EC8701 ANTENNAS AND MICROWAVE ENGINEERING

OBJECTIVES:

• To enable the student to understand the basic principles in antenna and microwave system design

• To enhance the student knowledge in the area of various antenna designs.

• To enhance the student knowledge in the area of microwave components and antenna for practical applications.

UNIT I INTRODUCTION TO MICROWAVE SYSTEMS AND ANTENNAS

Microwave frequency bands, Physical concept of radiation, Near- and far-field regions, Fields and Power Radiated by an Antenna, Antenna Pattern Characteristics, Antenna Gain and Efficiency, Aperture Efficiency and Effective Area, Antenna Noise Temperature and G/T, Impedance matching, Friis transmission equation, Link budget and link margin, Noise Characterization of a microwave receiver.

UNIT II RADIATION MECHANISMS AND DESIGN ASPECTS

Radiation Mechanisms of Linear Wire and Loop antennas, Aperture antennas, Reflector antennas, Micro strip antennas and Frequency independent antennas, Design considerations and applications.

UNIT III ANTENNA ARRAYS AND APPLICATIONS

Two-element array, Array factor, Pattern multiplication, Uniformly spaced arrays with uniform and non-uniform excitation amplitudes, Smart antennas.

UNIT IV PASSIVE AND ACTIVE MICROWAVE DEVICES

Microwave Passive components: Directional Coupler, Power Divider, Magic Tee, attenuator, resonator, Principles of Microwave Semiconductor Devices: Gunn Diodes, IMPATT diodes Schottky Barrier diodes, PIN diodes, Microwave tubes: Klystron, TWT, Magnetron.

UNIT V MICROWAVE DESIGN PRINCIPLES

Impedance transformation, Impedance Matching, Microwave Filter Design, RF and Microwave Amplifier Design, Microwave Power amplifier Design, Low Noise Amplifier Design, Microwave Mixer Design, Microwave Oscillator Design

OUTCOMES:

The student should be able to:

• Apply the basic principles and evaluate antenna parameters and link power budgets

• Design and assess the performance of various antennas

• Design a microwave system given the application specifications

Text Books:

1. John D Krauss, Ronald J Marhefka and Ahmad S. Khan, "Antennas and Wave Propagation: Fourth Edition, Tata McGraw-Hill, 2006. (Unit I, II, III)
2. David M. Pozar, "Microwave Engineering", Fourth Edition, Wiley India, 2012.(Unit I,IV,V)

References:

1. Constantine A.Balanis, Antenna Theory Analysis and Design, Third edition, John Wiley India Pvt Ltd., 2005.
2. R.E.Collin, "Foundations for Microwave Engineering", Second edition, IEEE Press, 2001

UNIT - I

Antennas & Microwave Engineering

Antennas :- Introduction to Microwave systems & Antennas:

The structure associated with the region of transition between a guided wave & a free-space wave or vice-versa

Transmission Line :-

This is a device for transmitting or guiding radio frequency energy from one point to another. usually it is desirable to transmit the energy with minimum attenuation, heat & radiation losses being as small as possible.

This means that while the energy is being conveyed from one point to another it is confined to the transmission line or is bounded closely to it. Thus the wave transmitted along the line is 1 dimensional in that it does not spread out into space but follows along the line.

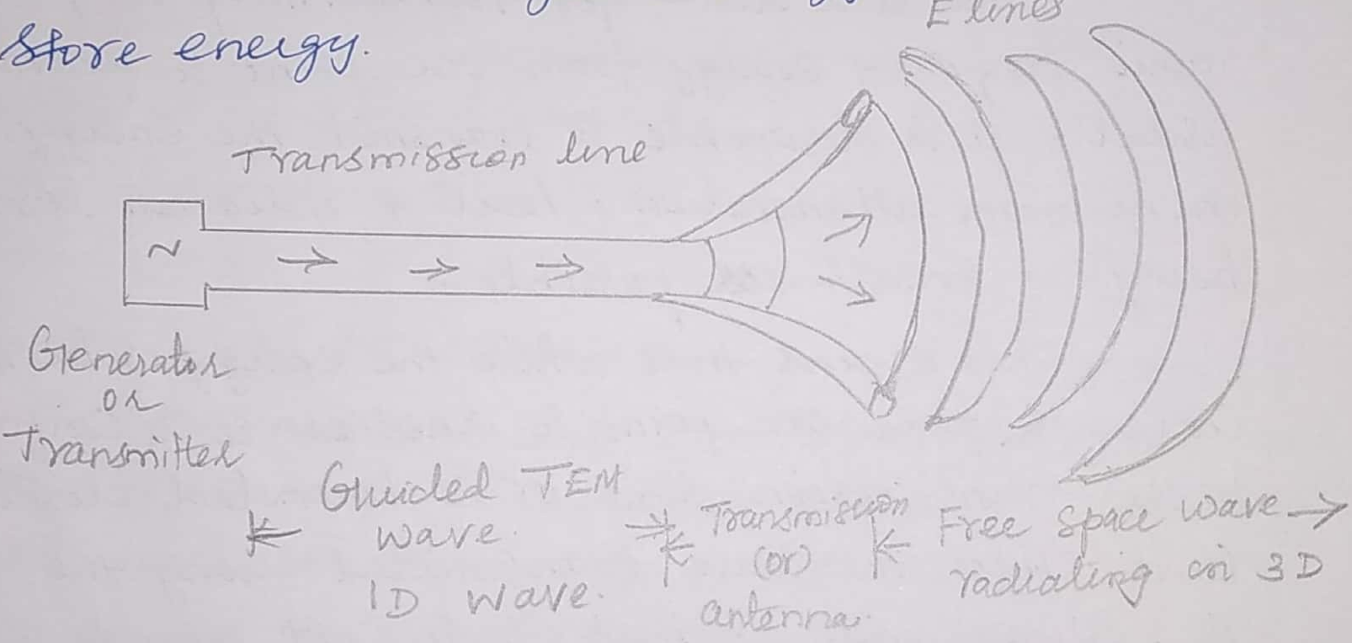
Example: Coaxial, 2 wire transmission lines & hollow pipes or waveguides.

A generator connected to an infinite, lossless transmission line produces a uniform traveling wave along the line. If the line is short circuited the outgoing traveling wave is reflected producing a standing wave on the line due to the interference between the outgoing & reflected waves.

A standing wave has associated with it local concentrations of energy.

If the reflected wave is equal to the outgoing wave, we have a pure standing wave. The energy concentrations in such a wave oscillate from entirely electric to entirely magnetic & back twice per cycle. Such a energy behavior is characteristics of a resonant circuit or resonator.

Thus, antennas radiate (or) receive energy transmission lines guide energy while resonators store energy.



A guided wave travelling along a transmission line which opens out & will radiate as a free space wave. The guided wave is plane wave while the free space wave is a spherically expanding wave. Along the uniform part of the line, energy is guided as a plane wave with little loss provided the spacing between the wires is small fraction of wavelength.

As the transmission line separation approaches a wavelength or more, the wave tends to be radiated so that the opened out line acts like an antenna which launches a free space wave.

The currents on the transmission line flow out on the transmission line flow out on the transmission line & end there; but the fields associated with them keep on going.

ie The region of transition between the guided wave & the free space wave may be defined as an antenna.

The antenna as a transmitting device. As a receiving device the definition is turned & the antenna is the region of transition between a free space wave & a guided wave.

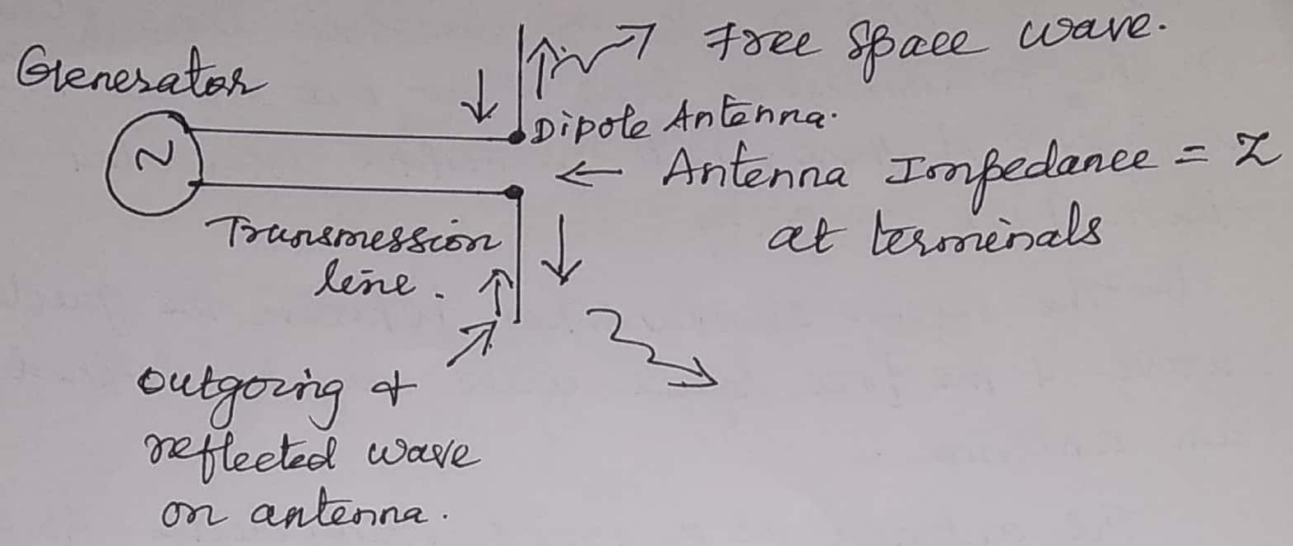
Thus an antenna is a transition device, or transducer or between guided wave & a free space wave or viceversa.

ie Transmission lines (or) waveguides are usually made as to minimize radiation, ~~and~~ but Antennas are designed to radiate (or) receive energy as effectively as possible.

The antenna, like eye is a transformation device converting electromagnetic photons into circuit currents but the antenna can also converts energy from a circuit into photons radiated into space.

Basic Antenna parameters :-

The antenna appears from the transmission line as a 2 terminal circuit element having an impedance Z with a resistive component called the radiation resistance R while from



Space, the antenna is characterized by its radiation pattern or patterns involving field quantities.

* The radiation resistance R_r is not associated with any resistance in the antenna proper but is a resistance coupled from the antenna & its environment to the antenna terminals.

* Antenna temperature, T_A. → Temperature which is related to distant regions of space coupled to the antenna. Via the radiation resistance.

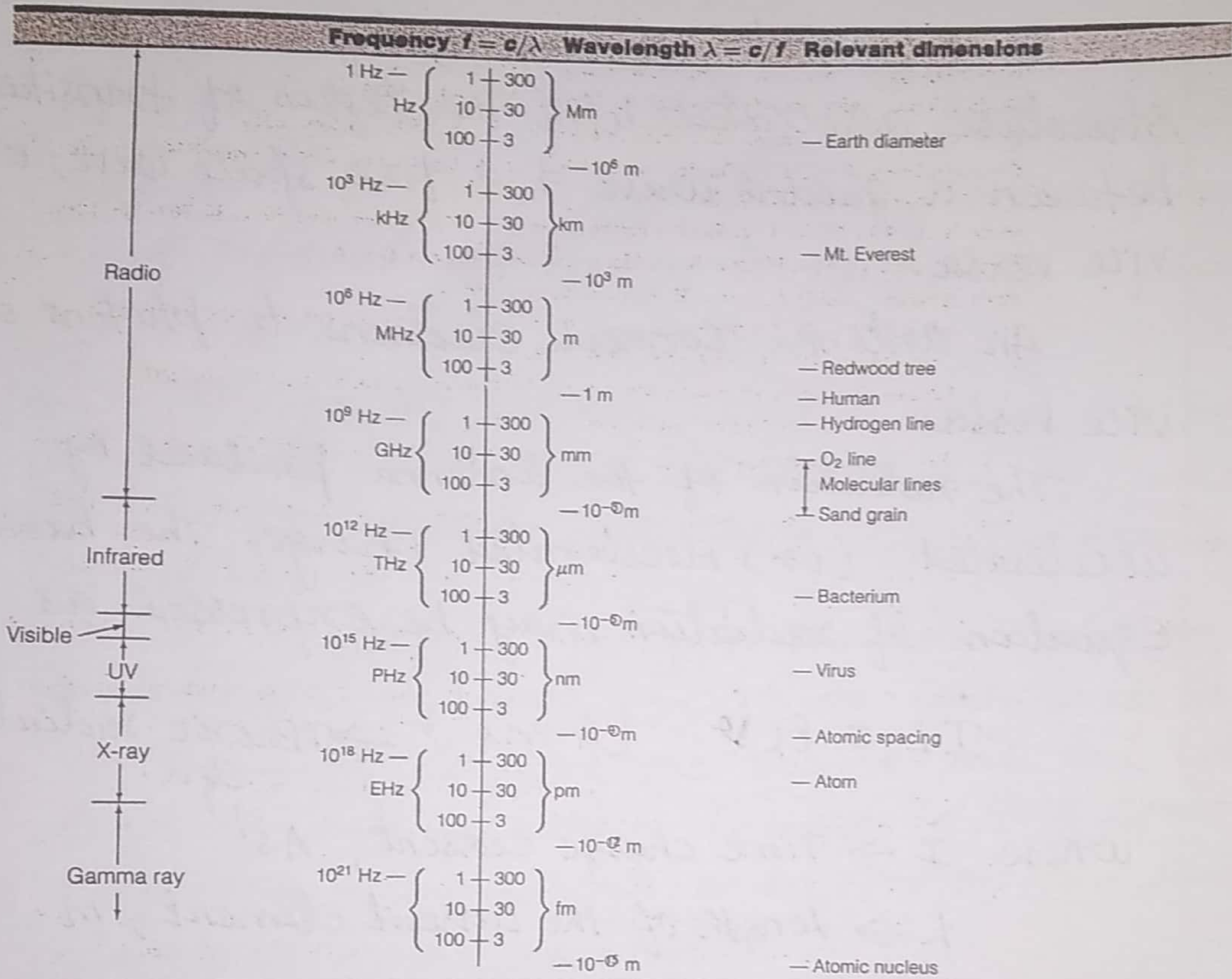
* Radiation resistance R_r, Temperature T_A are Scalar quantities.

* Radiation pattern → Vector quanty, varied with field or power as a function of the two spherical Coordinates θ & ϕ.

UNIT - I

Introduction to Microwave Systems & Antennas

Microwave Frequency Bands :-



Radio-frequency band names†

Name	Frequency	Principal use
ELF†	3–30 Hz	
SLF	30–300 Hz	Power grids
ULF	300–3000 Hz	
VLF	3–30 kHz	Submarines
LF	30–300 kHz	Beacons
MF	300–3000 kHz	AM broadcast
HF	3–30 MHz	Shortwave broadcast
VHF	30–300 MHz	FM, TV
UHF	300–3000 MHz	TV, LAN, cellular, GPS
SHF	3–30 GHz	Radar, GSO satellites, data
EHF	30–300 GHz	Radar, automotive, data

Microwave bands

"Old"	"New"	Frequency
L	D	1–2 GHz
S	E, F	2–4 GHz
C	G, H	4–8 GHz
X	I, J	8–12 GHz
Ku	J	12–18 GHz
K	J	18–26 GHz
Ka	K	26–40 GHz

Physical Concept of Radiation:-

Antenna:-

A Radio Antenna may be defined as the structure associated with the region of transition between a guided wave & a free space wave or vice versa.

An antenna convert electrons to photons or vice versa.

The radiation of the antenna produced by accelerated (or) decelerated charge. The basic equation of radiation may be expressed as

$$\dot{I}L = Q\dot{v} \quad (A\,m\,s^{-1}) \rightarrow \text{Basic radiation eqn.}$$

where $\dot{I}$ → Time charge current, As^{-1}

L → Length of the current element, m

Q → Charge, C

$\dot{v}$ → Time change of velocity which equals the acceleration of the charge, $m\,s^{-2}$

The time - changing current radiates & accelerated charge radiates.

* For steady state harmonic variation, the current would be focused.
* For transcients or pulses, the charge would be focused.

* The radiation is perpendicular to the acceleration, & the radiated power is proportional to the square of IL or Q i.e.

The two-wire transmission line is shown in fig 1.1a. is connected to a radio frequency generator (or) transmitter. Along the uniform part of the line, energy is guided as a plane TEM mode wave with little loss.

The spacing between wires is assumed to be a small fraction of a wavelength ($<1\lambda$). Further on, the transmission line opens out in a tapered transition.

As the separation approaches the order of a wavelength or more ($>1\lambda$) the wave tends to be radiated so that the opened out line acts like an antenna which launches a free space wave. Then the currents on the transmission line flow out on the antenna.

From fig, the transmission antenna, is a region of transition from a guided wave on a transmission line to free space wave.

The receiving antenna 1.1(b) is a region of transition from a space wave to a guided wave on a transmission line.

Thus an antenna is a transition device, or transducer, between a guided wave & a free space wave or vice versa.

The antenna is device which interfaces a circuit & space

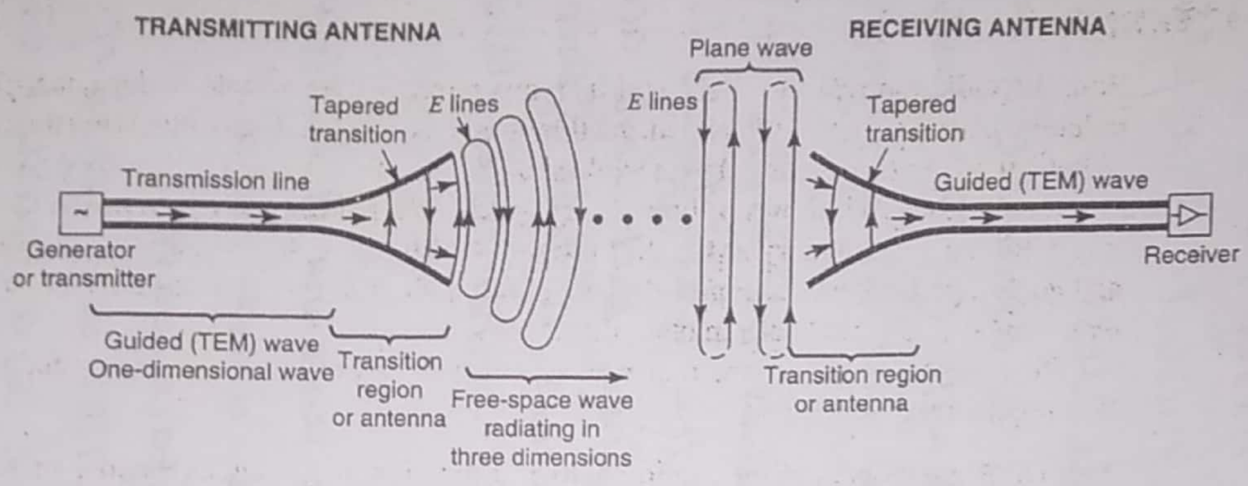


Figure 1-1
(*a*) Radio (or wireless) communication link with transmitting antenna and (*b*) receiving antenna. The receiving antenna is remote from the transmitting antenna so that the spherical wave radiated by the transmitting antenna arrives as an essentially plane wave at the receiving antenna.

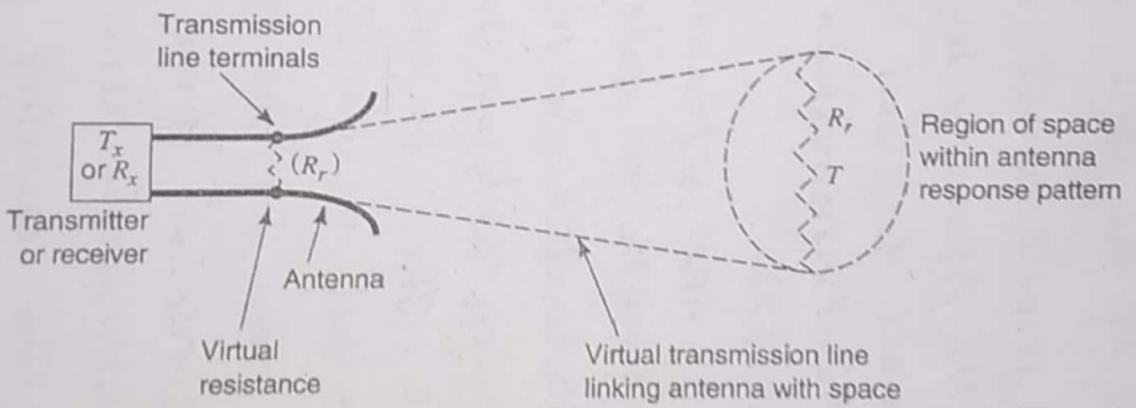


Figure 1-2
Schematic representation of region of space at temperature T linked via a virtual transmission line to an antenna.

From the circuit point of view, the antennas appear to the transmission lines as a resistance R_r, called the Radiation resistance. (ie) It is a resistance coupled from space to the antenna terminals.

In the transmitting case, the radiated power is absorbed by objects at a distance, ie trees, buildings, the ground, the sky + other antennas.

In the receiving case, passive radiation from distant objects or active radiation from other antennas raises the apparent temperature of R_r as shown in 1-2.

The receiving antenna, like the eye, converts electromagnetic photons, into circuit currents. (ie) The antenna converts photons to currents or vice-versa.

Near & Far field Regions :-

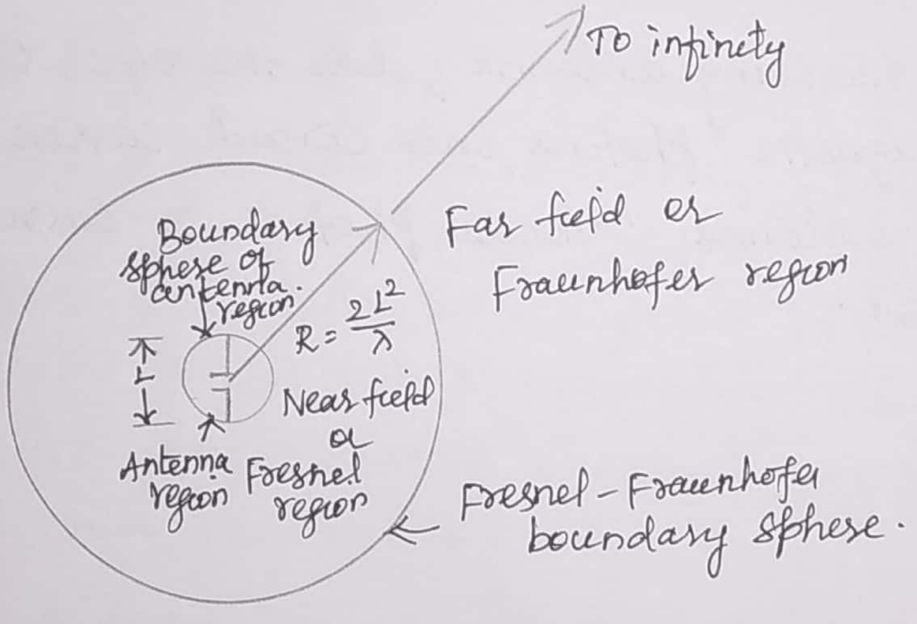
The fields around an antenna may be divided into two principal regions, one near antenna called the near field or Fresnel Zone & one at a large distance called the far field or Fraunhofer Zone.

The boundary between the two may be taken to be as

$$R = 2L^2/\lambda$$

where $L \rightarrow$ Maximum ~~distance~~ dimension of antenna, m.

$\lambda \rightarrow$ wavelength, m.



* In the far or Fraunhofer region, the measurable field components are transverse to the radial direction from the antenna & all power flow is directed radially outward.
* The shape of the field pattern is independent of the distance.

In

* The near or Freshel region, the longitudinal Component of the electric field may be significant & power flow is not entirely radial.
* The shape of the field pattern depends on the distance

Let us enclose the antenna in an imaginary boundary spher as shown. In the near region, the poles of the sphere acts as a reflector.

* The waves expanding perpendicular to the dipole in the equatorial region of the sphere results in power leakage through the sphere as if partially transparent in this region

This results in reciprocating (oscillating) energy flow near the antenna accompanied by outward flow in the equatorial region. The outflow accounts for the power radiated from the antenna, while the reciprocating energy represents reactive power that is trapped near the antenna like in a resonator.

* This field is more effective in the vicinity of the current only.
* It represents the energy stored in the magnetic field surrounding the current element.

Antenna Radiation ~~Pattern~~ Characteristics :-

(1) Radiation pattern.

   (a) Field Radiation pattern

   (b) Power Radiation pattern.

(2) Beam Solid angle (Beam width)

(3) Radiation Intensity

(4) Directive gain & Directivity

(5) Power gain

(6) Input impedance

(7) Polarization

(8) Bandwidth

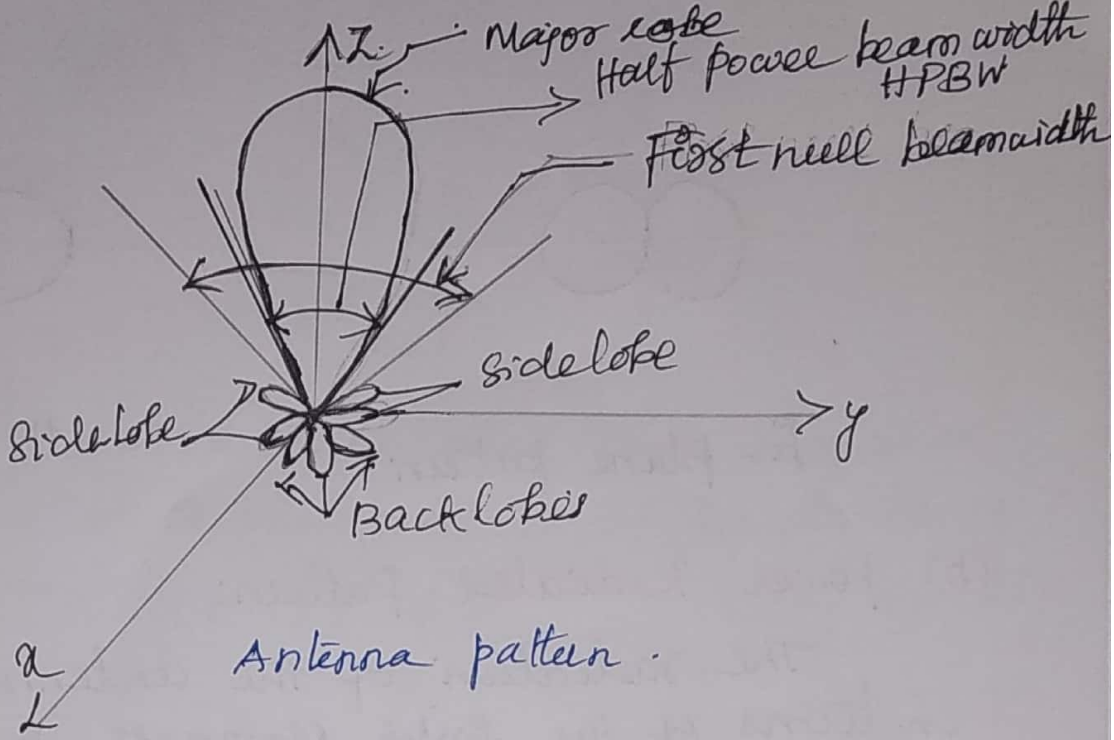
(9) Effective Aperture & Effective length

(10) Antenna temperature.

Radiation Pattern :-

* It indicates the distribution of energy radiated in the space.
* Practically the energy radiated from an antenna does not have same strength in all directions.
* It is more in one direction & less (or) zero in other direction.

An antenna radiation pattern is defined as a mathematical function or a graphical representation of the radiation properties of the antenna as a function of space coordinates. (r, θ, ϕ)



Antenna pattern.

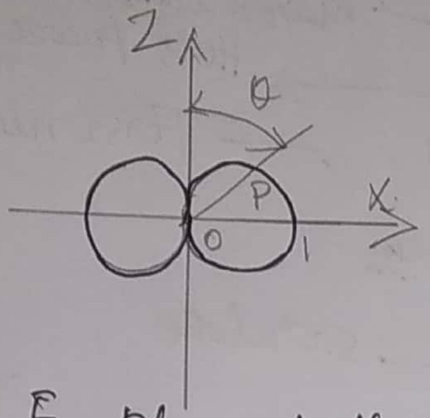
* The radiation properties include power flux density, radiation intensity, field strength directivity, phase or polarization.

(a) Field Radiation pattern:

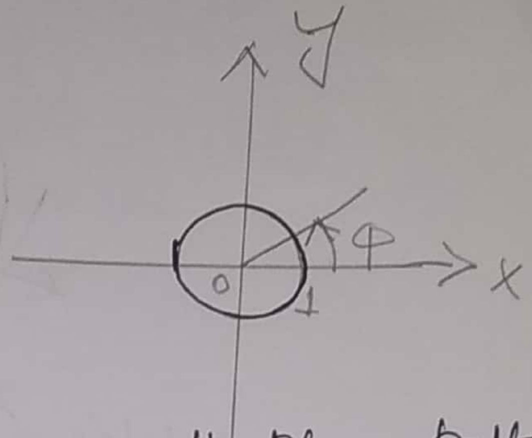
If the radiation of the antenna is expressed in terms of the field strength $\bar{E}$ (in V/m), then it is called field strength pattern or Field Radiation pattern.

* When the magnitude of the normalized field strength is plotted Vs θ with constant ϕ, the pattern is called E plane pattern or Vertical pattern.

* When the normalized field strength is plotted Vs ϕ for $\theta = \pi/2$, the pattern is called H-plane pattern or horizontal pattern.



E - Plane pattern



H Plane pattern

(b) Power Radiated Pattern:

The radiation of the antenna is expressed in terms of the ~~field strength~~ power per unit solid angle, then it is called power Radiation pattern or Power pattern

* The power density $P_d(\theta, \phi)$ is defined as power flow per unit area & is a function of the direction (θ, ϕ).

The power density can be expressed in terms of the magnitude of the electric field intensity as

$$P_d(\theta, \phi) = \frac{1}{2} \frac{|E(\theta,\phi)|^2}{\eta_0} = \frac{1}{2} \frac{|E(\theta,\phi)|^2}{120\pi}$$

$$\eta_0 = E/H = 120\pi \; ; \; \eta_0 \rightarrow \text{Intrinsic impedance of free space}$$

* In the direction in which $E(\theta,\phi)$ is maximum, $P_d(\theta,\phi)$ is also maximum. In this direction, the maximum value of power density is denoted by $P_d(max)$. Then the relative power flow per unit area in the direction of (θ, ϕ) is given by

$$G(\theta,\phi) = \frac{P_d(\theta,\phi)}{P_d(max)}$$

$$= \frac{|E(\theta,\phi)|^2}{E^2_{max}}$$

* The ratio $G(\theta,\phi)$ is called power radiation pattern & it is independent of the distance 'r' since both $P_d(\theta,\phi)$ & $P_d(max)$ vary inversely with r.

* The power radiation pattern is given by

$$G(\theta,\phi) = f^2(\theta,\phi).$$

Beam width :- various parts of a radiation pattern are referred to as lobes

a) Major lobe, b) Minor lobe, c) Sidelobe, & back lobe.

* Some lobes are having greater radiation intensity & some are having lesser radiation intensity

Major lobe :-

It is also called as main beam & is defined as the radiation lobe containing the direction of maximum radiation

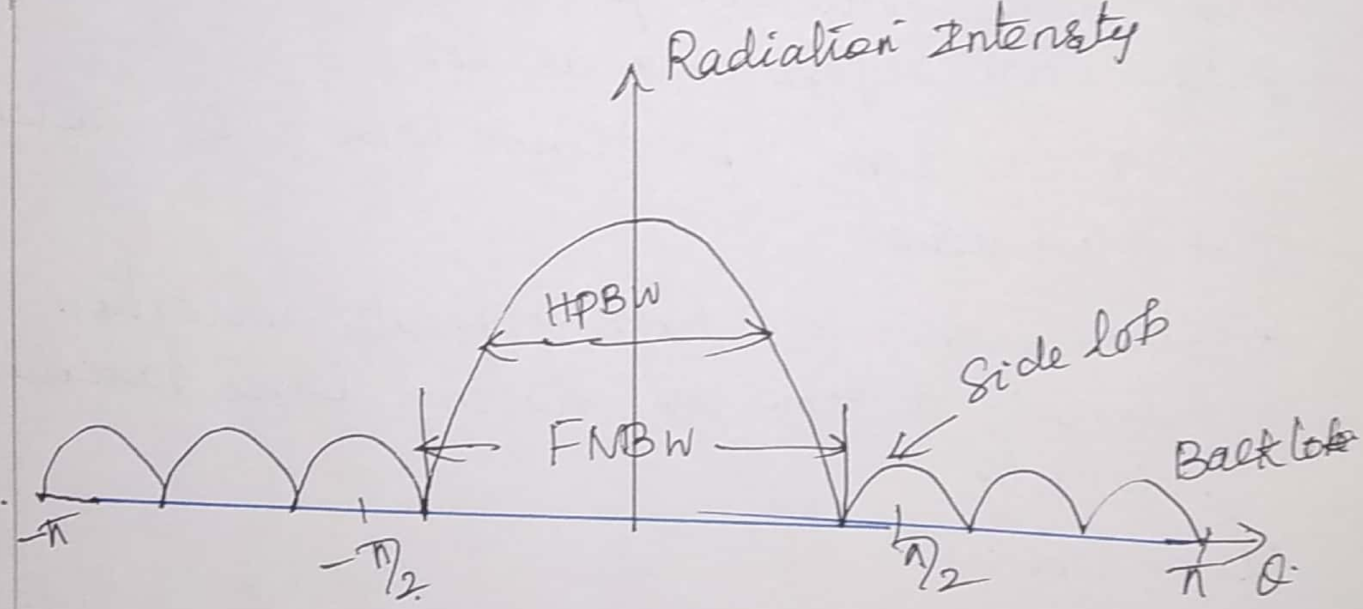
Minor lobe :-

Any lobe except a major lobe. i.e all the lobes except the major lobe are called minor lobe (Side lobe & Back lobe)

Sidelobe :- It is adjacent to the main lobe & occupies the hemisphere in the direction of the main lobe.

Back lobe:-
It occupies the hemisphere in a direction opposite to that of the major lobe. It's axis makes an angle of approximately 180° w.r. to beam of an antenna.

