = \frac{EV}{X} cos\delta - \frac{V^2}{X}$$

The maximum steady-state power transfers occur when δ = 0

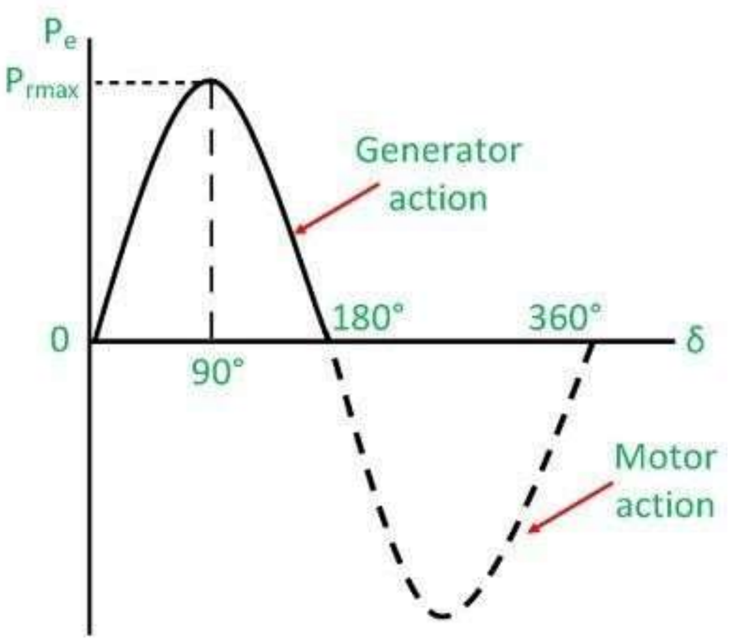
$$P_e = \frac{EV}{X} sin90^0$$

$$(\sin 90^0 = 1)$$

$$P = \frac{EV}{X}$$

$$P_e = P_{emax} sin\delta$$

The graphical representation of Pe and the load angle δ is called the power angle curve. It is widely used in power system stability studies. The power angle curve is shown below



Maximum power is transferred when δ = 90º. As the value of load angle δ is above 90, Pe decrease and becomes zero at δ = 180º. Above 180º, Pe becomes negative, which show that the direction of power flow is reversed, and the power is supplied from infinite bus to the generator. The value of Pe is often called pull out power. It is also called the steady-state limit.

The total reactance between two voltage sources E and X is called the transfer reactance. The maximum power limit is inversely proportion to the transfer reactance.

Equal area criterion

The equal area criterion is a simple graphical method for concluding the transient stability of two-machine systems or a single machine against an infinite bus. This principle does not require the swing equation for the determination of stability conditions. The stability conditions are recognized by equating the areas of segments on the power angle diagram between the p-curve and the new power transfer line of the given curve.

The principle of this method consists on the basis that when δ oscillates around the equilibrium point with constant amplitude, transient stability will be maintained.

Starting with swing equation

$$M\frac{d^2\delta}{dt^2} = P_s - P_E = P_A$$

where,

M = Angular Momentum

PE = Electrical Power

PS = Mechanical Power

δ= Load Angle

Multiplying both sides of the above equation by dδ/dt, we get

$$M\frac{d^2\delta}{dt^2}.\frac{d\delta}{dt} = P_s\frac{d\delta}{dt} - P_E\frac{d\delta}{dt} = \frac{d\delta}{dt}(P_s - P_E)$$

Or

$$\frac{1}{2}M\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = (P_S - P_E)\frac{d\delta}{dt}$$

Rearranging, multiplying by dt and integrating, we have

$$\left(\frac{d\delta}{dt}\right)^2 = \int_{\delta_o}^{\delta}\frac{2(P_S - P_e)}{M}d\delta$$

$$\frac{d\delta}{dt} = \sqrt{\int_{\delta_o}^{\delta}\frac{2(P_S - P_e)}{M}d\delta}$$

Where δo, is the torque angle at which the machine is operating while running at synchronous speed under normal conditions. Under the above conditions, the torque angle was not changing i.e. before the disturbance.

$$\frac{d\delta}{dt} = 0$$

Also, if the system has transient stability the machine will again operate at synchronous speed after the disturbances, i.e.,