Minor lobes usually represent radiation in undesired directions & they should be minimized.



Linear plot (2D) power pattern

* The normalized power pattern.

$$P_n(\theta,\phi) = \frac{S(\theta,\phi)}{S(\theta,\phi)_{max}} \quad ; \; S(\theta,\phi) \rightarrow \text{Poynting vector}$$

$$dB = 10 \log_{10} P_n(\theta,\phi).$$

Antenna Gain :- (G)

The gain of an antenna is defined as the ability of the antenna to concentrate the radiated power in a given direction.

$$\text{Gain (G)} = \frac{\text{Maximum Radiation Intensity from Subject or Test antenna}}{\text{Maximum Radiation Intensity from a Reference (Isotropic) Antenna with same power}}$$

* Since gain denotes concentration of energy, the high values of gain are associated with narrow beam width.
* Gain is equal to directivity provided antenna efficiency is 100 % . ie For antennas without any internal losses, gain & directivity are same. (Test antenna & Isotropic Antenna both are radiating the same total power).
* In terms of signal power received by a receiver at a distant point in the direction of maximum radiation, the gain of an antenna

$$\text{Gain} = \frac{\text{Maximum power received from subject antenna}}{\text{Maximum power received from reference antenna (isotropic)}}$$

* The gain of an antenna in terms of field strength is defined as the ratio of field strength at a given distance from test antenna in its

desired direction (E_1) to the field strength from an isotropic antenna at the same distance (E_2).

$$G = \frac{E_1}{E_2}.$$

* $G = \dfrac{\text{Voltage produced at a given point (test) Antenna by practical}}{\text{Voltage produced at a reference antenna.}}$

Types:— (i) Directive Gain (Gd):-

All practical antennas concentrate their radiated energy to more or less in certain preferred directions.

$$\text{Directive Gain (Gd)} = \frac{\text{Radiation Intensity in a Particular direction}}{\text{Average radiated power}}$$

$$G_d(\theta, \phi) = \frac{U(\theta, \phi)}{U_{ave}}.$$

Average Radiation Intensity $U_{ave} = \frac{W_r}{4\pi}$.

where $W_r \rightarrow$ radiated power, W/m^2

$$\therefore G_d(\theta, \phi) = \frac{U(\theta, \phi)}{W_r/4\pi} = \frac{4\pi U(\theta, \phi)}{W_r}.$$

$$G_d = \frac{\text{Maximum Radiation Intensity or Maximum power density}}{\text{Average power radiated}}$$

Gd → Directive gain or Directivity

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$$\therefore G_{d(max)} = \frac{P_{d\,max}}{\frac{P_{rad}}{4\pi r^2}} \qquad \frac{P_{rad}}{4\pi r^2} = W_r.$$

(or)

* Directivity can also be expressed interms of

$$G_{Dmax} = \frac{4\pi \, |E_{max}|^2}{\int_0^{2\pi}\int_0^{\pi} |E(\theta,\phi)|^2 \sin\theta \cdot d\theta \cdot d\phi}$$

(ii) Power gain (G_P) :-

* The test antenna & isotropic antenna, both are fed with same input power.

$$G_P = \frac{\text{Power density radiated in a particular direction by the subject antenna}}{\text{Power density radiated in that direction by an isotropic antenna}}$$

* In power gain, the gain takes into account the antenna losses.

*
$$G_P = \frac{\text{Radiation Intensity in a given direction}}{\text{Average total input power}}$$

$$G_P = \frac{U(\theta,\phi)}{W_T/4\pi}$$

$$W_T = W_r + W_l.$$

where $W_T \rightarrow$ Total i/p power
$W_r \rightarrow$ Radiated power
$W_l \rightarrow$ Ohmic losses in the antenna.

$$G_p = \frac{4\pi U(\theta, \phi)}{W_T}$$

* In terms of power input,

Power gain G_p = (Power input supplied to subject antenna in the direction of Maximum radiation) / (power input supplied to reference antenna.)

* The power gain depends on
  (i) Sharpness of lobe ; Sharper the lobe, higher will be the power gain
  (ii) Volume of the solid radiation

* power gain decibels

$$G_p \text{ in db} = 10 \log_{10} G_p = 10 \log_{10} \frac{P_1}{P_2}$$

$$= 10 \log \log_{10} (V_1/V_2)^2$$

$$= 20 \log_{10} (V_1/V_2)$$

Directivity :- 'D'

$$\text{Directivity} = \frac{\text{The maximum radiation intensity } U(\theta,\phi)_{max}}{\text{Average radiation Intensity } U_{ave}}$$

$$D = \frac{U(\theta,\phi)_{max}}{U_{ave}}$$

(OR:)

$$\text{Directivity} = \frac{\text{Max. poynting vector}}{\text{Average poynting vector}} = \frac{S(\theta,\phi)_{max}}{S(\theta,\phi)_{ave}}$$

Average poynting vector

$$S(\theta,\phi)_{ave} = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} S(\theta,\phi)\, d\Omega \quad W/m^2$$

$$\therefore D = \frac{S(\theta,\phi)_{max}}{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} S(\theta,\phi)\, d\Omega} = \frac{1}{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} \frac{S(\theta,\phi)}{S(\theta,\phi)_{max}}\, d\Omega}$$

$$D = \frac{1}{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} P_n(\theta,\phi)\, d\Omega}$$

where $\therefore \int_0^{2\pi}\int_0^{\pi} P_n(\theta,\phi)\, d\Omega = \Omega_A$

$$D = \frac{1}{\frac{1}{4\pi}\cdot\Omega_A}$$

$$D = \frac{4\pi}{\Omega_A}$$

$\therefore$ Smaller the beam angle, greater the directivity

Antenna Efficiency :-

The practical antenna is made up a conductor having finite conductivity, hence consider the ohmic power loss of the antenna here.

* If the practical antenna has ohmic losses (I^2R) represented by Ploss then the power radiated Prad is less than the input power Pin. Then Prad in terms of the P_{in}

$P_{rad} = \eta_r P_{in}$ where $\eta_r \rightarrow$ Radiation efficiency of antenna

$$\eta_r = \frac{P_{rad}}{P_{in}}$$

The total input power to the antenna

$$P_{in} = P_{rad} + P_{loss}.$$

∴ Radiation efficiency

$$\eta_r = \frac{P_{rad}}{P_{rad} + P_{loss}}.$$

$\therefore P_{rad} = I_{rms}^2 \cdot R_{rad} \; ; \; P_{loss} = I_{rms}^2 \cdot R_{loss}$

∴ The radiation Efficiency

$$\eta_r = \frac{R_{rad}}{R_{rad} + R_{loss}}$$

Effective Aperture :-

The total power extracted from a passing wave to the aperture or area of its mouth.

Types of Apertures :

(i) Effective aperture

(ii) Scattering aperture

(iii) Loss aperture

(iv) Collecting aperture

(v) Physical aperture.

1. Effective Aperture or Area :-

The antenna collects power from the wave & delivers it to the terminating or load impedance Z_L connected to its terminals.

The effective aperture is defined as the ratio of power received at the antenna load terminal to the power density (Poynting vector) in watts/metre² of the incident wave.

$$\text{Effective aperture, } A_e \,(m^2) = \frac{\text{Power received}}{\text{Power density of incident wave}}$$

$$A_e = \frac{P_T}{S} \,(m^2)$$

where $P_T \rightarrow$ Power received in watts

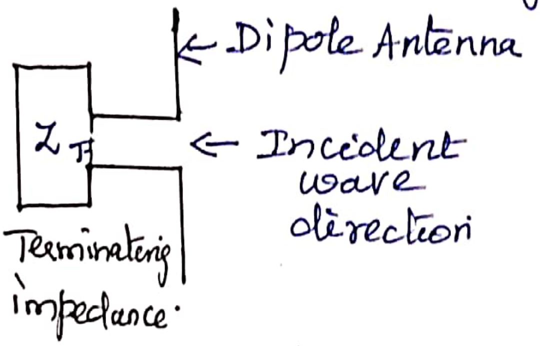
$S \rightarrow$ Power density or poynting vector of incident wave in watts/m²

The effective aperture is the area which when multiplied by the incident power density gives the power delivered to the load.

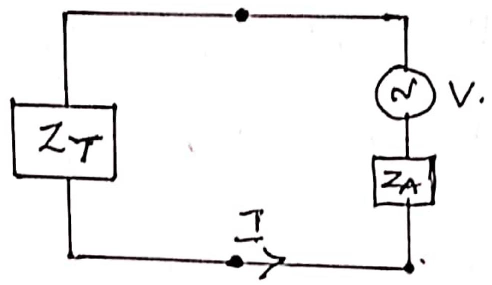
(ie) $P_T = A_e S$. (watts)

Effective Aperture of a Dipole Antenna :

Consider a dipole receiving antenna situated in the field of a passing electromagnetic waves.



Dipole antenna with plane wave incident on Antenna.



Equivalent circuit of figure.

In general the antenna collects power from the wave & delivers it to the terminating or load impedance Z_T connected to its terminals.

The poynting vector or power density of the wave is S watts / square meter.

The antenna may be replaced by an equivalent or Thevenin generator having an equivalent voltage V & internal or equivalent antenna impedance Z_A.

The voltage V is induced by the passing wave & produces a current through the terminating impedance Z_T

$$I = \frac{V}{Z_T + Z_A} \rightarrow (1)$$

I & V are rms or effective values

In general, the terminating & antenna impedances are complex, thus

$$Z_T = R_T + jX_T \rightarrow ②$$

$$Z_A = R_A + jX_A \rightarrow ③$$

The antenna resistance may be divided into two parts, a radiation resistance, R_r & a non radiative or loss resistance R_L ie

$$R_A = R_r + R_L \rightarrow ④$$

Let the power delivered by the antenna to the terminating impedance be P ie

$$P = I^2 R_T \rightarrow ⑤$$

By substituting ② & ③ in eqn ①

$$I = \frac{V}{R_A + jX_A + R_T + jX_T} \rightarrow ⑥$$

$\because R_A = R_r + R_L$

$$I = \frac{V}{R_r + R_L + R_T + j(X_A + X_T)}$$

The magnitude of current I

$$I = \frac{V}{\sqrt{(R_r + R_L + R_T)^2 + (X_A + X_T)^2}} \rightarrow ⑥a$$

Sub 6a in ⑤ $P = V^2 R_T / (R_r + R_L + R_T)^2 + (X_A + X_T)^2 \rightarrow ⑦$

The ratio of the power P in the terminating impedance to the power density of the incident wave is defined as the area A

$$A = P/S \rightarrow ⑧$$

where P → Power in watts ; A → area, m^2

S → Power density of incident wave, W/m^2

Substituting ⑦ into ⑧ gives

$$A = \frac{V^2 R_T}{S[(R_r + R_L + R_T)^2 + (X_A + X_T)^2]} \rightarrow (9)$$

The value of A in eqn ⑨ takes into account any antenna losses as given by R_L an any mismatch between the antenna & its terminating impedance.

Let us consider the situation where the terminating impedance is the complex conjugate of the antenna impedance so that maximum power is transferred

$$X_T = -X_A \rightarrow (10)$$

$$R_T = R_r + R_L \rightarrow (11)$$

Sub ⑩ & ⑪ in eqn ⑨ & it gives effective aperture A_e of the antenna

$$A_e = \frac{V^2 (R_r + R_L)}{S[(R_r + R_L + R_r + R_L)^2 + (X_A - X_A)^2]} \rightarrow (12)$$

$$A_e = \frac{V^2 (R_r + R_L)}{S[2(R_r + R_L)]^2}$$

Effective Aperture $A_e = \frac{V^2}{4S(R_r + R_L)} \, m^2$

If the antenna is lossless ($R_L = 0$), will get maximum effective aperture A_{em} of antenna.

Thus $= \frac{V^2}{4SR_r} \, m^2$ → Represents the area over which power is extracted from incident wave & delivered to the load.

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Aperture Efficiency :-

The ratio of effective aperture to physical aperture is aperture efficiency ϵ_{ap}.

$$\epsilon_{ap} = A_e / A_p \quad \text{dimensionless}$$

(between zero to ∞).

Antenna Temperatures (T_A) :-

* The antenna temperature is a parameter that depends on the temperature of the regions the antenna is 'looking at'.
* Both the antenna temperature (T_A) & radiation Resistance (R_r) are single valued scalar quantities.
* According to Nyquist relation, the noise power available from a resistor 'R' at absolute temperature $T°K$ is

$$P_a = KTB \rightarrow ①$$

where

$P_a \Rightarrow$ Noise power per unit band width in watts

$K \Rightarrow$ Boltzman's Constant $= 1.38 \times 10^{-23}$ J/K.

$T \Rightarrow$ Absolute temperature of resistor in K.

The power received from the source

$$P = S A_e B \rightarrow ②$$

where

$S \rightarrow$ Power density per unit bandwidth in W/m² Hz

$A_e \rightarrow$ Effective aperture in m².

$B \rightarrow$ Bandwidth in Hz

Equating these two eqn.

$$P = KTB = SA_eB$$

$$S = \frac{KT_A}{A_e} \quad W/m^2\ Hz \rightarrow (3)$$

$T_A \rightarrow$ Antenna temperature due to the source in degree K

$$T_A = \frac{SA_e}{K} \quad K^\circ \rightarrow (4)$$

In terms of Antenna beam solid angle Ω_A, & source solid angle Ω_S.

$$T_A = \frac{\Omega_S}{\Omega_A} T_S \rightarrow (5)$$

where

$\Omega_A \Rightarrow$ Antenna beam solid angle in steradian

$\Omega_S \Rightarrow$ Source solid angle in steradian.

$T_A \rightarrow$ Antenna noise Temperature

$T_S \rightarrow$ Source Temperature in K°

* In case, the receiver has a certain noise temperature T, due to thermal noise in the receiver components then the system noise power at the receiver terminals is given by

$$P_S = K(T_A + T_R)B$$

where $T_r \rightarrow$ Receiver noise temperature at receiver terminals

$B \rightarrow$ Bandwidth.

$T_A \rightarrow$ Antenna noise temperature at receiver terminals

$P_S \rightarrow$ System noise power at receiver terminals.

Then the output signal to noise ratio is

$$\frac{S}{N} = \frac{S_A}{(T_A + T_R) kB}$$

Equivalent Noise Temperature of Antenna (T_e) :-

* It is defined as that fictional temperature at the input of the network which would account for the noise ΔN at the output.

$\Delta N \rightarrow$ Additional noise introduced by the network itself.

The noise figure (F) related with effective noise temperature

$$F \rightarrow 1 + \frac{T_e}{T_0} \Rightarrow F - 1 = \frac{T_e}{T_0}.$$

$$T_e = T_0 (F-1)$$

where $F \rightarrow$ Noise figure (no dimension)

$T_0 = 290^\circ k$.

The noise figure F is decibel is

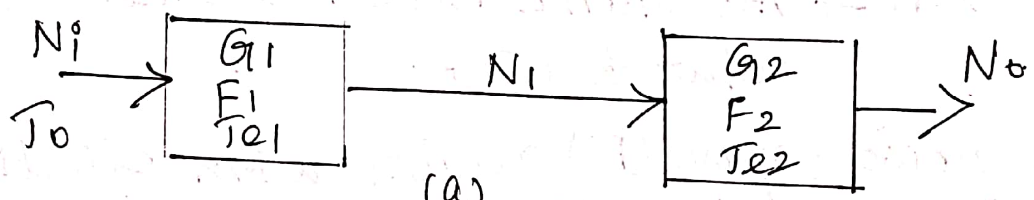
$$F(dB) = 10 \log_{10} F.$$

Noise Figure of a cascaded System :- for G/T

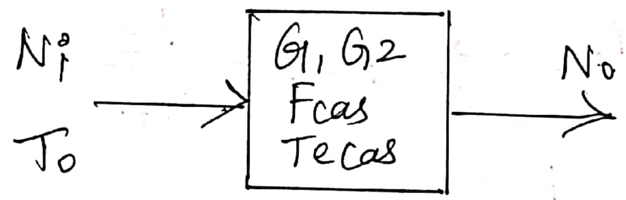
In a typical microwave system, the input signal travels through a cascade of many different components, each of which may degra degrade the signal to noise ratio to some degree.

If we know the noise figure (noise temperature) of the individual stages, we can determine the same for the cascaded connection.

Consider the cascade of two components having gains G_1, G_2 & noise figures F_1, F_2 & equivalent noise temperatures Te_1, Te_2 as shown.



(a) Two cascaded Nw



(b) Equivalent Nw

* using noise temperature, the noise power at the first stage

$$N_1 = G_1 K T_0 B + G_1 K Te_1 B \rightarrow (a)$$

Since $N_i = K T_0 B$

The noise power at the output of the 2nd stage is

$$N_0 = G_2 N_1 + G_2 K Te_2 B \rightarrow (b)$$

Sub (a) in (b)

$\therefore N_o = G_2 G_1 K T_o B + G_1 G_2 K T_{e1} B + G_2 K T_{e2} B$

$$N_o = G_1 G_2 K B \left(T_o + T_{e1} + \frac{1}{G_1} T_{e2}\right) \rightarrow (3)$$

where

$$\left(T_{e1} + \frac{1}{G_1} T_{e2}\right) \rightarrow T_{cas}$$

$$T_{cas} = T_{e1} + \frac{1}{G_1} T_{e2} \rightarrow (A).$$

||ly

$$F_{cas} = F_1 + \frac{1}{G_1}(F_2 - 1) \rightarrow (B)$$

Eqn (A) & (B) shows,

the noise characteristics of a cascaded system are dominated by the charateristics of the first stage since the effect of the second stage is reduced by the gain of the first (assuming $G_1 > 1$)

Thus for the best overall system noise ~~temperature~~ performance, the first stage should have a low noise figure & at least moderate gain.

If we increase the stage.

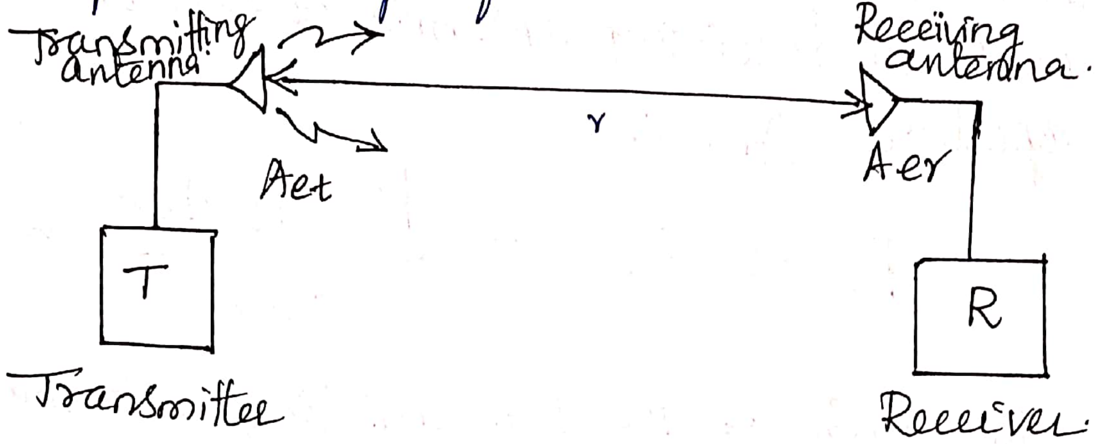
$$T_{cas} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_2} + \cdots$$

$$F_{cas} = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2} + \cdots$$

//

FRIIS Transmission formula :-

* FRIIS Transmission formula is derived from aperture concept of an antenna.



* FRIIS formula gives the power received over a radio communication circuit.
* Let the transmitter T feed a power P_t to a transmitting antenna of effective aperture A_{et}.
* At a distance r, a receiving antenna of effective aperture A_{er} intercepts some of the power radiated by the transmitting antenna & delivers it to the receiver R.
* Assume, the transmitting antenna is isotropic, the power per unit area at the receiving antenna is

$$S_r = \frac{P_t}{4\pi r^2} \quad W \rightarrow ①$$

* If the antenna has gain G_t, the power per unit area available at the receiving antenna will be increased in proportion as given by

$$S_r = \frac{P_t G_t}{4\pi r^2} \quad (W) \rightarrow (2)$$

* Now the power collected by the lossless, matched receiving antenna of effective aperture Aer is

$$P_r = S_r A_{er} = \frac{P_t G_t A_{er}}{4\pi r^2} \quad W \rightarrow (3)$$

* The gain of the transmitting antenna can be expressed as

$$G_t = \frac{4\pi A_{et}}{\lambda^2} \rightarrow (4)$$

* Sub (4) in (3)

$$P_r = \frac{P_t \cancel{4\pi} A_{et} A_{er}}{\cancel{4\pi} \lambda^2 r^2}$$

$$\boxed{\frac{P_r}{P_t} = \frac{A_{et} A_{er}}{\lambda^2 r^2}} \rightarrow \text{Friis transmission formula.}$$

where

P_r → received power, W

P_t → Transmitted power, W

A_{et} → Effective aperture of transmitting antenna, m^2

A_{er} → Effective aperture of receiving antenna, m^2

r → distance between antennas, m

λ → Wavelength, m

Link Budget & Link Margin :-

Link budget is figure of merit for an effective & reliable link between receiver & transmitter for terrestrial as well as satellite based communication link are frequency of operation, range requirement, antenna gain of transmitter & receiver antenna, data rate, receiver bandwidth, noise figure & system losses.

The main factors of terrestrial link contributing the signal losses are free space loss, rain, oxygen and antenna misalignment.

W.K.T.

The received power by a radio antenna having circular aperture antenna of diameter D is given by Friss radio link formula

$$P_r = \frac{P_t G_t A_e}{4\pi R^2} = \frac{G_t G_r \lambda^2}{(4\pi R)^2} P_t \cdot N\eta \left(\frac{\pi D^2}{\lambda}\right)^2$$

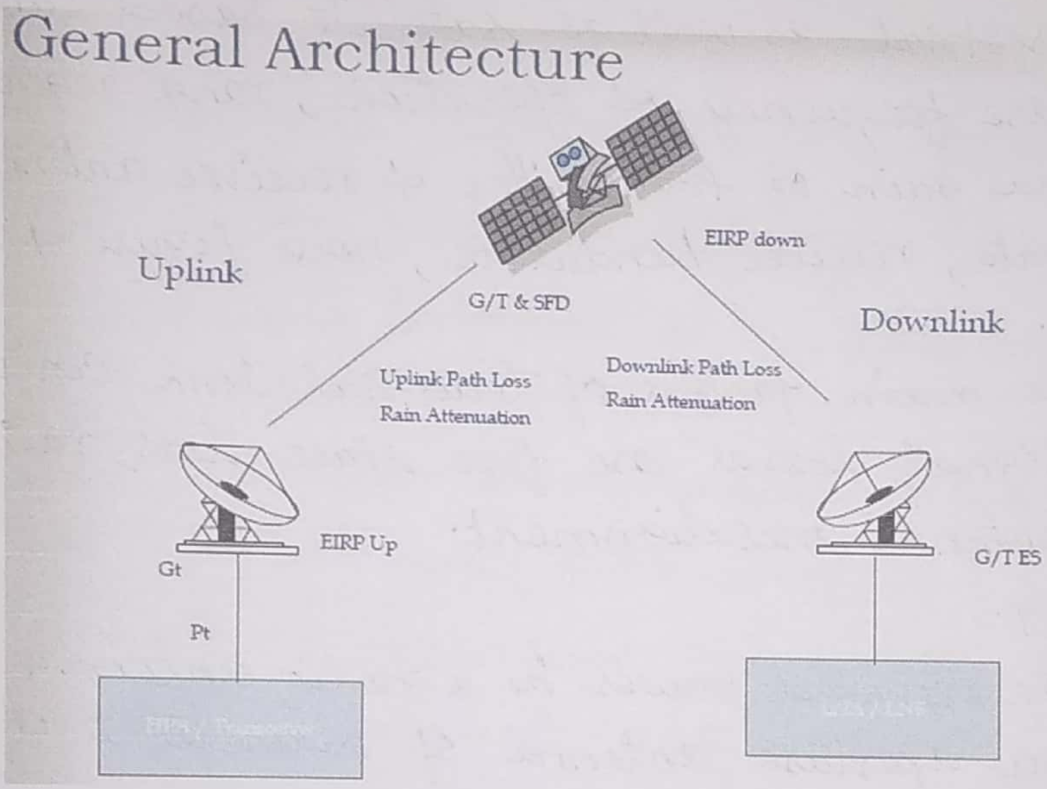
where $P_t G_t = EIRP$

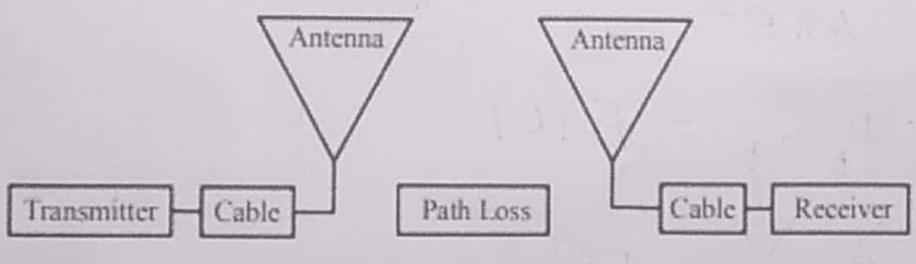
$$\left(\frac{\lambda}{4\pi R}\right)^2 = FSL$$

$$A_e = \eta \frac{\pi D^2}{4} = \frac{7160}{f^2 (MHz)}$$

$$G_r = \frac{4\pi A_e}{\lambda^2} \text{ dB}$$

$$P_r (dBW) = EIRP + G_r - L_p$$


General Architecture
Uplink
G/T & SFD
EIRP down
Downlink
Uplink Path Loss
Rain Attenuation
Downlink Path Loss
Rain Attenuation
EIRP Up
Gt
Pt
G/T ES


Antenna
Antenna
Transmitter
Cable
Path Loss
Cable
Receiver

Quality of a radio frequency communication link is a function of various parameters such as receiver sensitivity background noise level in the band, transmitted signal power level, transmitting / receiving antenna polarization as well as gain dissipation or propagation losses.

Link budget needs the following informations.

* Latitude & longitude of the uplink & downlink earth stations.
* Planned data or information rate.
* Modulation type (BPSK or QPSK)
* uplink & downlink frequencies
* Uplink & downlink antenna sizes
* Uplink & downlink antenna efficiency.
* Uplink & downlink transmit & receive gains at frequency.

Link budget → It is a commonly used metric to evaluate the performance of a communication system.

* It is accounting all power gains & losses that a communication signal experiences in a microwave system

Link budget equation is

Received power (dB) = Transmitted power (dB) + Gain (dB) − Losses (dB)

Link Margin: → Difference between minimum expected power received at receiver & receiver sensitivity.

* Various forms of Friss formula are used for link budget because of net effect on received power
* Additional loss factors: line losses or impedance mismatch, atmospheric ~~attenuation~~ attenuation & polarization mismatch.
* P_L (dB), free space reduction in signal strength with distance between Tx & Rx

$$L_0 (dB) = 20 \log \left(\frac{4 \pi R}{\lambda} \right) > 0$$

* Receiver power is

$$P_r (dBm) = P_t - L_t + G_t - L_0 - L_A + G_r - L_r$$

If Tx & /or Rx antenna is not impedance matched to Tx/Rx, impedance mismatch will reduce P_r by $(1 - |\Gamma|^2)$.

$$L_{imp} (dB) = -10 \log (1 - |\Gamma|^2) \geq 0$$

where

$P_t \rightarrow$ Transmit power

$L_t (-) \rightarrow$ Transmit antenna line loss

$G_t \rightarrow$ Transmit antenna gain

$L_o (-) \rightarrow$ path loss (free space)

$L_A (-) \rightarrow$ Atmospheric attenuation.

$G_r \rightarrow$ Receive antenna gain

$L_r (-) \rightarrow$ Receive antenna line loss.

$P_r \rightarrow$ Received power.

Maximum power transmission between Tx & Rx requires both antennas to be polarized in same manner.

Link Margin:

In a practical communication system it is desired to have received power level greater than threshold level required for minimum acceptable quality of service (Minimum SNR or CNR).

This design allowance for received power is referred to as link Margin. Approx: 3 to 20 dB

$$LM = P_r - P_r(min)$$

* It provides a level of robustness to the system to account for variables such as signal

fading due to weather, movement of a mobile user, multipath propagation problems & other unpredictable effects that can degrade system performance & QoS.

* Link budget & margin for a given Communication system can be improved by increasing received power (increasing P_t or G_t) or reducing minimum threshold power (by improving design of receiver, changing modulation method).

* Increasing LM, involves an increase in cost & complexity, so excessive increases in LM are usually avoided.

Microwave Receiver Noise characterization :

Radio Receiver :-

Radio receiver is the critical component of wireless system, having reliably recovering the desired signal from a wide spectrum of transmitting sources interference & noise.

* Different functions :

→ High gain → to restore the low power of the received signal to a level near its original baseband value.

→ Selectivity → To receive the desired signal while rejecting adjacent channels, image frequencies, & interference.

Down conversion → from the received RF frequency to a lower IF frequency for processing

Detection → of the received analog or digital Information.

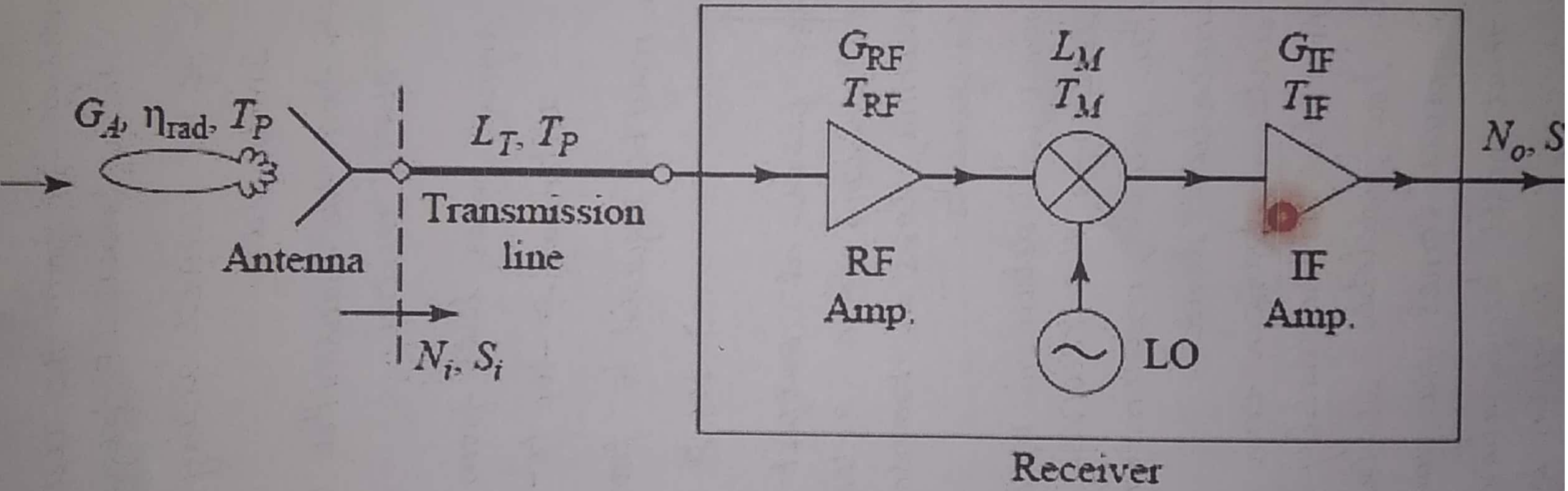
Isolation → from the transmitter to avoid saturation of the receiver.

Typical signal power level from receiving antenna → -100 to -120 dBm.

Receiver may be required to provide gain → 100 to 120 dB

* Total noise power at output of receiver N_o due to contributions from antenna pattern, loss in the antenna, loss in transmission line & receiver components.
* Noise power will determine minimum detectable signal level for the receiver & for a given transmitter power, maximum range of the communication link.
* Entire antenna pattern can collect noise power. If antenna has a reasonably high gain with relatively low sidelobes, we can assume that all noise power comes via main beam

So that noise temperature of antenna is given by



Noise analysis of a microwave receiver front end, including antenna and t
mission line contributions.

$$T_A = \eta_{rad} T_b + (1 - \eta_{rad}) T_b.$$

Noise power at the antenna terminals, which is also the noise power delivered to transmission line is

$$N_i = K T_A B = K B [\eta_{rad} T_b + (1 - \eta_{rad}) T_b]$$

If S_i is the received power at the antenna terminals, then i/p SNR at the antenna terminals is S_i / N_i.

The output signal power is

$$S_o = \frac{S_i G_{RF} G_{IF}}{L_T L_M} = S_i G_{sys}$$

output noise power is $K B G_{sys} T_{sys}$

O/p SNR is $\frac{S_o}{N_o} = \frac{S_i}{K B T_{sys}}$

$$\frac{S_o}{N_o} = \frac{S_i}{K B [\eta_{rad} T_b + (1 - \eta_{rad}) T_b + (L_T - 1) T_b + L_T T_{REC}]}$$

Where

G_{sys} → system power gain

T_{sys} → overall system temperature

EC8701 ANTENNAS AND MICROWAVE ENGINEERING

L	T	P	C
3	0	0	3

OBJECTIVES:

- To enable the student to understand the basic principles in antenna and microwave system design
- To enhance the student knowledge in the area of various antenna designs.
- To enhance the student knowledge in the area of microwave components and antenna for practical applications.

UNIT I INTRODUCTION TO MICROWAVE SYSTEMS AND ANTENNAS 9

Microwave frequency bands, Physical concept of radiation, Near- and far-field regions, Fields and Power Radiated by an Antenna, Antenna Pattern Characteristics, Antenna Gain and Efficiency, Aperture Efficiency and Effective Area, Antenna Noise Temperature and G/T, Impedance matching, Friis transmission equation, Link budget and link margin, Noise Characterization of a microwave receiver.

UNIT II RADIATION MECHANISMS AND DESIGN ASPECTS 9

Radiation Mechanisms of Linear Wire and Loop antennas, Aperture antennas, Reflector antennas, Microstrip antennas and Frequency independent antennas, Design considerations and applications.

UNIT III ANTENNA ARRAYS AND APPLICATIONS 9

Two-element array, Array factor, Pattern multiplication, Uniformly spaced arrays with uniform and non-uniform excitation amplitudes, Smart antennas.

UNIT IV PASSIVE AND ACTIVE MICROWAVE DEVICES 9

Microwave Passive components: Directional Coupler, Power Divider, Magic Tee, attenuator, resonator, Principles of Microwave Semiconductor Devices: Gunn Diodes, IMPATT diodes, Schottky Barrier diodes, PIN diodes, Microwave tubes: Klystron, TWT, Magnetron.

UNIT V MICROWAVE DESIGN PRINCIPLES 9

Impedance transformation, Impedance Matching, Microwave Filter Design, RF and Microwave Amplifier Design, Microwave Power amplifier Design, Low Noise Amplifier Design, Microwave Mixer Design, Microwave Oscillator Design

TOTAL: 45 PERIODS

OUTCOMES:

The student should be able to:

- Apply the basic principles and evaluate antenna parameters and link power budgets
- Design and assess the performance of various antennas
- Design a microwave system given the application specifications

TEXTBOOKS:

1. John D Krauss, Ronald J Marhefka and Ahmad S. Khan, "Antennas and Wave Propagation: Fourth Edition, Tata McGraw-Hill, 2006. (UNIT I, II, III)
2. David M. Pozar, "Microwave Engineering", Fourth Edition, Wiley India, 2012.(UNIT I,IV,V)

REFERENCES:

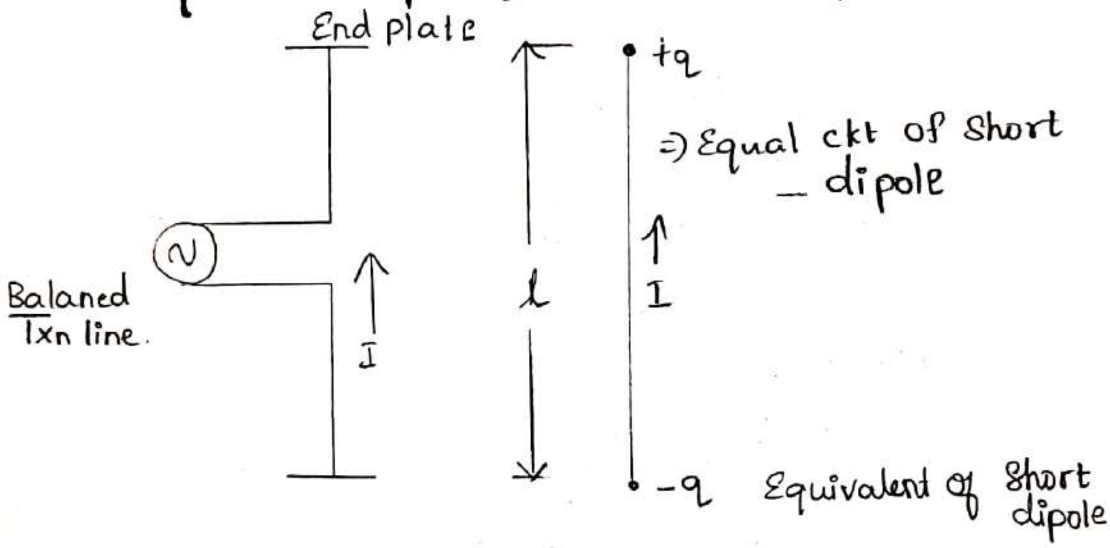
1. Constantine A.Balanis, “Antenna Theory Analysis and Design”, Third edition, John Wiley India Pvt Ltd., 2005.
2. R.E.Collin, "Foundations for Microwave Engineering", Second edition, IEEE Press, 2001

HERTIZIAN DIPOLE / SHORT ELECTRIC DIPOLE

[p.no → 414 (K.D prasad)]

Any linear antenna may be considered as a loop number of very short conductors connected in series (ie) End to End and hence it is important first to consider radiation properties of such short conductor.

A short linear conductor is so short that current may be assume to be constant throughout its length. Such short linear conductor is called as short dipole / Hertizian dipole.



Hertizian dipole is a hypothetical antenna and it is defined as a short isolated conductor carrying uniform alternating current.

The current carrying element

$$I = \frac{dq}{dt}$$

A short dipole that does not have a uniform current. so it is called as a Elemental dipole.

Such a dipole will generally, be considerably shorter than 1 tenth wavelength maximum specified for a short dipole. The other term called as Elementary doublet [1/10th shorter λ]

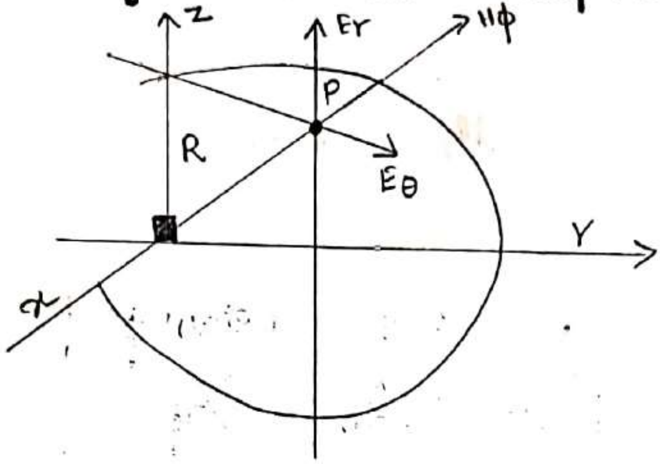
Hertizian dipole in which

$L << \lambda$

Basic elements of Antenna :-

5 types

1) Hertizian dipole
2) Short dipole
3) Short Monopole
4) Half wave dipole
5) quarter Wave Monopole.



The current distribution is constant for short dipole.

Triangular current distribution for short monopole.

Sinusoidal current distribution for $\lambda/2$ and $\lambda/4$ antenna.

The current distribution can be determined by Vector potential.

$$I = I_m \cos\omega t \qquad \therefore I_m = Idl$$

$$I = Idl \cos\omega t$$

$$A(r) = \frac{\mu}{4\pi} \int \frac{J(t - r/v)}{R} \, dv'$$

$$\therefore A_z = \frac{\mu}{4\pi} \frac{Idl \cos\omega(t - r/v)}{R}$$

$$A = A_x \hat{a_x} + A_y \hat{a_y} + A_z \hat{a_z}$$

($A_x \hat{a_x} = 0$, $A_y \hat{a_y} = 0$)

$$A = A_z \hat{a_z}$$

where A = magnetic Vector quantity.

$$\therefore A = \frac{\mu}{4\pi} \frac{I dl \cos\omega (t - r/v)}{R} a_z$$

Convert the Cartesian Coordinate System (x, y, z) to Spherical Co-ordinate System (r, θ, ϕ)

$$A_x = A_r = A_z \cos\theta$$

$$A_y = A_\theta = -A_z \sin\theta \qquad -A_z \sin\theta.$$

$$A_z = A_\phi = 0$$

normal formula:-

$$\nabla \times A = \frac{1}{r^2 \sin\theta} \begin{vmatrix} e_r & r e_\theta & r\sin\theta e_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

11/10/18

W.k.T $A_\phi = 0$

$\therefore$ So the fields are spherically symmetric

$$\nabla \times A = \frac{e_r}{r^2 \sin\theta}\left[\frac{\partial}{\partial\theta}(r\sin\theta A_\phi) - \frac{\partial}{\partial\phi}(rA_\theta)\right] - \frac{r e_\theta}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(r\sin\theta A_\phi) - \frac{\partial}{\partial\phi}(A_r)\right] + \frac{r\sin\theta e_\phi}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial\theta}(A_r)\right]$$

($r\sin\theta A_\phi = 0$ (since $A_\phi = 0$))

$$\therefore \nabla \times A = \frac{e_r}{r^2\sin\theta}\left[-\frac{\partial}{\partial\phi}(rA_\theta)\right] - \frac{e_\theta}{r\sin\theta}\left[-\frac{\partial}{\partial\phi}(A_r)\right] + \frac{e_\phi}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial\theta}(A_r)\right]$$

$$\nabla \times A = \frac{e_r}{r^2 \sin\theta}\left[-\partial/_{\partial\phi}(A_\theta \cdot r)\right] + \frac{e_\theta}{r\sin\theta}\left[\partial/_{\partial\phi}(A_r)\right]$$

$$+ \frac{e_\phi}{r}\left[\partial/_{\partial r}(rA_\theta) - \partial/_{\partial\theta}(A_r)\right]$$

According to formulae, we obtain

$$\nabla \times A = \frac{1}{r^2\sin\theta}\left[e_r(0) - r e_\theta(0) + r\sin\theta\, e_\phi \left[-A_z \sin\theta + A_z \sin\theta\right]\right]$$

The field is spherically symmetry

So $\partial/_{\partial\phi} = 0$ and $A_\phi = 0$

$(\nabla \times A)_r = \mu H_r = 0$

$(\nabla \times A)_\theta = \mu H_\phi = 0$

$(\nabla \times A)_\phi = \mu H_\phi = \frac{1}{r}\left[\partial/_{\partial r}(rA_\theta) - \partial/_{\partial\theta}(A_r)\right]$

$A_r = A_z \cos\theta$; $H_r = 0$

$A_\theta = -A_z \sin\theta$; $H_\theta = 0$

$A_\phi = 0$

$$\therefore \mu H_\phi = \frac{1}{r}\left[\partial/_{\partial r}(-A_z \sin\theta\, r) - \partial/_{\partial\theta}(A_z \cos\theta)\right] \rightarrow (1)$$

but

$$A_z = \frac{\mu}{4\pi r} I m\, dl \cos\omega(t - r/c)$$

multiply by $-r\sin\theta$ on both sides

$$\underbrace{-r\sin\theta\, A_z}_{A_\theta} = -\frac{\mu r}{4\pi r} I m\, dl \cos\omega(t - r/c)\sin\theta$$

$$rA_\theta = -\frac{\mu}{4\pi} I m\, dl \cos\omega(t - r/c)\sin\theta$$

$$\partial/_{\partial r}(rA_\theta) = -\frac{\partial}{\partial r}\left[\frac{\mu I m\, dl \cos\omega(t - r/c)\sin\theta}{4\pi}\right]$$

$$\frac{\partial}{\partial r}\left[-A_z \sin\theta \, r\right] = (-)\frac{\mu I m\, dl \sin\theta}{4\pi}\left\{\frac{\partial}{\partial r}\cos\omega\left(t - r/c\right)\right\}$$

$\cos[\omega t - \omega r/c]$

$$\frac{\partial}{\partial r}\left[-A_z \sin\theta \, r\right] = (-)\frac{\mu I m\, dl \sin\theta}{4\pi}\left[\sin\omega\left(t - r/c\right)\left(\omega/c\right)\right]$$

$$\frac{\partial}{\partial r}\left[-A_z \sin\theta \, r\right] = (-)\frac{\mu I m\, dl \sin\theta}{4\pi}\left[\left[\sin\omega\left(t - r/c\right)\right]\left(\omega/c\right)\right] \rightarrow (2)$$

$$\frac{\partial}{\partial \theta}\left[-A_z \cos\theta\right] = \frac{\partial}{\partial \theta}\left[\frac{\mu I m\, dl \cos\omega\left(t - r/c\right)}{4\pi r}\cos\theta\right]$$

$$\frac{\partial}{\partial \theta}\left[-A_z \cos\theta\right] = \frac{\mu I m\, dl \cos\omega\left(t - r/c\right)}{4\pi r}\left[\frac{\partial}{\partial \theta}\cos\theta\right]$$

$$\frac{\partial}{\partial \theta}\left[-A_z \cos\theta\right] = \frac{\mu I m\, dl \cos\omega\left(t - r/c\right)}{4\pi r}\left[\sin\theta\right]$$

$$\frac{\partial}{\partial \theta}\left[-A_z \cos\theta\right] = \frac{\mu I m\, dl \cos\omega\left(t - r/c\right)\left(\sin\theta\right)}{4\pi r} \rightarrow (3)$$

$$(\nabla \times A)_\phi = \mu H_\phi$$

substitute equ (2) & equ (3) in equ (1)

$$\mu H_\phi = \frac{1}{r}(-)\left[\left[\frac{\mu I m\, dl \sin\theta}{4\pi}\left[\frac{\omega \sin\omega\left(t - r/c\right)}{c}\right]\right] - \frac{\mu I m\, dl \sin\theta}{4\pi r}\cos\omega\left(t - r/c\right)\right]$$

$$\mu H_\phi = \frac{1}{r}(-)\left[\frac{\mu I m\, dl \sin\theta}{4\pi}\left(\frac{\omega \sin\omega\left(t - r/c\right)}{c} - \frac{\cos\omega\left(t - r/c\right)}{r}\right)\right]$$

let $t - r/c = t_1$

$$\mu H_\phi = \frac{1}{r}(-)\left[\frac{\mu I m\, dl \sin\theta}{4\pi}\left[\frac{\omega \sin\omega t_1}{c} - \frac{\cos\omega t_1}{r}\right]\right]$$

$$\therefore H_\phi = \frac{I_m dl \sin\theta}{4\pi}\left[\frac{\cos\omega t_1}{r^2 c} - \frac{\omega \sin\omega t_1}{rc}\right]$$

ELECTRIC FIELD COMPONENT [E] CAN BE DERIVED BY THIS ABOVE EQUATION:-

$$\nabla \times H = \frac{\partial}{\partial t} D = \varepsilon \frac{\partial}{\partial t} E \quad [D \rightarrow \text{displacement of current density}]$$

$$\nabla \times H = \varepsilon \frac{\partial E}{\partial t}$$

1) $\varepsilon \dfrac{\partial E_r}{\partial t} = (\nabla \times H)_r$

2) $\varepsilon \dfrac{\partial E_\theta}{\partial t} = (\nabla \times H)_\theta$

3) $\varepsilon \dfrac{\partial E_\phi}{\partial t} = (\nabla \times H)_\phi$

$$(\nabla \times H)_r = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}[H_\phi \sin\theta] - \frac{\partial}{\partial\phi}(H_\theta)\right]$$

$$(\nabla \times H)_\theta = \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\phi} - \frac{\partial}{\partial r}(H_\phi r)\right]$$

$$(\nabla \times H)_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(rH_\theta) - \frac{\partial}{\partial\theta}(H_r)\right]$$

$$\nabla \times H = \frac{1}{r^2\sin\theta}\begin{vmatrix} e_r & e_\theta & r\sin\theta\, e_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ H_r & rH_\theta & r\sin\theta H_\phi \end{vmatrix}$$

$$E_\phi = H_r = H_\theta = 0$$

$$H_\phi = \frac{I_m dl \sin\theta}{4\pi}\left[\frac{-\omega}{rc}\sin\omega t_1 + \frac{\cos\omega t_1}{r^2 c}\right]$$

$$\varepsilon\frac{\partial E_r}{\partial t} = (\nabla \times H)_r \text{ (from ①)}$$

$$= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(H_\phi \sin\theta)\right] \quad \boxed{H_\theta = 0}$$

$$\epsilon \frac{\partial E_r}{\partial t} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left[\frac{I_m dl \sin\theta}{4\pi}\left\{\frac{-\omega}{rc}\sin\omega t_1 + \frac{\cos\omega t_1}{r^2}\right\}\sin\theta\right]\right]$$

$\frac{\partial}{\partial\theta}\sin^2\theta$

$$\epsilon \frac{\partial E_r}{\partial t} = \frac{1}{r\sin\theta}\left[\frac{I_m dl}{4\pi}\left[\frac{\partial}{\partial\theta}(\sin^2\theta)\right]\left\{\frac{-\omega}{rc}\sin\omega t_1 + \frac{\cos\omega t_1}{r^2}\right\}\right]$$

$2\sin\theta\cos\theta$; $2\cos\theta\sin\theta$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$; $\sin 2\theta = 2\sin\theta\cos\theta$

$$\epsilon \frac{\partial E_r}{\partial t} = \frac{1}{r\sin\theta}\left[\frac{I_m dl}{4\pi}\left[\frac{\cos\omega t_1}{r^2} - \frac{\omega}{rc}\sin\omega t_1\right] \times 2\cos\theta\sin\theta\right]$$

$$\frac{\partial E_r}{\partial t} = \frac{2 I_m dl\cos\theta\sin\theta}{4\pi r\epsilon\sin\theta}\left[\frac{\cos\omega t_1}{r^2} - \frac{\omega}{rc}\sin\omega t_1\right]$$

$$\frac{\partial E_r}{\partial t} = \frac{2 I_m dl\cos\theta}{4\pi\epsilon}\left[\frac{\cos\omega t_1}{r^3} - \frac{\omega}{r^2 c}\sin\omega t_1\right]$$

$$\int\frac{\partial E_r}{\partial t} = \int\frac{2 I_m dl\cos\theta}{4\pi\epsilon}\left[\frac{\cos\omega t_1}{r^3} - \frac{\omega}{r^2 c}\sin\omega t_1\right]dt$$

$$E_r = \frac{2 I_m dl\cos\theta}{4\pi\epsilon}\int\left[\frac{\cos\omega t_1}{r^3} - \frac{\omega}{r^2 c}\sin\omega t_1\right]dt$$

$$E_r = \frac{2 I_m dl\cos\theta}{4\pi\epsilon}\left[\frac{\sin\omega t_1}{\omega r^3} - \frac{\omega}{r^2 c}\frac{(-\cos\omega t_1)}{\omega}\right]$$

$$E_r = \frac{2 I_m dl\cos\theta}{4\pi\epsilon}\left[\frac{\cos\omega t_1}{r^2 c} + \frac{\sin\omega t_1}{\omega r^3}\right]$$

$$\therefore E_r = \frac{2 I_m dl\cos\theta}{4\pi\epsilon}\left[\frac{\cos\omega t_1}{r^2 c} + \frac{\sin\omega t_1}{\omega r^3}\right]$$

$$\epsilon \frac{\partial E_\theta}{\partial t} = (\nabla\times H)_\theta = \frac{1}{r}\left\{\frac{-\partial}{\partial r}(rH_\phi)\right\}$$

$$= (-)\frac{1}{r}\frac{\partial}{\partial r}\left\{\frac{I_m dl\sin\theta}{4\pi}\left\{\frac{-\omega\sin\omega t_1}{rc}+\frac{\cos\omega t_1}{r^2}\right\}r\right\}$$

$$\because t_1 = t - r/c$$

$$= \frac{1}{r}\frac{I_m dl\sin\theta}{4\pi}\left\{\frac{\partial}{\partial r}\left\{\frac{\omega}{rc}\sin\omega t_1(r) - \frac{\cos\omega t_1}{r^2}r\right\}\right.$$

$$\epsilon\frac{\partial E_\theta}{\partial t} = \frac{I_m dl\sin\theta}{4\pi}\left\{\frac{\omega}{c}\cos\omega t_1(-\omega/c) - r\frac{\partial}{\partial r}\left[\frac{\cos\omega t_1 - \cos\omega t_1}{r^2}\right]\right\}$$

$$\frac{\partial E_\theta}{\partial t} = \frac{I_m dl\sin\theta}{4\pi\epsilon r}\left[\frac{-\omega^2}{c^2}\cos\omega t_1 - \left[\frac{r\sin\omega t_1(-\omega/c) - \cos\omega t_1}{r^2}\right]\right]$$

$$\frac{\partial E_\theta}{\partial t} = \frac{I_m dl\sin\theta}{4\pi r\epsilon}\left[\frac{-\omega^2\cos\omega t_1}{c^2} - \frac{\omega\sin\omega t_1}{rc} + \frac{\cos\omega t_1}{r^2}\right]$$

$$\frac{\partial E_\theta}{\partial t} = \frac{I_m dl\sin\theta}{4\pi\epsilon}\left[\frac{-\omega^2\cos\omega t_1}{rc^2} - \frac{\omega\sin\omega t_1}{r^2 c} + \frac{\cos\omega t_1}{r^3}\right]$$

$$\int\frac{\partial E_\theta}{\partial t} = \int\frac{I_m dl\sin\theta}{4\pi\epsilon}\left[\frac{-\omega^2\cos\omega t_1}{rc^2} - \frac{\omega\sin\omega t_1}{r^2 c} + \frac{\cos\omega t_1}{r^3}\right]dt$$

$$E_\theta = \frac{I_m dl\sin\theta}{4\pi\epsilon}\int\left(\frac{\cos\omega t_1}{r^3} - \frac{\omega\sin\omega t_1}{r^2 c} - \frac{\omega^2\cos\omega t_1}{rc^2}\right)dt$$

$$E_\theta = \frac{I_m dl\sin\theta}{4\pi\epsilon}\left[\frac{\sin\omega t_1}{\omega r^3} + \frac{\omega\cos\omega t_1}{\omega r^2 c} - \frac{\omega^2\sin\omega t_1}{\omega rc^2}\right]$$

$$E_\theta = \frac{I_m dl\sin\theta}{4\pi\epsilon}\left[\frac{\sin\omega t_1}{\omega r^3} + \frac{\cos\omega t_1}{r^2 c} - \frac{\omega\sin\omega t_1}{rc^2}\right]$$

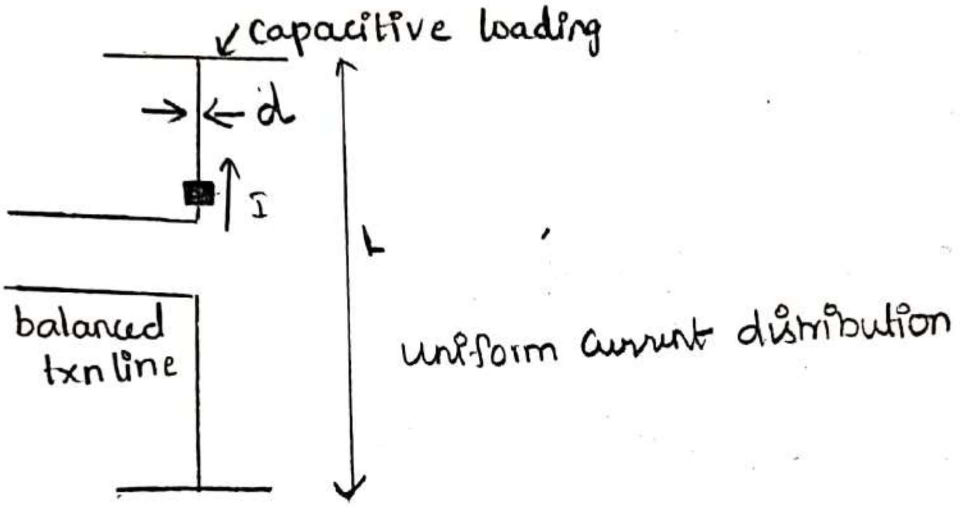
$$\boxed{E_\theta = \frac{I_m dl\sin\theta}{4\pi\epsilon}\left[\frac{\sin\omega t_1}{\omega r^3} + \frac{\cos\omega t_1}{r^2 c} - \frac{\omega\sin\omega t_1}{rc^2}\right]}$$

$$\therefore E_\theta = \frac{I_m dl \sin\theta}{4\pi \varepsilon}\left[\frac{\sin \omega t_1}{\omega r^3} + \frac{\cos \omega t_1}{r^2 c} - \frac{\omega \sin \omega t_1}{r c^2}\right]$$

$$E_r = \frac{2 I_m dl \cos\theta}{4\pi \varepsilon}\left[\frac{\cos \omega t_1}{r^2 c} + \frac{\sin \omega t_1}{\omega r^3}\right]$$

$$H_\phi = \frac{I_m dl \sin\theta}{4\pi}\left[\frac{\cos \omega t_1}{r^2} - \frac{\omega \sin \omega t_1}{r c}\right]$$

RETARDATION EFFECT:-



Definition:- If current is flowing in the short dipole, the effect of this current is not felt instantaneously at the point, but only after an interval is equal to time required for disturbance to propagate over 'r'. This is called as retardation effect.

Retardation Current : $I = I_m e^{j\omega(t - r/c)}$ A

Retardation density : $J = J_m e^{j\omega(t - r/c)}$ A/m²

Retardation Vector potential : $A = \frac{\mu}{4\pi}\int \frac{I}{r}\, dl$

$$A = \frac{\mu}{4\pi}\int \frac{I_m e^{j\omega(t - r/c)}}{r}\, dl$$

$I = I_m \cos\omega t$

The current Excited in short dipole

$$[A] = \frac{\mu}{4\pi} \frac{I_m \cos\omega(t - r/c)}{r} dl$$

(near field) (far field)

Induction of radiation field:-

$$H\phi = \frac{I_m dl \sin\theta}{4\pi}\left[\frac{\cos\omega t_1}{r^2} - \frac{\omega \sin\omega t_1}{rc}\right]$$

⇒ near field component ⇒ far field component

(The antenna musto only Excite with current source)

r = distance from source to destination.

The first term Varies inversely Square of the distance, when r is small $\frac{1}{r^2}$ is a predominant at points flows to the current Element.

The first term is responsible for Energy stored in the magnetic field and it cannot be trusted for reception.

The Second term is inversely proportional to the distance and this term is trusted becoz it is in far field.

Eθ and Er are the distance at which the radiation field = induction field.

$$\text{Induction field} = \frac{I_m dl \sin\theta}{4\pi}\left[\frac{\cos\omega t_1}{r^2}\right] \rightarrow (1)$$

$$\text{far field} = \frac{I_m dl \sin\theta}{4\pi}\left[\frac{-\omega \sin\omega t_1}{rc}\right] \rightarrow (2)$$

Taking modulus and Equating the above equations.

$$\left|\frac{I_m dl \sin\theta}{4\pi}\left[\frac{\cos\omega t_1}{r^2}\right]\right| = \left|\frac{I_m dl \sin\theta}{4\pi}\left[\frac{-\omega \sin\omega t_1}{rc}\right]\right|$$

(-) it becomes (+ve)

$$\frac{\cos\omega t_1}{r^2} = \frac{\omega \sin\omega t_1}{rc}$$

$$\frac{\cos\omega t_1}{\omega \sin\omega t_1} = \frac{r^2}{rc}$$

$$\frac{\cos\omega t_1}{\omega \sin\omega t_1} = \frac{r}{c}$$

$$\frac{\cos\omega t_1}{\sin\omega t_1} = \frac{\omega r}{c}$$

W.K.T $\frac{\cos\theta}{\sin\theta} = \cot\theta$

$$\cot\omega t_1 = \frac{r\omega}{c}$$

$$\frac{c}{\omega}\cot\omega t_1 = r$$

$\cot\omega t_1 = f(\omega + 90) = 1$ $(\omega = 90)$

$$r = c/\omega$$

(or) $$r = \frac{c}{2\pi f}$$

Where
$\omega = 2\pi f$
$c/f = \lambda$

$$r = \frac{\lambda}{2\pi}$$

$$r = \frac{\lambda}{6}$$

$$r = 0.159\lambda$$

Radiation pattern of Elemental dipole:-

1) field component $H_r = H_\theta = E_\phi = 0$

2) E_θ & H_ϕ are in time phase, in far field

$$\eta_0 = 120\pi = 377\,\Omega$$

3) E_θ & H_ϕ are proportional to $\sin\theta$

4) The Radiation pattern will be independ of ϕ

Power Radiated by Elemental dipole: [Hertizian dipole/ short dipole]

$$P = E \times H \quad \text{(power)}$$

$$P_r = E_\theta \times H_\phi \quad \text{(power radiated)}$$

$$P_r = \frac{I_m dl \sin\theta}{4\pi}\left[\frac{\sin\omega t_1}{\omega r^3} + \frac{\cos\omega t_1}{r^2 c} - \frac{\omega \sin\omega t_1}{r c^2}\right] \cdot$$

$$\frac{I_m dl \sin\theta}{4\pi}\left[\frac{\cos\omega t_1}{r^2} - \frac{\omega \sin\omega t_1}{r c}\right]$$

$$P_r = \frac{I_m^2 dl^2 \sin^2\theta}{16\pi^2}\left[\frac{\sin\omega t_1}{\omega r^3} \cdot \frac{\cos\omega t_1}{r^2} - \frac{\omega \sin^2\omega t_1}{\omega r^3 r c}\right.$$

$$+ \frac{\cos^2\omega t_1}{r^4 c^2} - \frac{\omega \sin\omega t_1 \cos\omega t_1}{r^3 c^2} - \frac{\omega \cos\omega t_1 \sin\omega t_1}{r^3 c^2}$$

$$\left. + \frac{\omega^2 \sin^2\omega t_1}{r^2 c^3}\right]$$

$$P_r = \frac{I_m^2 dl^2 \sin^2\theta}{16\pi^2}\left[\frac{\sin\omega t_1 \cos\omega t_1}{\omega r^5} - \frac{\sin^2\omega t_1}{r^4 c} + \frac{\cos^2\omega t_1}{r^4 c^2}\right.$$

$$\left. - \frac{\omega \sin\omega t_1 \cos\omega t_1}{r^3 c^2} - \frac{\omega \cos\omega t_1 \sin\omega t_1}{r^3 r^3} + \frac{\omega^2 \sin^2\omega t_1}{r^2 c^3}\right]$$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$= 2\cos^2\alpha - 1$

$2\cos^2\alpha = 1 + \cos 2\alpha$

$2\sin^2\alpha = 1 - \cos 2\alpha$

$$P_r = \left[\frac{I m\, dl \sin\theta}{4\pi}\right]^2 \left[\frac{\sin\omega t_1 \cos\omega t_1}{\omega r^5} - \frac{2\omega \sin\omega t_1 \cos\omega t_1}{r^3 c^2}\right.$$

$$\left. + \frac{\cos^2\omega t_1}{c r^4} - \frac{\sin^2\omega t_1}{c r^4} + \frac{\omega^2 \sin^2\omega t_1}{r^2 c^3}\right]$$

$$P_r = \left[\frac{I m\, dl \sin\theta}{4\pi}\right]^2 \left[\frac{\omega^2}{c^3 r^2}\left[\frac{1-\cos 2\omega t_1}{2}\right] - \right.$$

$$\left[\frac{2\omega \sin\omega t_1 \cos\omega t_1}{c^2 r^3}\right] - \left[\frac{\omega(1-\cos 2\omega t_1)}{2c\omega r^4}\right] +$$

$$\left.\left[\frac{1+\cos 2\omega t_1}{2 c r^4}\right] + \left[\frac{\sin\omega t_1 \cos\omega t_1}{\omega r^5}\right]\right]$$

Avg values of $\sin 2\omega t_1$, & $\cos 2\omega$ will be zero and also neglect high power of "r"

$$\therefore P_r = \frac{(I m\, dl \sin\theta)^2}{16\pi^2 \varepsilon} \left[\frac{\omega^2}{2c^3 r^2}\right]$$

$\omega = 2\pi f$

$\lambda = c/f$

$c = f\lambda$

$\eta = \frac{1}{\varepsilon c}$

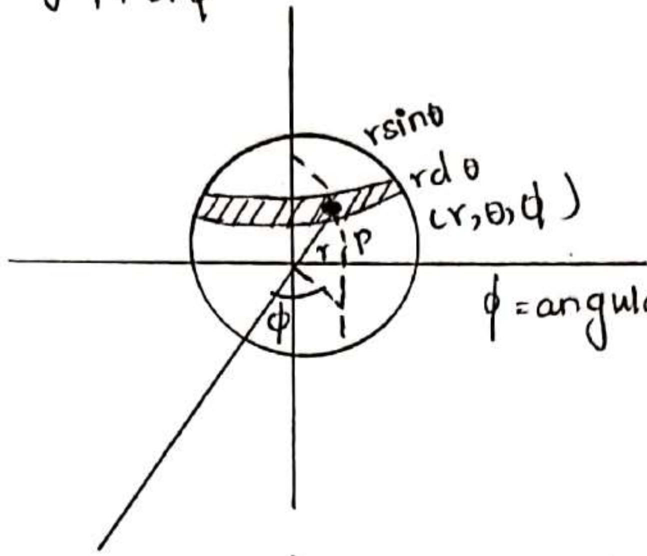
$$= \frac{(I m\, dl \sin\theta)^2}{16\pi^2 \varepsilon c} \left[\frac{4\pi^2 f^2}{2c^2 r^2}\right]$$

$$= \eta \frac{(I m\, dl \sin\theta)^2}{32\pi^2 r^2 c^2} \left[4\pi^2 f^2\right]$$

$$= \eta \frac{(I m\, dl \sin\theta)^2}{8\lambda^2 r^2}$$

$$P_r = \frac{377\,(I_m dl \sin\theta)^2}{8\lambda^2 r^2}\ \omega/m^2$$

$$W = \oint P_r\, ds$$



$$ds = 2\pi r^2 \sin\theta\, d\theta$$

$$ds = 2\pi[r\sin\theta]\, r\, d\theta$$

$$W = \oint \frac{377\,(I_m dl \sin\theta)^2}{8\lambda^2 r^2}[2\pi](r\sin\theta)\, r\, d\theta$$

$$W = \eta \frac{I_m^2 \pi}{4}\left[\frac{dl}{\lambda}\right)^2 2\int_0^{\pi} \sin^3\theta\, d\theta$$

$$W = \eta \frac{\pi I_m^2}{4}\left[\frac{dl}{\lambda}\right]^2 2\int_0^{\pi} \sin^3\theta\, d\theta$$

$$\int_0^{\pi} f(x)\,dx = 2\int_0^{a/2} f(x)\,dx$$

$$W = \eta \frac{\pi I_m^2}{4}\left[2\int_0^{\pi/2} \sin^3\theta\, d\theta\right]\left(\frac{dl}{\lambda}\right)^2$$

Hilli's formulae:-

$$\int_0^{\pi/2} \sin^3\theta\, d\theta = \frac{3-1}{3} = 2/3$$

$$W = \eta \frac{\pi I_m^2}{4} \cdot 2 \cdot \frac{2}{3} \left(\frac{dl}{\lambda}\right)^2$$

$$W = \eta \frac{\pi I_m^2}{3} \left(\frac{dl}{\lambda}\right)^2$$

$$W = \frac{120\pi (\pi) I_m^2}{3} \left(\frac{dl}{\lambda}\right)^2$$

$$W = 40\pi^2 I_m^2 \left[\frac{dl}{\lambda}\right]^2$$

$$W = 40\pi^2 (\sqrt{2} I_{rms})^2 \times \left(\frac{dl}{\lambda}\right)^2$$

$$W = 80\pi^2 I_{rms}^2 \left(\frac{dl}{\lambda}\right)^2$$

$$W = (I_{rms})^2 R_r$$

Where $R_r = 80\pi^2 \left(\frac{dl}{\lambda}\right)^2$

R_r = Radiated Resistance

Quality factor:-

$$Q \cdot f = 2\pi \times \frac{[\text{Total Energy stored by the antenna}]}{\text{Energy dissipated per cycle}}$$

$Q \cdot f \downarrow = \Omega_A \uparrow$

∴ The relationship between Q·f and Bandwidth

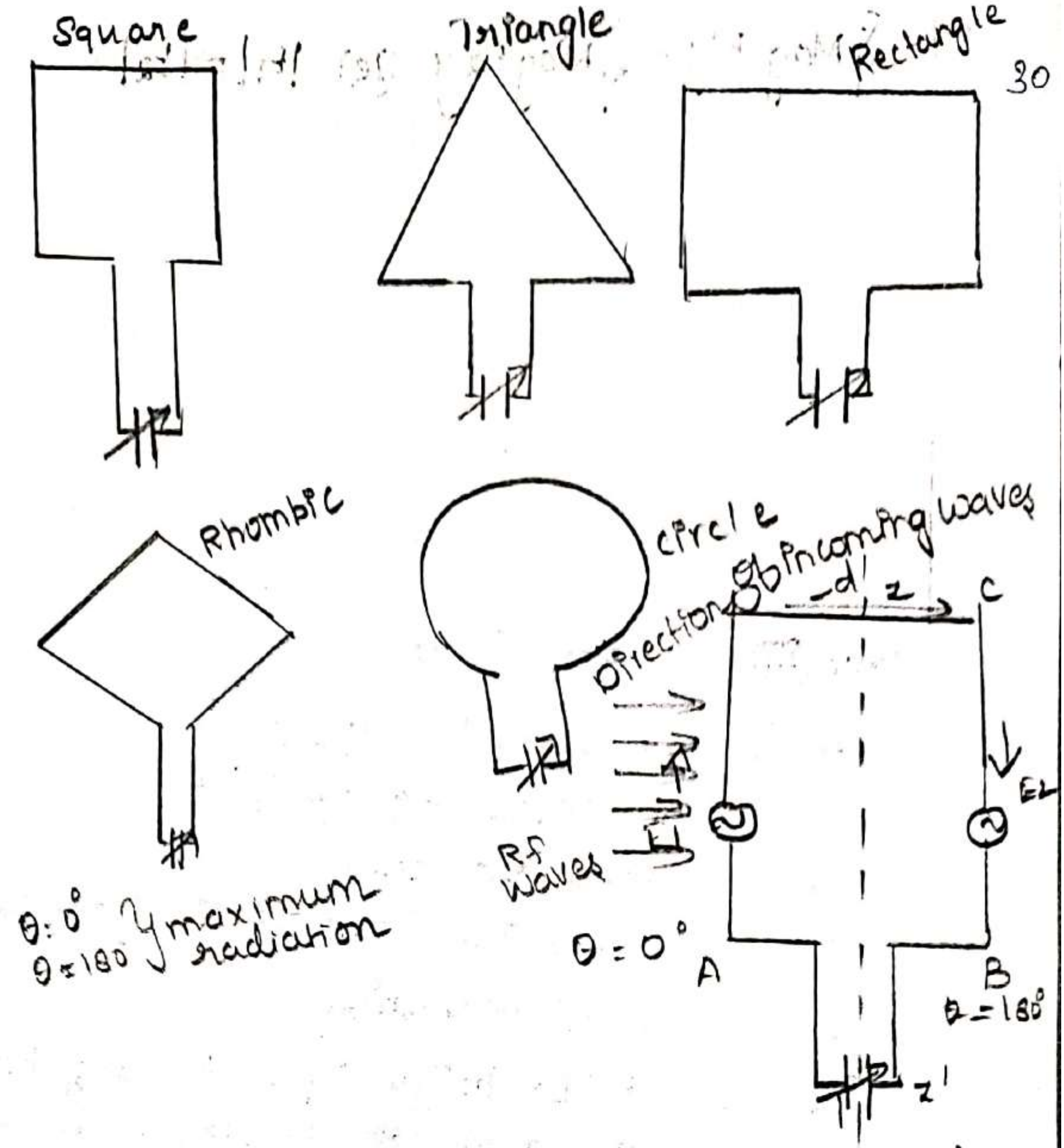
$$\Delta\omega \Rightarrow B \cdot W = \frac{\omega_r}{Q}$$

ω_r = resonant frequency

LOOP ANTENNA :-

It is a radiating coil of any convenient cross section of 1 or more turns carrying Rf current.