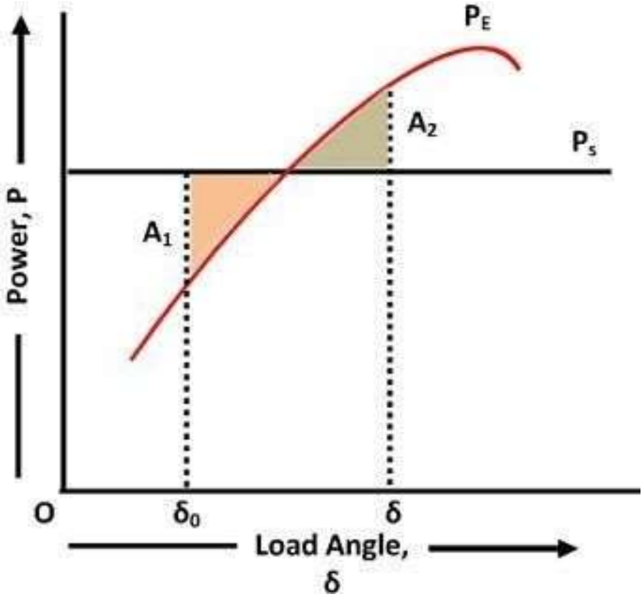
$$\frac{d\delta}{dt} = 0$$

Hence the condition for the transient state stability is given by the equation

$$\int_{\delta_o}^{\delta}\frac{2(P_S - P_e)}{M}d\delta = 0$$

$$\int_{\delta_o}^{\delta} \frac{2(P_S - P_e)}{M} d\delta = 0$$

$$\int_{\delta_o}^{\delta} P_A \, d\delta = 0$$



The area A1 represents the kinetic energy stored by the rotor during acceleration, and the A2 represents the kinetic energy given up by the rotor to the system, and when it is all given up, the machine has returned to its original speed.

The area under the curve PA should be zero, which is possible only when PA has both accelerating and decelerating powers, i.e., for a part of the curve PS> PE and for the other PE> PS. For a generation action, PS> PE for the positive area and A1> PS for negative areas A2 for stable operation. Hence the name equal area criterion.

The equal area criterion is also used for determining the maximum limit on the load that the system can take without exceeding the stability limit. This can happen only when the area between the PS line and the PE curve is equal to the area between the PS line, and the PE curve is equal to the area between the initial torque angle δ0 and the line PS. In this case, the area A2 is less than the area A1; the system will become unstable.

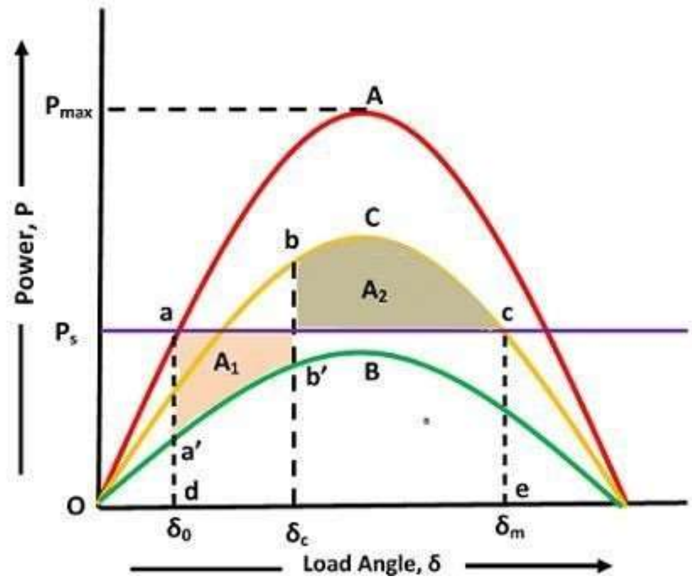
The problems associated with the transient stability of the system is because of the following reasons. These are as follows

- Sudden change in load
- Switching off one of the lines which cause a change in the reactance of the system and hence a change in load conditions.
- Sudden fault on the system which causes the reduction in output, requiring an arrangement for clearance for the clearance of the fault rapidly, and study of after fault condition which may cause part of the system outage.

In each case, the procedure will be to determine the power angle curve for the initial conditions of the system, for the conditions under fault, and for the after fault condition and plot the curve in per unit value. Then locate the points for the load initial conditions finding out δo. Then, using equal area criterion, determine the new angle of displacement δ. The maximum angle δmax which may be allowed and the corresponding maximum permissible load can also be determined.

Critical Clearing Angle and Time

The critical clearing angle is defined as the maximum change in the load angle curve before clearing the fault without loss of synchronism. In other words, when the fault occurs in the system the load angle curve begin to increase, and the system becomes unstable. The angle at which the fault becomes clear and the system becomes stable is called critical clearing angle.