It is used for direction finding radio receivers, aircraft and UHf Transmitter.



It has four arms, 2 horizontal arms and 2 Vertical arms. So ABDC act as horizontal antennas.

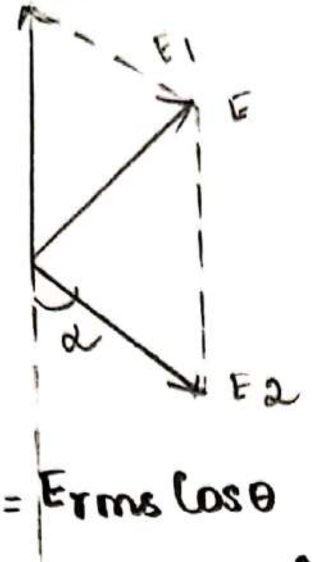
ADBC act as a Vertical antennas.

Case(i) If the plane of the Loop is perpendicular to the direction of Incoming Waves, the same Voltage will be Induced in each [E1 & E2] due to these Voltages the Vertical arms current flow in opposite direction.

Case(ii) If the plane of the loop is inline with the direction of the incoming waves then Voltage Induced at AD and BC is E1 and E2 respectively.

$\{Mag\ E_1\} = \{Mag\ E_2\}$ (ie) $|E_1| = |E_2|$

phase difference is α



Case (iii) $E_\theta = E_{rms} \cos\theta$

E_θ is max. when $\theta = 0°$ (or) $180°$

E_θ is mini when $\theta = 90°$ (or) $270°$

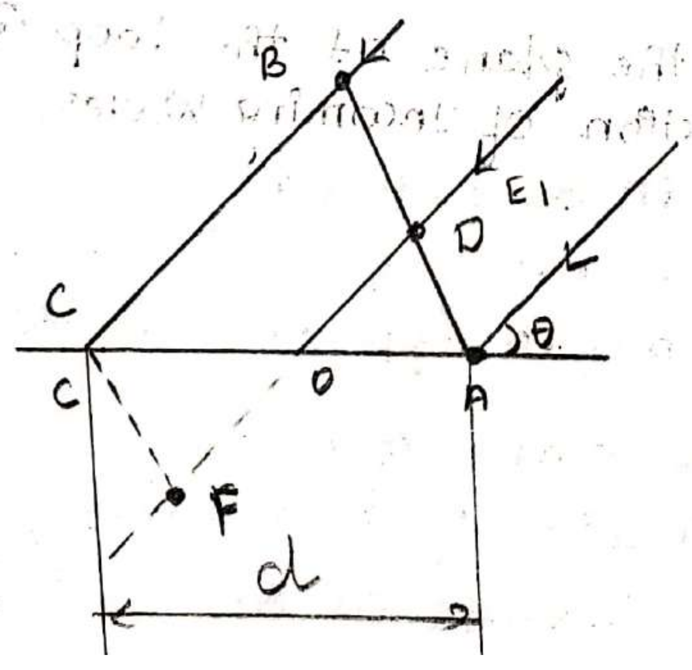
E_θ = max rms loop emf

θ = Angle b/w the plane of the loop antenna and direction of radiation.

E_θ depends on height h, d = spacing b/w

λ = wavelength, w = width, E = Electric field intensity.

EMF [I] Equation Of loop Antenna :-



The Wavefront passes through AC then the Path difference is OD.

Wavefront is received at Various points AOC.

Let At any instant Electric field at O

$$E = E_m \sin\omega t$$

with respect to O the D is leading by path difference α

At f its lagging by α

path diff, $OD = \frac{d}{2}\cos\theta$

phase diff, $\alpha = \frac{2\pi}{\lambda} \cdot \frac{d}{2}\cos\theta = \frac{\pi d\cos\theta}{\lambda}$

phase difference $= \frac{\pi d\cos\theta}{\lambda}$

at D, $E = E_m \sin(\omega t + \alpha)$ leading

at f, $E = E_m \sin(\omega t - \alpha)$ lagging

$E_1 = E_m h \sin(\omega t + \alpha)$ . . . in AD $\rightarrow$ (1)

$E_2 = E_m h \sin(\omega t - \alpha)$ . . . in BC $\rightarrow$ (2)

$e_\theta = E_1 - E_2$

$= E_m h \sin(\omega t + \alpha) - E_m h \sin(\omega t - \alpha)$

$= E_m h [\sin(\omega t + \alpha) - \sin(\omega t - \alpha)]$

$= 2 E_m h \cos\omega t \sin\alpha$

$\sin A \cos B + \cos A \sin B$

$\sin(A+B) - \sin(A-B) = 2\cos A \sin B$

$e_\theta = 2 E_m h \cos\omega t \sin\alpha \rightarrow$ (2a)

Substitute $\alpha = \frac{\pi d\cos\theta}{\lambda}$

$e_\theta = 2 E_m h \cos(\omega t) \sin\left(\frac{\pi d\cos\theta}{\lambda}\right) \rightarrow$ (3)

Assume α = small value then d is much much smaller than λ, then $\sin\alpha = \alpha$

$\therefore e_\theta = 2 E_m h \cos\omega t \left(\frac{\pi d\cos\theta}{\lambda}\right)$

$$e_\theta = \frac{2\pi d E_m h}{\lambda} \cos\omega t \cos\theta$$

$$e_\theta = \frac{2\pi h d \cos\theta}{\lambda} [E_m \cos\omega t] \rightarrow (4)$$

$hd = A$ (Area $= l \times b$)

$$e_\theta = \frac{2\pi A \cos\theta}{\lambda} [E_m \cos\omega t]$$

This is for single turn loop antenna

$$e_\theta = \frac{2\pi A N}{\lambda} \cos\theta [E_m \cos\omega t] \text{ for N turn}$$

Loop antenna. $\rightarrow (5)$

rewritten as

$$e_\theta = \frac{2\pi A N}{\lambda} \cos\theta \left[E_m \sin(\omega t + \pi/2)\right]$$

$N=1$

$$e_\theta = \frac{2\pi A}{\lambda} \left[E_m \cos\omega t \sin(\theta + \pi/2)\right]$$

$$e_\theta = \frac{2\pi A}{\lambda} E_m \cos\omega t \sin(\theta + \pi/2)$$

Equation (5) is the general expression for the instantaneous value of emf at the centre of the loop.

$$V_m = \frac{2\pi A N}{\lambda} E_m \cos\theta \rightarrow (6)$$

$$\frac{V_m}{\sqrt{2}} = \frac{2\pi A N}{\lambda} \left[\frac{E_m}{\sqrt{2}}\right] \cos\theta$$

$$V_{rms} = \frac{2\pi A N}{\lambda} E_{rms} \cos\theta \rightarrow (7)$$

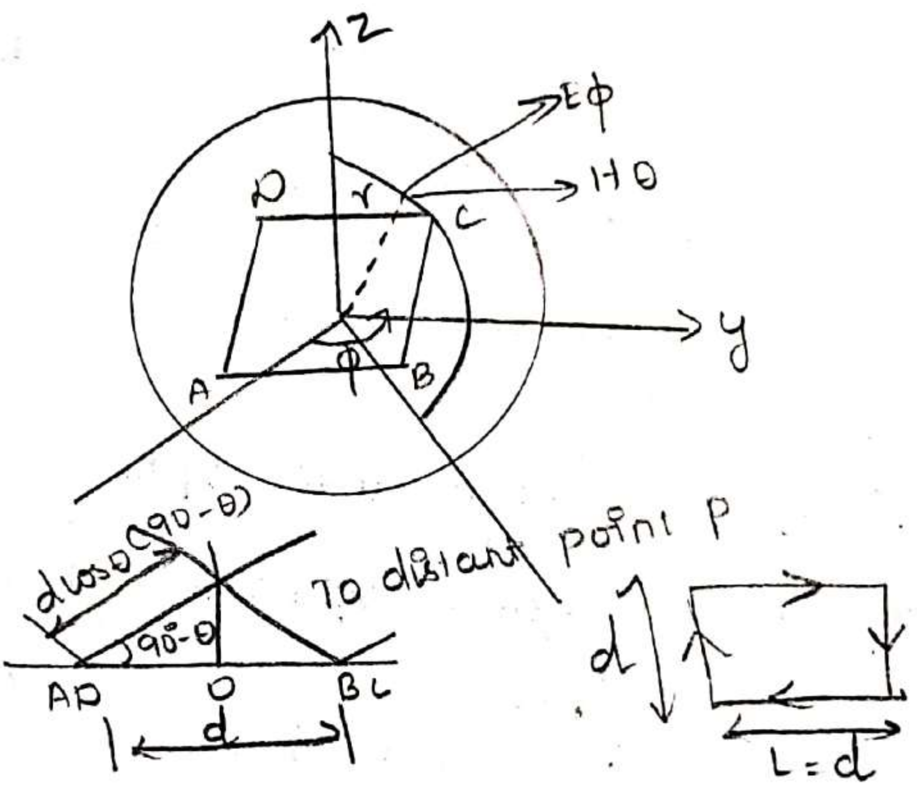
where V_{rms} = rms value of induced emf in the loop [in Volts]

E_{rms} = rms value of electric field strength of the wave in Volts.

λ = Wavelength in meters
A = Area of the loop [m^2]
N = No. of. turns
θ = Angle b/w plane and direction of incoming waves.

$\frac{2\pi AN}{\lambda}$ = Effective length/height of an antenna

(ii) Transmitting the loop antenna :-



$2\pi r = c^2$

field pattern of the circular loop (or)

$d^2 = \pi a^2$

d → side length of the square loop.

$E\phi$ = {field amp due to dipole AD} + {field component due to dipole BC}

$E\phi = (-) E_0 e^{j\psi/2} + E_0 e^{-j\psi/2}$ → (9)

$E_\phi = (-) 2j E_0 \sin(\psi/2)$

$\psi = \beta d \cos(90° - \theta)$

$\psi = \beta d \sin\theta \rightarrow (10)$

$E_\phi = (-) 2j E_\theta \sin\left(\beta d \sin\theta / 2\right) \rightarrow (11)$

(i) Efficiency is very low Used as transmitter

(ii) Current in phase throughout the loop

r is much smaller than d

from Equ (11) E_ϕ = far field of two dipoles

E_0 ⇒ Individual dipole.

The term j indicates the total field E_ϕ is inphase Quadrature w.r.to individual field Component E_0.

fields of the short electric dipole.

Components	General Expression	far field
E_r	$\frac{I L \cos\theta}{2\pi\varepsilon_0}\left[\frac{1}{r^2} + \frac{1}{j\omega r^3}\right]$	0
E_θ	$\frac{I L \sin\theta}{4\pi\varepsilon_0}\left[\frac{j\omega}{c^2 r} + \frac{1}{c r^2} + \frac{1}{j\omega r^3}\right]$	$\frac{I L j\omega \sin\theta}{4\pi\varepsilon_0 c^2 r}$ (or) $\frac{j 60\pi [I] \sin\theta}{r\lambda}$
H_ϕ	$\frac{I L \sin\theta}{4\pi}\left[\frac{j\omega}{c r} + \frac{1}{r^2}\right]$	$\frac{I L j\omega \sin\theta}{4\pi c r}$ (or) $\frac{j I \sin\theta L}{2 r \lambda}$

$$E_\theta = \frac{j60\pi[I]\sin\theta \, L}{r\lambda} \longrightarrow (12)$$

$$E_\theta = \frac{j60\pi[I]L}{r\lambda} \longrightarrow (13)$$

From Equ (12) θ is measured in X axis which is perpendicular to yz plane

Equ (13) becomes retarded current

$$I = I_m e^{j\omega(t - r/c)} \longrightarrow (6')$$

$r << \lambda$ / $d << \lambda$ then $\sin \psi/2 = \psi/2$

$$\psi = \beta d \sin\theta$$

Defn :- Retarded Current.

From Equ (11) we get

$$E_\phi = (-)j E_\theta \sin\left(\frac{\beta d \sin\theta}{2}\right)$$

$$= (-j)\left[\frac{j60\pi[I]L}{r\lambda}\right]\sin\theta$$

$$= (-)j\left[\frac{j60\pi[I]L}{r\lambda}\right]\sin\theta$$

$$E_\phi = \frac{60\pi[I]L\beta d \sin\theta}{r\lambda} \longrightarrow (14)$$

$\therefore L = d \quad d^2 = A \quad \beta = 2\pi/\lambda$

$$E_\phi = \frac{60\pi[I]d^2 \times 2\pi\sin\theta}{r\lambda^2}$$

$$E_\phi = \frac{120\pi^2[I]A\sin\theta}{r\lambda^2} \longrightarrow (15)$$

This is the instantaneous value of the E_ϕ component of the far field of a small loop area A from $E\phi \rightarrow H\theta$

$$\eta = \frac{E\phi}{H\theta}$$

$$\eta = 120\pi$$

$$\eta = \frac{E\phi}{H\theta}$$

$$120\pi = \frac{120\pi^2 [I] A \sin\theta}{H\theta}$$

$$H\theta = \frac{\pi [I] A \sin\theta}{r d^2}$$

Comparison of far field of small loop antenna and Short dipole antenna

field	Electric dipole	loop
EF	$E_\theta = \frac{j60\pi [I] \sin\theta L}{r d}$	$\frac{120\pi^2 [I] A \sin\theta}{r d^2}$
MF	$\frac{jI \sin\theta L}{2 r d}$	$\frac{\pi I A \sin\theta}{r d^2}$

Radiation Resistance of the loop antenna:-

$$P = I^2_{rms} \cdot R_r$$

$$P = \left(\frac{I_m}{\sqrt{2}}\right)^2 R_r$$

$$P = \frac{1}{2} I^2_m \cdot R_r$$

$$P = \frac{1}{2} Re [E \times H^*]$$

$$R_r = 31200 \left[\frac{NA}{d}\right]^2 \Omega$$

$$R_r = 20\pi^2 [C/\lambda]^4 \Omega$$

$$R_r = 197 [C/\lambda]^4 \Omega$$

$$W = 0.682 [C/\lambda]$$

$$A_e = 0.0543 C\lambda$$

Log Periodic Antenna

(Log Periodic Dipole Array Antenna)

[LPDA]

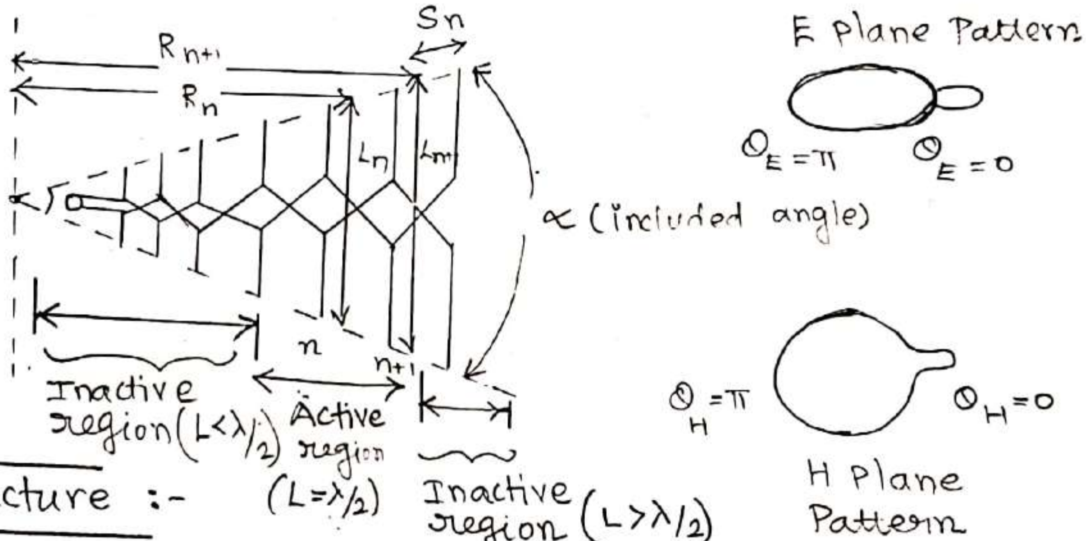
Frequency Independent Antennas.

* LPA are broad-band antennas.

* The electrical properties must repeat periodically with the log of the frequency.

* Freq independence can be obtained when the variation of th properties over one period(over all the periods) is small.

H-Plane View



Design & structure :-

The Design of LPA involves a basic geometric structu that is repeated with a changing size of the structure. The structure size changes with each repetition by a "constant scale factor". i.e the structure expands or contracts by a constant scale factor.

LPDA Structure :-

(1) It has no. of dipoles of diff lengths and spacing.

(2) It is fed by a 2wire balanced tx. line which is transposed betwee each adj pair of dipoles. It is usually fed at narrow enc

(3) All the dimensions increase in proportion to the distance from the origin.

(4) The dipole length also increases along the length of the antenna.

α = included angle

L_n = length of the n^{th} dipole antenna

R_n = distance of the n^{th} antenna from origin.

S_n = spacing between n^{th} antenna & $(n+1)^{th}$ antenna.

L, R, S are related as

$$\frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_4} = \cdots = \frac{R_n}{R_{n+1}} = \tau$$

$$\frac{L_1}{L_2} = \frac{L_2}{L_3} = \frac{L_3}{L_4} = \cdots = \frac{L_n}{L_{n+1}} = \tau$$

i.e

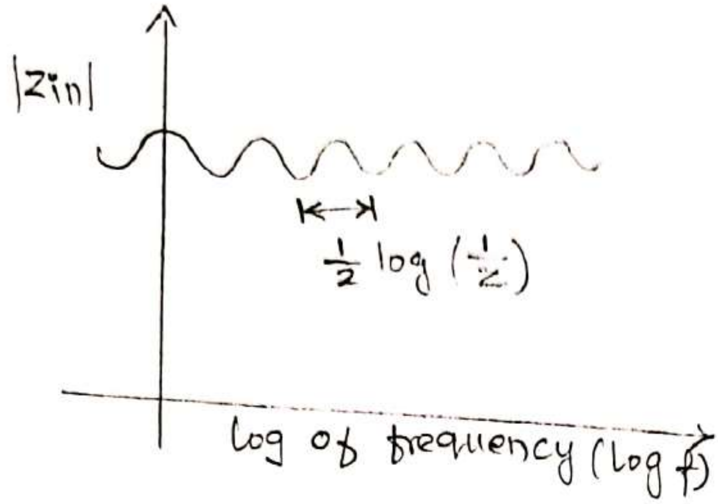
$$\boxed{\frac{R_n}{R_{n+1}} = \frac{L_n}{L_{n+1}} = \frac{S_n}{S_{n+1}} = \tau}$$

τ is called as scale factor (or) design ratio (or) periodicity factor. usually $0 < \tau < 1$.

* usually the ends of the dipoles lie along 2 st lines. These st lines meet at angle α at one end and converge at other end.

* Typical value of $\alpha = 30°$; $\tau = 0.7$.

* In the plot of Z_{in} vs f, a repetitive variation can be observed. If the plot is made against $\log f$, then this variation will be periodic. (i.e) the i/p impedance Z_{in} will go through identical cycles of variations. It is shown below.



* all the electrical properties like radiation pattern, directive gain, side lobe level, beam width undergo simil periodic variation.

If impedance variation occurs at 2 frequecies f_1, f_2 then

$$\log \frac{f_2}{f_1} = \log \frac{1}{\tau}$$

(or)

$$\frac{f_2}{f_1} = \frac{1}{\tau}$$

i.e $f_1 = \tau f_2$ $\quad f_2 > f_1$

whatever properties a log periodic antenna is having at freq f_1, the same properties will be repeated at freq given by $(\tau^n f)$ or at $\frac{f}{\tau^n}$.

[assuming these freq are within cutoff limits of antenna]

* practically LPDA will have cut off freq due to limitations in size, spacing of conductors.

Design of LPDA

1) design ratio (τ) :-

$$\frac{L_n}{L_{n+1}} = \frac{R_n}{R_{n+1}} = \frac{S_n}{S_{n+1}} = \frac{d_n}{d_{n+1}} = \frac{a_n}{a_{n+1}} = \tau \quad \ldots\ldots (A)$$

L_n : length of dipole antenna (nth)

R_n : distance of " " from the origin.

S_n : spacing between antenna n & $(n+1)$.

d_n : diameter of antenna (nth)

a_n : gap spacing at dipole centre of n^{th} antenna.

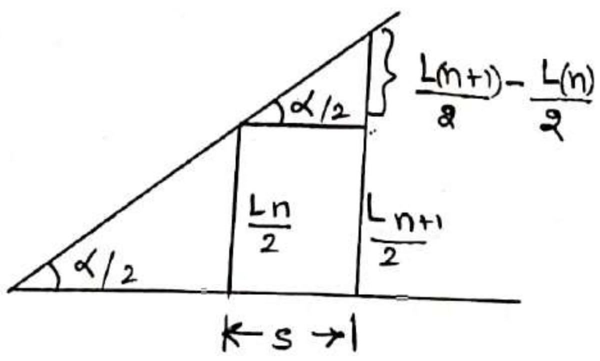
where $n = 1, 2, 3 \ldots\ldots$

2) Spacing factor (σ) :-

$$\sigma = \frac{R_{(n+1)} - R_n}{2L_n} = \frac{S_n}{2L_n} \quad \ldots\ldots (B)$$

i.e $\frac{L_{n+1}}{L_n} = \frac{S_{n+1}}{S_n} = k = \frac{1}{\tau}$ k is a constant.

[at any given frequency, only the fraction of antenna is used. (i.e antenna in active region



Section of LPDA

from the above fig

$$\tan\left(\frac{\alpha}{2}\right) = \frac{\text{opp. side}}{\text{Adj. side}} = \frac{\frac{L_{n+1} - L_n}{2}}{S} = \frac{L(n+1) - L_n}{2S} \quad \ldots (c)$$

from eqn(b),

$$\frac{L_{n+1}}{K} = L_n$$

for active region $L_{n+1} \simeq \lambda/2$

from eqn(c), $$\tan(\alpha/2) = \frac{L_{n+1} - \frac{L_{n+1}}{K}}{2S} = \frac{L_{n+1}\left[1 - \frac{1}{K}\right]}{2S}$$

$$= \frac{\frac{\lambda}{2}\left[1 - \frac{1}{K}\right]}{2S} = \frac{\left[1 - \frac{1}{K}\right]}{4S/\lambda}$$

$$\therefore \tan\left(\frac{\alpha}{2}\right) = \frac{1 - \tau}{4 \cdot (S/\lambda)}$$

$$\Rightarrow \left(\frac{S}{\lambda}\right) = \frac{1 - \tau}{4 \tan(\alpha/2)}$$

i.e $$\boxed{\sigma = \frac{1 - \tau}{4 \tan(\alpha/2)}}$$

(or) $$\tan \alpha/2 = \frac{1 - \tau}{4\sigma}$$

$$\alpha/2 = \tan^{-1}\left[\frac{1 - \tau}{4\sigma}\right]$$

$$\boxed{\alpha = 2\tan^{-1}\left[\frac{1 - \tau}{4\sigma}\right]}$$

α : apex angle

K : Scale factor

$S/\lambda (\sigma)$: spacing in wavelength

also $$\boxed{\tau = 1/K = \frac{S_n}{S_{n+1}} = \frac{L_n}{L_{n+1}}}$$

if out of (σ, α, τ) any 2 are specified, third canbe found.

We know,

$$\frac{L_2}{L_1} = K \quad ; \quad \frac{L_3}{L_2} = K$$

$$\therefore \frac{L_3}{L_1} = K.K = K^2$$

$$\therefore \frac{L_n}{L_1} = K^{(n-1)} \Rightarrow \frac{L_{n+1}}{L_1} = K^n$$

$$\frac{L_{n+1}}{L_1} = F$$ (Frequency ratio or BW.)

Ex:- For optimum Design, for $n=4$

$$K = 1.19$$

$$\therefore F = K^n = (1.19)^4 = 2.0053$$

$$\therefore F \simeq 2$$

$\therefore$ no of elements $= n+1 = 4+1 = 5$.

Hence for 5 elements dipole array & $K = 1.19$, the BW is 2:1.

Analysis of LPDA

There are 3 regions exist for LPDA.

(i) Inactive transmission line region ($L < \lambda/2$)

(ii) Active region ($L \simeq \lambda/2$)

(iii) Inactive reflective region ($L > \lambda/2$)

* radiation from LPDA is always in backward direction.

General characteristics

1) LPDA is fed by a balanced 2-wire tx.line. always excited from the shorter length side or high fre side.

2) Broadband will be with those LPDA which have small variation in periodicity properties.

3) Unidirectional LPDA - radiation is in backward dir. towards shorter element.

Bidirectional LPDA - maximum radiation is in Broad side direction.

4) Tx.line inactive region (between active and vertex) must have proper impedance with negligible radiation.

5) In active region, the currents magnitude and phase should be proper so that pt(3) is satisfied.

Typical value : $\lambda/4$ spacing, 90° phase (unidirectional)
0° phase (bidirectional)

6) In ~~inner~~ inactive reflective region, there should be rapid decay of current. (with in the reflective region).

Applications

(i) HF communication. No power is wasted in terminating resistance.

(ii) LPDA in TV reception. only one LPDA is enough upl UHF band.

(iii) If the cost of installation is not considered, then all round monitoring can be done. [one LPDA will cover all the higher frequency bands].

Co-Ordinate System:

- Cartesian Coordinate system (x, y and z)
- Cylindrical Coordinate system (r, θ and z)
- Spherical Coordinate system (ρ, θ and φ)

Aperture Antenna (Horn Antenna)

Horn antennas are very popular at UHF (300 MHz-3 GHz) and higher frequencies (I've heard of horn antennas operating as high as 140 GHz). Horn antennas often have a directional radiation pattern with a high antenna gain, which can range up to 25 dB in some cases, with 10-20 dB being typical. Horn antennas have a wide impedance bandwidth, implying that the input impedance is slowly varying over a wide frequency range (which also implies low values for S11 or VSWR). The bandwidth for practical horn antennas can be on the order of 20:1 (for instance, operating from 1 GHz - 20 GHz), with a 10:1 bandwidth not being uncommon.

The gain of horn antennas often increases (and the beamwidth decreases) as the frequency of operation is increased. This is because the size of the horn aperture is always measured in wavelengths; at higher frequencies the horn antenna is "electrically larger"; this is because a higher frequency has a smaller wavelength. Since the horn antenna has a fixed physical size (say a square aperture of 20 cm across, for instance), the aperture is more wavelengths across at higher frequencies. And, a recurring theme in antenna theory is that larger antennas (in terms of wavelengths in size) have higher directivities.

Horn antennas have very little loss, so the directivity of a horn is roughly equal to its gain. Horn antennas are somewhat intuitive and relatively simple to manufacture. In addition, acoustic horn antennas are also used in transmitting sound waves (for example, with a megaphone). Horn antennas are also often used to feed a dish antenna, or as a "standard gain" antenna in measurements. Popular versions of the horn antenna include the E-plane horn, shown in Figure 1. This horn antenna is flared in the E-plane, giving the name. The horizontal dimension is constant at **w**.

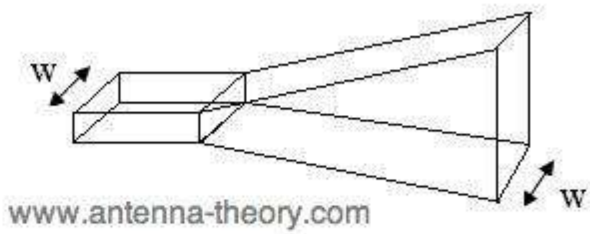


Figure 1. E-plane horn antenna.

Another example of a horn antenna is the H-plane horn, shown in Figure 2. This horn is flared in the H-plane, with a constant height for the waveguide and horn of *h*.

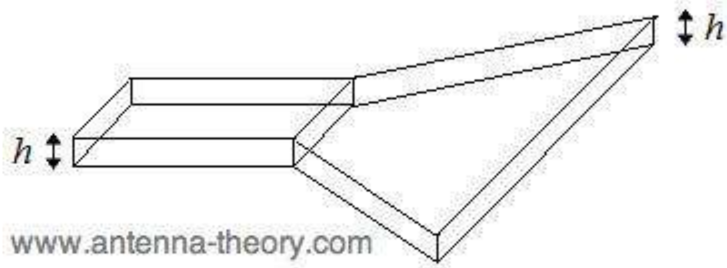


Figure 2. H-Plane horn antenna.

The most popular horn antenna is flared in both planes as shown in Figure 3. This is a pyramidal horn, and has a width *B* and height *A* at the end of the horn.

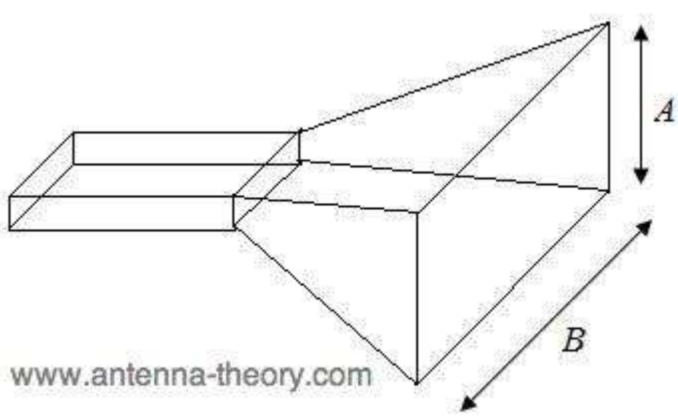


Figure 3. Pyramidal horn antenna.

Horn antennas are typically fed by a section of a waveguide, as shown in Figure 4. The waveguide itself is often fed with a short dipole, which is shown in red in Figure 4. A waveguide is simply a hollow, metal cavity (see the waveguide tutorial). Waveguides are used to guide electromagnetic energy from one place to another. The waveguide in Figure 4 is a rectangular waveguide of width *b* and height *a*, with *b*>*a*. The E-field distribution for the dominant mode is shown in the lower part of Figure 1.