When the initial load is given, then there is a critical clearing angle, and if the actual clearing angle exceeds a critical clearing angle, the system becomes unstable otherwise it is stable. Let the curve A represents the power angle curve for a healthy condition; curve B represents the power angle curve for faulty condition and curve C represents the power angle curve after isolation of fault as shown below.

Where γ1 is the ratio of system reactance in healthy condition to that of during the fault and γ2 is the ratio of steady state limit of the system after the isolation of fault and that of system under the initial condition.

For transient stability limit, two areas $A_1 = A_2$ or in other words the area under curve **ADEC** (rectangle) is equal to the area under the curve **DA'B'BCE.**

$$P_S(\delta_m - \delta_n) = \int_{\delta_0}^{\delta} \gamma_1 P_{max} sin\delta_{max} d\delta + \int_{\delta_0}^{\delta} \gamma_2 P_{max} sin\delta_{max} d\delta$$

$$= P_{max}(cos\delta_0 - cos\delta_c) + \gamma_2 P_{max}(cos\delta_c - cos\delta_m)$$

Now substituting,

$$P_S = P_{max} sin\delta_0$$

we have,

$$(\delta_m - \delta_0) P_{max} sin\delta_0 = \gamma_1 P_{max}(cos\delta_0 - cos\delta_c) + \gamma_2 P_{max}(cos\delta_c - cos\delta_m)$$

$$(\delta_m - \delta_0) sin\delta_0 = cos\delta_c(\gamma_1 - \gamma_1) + \gamma_1 cos\delta_0 - \gamma_2 cos\delta_m$$

or

$$cos\delta_c = \frac{(\delta_m - \delta_0) sin\delta_0 - \gamma_1 cos\delta_0 + \gamma_2 cos\delta_m}{(\gamma_1 - \gamma_1)}$$

Also from the curves

$$P_S = P_{max} sin\delta_0 = \gamma_2 P_{max} sin\delta_m = \gamma_2 P_{max} sin(\pi - \delta_m)$$

$$sin\delta_0 = \gamma_2 sin(\pi - \delta_m)$$

Or

$$\delta_m = \pi - sin^{-1}\left(\frac{sin\delta_0}{\gamma_2}\right)$$

Thus if γ1, γ2, and δ0 are known, the critical clearing angle δc can be determined.

Critical Clearing Time

CCT is defined as the maximum time that is allowed to remove the disturbance without interrupting the system's performance. The system will be stable if the disturbance can be cleared before the time allowed.

Classical Step-By-Step Solution of the Swing Equation

The swing equation is

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} = \frac{1}{M}(P_i - P_m \sin\delta).$$

Its solution gives a plot of δ versus t. The swing equation indicates that δ starts decreasing after reaching maximum value, the system can be assumed to be stable. The swing equation is a non-linear equation and a formal solution is not feasible. The step by step solution is very simple and common method of solving this equation. In this method the change in δ during a small time interval Δt is Pa calculated by assuming that the accelerating power calculated at the beginning of the interval is constant from the middle of the preceding interval to the middle of the interval being considered.

Let us consider the nth time interval which begins at t = (n-1) Δt. The angular position of the rotor at this instant is δn-1. The accelerating power Pa(n-1) and hence, acceleration αn-1 as calculated at this instant is assumed to be constant from t = (n-3/2) Δt to (n-1/2) Δt.

During this interval the change in rotor speed can be written as

$$\Delta\omega_{n-\frac{1}{2}} = (\Delta t)\alpha_{n-1} = \frac{\Delta t}{M}P_{a(n-1)}$$

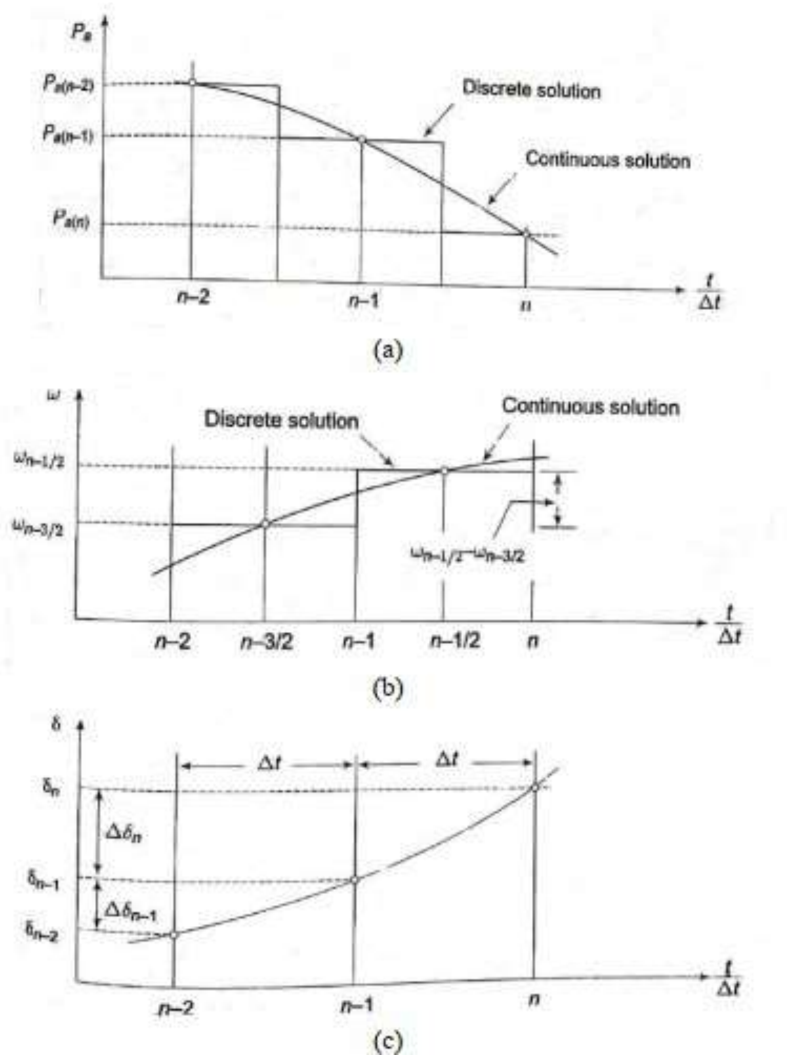
Thus, the speed at the end of nth interval is

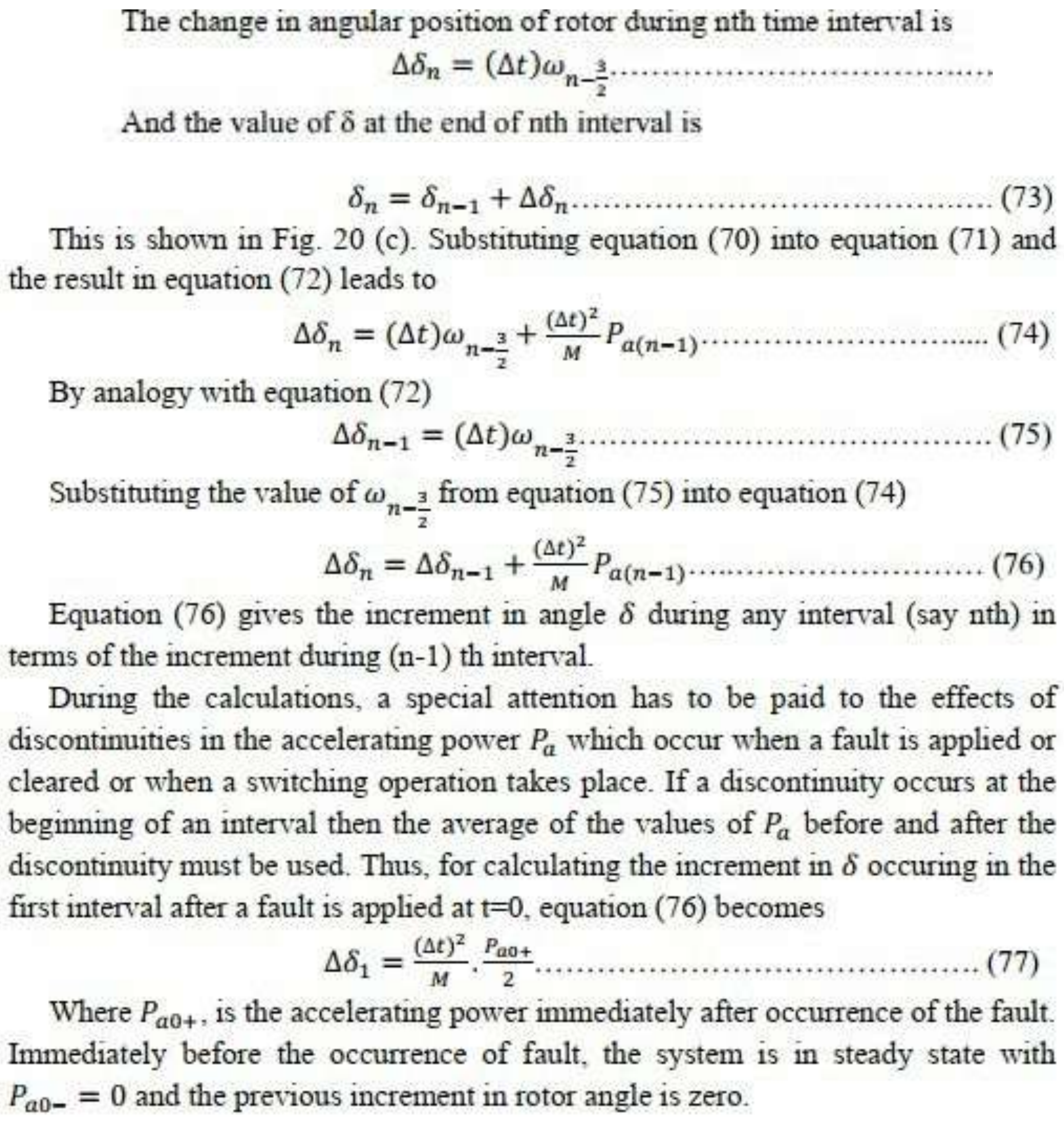
$$\omega_{n-\frac{1}{2}} = \omega_{n-\frac{3}{2}} + \Delta\omega_{n-\frac{1}{2}}\ldots..$$