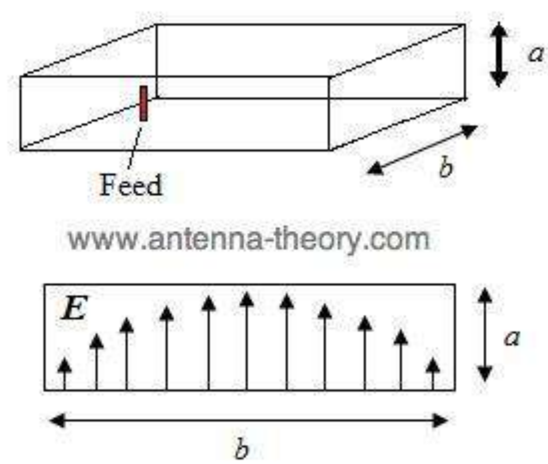


Figure 4. Waveguide used as a feed to horn antennas.

Fields and Geometrical Parameters for Horn Antennas

Antenna texts typically derive very complicated functions for the radiation patterns of horn antennas. To do this, first the E-field across the aperture of the horn antenna is assumed to be known, and the far-field radiation pattern is calculated using the radiation equations. While this is conceptually straight forward, the resulting field functions end up being extremely complex, and personally I don't feel add a whole lot of value. If you would like to see these derivations, pick up any antenna textbook that has a section on horn antennas. (Also, as a practicing antenna engineer, I can assure you that we never use radiation integrals to estimate patterns. We always go on previous experience, computer simulations and measurements.)

Instead of the traditional academic derivation approach, I'll state some results for the horn antenna and show some typical radiation patterns, and attempt to provide a feel for the design parameters of horn antennas. Since the pyramidal horn antenna is the most popular, we'll analyze that. The E-field distribution across the aperture of the horn antenna is what is responsible for the radiation.

The radiation pattern of a horn antenna will depend on B and A (the dimensions of the horn at the opening) and R (the length of the horn, which also affects the flare angles of the horn), along with b and a (the dimensions of the waveguide). These parameters are optimized in order to taylor the performance of the horn antenna, and are illustrated in the following Figures.

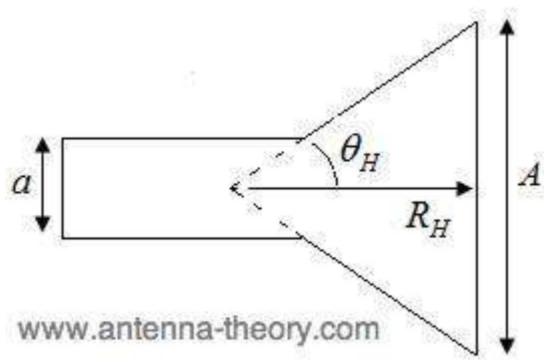


Figure 5. Cross section of waveguide, cut in the H-plane.

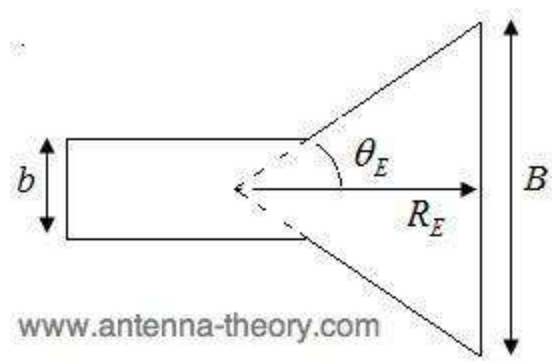
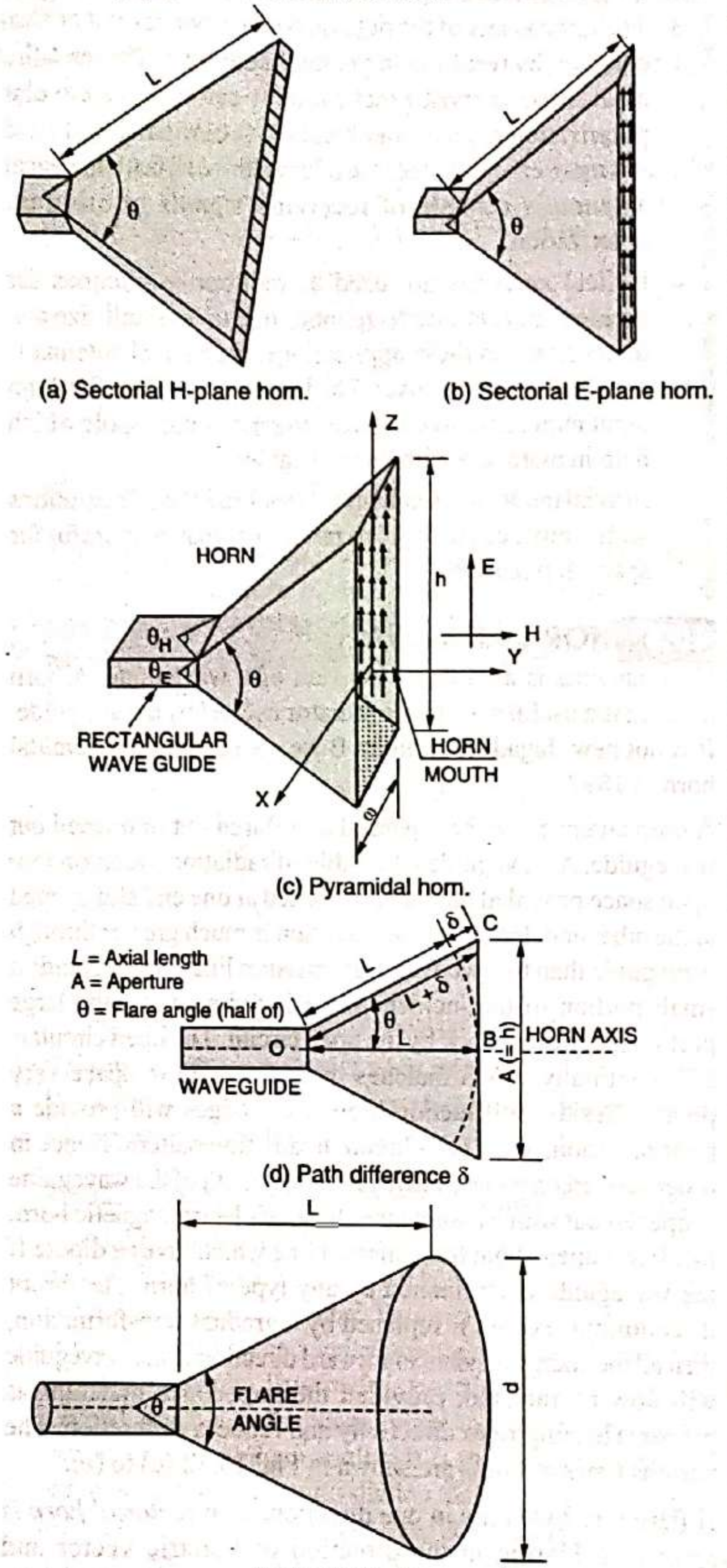


Figure 6. Cross section of waveguide, cut in the E-plane.

Observe that the flare angles (θ_E and θ_H) depend on the height, width and length of the horn antenna.



(a) Sectorial H-plane horn. (b) Sectorial E-plane horn.

(c) Pyramidal horn.

(d) Path difference δ

(e) Conical horn

Fig. 10.32. (a to e) Important horn shapes.

However, this may be treated as transition region where the change over from the guided propagation to free space propagation occurs. Since the waveguide impedance and free space impedance are not equal, *hence to avoid standing wave ratio, flaring of walls of waveguide is done which besides matching of impedance, also provides concentrated radiation pattern i.e. greater directivity and narrower beamwidth. It is the flared structure that is given the name electromagnetic horn radiator.*

The function of the electromagnetic horn is to produce a uniform phase front with a larger aperture in comparison to waveguide and thus the directivity is greater. Although the principle of equality of path length is applicable to horn design but in different sense *i.e.* instead of specifying that the wave over the *plane of the horn mouth is in phase exactly, we allow that phase may deviate but by an amount less than specified amount.* From the geometry of the Fig. 10.32 (*d*), we have

$$\cos\theta = \frac{L}{L+\delta} \text{ and } \tan\theta = \frac{h/2}{L} \text{ or } \tan\theta\, \frac{h}{2L}$$

$$\theta = \tan^{-1}\left(\frac{h}{2L}\right) = \cos^{-1}\left(\frac{L}{L+\delta}\right) \quad ...(10.59)$$

where δ = Permissible phase angle variation expressed as fraction of 360°.

From right angled triangle *OBC* [Fig. 10.30(*d*)]

$$(L+\delta)^2 = L^2 + \left(\frac{h}{2}\right)^2 \text{ or } L^2 + \delta^2 + 2L\delta = L^2 + \frac{h^2}{4}$$

If δ is small, then δ^2 can be neglected

$$2L\delta = \frac{h^2}{4}$$

or

$$L = \frac{h^2}{8\delta} \quad ...(10.60)$$

Eqns. (10.59) and (10.60) give the design equations of the horn antenna. If flare angle (2θ) is very large, the wave front on the mouth of the horn will be curved rather than plane. This will result in non uniform phase distribution over the aperture, resulting increased beam width and decreased directivity, and *vice-versa* occurs directivity is proportional to the aperture size for a given aperture distribution. Thus there is optimum aperture angle given by Eqn. (10.59). The maximum directivity is achieved at the largest flare angle for which in δ are 0.25, 0.32, 0.40 for plane horn, conical horn and H-plane horn respectively.

As customary for E-plane horn, phase difference upto 72° (*i.e.* ± 36° variation) for δ less than 0.20 λ and for H-plane phase difference upto 135° for δ less than 0.375 λ are allowed. In practice 2θ varies from 40° to 15° which gives beamwidth 66°. Directivity 40 for $L = 6\lambda$ and beamwidth 23°, gain 120 for $L = 50\lambda$. Directivity with pyramidal or conical horn antenna increases as they have more than one flare angle. However, the directivity of parabolic antenna is more than the horn antenna. As there is no resonant element involved in the horn antennas hence they can be operated over a broad band of frequency.

Although derivation of exact relation for beam width of horn antenna is possible yet approximate formulae for the half power beamwidth of optimum flare horns are as follow [refer Fig. 10.32 (*c*)].

$$\theta_E = \frac{56\lambda}{h} \text{ degree} \quad ... [10.61\,(a)]$$

and

$$\theta_H = \frac{67\lambda}{\omega} \text{ degree} \quad ... [10.61\,(b)]$$

where θ_E and θ_H are HPBW in *E* and *H* directions. Thus the directivity is given by

$$D = \frac{7.5\, h\cdot\omega}{\lambda^2} = \frac{7.5\, A}{\lambda^2} \quad ... [(10.62)]$$

where $A = h \times \omega$
= area of horn mouth opening (aperture).

and power gain

$$G_P = \frac{4.5\, h \cdot \omega}{\lambda^2} = \frac{4.5\, A}{\lambda^2} \quad \text{... [(10.63)]}$$

10.6.1. Uses of Horn Antenna

Horn antennas are extensively used at microwave frequencies under the condition that power gain needed is moderate. For high power gain, since the horn dimensions becomes large, so the other antenna like lens or parabolic reflector etc. are preferred rather than horns.

10.6.2. Application of Horn Antennas

1. Horn antennas are used as feed element for parabolic reflectors and lenses.
2. Most suitable antennas for various applications in microwave frequency range where moderate gains are sufficient enough.
3. Most widely used for measurement of different parameters in the laboratories like gain etc. It is used for calibration and gain measurement of other antennas and such horn antennas are known as 'Standard gain horn antennas'.
4. The horn is widely used as a feed element for parabolic dishes which are used for large radio astronomy, satellite tracking, and communication dishes found installed all over the world.
5. It is widely used for microwave frequencies (3 GHz and above) because of their moderate gain and low USWR.

10.7. BABINET'S PRINCIPLE AND COMPLEMENTARY ANTENNAS

One may enquire whether there is any relation between wire antenna and aperture antenna, the same can be answered better by first introducing Babinet's Principle of optics. The Babinet's (Ba-bi-nay's) in optics states that "*when the field behind a screen with an opening is added to the field of a complementary structure, the sum is equal to the field when there is no screen*". Babinet principle in optics does not consider polarization, which is so vital in antenna theory. It deals primarily with absorbing screens. An extension of Babinet's principle, which induces polarization and the more practical conducting screens, was introduced by Booker. By introduction of Babinet's principle many of the problems of slot antennas can be reduced to situation involving complementary linear antennas for which solutions have already been obtained.

The Baninet's principle may be illustrated by considering the following example with three cases. Let a source and two imaginary planes be arranged as shown in Fig. 10.35 in which the first plane is a plane of screens S_1 and the plane is a plane of observation S_2. Now three cases arise.

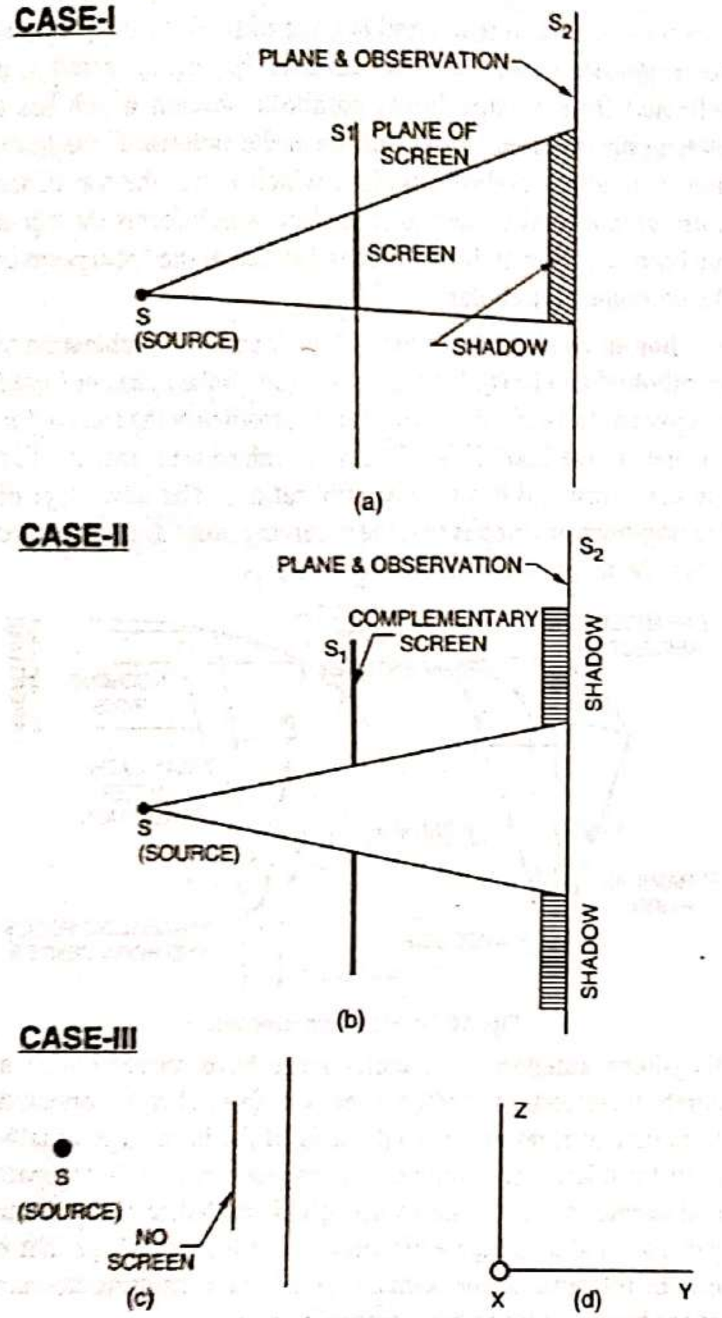


Fig. 10.35. Babinet's principle.

Case I. Let a perfectly absorbing screen be placed in plane S_1 then in plane, there is a region of shadow as shown. Let the field behind this screen be some function of f_1 (x, y, z) *i.e.* be replaced by its complementary screen and the field behind it be given by

$$F_1 = f_1(xyz) \qquad ...(10.64)$$

Case II. Let the first screen S_1 be replaced by its complementary screen and the field behind it be given by

$$F_2 = f_2(xyz) \qquad ...(10.65)$$

Case III. Let there is no screen present, then the field is given by

$$F_3 = f_3(xyz) \qquad ...(10.66)$$

Babinet's principle then states that at the same point $(x\,y\,z)$

$$F_3 = (x\,y\,z) = F_1\,(x\,y\,z) + F_2\,(x\,y\,z) \qquad ...(10.66)$$

or

$$F_3 = F_1 + F_2 \qquad ...(10.67)$$

The source may be a point as in the above example or a distribution of sources. The principle applies not only to point in the plane of observation S_2 as outlined in Fig. 10.34 but also to any point behind screen S_1. The principle is obvious enough for shadow (Case I), it is also true when diffraction is taken into account.

The correctness of this valid statement [Eqn. (10.67)] can be verified easily for the simple cases of complementary screens consisting of semi-infinite absorbing planes.

In electromagnetics at ratio frequencies, thin perfectly absorbing screens are not available, even approximately and one is concerned with *conducting screens and vector fields* for which polarization plays an important role. As such the simple statement of optics could not be expected to apply but an extension of the principle, valid for conducting screens and polarized fields has been formulated by H.G. Booker.

As an illustration of Booker's extension of Babinet's principle, let us consider the following three cases shown in Fig. 10.36. The source (s) in all the three cases is a short dipole, theoretically infinitesimal dipole.

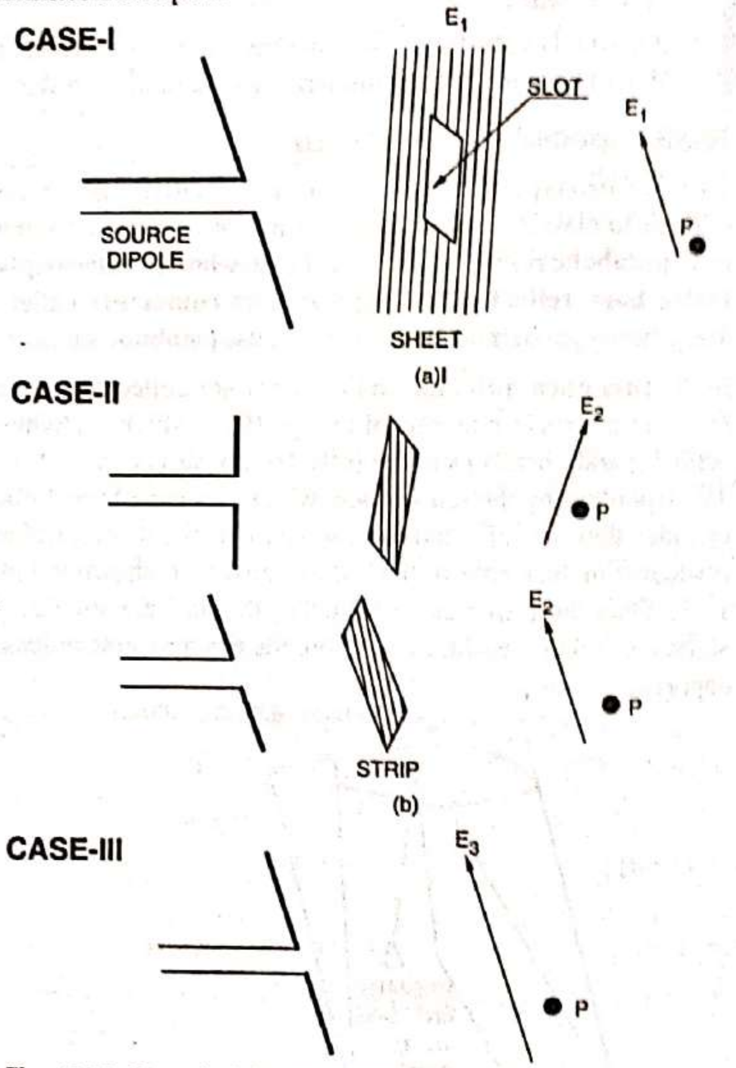


Fig. 10.36. Extension of Babinet principle for slot of infinite metal sheet and the complementary metal strip.

Case I. The dipole is horizontal and original screen is an infinite, perfectly conducting, plane, infinitesimally thin sheet with a vertical slot cut out. At a point P behind the screen the field is E_1.

Case II. In this case the original screen is replaced by the complementary screen consisting of a perfectly conducting, plane infinitesimally thin strip of the **same dimensions as the slot in the original screen.** Besides, the dipole is source and is turned vertical so that $\vec{E}$ and $\vec{H}$ are interchanged. At the same point P, behind the screen the field is E_2.

Alternatively, the dipole source is turned horizontal and so also the strip.

Case III. In this case, no screen is placed and the field at point P is E_3.

According to Baninet's principle

$$E_1 + E_2 = E_3$$

or

$$\frac{E_1}{E_3} + \frac{E_2}{E_3} = 1 \qquad ...(10.68)$$

The principle may also be applied to points in front of the screens, In Case-I, a large amount of energy may be transmitted through the slot so that $E_1 \simeq E_3$. In such situation the complementary dipole (Case-II) acts like a reflector and E_2 is very small.

Using Booker's extension, it can be shown that if a screen and its complementary are immersed in a medium with an intrinsic impedance η and have terminal impedances of Z_S (screen) and Z_C (complementary) respectively, then the impedances are related by

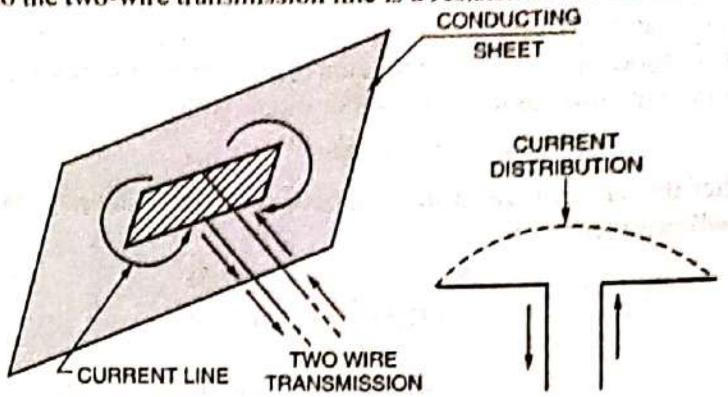
$$Z_S Z_C = \frac{\eta^2}{4} \qquad ...(10.69)$$

In order to obtain the impedance Z_C of the complementary dipole in practical arrangement a gap must be introduced to represent the feed points.

10.8. SLOT ANTENNAS

The slot antenna, as its name suggests, is a simply an opening cut in a sheet of conductor which is energized in some appropriate manner, such as via a coaxial cable or waveguide. One simple type of slot antenna is a half wavelength long with narrow width and excited via a 50 ohm coaxial cable normally connected about 0.05λ from one end of the slot to achieve reasonable matching conditions. A horizontal slot so energized produces vertical polarization in the direction normal to the slot, and a vertical slot produces horizontal polarization. Radiation occurs from both sides of the conductive sheet but if the slot is "boxed" with internal dimension of depth $d = \lambda/4$, the radiation is outwards from the opening of the box. A single half wavelength slot in many ways resembles the half wave dipole in terms of gain and radiation except that there is a difference in polarization. In order to enhance the gain and directive properties of the basic slot antenna, it is common to have arrays of slots in a manner similar to the arrays of dipoles. Some VHF transmitters employ cylindrical arrays of slots to produce omni-directional radiation in the horizontal plane with horizontal polarization.

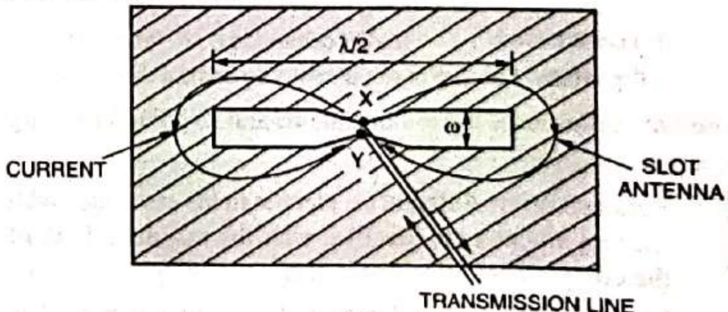
The slot antenna makes use of the fact that energy is radiated when a high frequency field exists across a narrow slot in a conducting plane. A typical slot antenna is shown in Fig. 10.37. Here the fields are excited by a two-wire transmission line. The electric field across the slot is maximum at the centre and tapers off towards the edges as indicated, while at sometime currents flow in the conducting plane in the general manner indicated when the slot is exactly half wavelength long, the electric field distribution is sinusoidal and the impedance offered by the slot to the two-wire transmission line is a resistance of 365 ohm.



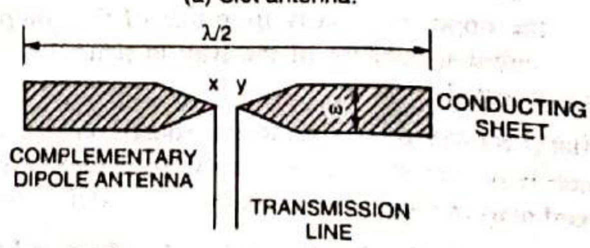
(a) Slot antenna, (b) Complementary wire antenna.

Fig. 10.37.

If a λ/2 slot is cut in a large metal sheet and a transmission line connected to the point XY as shown in Fig. 10.38 (*a*), the arrangement will radiate effectively due to currents flowing on the sheet. The analysis of *such a slot antenna is greatly facilitated by considering the slot's complementary antenna.* Therefore, the antenna which is complementary to the slot of Fig. 10.38 (*a*) is the dipole of Fig. 10.38 (*b*). The metal and air regions of the slot are interchanged for the dipole. According to the G.Booker's theory the pattern of the slot of Fig. 10.38 (*a*) is identical in shape to that of the dipole of Fig. 10.38 (*b*) **except that the electric field will be vertically polarized for the slot and horizontally polarized for the dipole.** Besides, the terminal impedance Z_s of the *slot* is related to the terminal impedance of *dipole* (Z_d) by intrinsic impedances η_0 of free space by the relation



(a) Slot antenna.



(b) Complementary dipole antenna.

Fig. 10.38. Slot and complementary dipole antenna of long λ/2 and width w, fed at point XY.

$$Z_s Z_d = \frac{\eta_0^2}{4} \quad ...(10.70)$$

$$Z_s Z_d = \frac{(376.7)^2}{4} = \frac{141902.89}{4} = 35475.722$$

or
$$Z_s \simeq \frac{35476}{Z_d} \text{ Ohms} \quad ...(10.71)$$

Hence by knowing the properties of dipole antennas, the properties of the complementary slot antenna can be determined. For example, let the width of the dipole and slot of Fig. 10.38 be reduced to a very small fraction of a wavelength so that the dipole qualities as a thin λ/2 linear dipole with

$$Z_d = 73 + j\,(42.5) \text{ Ohms}$$

then the terminal impedance of the complementary slot antenna will be given by

$$Z_s = \frac{35476}{73 + j\,(42.5)} \times \frac{73 - j\,(42.5)}{73 - j\,(42.5)}$$

$$= \frac{35476}{(73)^2 + (42.5)^2}(73 - j\,42.5)$$

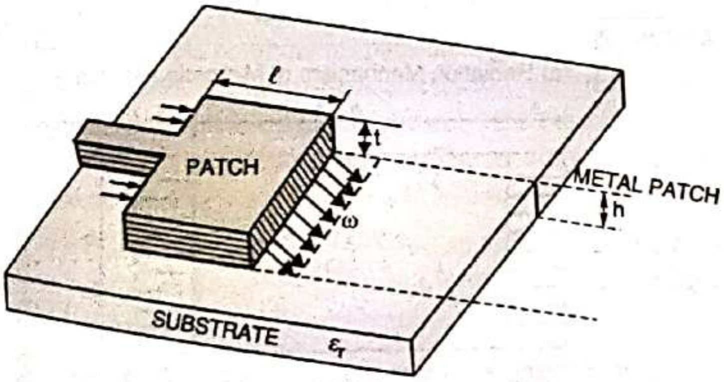
$$= 4.9719\,(73 - j\,42.5)$$

$$Z_s \simeq 363 - j\,211 \text{ Ohms}$$

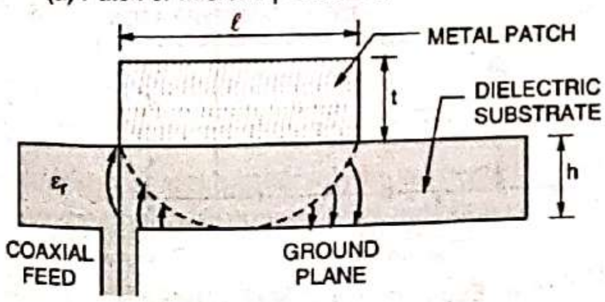
10.10. MICROSTRIP OR PATCH ANTENNAS

In spacecraft or aircraft applications, where size, weight, cost, performance, ease of installation, and aerodynamic profile are constraints, low profile antennas are required. In order to meet these specifications microstrip or patch antennas are used. These antennas can be flush-mounted to metal or other existing surfaces and they only require space for the feed line which is normally placed behind the ground plane. The major disadvantages of patch or microstrip antennas are their inefficiency and very narrow frequency bandwidth which is typically only a fraction of a percent or at the most a few percent.

Microstrip or patch antennas are popular for low profile applications at frequencies above 100 MHz (or $\lambda_0 < 3$ m). They usually consist of a rectangular metal patch on a dielectric-coated ground plane (circuit board). Microstrip antennas consist of a very thin metallic strip (patch) ($t << \lambda$) placed on a small fraction of wavelength ($h << \lambda$) above a ground plane. The strip (patch) and the ground plane are separated by a dielectric sheet referred to as the substrate, Fig. 10.49. The radiating element and the feed lines are normally photoetched on the dielectric substrate. The radiating patch may be square, circular, elliptical, rectangular or any shape. However, square, circular or rectangular are mostly preferred because of the ease of analysis and fabrication and their attractive radiation characteristics, especially low cross polarization radiation.. The feed line is also a conducting strip normally of smaller width. Coaxial line feeds where the inner conductor of the coaxial line is attached to the radiating patch are widely used. Linear and circular polarization can be achieved with microstrip or patch antennas and arrays of microstrip elements with single or multiple feeds may be used for greater directivity.



(a) Patch or Microstrip antenna.



(b) Side view of patch antenna with feed at the left edge.

Fig. 10.49.

As the thickness of the Microstrip is normally very small, the waves generated within the dielectric (substrate between the patch and the ground plane) undergo reflections to some extent when they arrive at the edge of the strip, resulting in radiation of only small fraction of the incident energy. Therefore, the antenna is considered to be very inefficient and it behaves more like a cavity rather than a radiator.

The patch antenna acts as a resonant $\lambda/2$ parallel plate microstrip transmission line with characteristic impedance equal to the reciprocal of the number n of ***parallel field cell transmission lines.*** Each field transmission line has a characteristic impedance Z_0 equal to intrinsic impedance of the medium *i.e.*

$$Z_0 = \eta_i = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$$[\because \mu = \mu_0 \mu_r, \quad \varepsilon = \varepsilon_0 \varepsilon_r]$$

$$Z_0 = 120\pi \sqrt{\frac{\mu_r}{\varepsilon_r}} \qquad \text{... [10.82 (a)]}$$

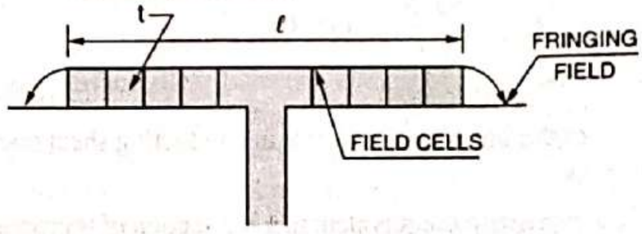


Fig. 10.50. Microstrip antenna with gap or slot divided into n-square field cells.

It is obvious from the patch (from left to right, Fig. 10.50), that the cross-section has 10 field cells transmission lines, hence for $\varepsilon_r = 2$ the characteristic impedance of patch antenna is given by

$$Z_c = \frac{Z_0}{\eta\sqrt{\varepsilon_r}} = \frac{376.7}{10\sqrt{2}} = 26.63 \text{ ohm} \qquad \text{...(10.83)}$$

Eqn. (10.83) can be written as

$$Z_c = \frac{Z_0\, t}{\ell\sqrt{\varepsilon_r}} \qquad \left[\because n = \frac{\ell}{t}\right] \qquad \text{...(10.84)}$$

which is he general expression for Z_c.