Assume the change in speed occur at the middle of one interval, i.e., t=(n-1)Δt which is same the same instant for which the acceleration was calculated. Then the speed is assumed to remain constant till the middle of the next interval as shown in Fig. In other words, the speed assumed to be constant at the value

$$\omega_{n-\frac{1}{2}}$$

throughout the nth interval from t = (n-1) Δt to t = n Δt.





The change in angular position of rotor during nth time interval is

$$\Delta\delta_n = (\Delta t)\omega_{n-\frac{3}{2}} \dots\dots\dots\dots\dots\dots$$

And the value of δ at the end of nth interval is

$$\delta_n = \delta_{n-1} + \Delta\delta_n \dots\dots\dots\dots\dots\dots (73)$$

This is shown in Fig. 20 (c). Substituting equation (70) into equation (71) and the result in equation (72) leads to

$$\Delta\delta_n = (\Delta t)\omega_{n-\frac{3}{2}} + \frac{(\Delta t)^2}{M} P_{a(n-1)} \dots\dots\dots\dots\dots\dots (74)$$

By analogy with equation (72)

$$\Delta\delta_{n-1} = (\Delta t)\omega_{n-\frac{3}{2}} \dots\dots\dots\dots\dots\dots (75)$$

Substituting the value of $\omega_{n-\frac{3}{2}}$ from equation (75) into equation (74)

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{(\Delta t)^2}{M} P_{a(n-1)} \dots\dots\dots\dots\dots\dots (76)$$

Equation (76) gives the increment in angle δ during any interval (say nth) in terms of the increment during (n-1) th interval.

During the calculations, a special attention has to be paid to the effects of discontinuities in the accelerating power P_a which occur when a fault is applied or cleared or when a switching operation takes place. If a discontinuity occurs at the beginning of an interval then the average of the values of P_a before and after the discontinuity must be used. Thus, for calculating the increment in δ occuring in the first interval after a fault is applied at t=0, equation (76) becomes

$$\Delta\delta_1 = \frac{(\Delta t)^2}{M} . \frac{P_{a0+}}{2} \dots\dots\dots\dots\dots\dots (77)$$

Where P_{a0+}, is the accelerating power immediately after occurrence of the fault. Immediately before the occurrence of fault, the system is in steady state with $P_{a0-} = 0$ and the previous increment in rotor angle is zero.

Factors Affecting Transient Stability:-

Various methods which improve power system transient stability are

1. Improved steady-state stability
    - a) Higher system voltage levels
    - b) Additional transmission line
    - c) Smaller transmission line series reactance
    - d) Smaller transfer leakage reactance
    - e) Series capacitive transmission line compensation

    f) Static var compensators and flexible ac transmission systems (FACTs)

2. High speed fault clearing
3. High speed reclosuer of circuit breaker
4. Single pole switching
5. Large machine inertia, lower transient reactance
6. Fast responding, high gain exciter
7. Fast valving
8. Breaking resistor