In the Eqn. (10.84), fringing effects of the field at the edges has been neglected. As ω is typically even much larger than the t, the fringing effect is small for a patch. However, for a microstrip transmission line, where the ratio (ℓ/t) is smaller, the fringing effect can be accounted for by adding 2-cells, giving a more accurate formula for microstrip line impedance,

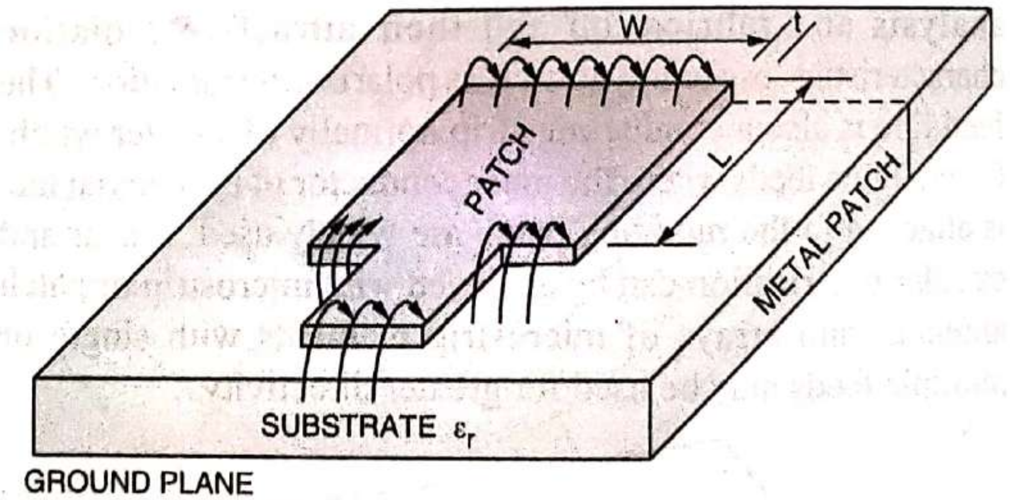
$$Z_c = \frac{Z_0 t}{\sqrt{\varepsilon_r}\,\ell} = \frac{Z_0}{\sqrt{\varepsilon_r}\,\ell/t} \qquad \text{...[10.84 (a)]}$$

$$Z_c = \frac{Z_0}{\sqrt{\varepsilon_r}\,[\ell/t + 2]}$$

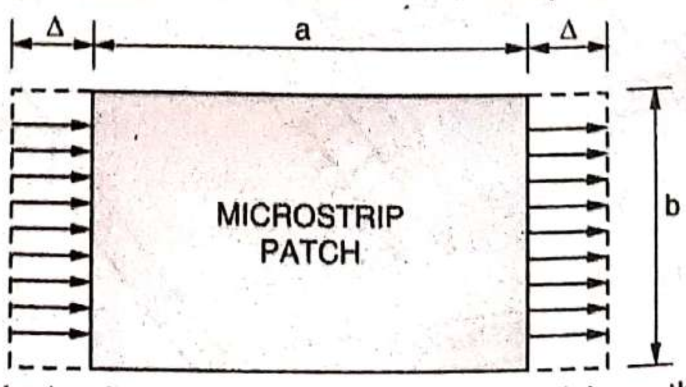
The resonant length l of the patch is critical and typically a couple of percentage less than $\lambda/2$, where λ is the wavelength in the dielectric $\left(\lambda = \frac{\lambda_0}{\sqrt{\varepsilon_r}}\right)$. Radiation from the patch occurs as if from 2-slots [Fig. 10.49 (*b*)]. The impedance can be calculated for a case where dielectric constant is air ($\varepsilon_r = 1$). It

Discontinuities change the electric and magnetic field distributions. Therefore, these result in energy storage and sometimes radiation at the discontinuities. As long as the physical dimensions and relative dielectric constant (ε_r) of the line remain constant, virtually there is no radiation. **However, the discontinuity introduced by the rapid change in line width at the junction between the feed line and patch radiates. Not only this, the other end of the patch where the metallization abruptly ends also radiates.**

When the fields on a microstrip line encounter an abrupt change in width at the input to the patch, electric fields spread out. It creates fringing fields at this edge as shown in Fig. 10.49. After this transition the patch looks like another microstrip line. The fields propagate down this transmission line until the other edge is reached. At this point, the abrupt ending of the line again creates fringing fields as for the open end discontinuity. The fringing fields store energy. The fringing fields store energy. The edge appear as capacitors to ground as the changes in the electric field are greater than that for the magnetic field. As the patch is much wider than a typical microstrip line, the fringing fields also radiate, which is represented by conductance in shunt with the edge capacitance. This accounts for power lost due to radiation as shown in Fig. 10.51.



(a) Radiation Mechanism of Microsrip antenna



(b) Fringing fields at the input and output of the path

Realization of a microstrip like antenna integrated with microstrip transmission line was developed in 1953 by **Deschamps.** Microstrip antenna design was patented by **Gutton and Baissinot** by 1955. Development of microstrip transmission line analysis and design continued in the mid to late 1960's by **Wheeler and Purcel *et al.* Denlinger in 1969 noted that rectangular and circular microstrip resonator could radiate efficiently.** Microstrip antenna concept atlast began to receive closer attention in the early 1970s when aerospace applications, such as space craft and missiles, produced the impetus for researchers to investigate the utility of conformal antenna designs.

The geometry of microstrip antenna is shown in Fig. 10.53 and Fig. 10.54. A conductive patch exists along the plane of the upper surface of a dielectric slab. **This area of conductor, which forms the radiating element, is generally rectangular or circular but it may be of any shape.** The dielectric substrate has ground plane on its bottom surface. The widespread use of printed circuits led to the idea of constructing radiating elements and interconnecting transmission lines using the same technology. **Thus, antenna made from patches of conducting material on a dielectric substrate above a ground plane is referred to as microstrip antenna. Microstrip antenna is also often sometimes referred to as Patch antenna.**

The patch is typically of rectangular or circular shape with dimensions of order of one-half wavelength. The radiating patch may also be square, diamond, triangle, ring, thin strip (dipole), circular, elliptical or any configuration (Fig. 10.53.) Microstrip dipoles are attractive because they possess inherently a large bandwidth and occupy less space, which make them attractive for arrays. Arrays of microstrip elements, with single or multiple feeds may also be used to introduced scanning capabilities and achieve greater directivities.

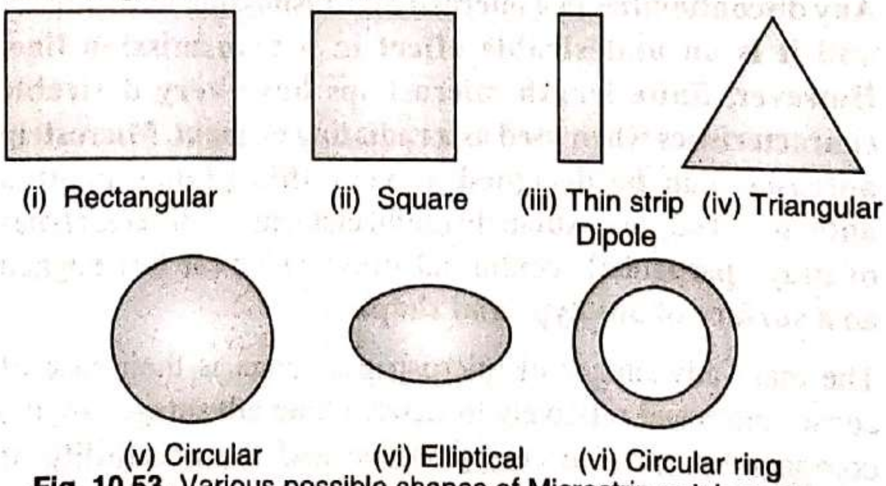


Fig. 10.53. Various possible shapes of Microstrip patch antenna.

The antenna may be fed with microstrip transmission line or a coaxial line as shown in Fig. 10.54. The feed is positioned away from the end by an amount that will give a good impedance match.

10.10.3. Feeding Methods of Microstrip Patch Antennas

1. Microstrip antennas can be fed in a number of ways. These feeding methods can be classified as

   (*a*) Contacting feed

   (*b*) Non-contacting feed.

   In the former method, the RF power is directly fed to **radiating patch with the help of a microstrip** or coaxial **line.**

   In the latter method, electromagnetic coupling is done to transfer the power between the feedline and the radiating patch.

2. There are many configurations that can be used to feed microstrip antenna. The most popular feed techniques are:

   (*a*) Microstrip line
   (*b*) Co-axial probe } ...(contacting scheme)

   (*c*) Aperture coupling
   (*d*) Proximity coupling } ...(non-contacting scheme)

3. **Microstrip feed line** is also a conducting strip, normally of much smaller width as compared to the width of patch. Microstrip feed line is easy to fabricate, simple to match by controlling the inset position. This has the advantages that the feed can be etched on the same substrate to provide planar structure. However as the substrate thickness increases, surface waves and spurious feed radiation increases which for practical design limit the bandwidth (typically 2% to 5%). There are many versions of microstrip feeds

   (*a*) centre feed

   (*b*) offset feed

   (*c*) inset feed.

4. **Co-axial feed or probe feed** is a very common technique employed for feeding microstrip patch antenna. In this the inner conductor of the coax is attached to the radiation patch and the outer conductor is connected to the ground

plane. It is also widely used. The position of the feed can be changed to control the input impedance. The inner conductor of the coaxial connector extends through dielectric and is soldered to the radiating patch.

There are many configurations that can be used to feed microstrip antenna. The most popular feeds are the

Contacting feed

1. Microstrip line feed,
2. Coaxial probe feed,

Non-contacting feed

3. Aperture coupling feed and
4. Proximity coupling feed.

Fig. 10.55 shows several popular feed mechanisms that can be utilized with microstrip antenna. One set of equivalent circuits for each one of the above facts have also been shown in Fig. 10.56. Each feed configuration has its own advantages and disadvantages.

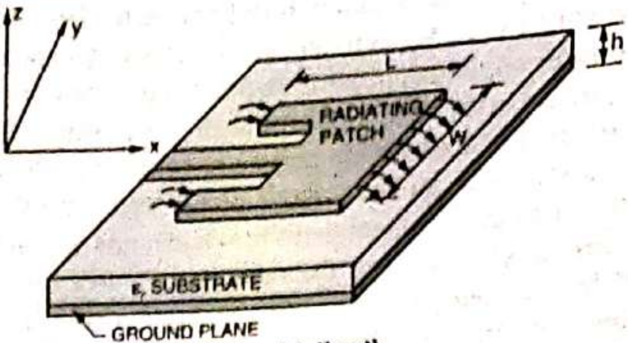
1. Microstrip feed line. [Fig. 10.55 (a). For impedance matching purposes, the offset microstrip line feed is the easiest to use as the offset depth controls the input impedance of the antenna. Moreover, this configuration is simple to fabricate and analyse. Microstrip feedline is also a conducting strip, usually of much smaller width compared to the patch. The microstrip feedline is easy to fabricate, simple to match by controlling the inset position and also simple to the model. However, as the substrate thickness increases surface waves and spurious feed radiation increase, which for practical design limit the bandwidth by 2 to 5%. There are many versions of microstrip feeds (*a*) centre feed (*b*) off-feed, (*c*) Inset feed. It is very common technique employed for feeding microstrip antenna.

2. Coaxial line or probe feed [Fig. 10.55 (*b*)], where the inner conductor of the coaxial is attached to the radiation patch while outer conductor is connected to the ground plane, are also widely used.

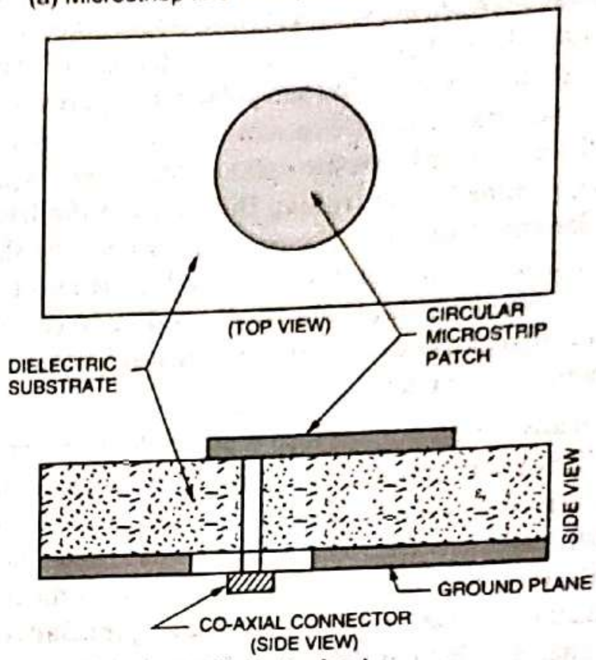
Coaxial probe feed is also easy to fabricate and match and it has low spurious radiation. However, it also has narrow bandwidth and it is more difficult to model, especially for thick substrate ($h > 0.02\ \lambda_0$). Both the **microstrip feed line and the probe** have inherent asymmetries which generate higher order modes leading to produce cross-polarized radiation. To overcome the problem, **non-contacting aperture coupling feeds** as shown in [Fig. 10.55 (*c*), (*d*)] have been used.

The **main advantage** of coaxial feed is that the feed can placed at any desired location inside the patch to match with its input impedance. Its **major disadvantage** is that ground plane coaxial provide a narrow band-width and difficult to model because a hole has to be drilled in the substrate.

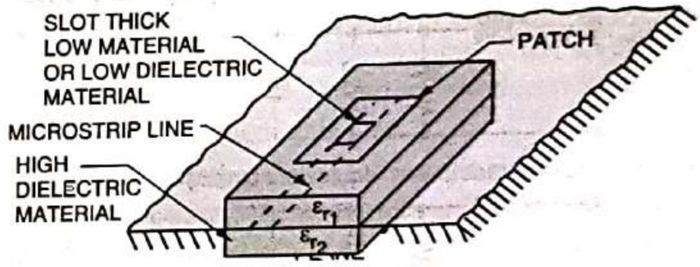
It is also called as electro magnertic coupling scheme. In this two dielectric substances are used such that the feed line is sandwiched between the two and the radiating patch is on the top of upper subatrate as shown in Fig. 10.55(c). The feed is shielded from the antenna by a conducting plane with a hole/slot ti transmit the energy to the antenna.



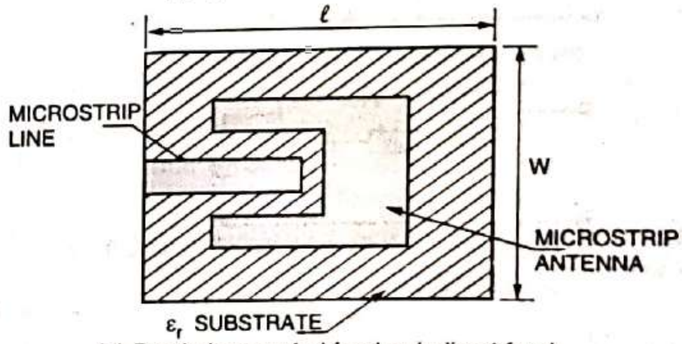
(a) Microstriop line feed (off-set).



(b) Probe-feed.



(c) Aperture-coupled feed.



(d) Proximity-coupled feed or indirect feed.

Fig. 10.55. Various feeds for microstrip antennas.

3. The aperture coupled feed [Fig. 10.55 (*c*)]. It is also called as electromagnetic coupling scheme. In this, two dielectric substrates are used such that the field line is sandwiched between the two and the radiating patch is on the top the upper substrate as shown in Fig. 10.55 (*c*). The feed circuitry is shielded from the antenna by a conducting plane with a slot/hole to transmit energy to the antenna. Aperture coupling of Fig. 10.55 (*c*) is the most difficult methods of all four to fabricate. It has also narrow banwidth. However, it is somewhat easier to model and

0.10.4. Advantages of Microstrip Antenna

'he main advantage of microstrip antenna are

1. Low cost fabrication.
2. Can easily conform to a curved surface of a vehicle or product.
3. Many designs readily produce linear or circular polarization.
4. Considerable range of gain and pattern options (2.5 to 10 dB) available.
5. Antenna thickness (profile) is small.
6. Other microwave devices in microstrip may be integrated with a microstrip antenna with no extra fabrication steps ($v|z$ branch line hybrid to produce circular polarization or corporate feed network for an array of microstrip antenna).
7. Microstrip antennas meet the prime needs *i.e.* small size, low weight and hence are easy to manufacture on mass scale with low manufacturing cost. These can be directly applied to metallic surface on aircraft, missile and do not disturb aerodynamic flow and thus have better aerodynamic properties. Thus, these antennas are replacing of old and bulky aerospace vehicles.

0.10.5. Disadvantage of Microstrip Antenna

he main disadvantages of microstrip antennas are :

1. Narrow bandwidth (5% to 10%, VSWR 2:1) is typically without special techniques.
2. Sensitivity to environmental factors like temperature and humidity.
3. Dielectric and conductor losses can be large for thin patches leading to poor antenna efficiency.
4. Low power handling capability.
5. Poor end-fire radiation characteristics and limited gain.

Characteristics:-

Different dielectric substrates can be Used in the microstrip antennas.

The Value of dielectric constant Varies from $1 \leq \varepsilon_r \leq 13$.

Substrate Material	ε_r
1) Air	1
2) PTFE / glass	2.2
3) Roger's RT Duroid	2.26
4) FR-4	4.0 – 4.8
5) Alumina	9.6 - 10
6) Sapphire	9.4
7) GaAs Gallium Arsenoid	11 - 13
8) Silicon (Si)	12

It provides larger bandwidth, better efficiency.

The thin substrate are Used to small size of antenna.

Definition :-

The antenna which is made up of metal patches placed on dielectric and fed by microstrip (or) Coplanar Transmission line is called microstrip antenna (or) patch antennas

Types of planar Transmission line:-

(i) Slot line
(ii) Strip line
(iii) Coplanar Wave Guides (cpw)

Construction:-

(i) Thickness of microstrip is very small compared to free space wavelength ($t \ll \lambda_0$)

(ii) The height of the substrate is very small [$h \ll \lambda_0$], the typical value is [$0.003\lambda_0 \leq h \leq 0.05\lambda_0$]

(iii) The substrate in b/w the patch and the ground plane it is a dielectric sheet.

(iv) The Typical length of the patch is in b/w $\frac{\lambda_0}{3} < L < \frac{\lambda_0}{2}$.

10.10.10. Limitations of Microstrip Antennas

1. The bandwidth of a square or circular patch antenna for a VSWR S can be given by

$$\text{Bandwidth} = \frac{100(S-1)}{\sqrt{S}} \cdot \frac{8}{\varepsilon_r} \cdot \frac{h}{\lambda_0}\% \qquad \text{... (10.99)}$$

This shows that the bandwidth decreases with increase of h *i.e.* **thinner antennas have lesser bandwidth.**

2. The feed structure of these antennas is usually printed on the substrate substance together with radiating elements. The feeder lines, therefore, introduces additional loss, thereby reducing the efficiency.
3. The mechanical tolerances of thin microstrip antenna normally place a limit on the precision with which the aperture phase and amplitude distribution can be controlled in manufacture.
4. Practical limitations on maximum gain (nearly 20 dB)
5. Poor end-fire radiation performance.
6. Low power handling capability.
7. Possibilities of excitation of surface waves.

Some of the above limitations may be overcome by

1. Using thick sustrate
2. Cutting slots in the metallic patch
3. Introducing parasitic patches either on the top of the main patch on the same layer
4. Using aperture coupled stacked patch antenna.

10.10.11. Applications of Microstrip Antennas

1. Microstrip antenna (MSA) are gaining popularity for use in wireless applications because of their low-profile structure.
2. They are extremely compatible for embedded antenna in handheld wireless devices like mobile phone. (cellular) and pagers.
3. Telemetry and communications antennas on missiles required to be thin and conformal and are usually microstrip antennas.
4. It is used in satellite communication because of their small size and low profile features.
5. It has widespread use in microwave and millimeter wave systems.
6. These are employed in airborne and spacecraft systems because of their low profile and conformal nature.
7. In phased arrays radars, where low profile antennas are needed and bandwidths less than a few percent are tolerable, microstrip antennas are quite popular.
8. A large number of commercial requirements are met by the use of microstrip and printed antennas. The most popular microstrip antenna is certainly the rectangular patch. **The Global Positioning System (GPS) has** become abiquitous in its applications.
9. GPS applications such as the asset tracking of vehicles and marine use have created a large demand for these antennas.
10. Satellite Digital Auto Radio Services (SDARS) have become a viable alternative to AM and FM commercial broadcasts in automobiles.

10.21. ANTENNA WITH PARABOLIC REFLECTORS

10.21.1. Beam Formation by Parabolic Reflectors

A parabola may be defined *as the locus of a point which moves in such a way that its distance from the fixed point (called focus) plus its distance from a straight line (called directrix) is constant.* A parabola with focus F and vertex O is shown in Fig. 10.85. The Parabola is a two-dimensional plane curve.

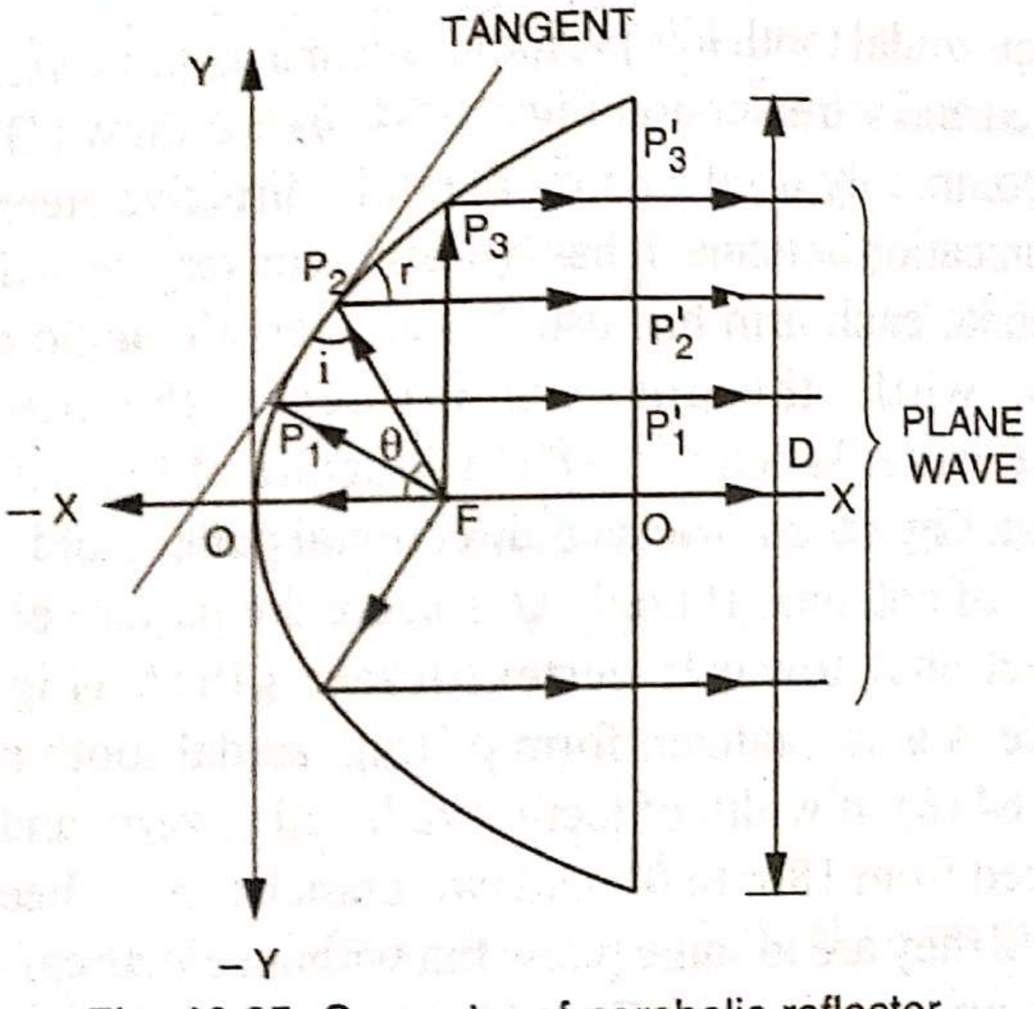


Fig. 10.85. Geometry of parabolic reflector.

OF = Focal length = f

K = A constant which depends on the shape of Parabola curve

F = Focus

O = Vertex

OO' = Axis of parabola.

By definition of parabola, apparently,

$$FP_1 + P_1P_1' = FP_2 + P_2P_2' = FP_3 + P_3P_3'$$
$$= \text{constant (say, } K) \quad ...(10.152)$$

The equation of Parabola curve interms of its coordinate is given by

$$Y^2 = 4fx \quad ...[(10.153(a)]$$

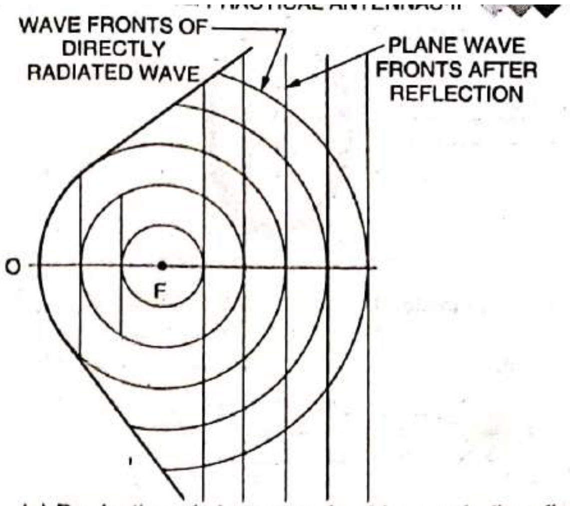
The open mouth (D) of the parabola is known as the **Aperture.** The ratio of focal length to Aperture size (*i.e.* f/D) known as "**f over D ratio**" is an important characteristic of parabolic reflector and its value usually varies between 0.25 to 0.50.

Focussing or beam formation action of parabolic reflector can be understood by considering a source of radiation at the focus. Let a ray start from the focus (F) at an angle θ w.r.t. parabolic axis (OO'). The curve strikes at point P_2 on the parabola curve. Let a tangent is drawn at P_2 on the curve. According to law of reflection, the angle of incidence ($\angle i$) and angle reflection ($\angle r$) will be equal as shown. This results the reflected ray in the reflected ray being is parallel to the parabolic axis, regardless of the particular value of θ that may be considered. *In other words, all the waves originating from focus will be reflected parallel to the parabolic axis. This implies that all the waves thus, reaching at the aperture plane are in phase.* This shows that a wavefront–a surface of constant phase–is created in the aperture plane. Therefore, the rays are parallel to the parabolic axis, because rays are always perpendicular to a wavefront. *Since all the waves are in phase, so a very strong and concentrated beam of radiation is there along the parabolic axis.*

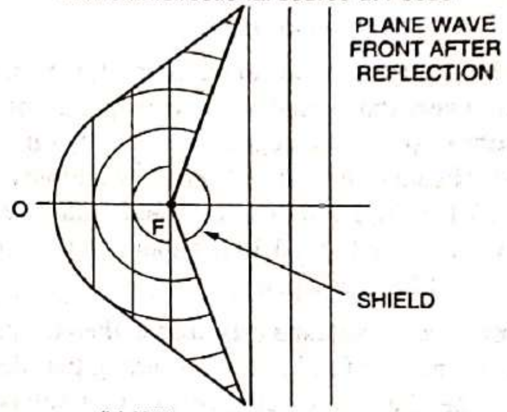
Alternatively, all the waves emanating from the source at focus and reflected by parabola are travelling the same distance (because distances are equal by Eqn. 10.152) in same time in reaching the directrix and hence they are in phase. The principle of equality of path length is maintained between all rays of two wavefronts. Putting in another way where there is path length difference between the two rays cancellation action will take place. *Hence the geometrical properties of parabola provide excellent microwave reflectors that lead to the production of concentrated beam of radiation.*

In fact, parabola converts a spherical wavefront coming from the focus into a plane wavefront at the mouth of the parabola as illustrated in Fig. 10.86. The part of the radiation from the focus which is not striking the parabolic curve as spherical wave appears as minor lobes. Obviously this is a waste of power. This is minimized by partially shielding the source as shown in Fig. 10.86 (b).

Further if a beam of parallel rays is incident on the parabolic surface, they will be focussed at a point *i.e.* Focus. This is in effect due to the principle of reciprocity theorem already discussed which says that properties of an antenna are independent whether it is for transmission or reception, the parabolic reflector is directional for reception case also as only rays coming perpendicular to directrix will be focussed at the focus and not others due to path length difference (Fig. 10.87). Parallel rays are known as *collimated.*



(a) Production of plane wavefront by parabolic reflector with omnidirectional source at Focus



(b) With partially shield source

Fig. 10.86.

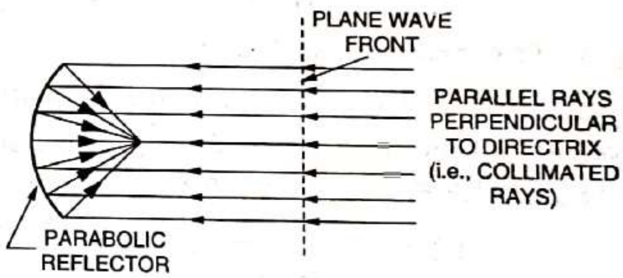


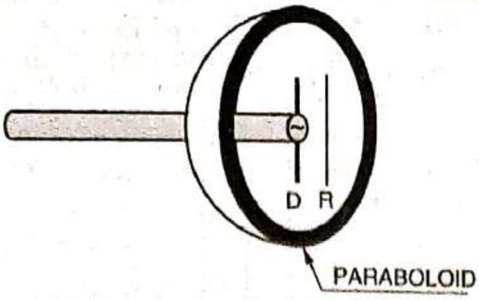
Fig. 10.87. Focussing by a parabolic reflector (Receiving Case.)

10.21.2. Paraboloidal Reflector or Microwave Dish

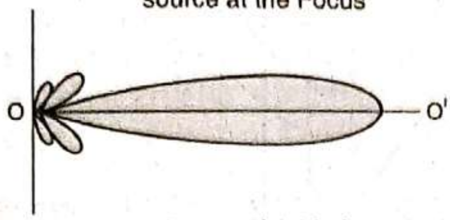
A parabola is a two-dimensional plane curve. A practical reflector is a three-dimensional curved surface. Therefore a practical reflector is formed by rotating a parabola about its axis (OO'). The surface so generated is known as ***Paraboloid*** which is often called ***microwave dish*** or ***Parabolic reflector*** (Fig. 10.88). Paraboloid produces a parallel beam of circular cross-section, because the mouth of the paraboloid is circular. If a third Cartesian coordinate z has its axis perpendicular to both x-axis and y-axis in Fig. 10.88, then equation of paraboloid will be

$$y^2 + z^2 = 4fx \quad ... [10.153 (b)]$$

The intersection of any plane perpendicular to x-axis with the paraboloid surface is a circle. In the conventional automobile, (*e.g.* motor-car headlight, or in search light), this beam forming property is utilized.



(a) Full paraboloidal reflector with dipole source at the Focus



(b) Radiation pattern of a paraboloid of aperture $D = 10\ \lambda$.

Fig. 10.88.

The radiation pattern on an antenna employing paraboloid reflector has a very sharp major lobe accompanied by a number of minor lobes which, of course, are smaller in size. The narrow major beam is in the direction of paraboloid axis shown in Fig. 10.88 (*b*). The three-dimensional pattern is a figure obtained by revolving Fig. 10.88 (*b*) about OO' and the actual shape would be like a fat cigar.

If the *feed* or *primary* antenna is isotropic, then the paraboloid will produce a beam of radiation. Assuming that the circular aperture is large, the Beamwidth between first null is given by

$$\text{BWFN} = \frac{140\lambda}{D}\ \text{degree} \quad \text{...[10.154 (a)]}$$

where λ = Free space wavelength, in m.

D = Diameter of aperture, in m *i.e.* mouth diameter.

The beamwidth between first nulls for a large uniformly illuminated rectangular aperture is given by

$$\text{BWFN} = \frac{115\lambda}{L}\ \text{degree} \quad \text{... [10.154 (b)]}$$

where L = Length of Aperture, in λ

Also width between Half-power points for a large circular aperture is given by

$$\text{HPBW} = \frac{58\lambda}{D}\ \text{degree} \quad \text{... [10.154 (c)]}$$

Further, the directivity D of a large uniformly illuminated aperture is

$$D = \frac{4\pi A}{\lambda^2} \quad \text{... [10.154 (d)]}$$

and for a circular, aperture

$$D = \frac{4\pi}{\lambda^2}\left(\frac{\pi D^2}{4}\right) = \pi^2\left(\frac{D}{\lambda}\right)^2$$

$$D = 9.87\left(\frac{D}{\lambda}\right)^2 \quad \text{... [10.154 (e)]}$$

where D = Diameter of the aperture, in λ.

10.21.4. Primary and Secondary Pattern

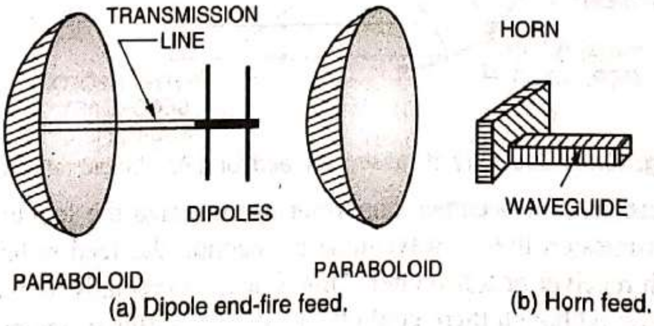
The antenna placed at the **focus** of a paraboloid is known as ***Feed radiator*** or ***primary radiator*** or simply ***feed*** and its radiation pattern is known as **primary pattern**. The parabolic reflector is known as **Secondary radiator** and the radiation pattern of entire antenna system (*e.g.* Reflector and primary radiator) is called as *Secondary pattern.* Sometimes *Antenna pattern* is used for secondary pattern and *Feed pattern* for *Primary pattern.*

10.21.5. Feed Systems

The entire Parabolic reflector antenna consists of two basic components *e.g.* the reflector and a source of primary radiation at the focus. The source is called the primary radiator or feed radiator or simply feed while the reflector, the secondary radiator. Now detailed design of feed is discussed.

An ideal *feed* would be that radiator which radiates towards reflector in such a way that it illuminates the entire surface of reflector and no or zero energy is radiated in any other direction. Of course, such an ideal radiator is not available in practice. Clearly an isotropic antenna as feed would not be a better choice. As far as the secondary radiator is concerned, the best choice is the Paraboloid which is inherited with compactness and simplicity. However, there are a number of choices for primary feed.

Similarly, a dipole antenna is also not very much suitable for the feed but occasionally used. The simplest and generally used is a dipole with parasitic reflector (*i.e.* Yagi-Uda) or a small plane reflector, which is fed with a coaxial line (Fig. 10.88). Typically the spacing between driven element and parasitic element is 0.125 λ and for a plane reflector it may be around 0.4 λ. Besides end fire arrays of dipoles are also used in front of reflector as shown in Fig. 10.95 (*a*). The double diploes are so spaced and phased that end-fire pattern is produced which illuminates the paraboloid reflector. It may be noted, however, that feeding with a dipole involves changing from unbalanced system to a balanced system.



(a) Dipole end-fire feed. (b) Horn feed.

Fig. 10.95.

A most common feed radiation for paraboloid reflector antenna is a '*waveguide horn*' [Fig. 10.95 (*b*)].

The horn feed is waveguide feed. As shown horn antenna (*i.e.* feed antenna) is pointing the paraboloid and radiation pattern of horn antenna is mild, in the same direction. Thus, the direct radiation from the horn (*i.e.* feed) antenna is minimum. Further, if circular polarization is required then, conical horn antenna or helix, antenna can be used as feed at the focus of paraboloid. For getting maximised beam pattern along the parabolic axis, feed is placed at the focus. But if the feed is moved laterally from the focus *i.e.* perpendicular to axis, then beam deteriorate *i.e.*, limited beam motion can be obtained. On the other hand, if the feed is moved along the axis, then the pattern is broadened. Thus important position of feed is the focus and for the small reflector of short focal length the position of feed is as shown in Fig. 10.96.

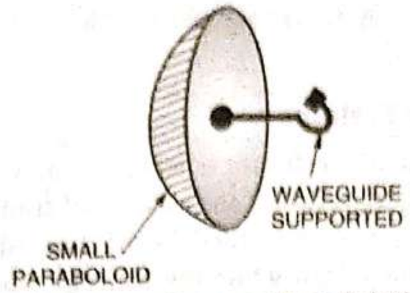


Fig. 10.96. Feed support for a small paraboloid reflector.

10.21.6. Cassegrain Feed

It is named after the name of 18th Century Astronomer and is illustrated in Fig. 10.97 in which the primary feed radiator is positioned around an opening near the vertex of the paraboloid instead of at focus. Cassegrain feed system employs a hyperboloid secondary reflector whose one of the foci coincides with the focus of paraboloid.

The feed radiator is aimed at the *secondary hyperboloid reflector or sub-reflector.* As such, the radiations emitted from feed radiator are reflected from cassegrain secondary reflector which illuminates the main Paraboloid reflector as if they had originated from the focus. Then the paraboloid reflector colliminates the rays (renders parallel) as usual.

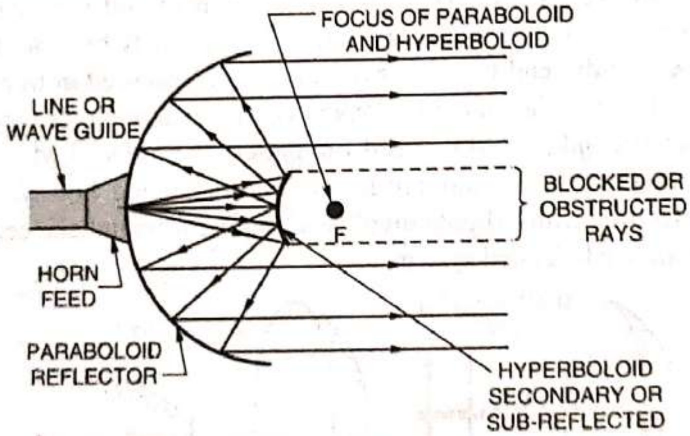


Fig. 10.97. Geometry of cassegrain feed for a paraboloid reflector.

Sometimes, it becomes important to minimize the length of transmission line or waveguide connecting the feed radiator with receiver or transmitter. This is needed specially to avoid losses. Although there could be a solution of this problem by placing the RF Amplifier stage of R_x near the focus which minimizes the losses on reception, but this is not practicable for transmitters, as the RF amplifier of a transmitter is bulky, heavy and having enough power so not possible to place at feed point. *Hence the practical solution in such cases is cassegrain feed when the transmission line or waveguide length between feed and transmitter and receiver, is required to be short.*

The disadvantage of the cassegrain feed is that some of the radiation from the Paraboloid reflector is obstructed. This is tolerable in greater dimension paraboloid but becomes problem with small dimension paraboloid. The dimension of secondary reflector depends on the distance between horn feed and sub-reflector, mouth of horn which in turn depends on frequency. This aperture blocking defect can be avoided by using an *off set reflector* which is applicable to focal point feed shown in Fig. 10.98. The other method is to use a polarization twisting scheme in which hyperboloid reflector is made of wire grating (transparent) instead of solid.

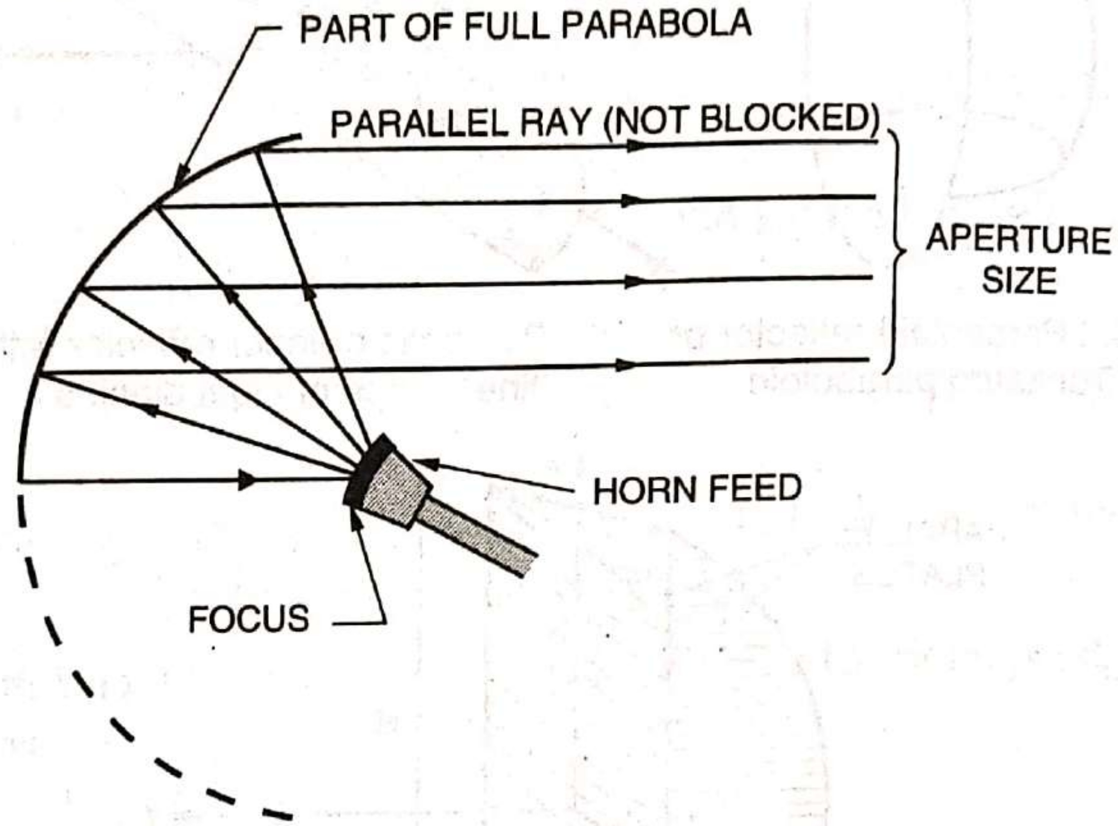


Fig. 10.98. Off-set Paraboloid reflector showing no blocking of rays.

10.21.7. Advantages of Cassegrain Feed

The following are the advantages of cassegrain feed arrangements in general :

1. Reduction in spillover and minor lobe radiation
2. Ability to get an equivalent focal length much greater than the physical length
3. Ability to place the feed in a convenient location
4. Capability for scanning or broadening of the beam by moving one of the reflecting surfaces.

UNIT - III

Antenna Arrays & Applications

Introduction:

In the point to point communication, it is desired to have most of the energy radiated in one particular direction.

A single small antenna like short dipole will not meet this requirement since their radiation is not uniform.

Therefore several antennas of similar type are arranged in a system to radiate more in desired with high gain. This can be achieved by combining the individual antenna radiations in desired direction & canceling the radiation in undesired direction. Such as system is called an antenna array.

The antenna array is a system of similar antennas oriented similarly to get greater directivity in a desired direction

It is defined as " A radiating system consisting of several spaced and properly phased radiators.

* The total field produced by an antenna array system is the vector sum of the fields produced by the individual antennas of the array system.
* The individual antenna of an antenna array system is also termed as Elements.

* The antenna array is said to be linear if the elements of the antenna array are equally spaced along a straight line.

* The linear antenna array is said to be uniform linear array if all the elements are fed with a current of equal magnitude with progressive uniform phase shift along the line.

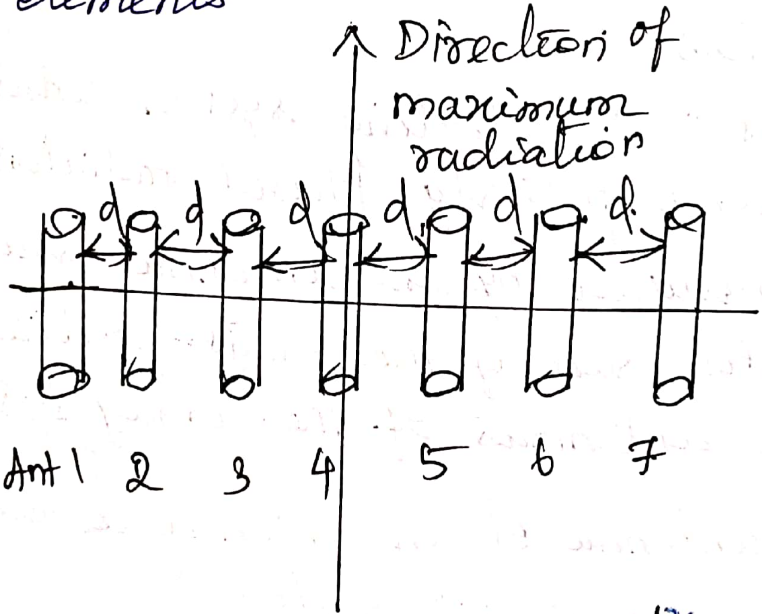
Various forms of Antenna Arrays

(i) Broadside array (ii) End fire array

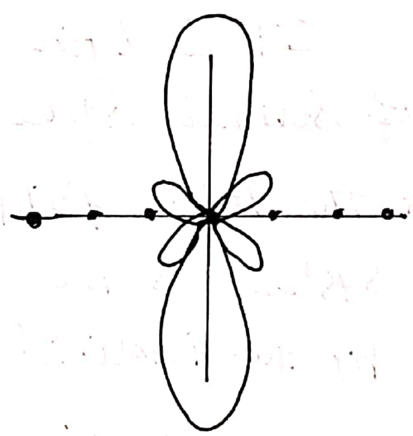
(iii) Collinear array (iv) Parasitic array.

Broadside Array :-

Here all the elements are placed parallel to each other & the direction of maximum radiation is always perpendicular to the plane consisting elements.



Broadside array of antennas.



Radiation pattern of broadside.

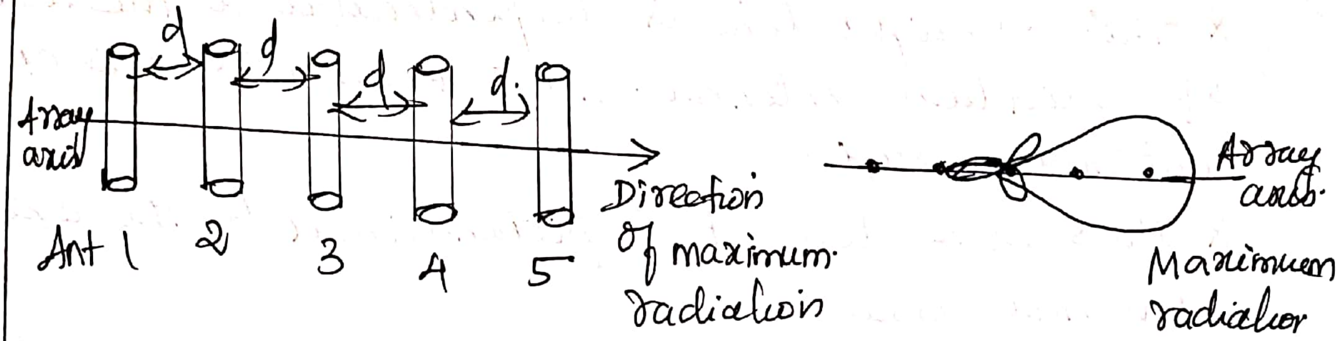
* A broadside array consists of number of identical antennas placed parallel to each other along a straight line.
* This straight line is perpendicular to the axis of individual antenna. It is known as axis of antenna array.
* Thus each element is perpendicular to the axis of antenna array.
* All the individal antennas are spaced equally along the axis of antenna array, denoted by 'd'.
* All the elements are fed with currents of equal magnitude & same phase.

As the maximum radiation is directed in broadside direction ie perpendicular to the line of axis of array, the radiation pattern for the broadside array is bidirectional.

Thus we can defined broadside array as the arrangement of antennas in which maximum radiation is in the direction perpendicular to the axis of array & plane containing the elements of array.

Endfire array:-

The direction of the maximum radiation is along the axis of the array.



Thus in the end fire array number of identical antennas are spaced equally along a line. All the antennas are fed individually with currents of equal magnitudes but their phases vary progressively along the line to make entire arrangement to get unidirectional radiation along the axis of the array.

Thus end fire array can be defined as an array with direction of maximum radiation coincides with the direction of the axis of the array to get unidirectional radiation.

Collinear array:-

Antennas are arranged coaxially ie antennas are arranged end to end along a single line.

The individual elements in the collinear array are fed with ~~constant~~ currents equal in magnitude & phase.

Simillar to ~~array~~. broadside array

(IV) Parasitic array:-

In order to overcome feeding problems of the ~~endo~~ antenna, sometimes the elements of the array are fed through the radiation from the nearby elements.

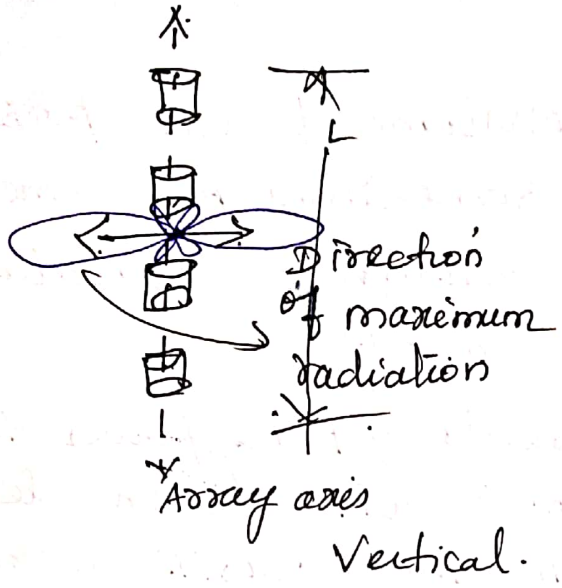
* The parasitic elements get the power through electromagnetic coupling with driven element which is in proximity with the parasitic element is known as parasitic array.

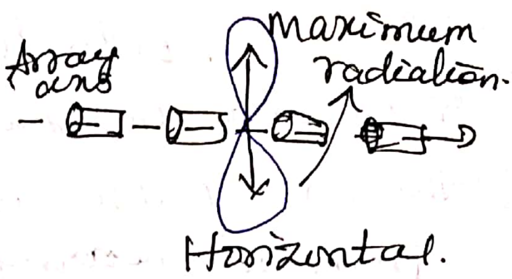
Example : Yagi Uda Antenna.

The amplitude & phase of the current induced in the parasitic element depends on the spacing between the driven element & parasitic element.

To make the radiation pattern unidirectional, the relative phases of the currents are changed by adjusting the spacing between the elements. This is called tuning of array.

For a $\lambda/4$ spacing between the driven & parasitic element, with phase difference of $\pi/2$ radian, unidirectional radiation pattern is obtained





* Therefore in collinear array, the direction of maximum radiation is perpendicular to the axis of array.

* The radiation pattern has circular symmetry with main lobe perpendicular everywhere to the principal axis. Thus the linear arrays are called omnidirectional or broadcast array.

The gain of the collinear array is maximum if the spacing ~~random~~ between the elements are of the order of 0.3λ to 0.5λ

But this small spacing introduces constructional of feeding problems

To overcome this difficulty, the elements of the array are operated with their ends very close to each other by connecting ends of by an insulator

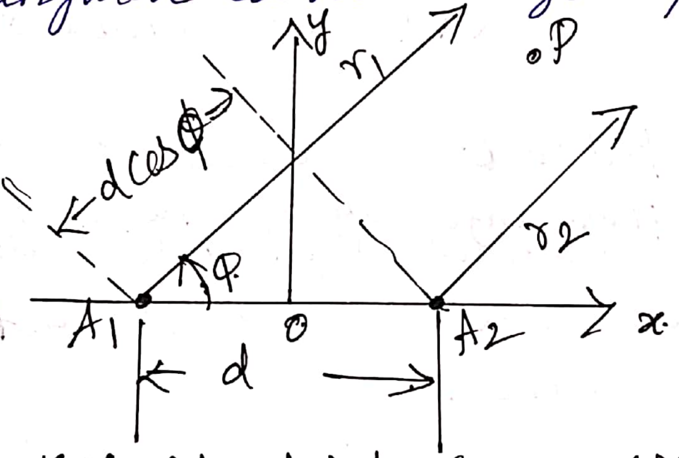
Array of two point sources:-

Array of two point sources is the simplest form of isotropic point sources.

Assume that two point sources are separated by a distance (d) & have the same polarization. There are 3 different cases.

Case (i) Arrays of two point sources with equal amplitude & phase.

Let us Consider, two isotropic point sources are symmetrically situated w.r.to the origin in the RCS. (Rectangular coordinate system).



Two isotropic point sources situated symmetrically w.r.to origin with same amplitude & phase.

* To Calculate fields at a great distant point 'P' at distance R from the origin O & the origin is taken as reference point for phase calculation
* The waves from source 1 reaches the point P at a latter time than the

waves from source 2 because of path difference (1,2) involved between the 2 waves.

Thus the fields due to source 1 lags while that due to source 2 leads. Path difference between the two waves (1,2) & is given by.

Path difference $= d\cos\theta$.

Path difference in terms of λ $\Big\} = \frac{d\cos\theta}{\lambda}$.

$\therefore$ Phase angle(φ) $= 2\pi \times$ (Path diff)

$$\varphi = 2\pi \frac{d\cos\theta}{\lambda}$$

$$= \frac{2\pi d}{\lambda}\cos\theta \text{ radians}$$

$\varphi = \beta d\cos\theta$. where $\beta = \frac{2\pi}{\lambda}$

$\Rightarrow$ Phase angle diff b/w the fields of the two sources measured at an angle θ along radius vector.

Let

$E_1 \rightarrow$ Far electric field at distant point 'P' due to source 1.

$E_2 \rightarrow$ Far electric field at distant point 'P' due to source 2.

$E \rightarrow$ Total electric field at distance point

$E = E_1 e^{-j\varphi/2} + E_2 e^{j\varphi/2}$ → field component due to source 2.

↳ field component due to source 1

E_1 & E_2 → Both amplitudes are same.

ie $E_1 = E_2 = E_0$.

$$E = E_0 \left(e^{j\varphi/2} + e^{-j\varphi/2} \right)$$

Let $\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2} \Rightarrow 2\cos\theta = e^{+j\theta} + e^{-j\theta}$.

$$\therefore E = 2E_0 \cos \varphi/2.$$

$$E = 2E_0 \cos\left(\frac{\beta d \cos\theta}{2}\right)$$

This is the equation of far field pattern of two isotropic point sources of same amplitude & phase.

Here the total amplitude is $2E_0$, whose maximum value may be 1.

$$2E_0 = 1 \text{ or } E_0 = 1/2.$$

Then $E = \cos\left(\dfrac{\beta d \cos\theta}{2}\right)$

$$E = \cos\left(\frac{2\pi}{\lambda} \times \frac{\lambda}{2} \frac{\cos\theta}{2}\right) \qquad d = \lambda/2.$$

$$E = \cos\left(\pi/2 \cos\theta\right)$$

Field pattern → The ~~different~~ directions of Maxima, Minima & half power points must be known.

Maxima Direction :-

E is maximum, when $\cos(\frac{\pi}{2}\cos\theta)$ is maximum & its maximum value is ± 1.

$\therefore \cos(\frac{\pi}{2}\cos\theta) = \pm 1.$

$\frac{\pi}{2}\cos\theta_{max} = \pm n\pi$ where $n = 0, 1, 2 \ldots$

$\frac{\pi}{2}\cos\theta_{max} = 0$ if $n = 0.$

$\cos\theta_{max} = 0$ $\Rightarrow \theta_{max} = \cos 0$

$\theta_{max} = 90^\circ$ & 270°.

Minimum direction :-

E is minimum when $\cos(\frac{\pi}{2}\cos\theta)$ is minimum & its minimum value is 0

$\cos(\frac{\pi}{2}\cos\theta) = 0$

$\frac{\pi}{2}\cos\theta_{min} = \pm(2n+1)\frac{\pi}{2}$, $n = 0, 1, 2 \ldots$

$\frac{\pi}{2}\cos\theta_{min} = \pm\frac{\pi}{2}$ if $n = 0$

$\cos\theta_{min} = \pm 1.$

$\theta_{min} = 0^\circ$ & 180°.

Half power point direction :-

At half power points, power is 1/2 or voltage or current is $1/\sqrt{2}$ times the maximum value of voltage or current.

$$\cos\left(\frac{\pi}{2}\cos\theta\right) = \pm\frac{1}{\sqrt{2}}$$

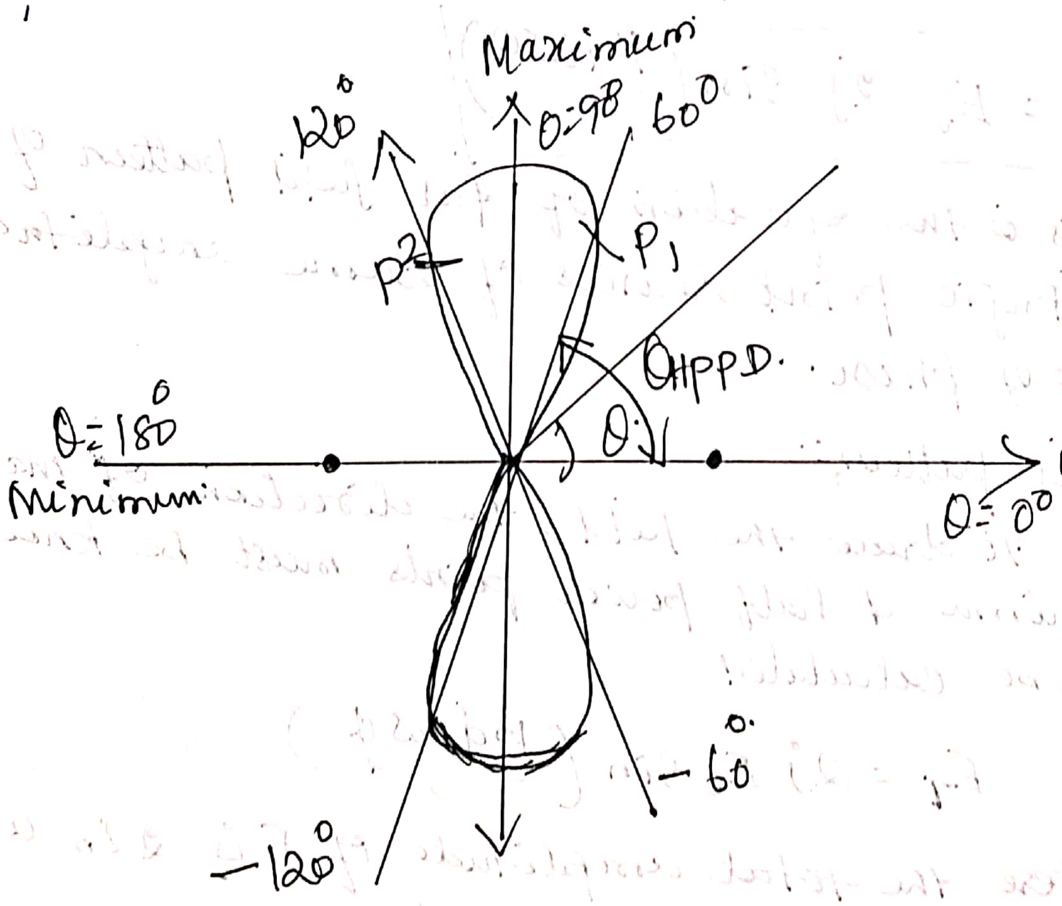
$$\frac{\pi}{2}\cos\theta_{HPPD} = \pm(2n+1)\frac{\pi}{4} \quad \text{where } n = 0, 1, 2 \ldots$$

$$\frac{\pi}{2}\cos\theta_{HPPD} = \pm\frac{\pi}{4} \quad n = 0$$

$$\cos\theta_{HPPD} = \pm\frac{1}{2}$$

$$\theta_{HPPD} = \pm 60^{\circ} \text{ or } \pm 120^{\circ}$$

In broadside array, two isotropic radiators are in phase, it gives bidirectional pattern.



Two point sources with currents equal in magnitude But opposite in phase:

Consider 2 point sources separated by distance 'd' & supplied with currents of equal magnitude but opposite in phase.

Total far field at distance point 'p' is given by

$$E_1 = -E_1 e^{-j\varphi/2} + E_2 e^{j\varphi/2}$$

let $E_1 = E_2 = E_0$.

$$E_T = E_0 \left(e^{j\varphi/2} - e^{-j\varphi/2}\right)$$

$$\frac{e^{j\theta/2} - e^{-j\theta/2}}{2j} = \sin\theta/2$$

$$E_T = E_0 \, 2j \sin\varphi/2 \qquad (\varphi = \beta d \cos\theta)$$

$$\boxed{E_T = E_0 \, 2j \sin\left(\frac{\beta d \cos\theta}{2}\right)}$$

This is the equation of far field pattern of 2 isotropic point sources of same amplitude & out of phase.

Field pattern:

To draw the field, the direction of maxima, minima & half power points must be known which can be calculated

$$E_T = 2j \, E_0 \sin\left(\frac{\beta d \cos\phi}{2}\right)$$

Here the total amplitude of E is $2E_0$ whose maximum value may be 1.

Scanned with CamScanner

$|2E_0| = 1$; the pattern is said to be normalized.

$$E = \sin\left(\frac{\beta d \cos\phi}{2}\right) \quad ; \; d = \lambda/2 \; ; \; \beta = 2\pi/\lambda$$

$$E = \sin\left(\frac{2\pi}{\lambda} \times \lambda/2 \, \frac{\cos\phi}{2}\right)$$

$$\boxed{E = \sin(\pi/2 \cos\phi)}$$

Maxima Directions:-

The direction through which maximum radiation occurs is called as maxima direction or maxima. → Electric field is maximum obviously.

$$E = \pm 1$$

$$E = \sin(\pi/2 \cos\phi) = \pm 1$$

$$\pi/2 \cos\phi_{max} = \sin^{-1}(\pm 1) = \pm (2n+1)\pi/2$$

where $n = 0, 1, 2 \ldots$

If $n = 0$ then $\pi/2 \cos\phi_{max} = \pm \pi/2$

i.e. $\cos\phi_{max} = \pm 1$.

$$\phi_{max} = 0^\circ \; \& \; 180^\circ$$

Minima Direction:

The total field strength E is minimum i.e. zero.

$$E = \sin(\pi/2 \cos\phi) = 0$$

$$\pi/2 \cos\phi_{max} = \sin^{-1}(0) = \pm n\pi$$

where $n = 0, 1, 2 \ldots$

If $n=0$, $\frac{\pi}{2}\cos\phi_{min} = 0$.

$\cos\phi_{min} = 0$

$\phi_{min} = \pm 90°$.

Half power point Direction (HPPD):-

At half power points, power is 1/2 (or) voltage & current is $\frac{1}{\sqrt{2}}$ times the maximum value

At half power direction, the electric field is $= \pm 1/\sqrt{2}$

$$E = \pm \frac{1}{\sqrt{2}}$$

$$E = \sin\left(\frac{\pi}{2}\cos\phi\right) = \pm\frac{1}{\sqrt{2}}.$$

$$\frac{\pi}{2}\cos\phi = \sin^{-1}\left(\pm\frac{1}{\sqrt{2}}\right)$$

$$= \pm(2n+1)\frac{\pi}{2} \quad \text{where } n = 0, 1, 2 \ldots$$

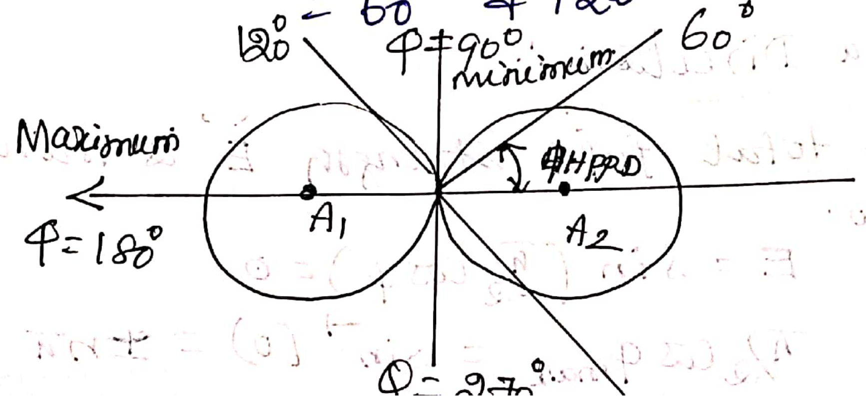
If $n=0$ then

$$\frac{\pi}{2}\cos\phi_{HPPD} = \pm\frac{\pi}{4}$$

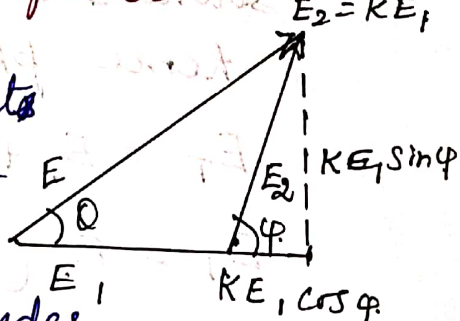
$$\cos\phi_{HPPD} = +\frac{1}{2}$$

$$\phi_{HPPD} = \cos^{-1}\left(\pm\frac{1}{2}\right)$$

$$= 60° \text{ \& } 120°$$



Two points Sources with currents unequal in Magnitude & with Any phase.



Let us consider that the 2 points sources are separated by distance 'd' & supplied with currents which are different in magnitudes & with any phase difference say α.

Source A_1 is assumed to be reference for phase & amplitude of the fields E_1 & E_2 which are due to source A_1 & source A_2 respectively at the distant point 'p'.

Let us assume the $|E_1| > |E_2|$.

Now the total phase difference between the radiations by the 2 point sources at any far point p is

$$\varphi = \frac{2\pi}{\lambda} \cos\varphi + \alpha.$$

where $\alpha \rightarrow$ phase angle with which current I_2 leads current I_1.

If $\alpha = 0$, (then two point sources with currents equal in magnitude & phase).

If $\alpha = 180^\circ$; (then 2 point source with currents equal in magnitude but opposite in phase.)

Assume the value of α & $0 < \alpha < 180^\circ$, then the resultant field at point 'p' is given by

$$E_T = E_1 e^{j\theta} + E_2 e^{j\varphi}.$$

Source 1 is assumed to be reference hence phase angle is 0.

$$E_T = E_1 + E_2 e^{j\varphi}.$$

$$E_T = E_1 \left(1 + \frac{E_2}{E_1} e^{j\varphi}\right)$$

$$\frac{E_2}{E_1} = k.$$

Since $E_1 > E_2$ the value of k is less than unity.

$$(0 \leq k \leq 1)$$

$$E_T = E_1 (1 + k e^{j\varphi})$$

$$E_T = E_1 (1 + k(\cos\varphi + j\sin\varphi))$$

~~This is~~ The magnitude of the resultant field at point sources of unequal amplitude & any phase

The magnitude of the resultant field at point 'p' is

$$|E_T| = \{E_1 (1 + k\cos\varphi + jk\sin\varphi)\}$$

$$|E_T| = E_1 \sqrt{(1+k\cos\varphi)^2 + (k\sin\varphi)^2}$$

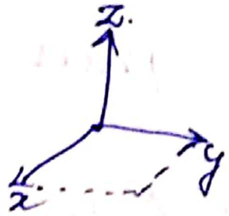
The phase angle between 2 fields at the far point 'p' is given

$$\boxed{\theta = \tan^{-1} \frac{k\sin\varphi}{1+k\cos\varphi}}$$

1. Calculate the approximate 3 dB beamwidth of the x-z plane radiation pattern of an aperture $(a = \lambda)$ with uniform current distribution.

Half power beamwidth

$$|HPBW|_{\phi=0} = 2\sin^{-1}\left(\frac{1.391\lambda}{\pi\lambda}\right)$$



2. Find out the power gain in dB of a paraboloidal reflector of open mouth aperture 10λ

Diameter $D = 10\lambda$

Power gain of a paraboloid,

$$G_p = 6\left(D/\lambda\right)^2 =$$

$$\text{Power gain in dB} = 10\log_{10} G_p$$

$$=$$

3. Findout the beam width between first nulls & power gain of 2-m paraboloid reflector operating at 6000 MHz

$$BWFN = \frac{140\lambda}{D} \quad ; \quad G_p = 6\left(\frac{D}{\lambda}\right)^2$$

D – Diameter of aperture

$D = 2$ meter ; $f = 6000 \times 10^6$

$\lambda = c/f = \frac{3 \times 10^8}{6000 \times 10^6} = \frac{300}{6000} = 0.05$ meter

$BWFN = \frac{140\lambda}{D} = \frac{140 \times 0.05}{2} =$

Power gain $G_p = 6 \times (D/\lambda)^2 =$

$=$

4. A paraboloidal - reflector antenna is designed for operation of 3000 MHz. Its largest aperture dimension is 20ft. For measurement of radiation pattern, what should be the minimum distance between primary & secondary antenna (one feet = 0.3048 m)

$f = 3000\,MHz \Rightarrow \lambda = c/f$

$=$

$D = 20$ feet $= 20 \times 0.3048 =$

The distance b/w 1° & 2° antenna is

$r \geq 2D^2/\lambda$

$\simeq$

5. Find the gain, beamwidth & capture area for a parabolic antenna with 20 m diameter dish & dipole feed at 20 GHz

$$D = 20m. \; ; \; f = 20 \, GHz$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{20 \times 10^9} = 0.015 \, m.$$

$$\text{Gain} = 6 \left(D/\lambda \right)^2$$

$$\text{Directivity } (D) = 4\pi/\lambda^2$$

$$\text{Capture Area } A = \frac{\lambda^2 D}{4\pi}$$

$$BWFN = \frac{140}{D/\lambda} = \frac{140}{\frac{20}{0.015}}$$

n- Element uniform linear Array:-

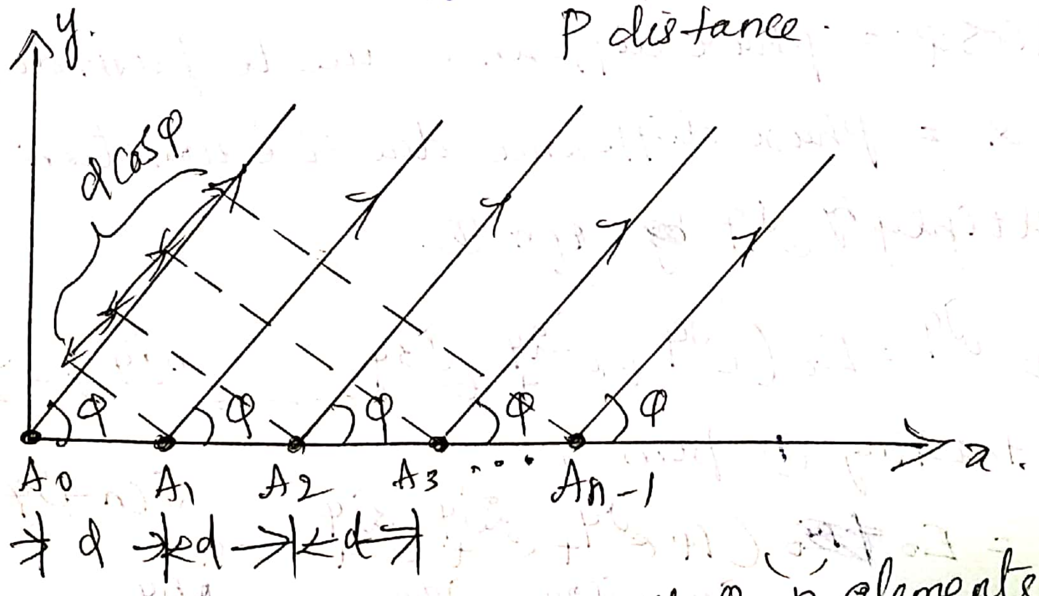
At higher frequencies, for point to point communications, it is necessary to have pattern with single beam radiation. Such a highly ~~directive~~ directive single beam pattern can be obtained by increasing the point sources in the array from 2 to 'n' number of sources.

Linear Array:-

An array of 'n' elements are said to be linear array if all the individual elements are spaced equally along a line.

Uniform Array:-

An array is said to be uniform array if the elements in the array are fed with currents of equal magnitudes & uniform progressive phase shift along a line.



Uniform linear array of 'n' elements

Consider a general 'n' element uniform linear array.

These point sources are equally spaced & fed with a current of equal amplitude & phase shift is uniform progressive phase shift.

* Total field at a distant point 'p' is obtained by adding the fields due to 'n' individual sources vectorically.

$$E_t = E_0 e^{0j\varphi} + E_0 e^{j\varphi} + E_0 e^{j2\varphi} + E_0 e^{j3\varphi} + \dots E_0 e^{j(n-1)\varphi}$$

$$E_t = E_0 (1 + e^{j\varphi} + e^{2j\varphi} + e^{3j\varphi} + \dots e^{j(n-1)\varphi}) \dots \text{①}$$

$\varphi \Rightarrow \beta d \cos\phi - \alpha$ radian

$\varphi \Rightarrow$ Total phase difference of the fields at distant point 'p' from adjacent sources

$\alpha \Rightarrow$ phase difference in adjacent point sources (or) progressive phase shift b/w 2 pt sources.

$\beta d \cos\phi$ = phase difference due to path difference

α = phase difference due to excitation

Multiply by $e^{j\varphi}$ ~~by~~ eqn ①

$$E_t e^{j\varphi} = E_0 (e^{j\varphi} + e^{j2\varphi} + e^{j3\varphi} + \dots e^{nj\varphi}) \rightarrow \text{②}$$

Subtracting ② from ①

$$E_t = E_0 (1 + e^{j\varphi} + e^{2j\varphi} + e^{3j\varphi} + \dots e^{j(n-1)\varphi}$$

$$(-)\ E_t \cdot e^{j\varphi} = E_0 (e^{j\varphi} + e^{j2\varphi} + e^{j3\varphi} + \dots e^{jn\varphi}$$

Scanned with CamScanner

$$E_t(1-e^{j\varphi}) = E_0(1-e^{jn\varphi})$$

$$E_t = E_0 \frac{(1-e^{jn\varphi})}{(1-e^{j\varphi})}$$

$$E_t = E_0 \frac{(1-e^{jn\varphi/2} \cdot e^{jn\varphi/2})}{(1-e^{j\varphi/2} \cdot e^{j\varphi/2})}$$

$$\times \text{ \& } \div \; \frac{-e^{jn\varphi/2} \cdot e^{jn\varphi/2}}{-e^{jn\varphi/2} \cdot e^{jn\varphi/2}}$$

$$= E_0 \frac{e^{jn\varphi/2} \cdot e^{-jn\varphi/2} - e^{jn\varphi/2} \cdot e^{jn\varphi/2}}{e^{j\varphi/2} \cdot e^{-j\varphi/2} - e^{j\varphi/2} \cdot e^{j\varphi/2}}$$

$$= E_0 \frac{-e^{jn\varphi/2}(e^{jn\varphi/2} - e^{-jn\varphi/2})/2j}{-e^{j\varphi/2}(e^{j\varphi/2} - e^{-j\varphi/2})/2j}$$

$$E_t = E_0 \, e^{j(n-1)\varphi/2} \cdot \frac{\sin n\varphi/2}{\sin \varphi/2}$$

$$E_t = E_0 \, e^{j\varphi} \cdot \frac{\sin \frac{n\varphi}{2}}{\sin \varphi/2}$$

$(n-1)\varphi/2 = \varphi'$

→ if the reference pt is shifted to the center of the array then $(n-1)\varphi/2$ is automatically eleminated

$$E_t = E_0 \frac{\sin \frac{n\varphi}{2}}{\sin \varphi/2} (\cos \varphi + j \sin \varphi)$$

$$\boxed{E_t = E_0 \left(\frac{\sin n\varphi/2}{\sin \varphi/2}\right) \angle \varphi}$$

$$\boxed{E_t = E_0 \frac{\sin \frac{n\varphi}{2}}{\sin \varphi/2}}$$

According to multiplication of pattern

E_0 → Individual source pattern

Array pattern

Broadside Array :-

* It is a uniform linear array.
* The maximum radiation occurs in the directions normal to the line of array.

$$\psi = \beta d \cos\phi + \alpha = 0$$

$\alpha \to$ Sources are fed in phase.

$$\beta d \cos\phi_{max} = 0$$

$$\cos\phi_{max} = 0$$

ie $\phi_{max} = 90^\circ$ or 270° ← Principle maxima.

∴ Hence the maximum radiation of broadside array is in 90° & 270°.

(1) Pattern maxima (Minor lobe maxima $(\phi_{max})_{minor}$)

* Sometimes, the antenna in broadside radiates the power along the direction also apart from maximum radiation directions. This is called as Minor lobe maxima or pattern maxima.

* The minor lobe maxima occurs between first nulls & higher order nulls.

* Nulls are the directions through which an array radiate zero power.

* The total field strength of n element uniform linear array is

$$E_t = E_0 \frac{\sin\frac{n\psi}{2}}{\sin\psi/2}$$

This eqn is maximum, when numerator is maximum i.e. $\sin n\psi/2$ is maximum provided $\sin \psi/2 \neq 0$

$$\therefore \sin \frac{n\psi}{2} = 1.$$

$$\frac{n\psi}{2} = \pm (2N+1)\pi/2 \quad \text{where } N = 1, 2, 3, 4 \ldots$$

N=0 corresponds to major lobe maxima.

$$\frac{\psi}{2} = \pm (2N+1)\pi/2n$$

$$\psi = \pm (2N+1)\frac{\pi}{n}$$

$$\psi = \beta d \cos(\phi_{max})_{min} + \alpha$$

$$\beta d \cos(\phi_{max})_{minor} + \alpha = \pm(2N+1)\pi/n - \alpha$$

$$\cos(\phi_{max})_{minor} = \pm \frac{(2N+1)\pi/n - \alpha}{\beta d}$$

$$(\phi_{max})_{minor} = \cos^{-1}\left\{\frac{1}{\beta d}\left[\pm \frac{(2N+1)\pi}{n} - \alpha\right]\right\}$$

For a broadside array $\alpha = 0$

$$(\phi_{max})_{minor} = \cos^{-1}\left\{\frac{1}{\beta d}\left[\pm \frac{(2N+1)\pi}{n}\right]\right\}$$

Sub $\beta = 2\pi/\lambda$

$$= \cos^{-1}\left\{\frac{1}{\frac{2\pi}{\lambda} d}\left[\pm \frac{(2N+1)\pi}{n}\right]\right\}$$

$$(\phi_{max})_{minor} = \cos^{-1}\left\{\pm \frac{(2N+1)\lambda}{2nd}\right\}$$

For example

if $n=4$, $d=\lambda/2$, $N=1$.

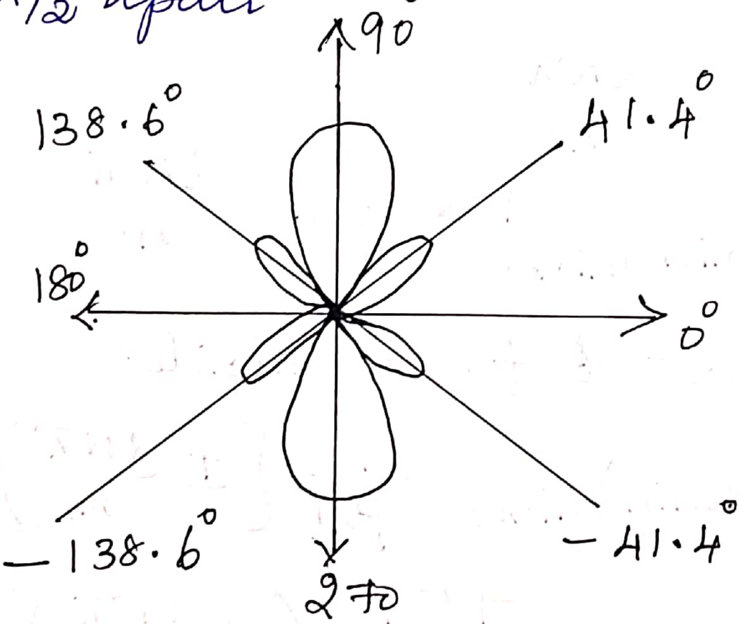
$$(\phi_{max})_{minor} = \cos^{-1}\left\{\pm \frac{(2(1)+1)\lambda}{2\times 4\times \lambda/2}\right\}$$

$$= \cos^{-1}\left\{\pm\left(\frac{3}{4}\right)\right\}$$

$$= \cos^{-1}(\pm 3/4)$$

$$(\phi_{max})_{minor} = \pm 41.4^0 \text{ or } \pm 138.6^0$$

These are the four minor lobe maxima of the array of 4 isotropic sources fed in phase & spaced $\lambda/2$ apart.



Note:

No other maxima exists for $N \geq 2$ because for $N=2$ $\cos(\phi_{max})_{minor} = \pm 5/4$ which is $>> 1$. where as cosine value is always $<< 1$.

(ii) Pattern minima [Minor lobe minima $(\phi_{min})_{minor}$]

Minima is the direction through which the array radiate zero power. It is called as null direction.

* The electric field intensity is zero along the null direction.

$$E = E_0 \frac{\sin \frac{n\phi}{2}}{\sin \phi/2} = 0.$$

Minima occurs when $\sin \frac{n\phi}{2} = 0$ provided $\sin \phi/2 \neq 0$.

$$\frac{n\phi}{2} = \pm N\pi \quad \text{where } N = 1, 2, 3 \ldots$$

$$\phi = \pm \frac{2N\pi}{n}$$

$$\beta d (\cos \phi_{min})_{minor} + \alpha = \pm \frac{2N\pi}{n}$$

$\alpha = 0$ for broadside.

$$\cos (\phi_{min})_{minor} = \frac{1}{\beta d} \left[\pm \frac{2N\pi}{n} \right]$$

$$(\phi_{min})_{minor} = \cos^{-1} \left[\frac{1}{\beta d} \left(\pm \frac{2N\pi}{n} \right) \right]$$

$$= \cos^{-1} \left[\frac{1}{\frac{2\pi}{\lambda} d} \left(\pm \frac{2N\pi}{n} \right) \right]$$

$$(\phi_{min})_{minor} = \cos^{-1}\left[\pm \frac{N\lambda}{nd}\right]$$

If $n = 4$; $d = \lambda/2$ & $N = 1$.

$$(\phi_{min})_{minor} = \cos^{-1} \pm \frac{(1)\lambda}{4 \times \lambda/2}$$

$$= \cos^{-1}\left[\pm \tfrac{1}{2}\right]$$

$$(\phi_{min})_{minor} = \pm 60^\circ, \pm 120^\circ$$

If $N = 2$.

$$(\phi_{min})_{minor} = \cos^{-1}\left[\pm \frac{2 \cdot \lambda}{4 \cdot \lambda/2}\right]$$

$$= \cos^{-1}[\pm 1]$$

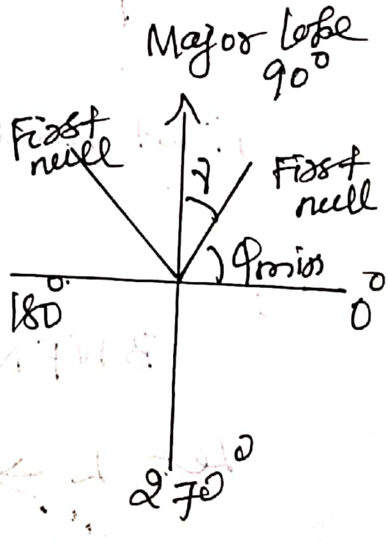
$$= 0^\circ, 180^\circ$$

$0^\circ, 60^\circ, 120^\circ, 180^\circ, -60^\circ, -120^\circ$ are six minor lobe minima of the array of 4 isotropic sources spaced $\lambda/2$ apart.

(iii) Beam width of major lobe :- It is defined as

(a) The angle between first nulls

(b) Double the angle between first nulls & major lobe maxima directions



If $\gamma = 90 - \phi_{min}$

$\therefore \phi_{min} = 90^\circ - \gamma \rightarrow (A)$

Beamwidth (BW) = $2 \times$ {Angle between first nulls & maximum of major lobe}

$BW = 2 \times \gamma$

then

$$(\phi_{min})_{minor} = \cos^{-1}\left[\pm \frac{N\lambda}{nd}\right] \rightarrow (B)$$

Compare (A) & (B)

$$90^\circ - \gamma = \cos^{-1}\left[\pm \frac{N\lambda}{nd}\right]$$

$$\cos(90^\circ - \gamma) = \pm \frac{N\lambda}{nd}$$

$\sin\gamma = \gamma$ when γ is very small.

$$\sin\gamma = \pm \frac{N\lambda}{nd}$$

$$\boxed{\gamma = \pm \frac{N\lambda}{nd}}$$

First null occurs when $N = 1$.

$$\gamma_1 = \pm \frac{\lambda}{nd}$$

$$\therefore BWFN = 2 \times \gamma_1 = \frac{2\lambda}{nd}$$

Let $L \rightarrow$ Total length of the array in meter

$L \simeq nd$ (if n is large)

$$2\gamma_1 = \frac{2\lambda}{L} = \frac{2}{L/\lambda} \text{ radian.}$$

$$= \frac{2}{L/\lambda} \times 57.3^\circ \text{ degree.}$$

$$2\gamma_1 = \frac{114.6^\circ}{L\lambda.}$$

$$\boxed{BWFN = \frac{114.6^\circ}{L/\lambda.}}$$

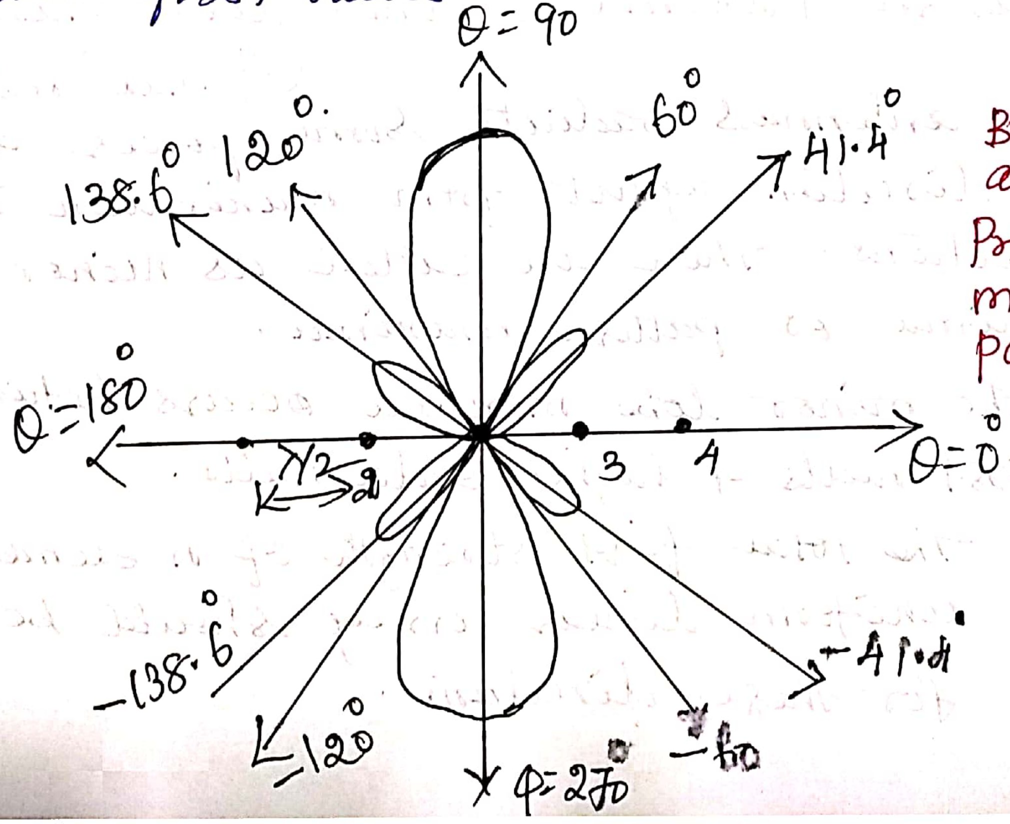
Half power beam width $= \frac{1}{2}$ BWFN.

$$= \frac{1}{2} \text{ BWFN}$$

$$= \frac{1}{2} \times \frac{114.6^\circ}{L/\lambda}$$

$$\boxed{HPBW = \frac{57.3^\circ}{L/\lambda.}}$$

Half power beam width is half of beam width between first nulls.



Broadside array with Principal maxima & Pattern maxima & nulls.

End fire Array :-

* For an array to be End fire, the phase angle is such that it makes the maximum radiation in the line of array ie $\phi = 0^\circ$ or 180°. Thus $\phi = 0^\circ$ or 180°.

$$\psi = \beta d \cos\phi + \alpha = 0$$

$$\beta d \cos 0^\circ = -\alpha$$

$$\alpha = -\beta d = \frac{-2\pi}{\lambda} d$$

$-\alpha \rightarrow$ indicates that the phase difference between the sources of an end fire is retarded progressively by some amount (2 lags behind 1).

If spacing between 2 sources is $\lambda/2$ or $\lambda/4$ then

$$\frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi \quad \text{or} \quad \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2} \text{ radian}$$

(i) Pattern Maxima (Minor lobe maxima $(\phi_{max})_{minor}$)

The antennas radiate some power along the direction apart from maximum radiation directions. These are called as minor lobe maxima or pattern maxima.

* The minor lobe maxima occurs between first nulls & higher order nulls.
* The total field strength of n element uniform linear array should be maximum for these directions.

$$E = E_0 \frac{\sin \frac{n\varphi}{2}}{\sin \varphi/2}$$

should be maximum provided $\sin\varphi/_2 \neq 0$

ie $\sin n\varphi/_2 = 1$

$$\sin \frac{n\varphi}{2} = 1 \quad (\text{Max})$$

$$\frac{n\varphi}{2} = \sin^{-1}(1) = \pm (2N+1)\pi/2$$

where $N = 1, 2, 3 \ldots$

$N = 0$, → major lobe maxima.

$$\Rightarrow \quad \varphi = \pm \frac{(2N+1)}{n} \frac{\pi 2}{2} = \pm \frac{(2N+1)\pi}{n} \rightarrow (1)$$

$$\varphi = \beta d \cos\phi + \alpha \quad ; \quad \alpha = -\beta d$$

$$\varphi = \beta d \cos\phi - \beta d$$

$$= \beta d (\cos\phi - 1) \rightarrow (2)$$

Compare eqn (1) + (2)

$$\beta d (\cos\phi - 1) = \pm \frac{(2N+1)\pi}{n}$$

$$(\cos\phi - 1) = \pm \frac{(2N+1)\pi}{n\beta d}$$

$$\cos\phi = 1 \pm \frac{(2N+1)\pi}{n\beta d}$$

$$(\phi_{max})_{minor} = \cos^{-1}\left[1 \pm \frac{(2N+1)\pi}{n\beta d}\right]$$

Sub $\beta = 2\pi/\lambda$.

$$(\phi_{max})_{minor} = \cos^{-1}\left[1 + \frac{(2N+1)\lambda}{2nd}\right]$$

For example $n=4$; $d = \lambda/2$.

$$(\phi_{max})_{minor} = \cos^{-1}\left[1 + \frac{(2N+1)\lambda}{2\times 4\times \lambda/2}\right]$$

$$\boxed{(\phi_{max})_{minor} = \cos^{-1}\left(1 \pm \frac{(2N+1)}{4}\right)}$$

If $N=1$,

$$(\phi_{max})_{minor} = \cos^{-1}(1 \pm 3/4)$$

$$= \cos^{-1}(1+3/4) \text{ (or) } (1-3/4)$$

$$= \cos^{-1}\left(\frac{4+3}{4}\right) \text{ or } (1/4)$$

$$= \cos^{-1}(1/4) \text{ or } \cos^{-1}(7/4)$$

Invalid.

$$\boxed{(\phi_{max})_{minor} = 75.5^0}$$

If $N=2$

$$(\phi_{max})_{minor} = \cos^{-1}(1 \pm 5/4)$$

$$= \cos^{-1}(-1/4) \text{ or } \cos^{-1}(9/4)$$

Invalid

$$= \cos^{-1}(-1/4)$$

$$\boxed{(\phi_{max})_{minor} = \pm 104.5^0}$$

If N=3

$$(\varphi_{max})_{minor} = \cos^{-1}(1 \pm 7/4)$$

$$= \cos^{-1}(-3/4) \text{ or } \cos^{-1}(11/4)$$

Invalid.

$$= \cos^{-1}(-3/4)$$

$$\boxed{(\varphi_{max})_{minor} = \pm 138.6^{\circ}}$$

N=4, $(\varphi_{max})_{minor} = \cos^{-1}(1 \pm 9/4)$

$$= \cos^{-1}(-5/4) \text{ or } \cos^{-1}(13/4) \text{ invalid.}$$

∴ For $N \geq 4$, $\cos\varphi$ is invalid.

Therefore for an end fire array of 4 isotropic sources spaced $\lambda/2$

(ii) pattern Minima (Minor lobe minima $(\varphi_{min})_{minor}$)

Minima is the direction through which the array radiate zero power. It is otherwise called as Null direction.

The electric field intensity is zero along the null direction

i.e. $E = 0$

W.K.T

$$E = E_0 \frac{\sin \frac{n\varphi}{2}}{\sin \varphi/2} = 0$$

$$\sin \frac{n\varphi}{2} = 0$$

$$\frac{n\varphi}{2} = \sin^{-1}(0) = \pm N\pi$$

where $N = 1, 2, 3 \ldots$; $N=0$ Corresponds to major lobe.

$$\varphi = \pm \frac{2N\pi}{n} \rightarrow (1)$$

but $\varphi = \beta d(\cos\phi - 1) \rightarrow (2)$

equate

$$\beta d(\cos\phi - 1) = \pm \frac{2N\pi}{n}$$

$$(\cos\phi - 1) = \pm \frac{2N\pi}{\beta n d}$$

$$\cos\phi = 1 \pm \frac{2N\pi}{n\beta d}$$

$$(\phi_{min})_{minor} = \cos^{-1}\left(1 \pm \frac{2N\pi}{n\beta d}\right)$$

$$(\phi_{min})_{minor} = \cos^{-1}\left(1 \pm \frac{2N\pi}{n \cdot \frac{2\pi}{\lambda} \cdot d}\right)$$

$$(\phi_{min})_{minor} = \cos^{-1}\left(1 \pm \frac{N\lambda}{nd}\right)$$

Let the array have 4 elements. i.e. $n=4$, $d=\lambda/2$

$$(\phi_{min})_{minor} = \cos^{-1}\left\{1 \pm \frac{N\lambda}{4 \cdot \lambda/2}\right\}$$

$$(\phi_{min})_{minor} = \cos^{-1}\left(1 + \frac{N}{2}\right)$$

If $N = 1$.

$$(\varphi_{min})_{minor} = \cos^{-1}(1 \pm 1/2)$$

$$= \cos^{-1}(1/2) \text{ or } \cos^{-1}(3/2) \leftarrow \text{invalid}.$$

$$(\varphi_{min})_{minor} = \cos^{-1}(1/2).$$

$$\boxed{(\varphi_{min})_{minor} = \pm 60^\circ}$$

If $N = 2$,

$$(\varphi_{min})_{minor} = \cos^{-1}(1 \pm 1)$$

$$= \cos^{-1}(0) \text{ or } \cos^{-1}(2)$$

↑invalid

$$= \cos^{-1}(0).$$

$$\boxed{(\varphi_{min})_{minor} = \pm 90^\circ}$$

If $N = 3$

$$(\varphi_{min})_{minor} = (1 \pm 3/2).$$

$$= \cos^{-1}(-1/2) \text{ or } \cos^{-1}(5/2) \leftarrow \text{invalid}.$$

$$= \cos^{-1}(-1/2)$$

$$(\varphi_{min})_{minor} = \pm 60^\circ \; ; \; \pm 120^\circ. \; *$$

If $N = 4$

$$(\varphi_{min})_{minor} = \cos^{-1}(1 \pm 4/2)$$

$$= \cos^{-1}(-1) \text{ or } \cos^{-1}(3) \leftarrow \text{invalid}$$

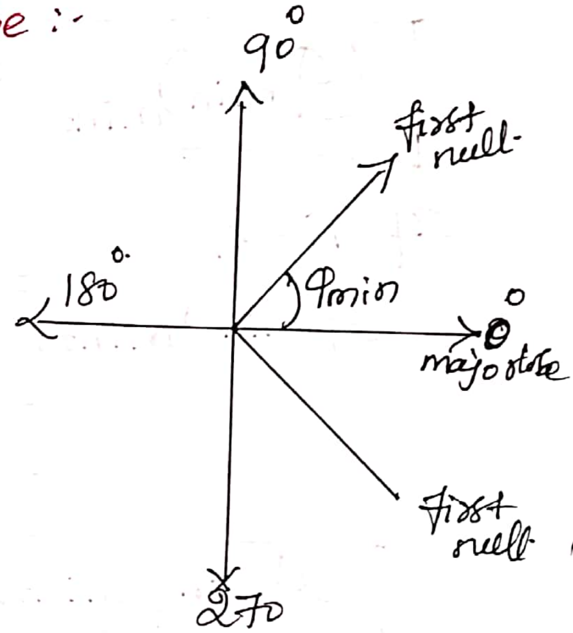
$$= \cos^{-1}(-1)$$

$$(\varphi_{min})_{minor} = \pm 180^\circ.$$

For $N \geq 5$, $\cos\varphi$ is invalid.

Therefore for end fire array of 4 isotropic sources spaced $\lambda/2$ apart, there are six null directions along $\pm 60°$, $\pm 90°$ & $\pm 180°$

(iii) Beam width of major lobe :-



From

$$(\varphi_{min}) = \cos^{-1}\left(1 \pm \frac{N\lambda}{nd}\right)$$

$$\cos\varphi_{min} = 1 \pm \frac{N\lambda}{nd}$$

$$\cos\theta = 1 - 2\sin^2\theta/2$$

$$\therefore 1 - 2\sin^2\frac{\varphi_{min}}{2} = 1 \pm \frac{N\lambda}{nd}$$

$$2\sin^2\frac{\varphi_{min}}{2} = \pm\frac{N\lambda}{nd}$$

$$\sin^2\frac{\varphi_{min}}{2} = \pm\frac{N\lambda}{2nd} = \pm\frac{N\lambda}{2L}$$ $nd \simeq L$

$$\sin\frac{\varphi_{min}}{2} = \pm\sqrt{\frac{N\lambda}{2L}}$$

$$\frac{\varphi_{min}}{2} = \sin\left(\pm\sqrt{N\lambda/2L}\right)$$ for small angle $\sin\theta \simeq \theta$.

$$\boxed{\varphi_{min} = \pm\sqrt{\frac{2N\lambda}{L}}}$$

Beam width between first nulls

$$BWFN = 2 \times \varphi_{min} = \pm 2\sqrt{\frac{2N\lambda}{L}}$$

for $N=1$:

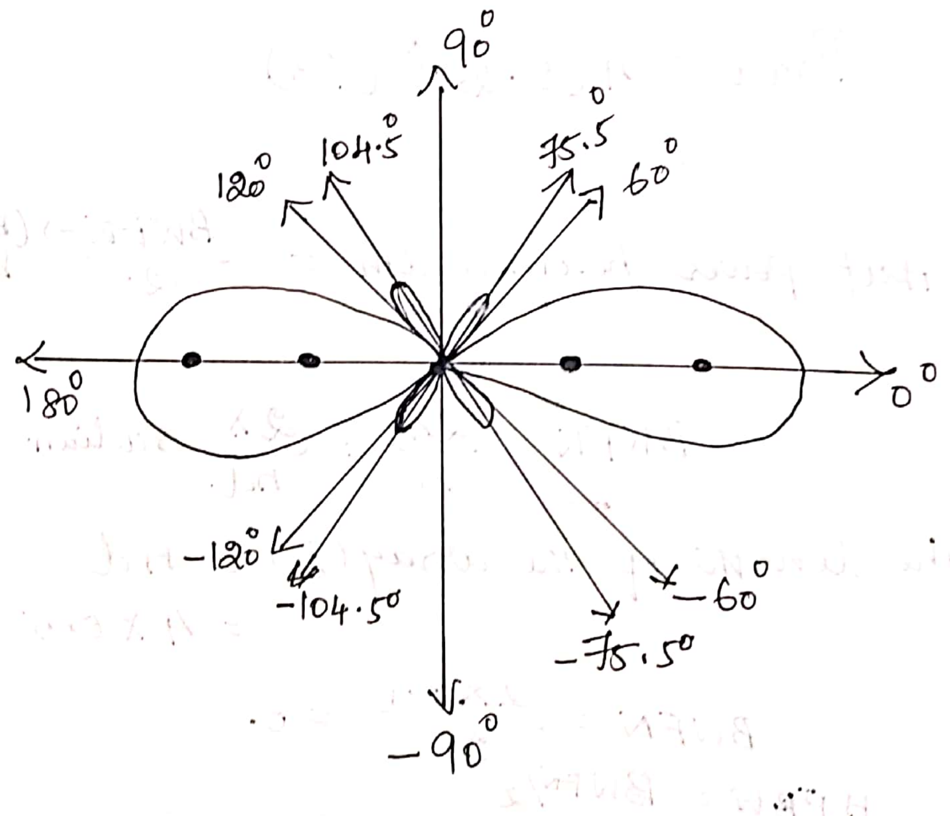
$$BWFN = \pm 2\sqrt{\frac{2(1)}{L/\lambda}}$$

$$BWFN = \pm \frac{2\sqrt{2}}{\sqrt{L/\lambda}} \text{ radians}$$

$$BWFN = \frac{162.2}{\sqrt{L/\lambda}} \text{ degrees}$$

Half power beam width $HPBW = \frac{BWFN}{2}$.

$$HPBW = \frac{81.1}{\sqrt{L/\lambda}} \text{ degree}$$



1. A Broadside array consists of four identical half wave dipoles spaced 50 cm apart. If the wavelength is 0.1 m & each element carries r.f current of equal magnitude of 0.25 A. & same phase, calculate power radiated & half power beamwidth of the major lobe.

Given:

$n \rightarrow$ no. of elements $= 4$.

$\lambda \rightarrow 0.1\ m$.

d = spacing b/w any two elements

0.5 m. or 50 cm.

$I = 0.25 A$

(i) Power radiated $P_{rad} = n(I^2 R_{rad})$

R_{rad} for halfwave dipole $= 73\Omega$.

$$P_{rad} = 4(0.25)^2 (73)$$

$=$

(ii) Half power beam width $= \frac{BWFN}{2}$ → (Beam width b/w first nulls)

$$BWFN = \frac{2\lambda}{L} = \frac{2\lambda}{nd} \text{ radian}$$

The length of the array $(L) = nd$

$= 4 \times 0.5$

$$BWFN = \frac{2 \times 0.1}{2} = 0.$$

$HPBW = BWFN/2 =$

2. Find the minimum spacing between the elements in a broadside array of 10 isotropic radiators to have directivity of 7 dB

$G_{Dmax} = 7$ dB.

$n \rightarrow 10$

G_{Dmax} in dB $= 10 \log_{10} |G_{Dmax}|$

$7 = 10 \log_{10} |G_{Dmax}|$

$G_{Dmax} =$

The directivity of the broadside array is

$G_{Dmax} = 2(L/\lambda) = 2\left(\frac{nd}{\lambda}\right)$

5. .

$d =$

3. Calculate the directivity in dB for the broadside as well as end fire array consisting of 8 isotropic elements separated by $\lambda/4$ distance.

$n = 8$; $d = \lambda/4$ m.

(i) For broadside array:

The directivity $G_{Dmax} = 2\left(\frac{nd}{\lambda}\right) = \frac{2 \times 8 \times \lambda/4}{\lambda}$

G_{Dmax} in dB $= 10 \log_{10} (G_{Dmax})$

$=$

4. Find the length & BWFN for broadside & end fire array if the directive gain 15

$G_{Dmax} = 15$

(i) For broadside array

$G_{Dmax} = 2(L/\lambda)$

$15 = 2(L/\lambda)$

$L = 7.5\lambda$ meter

(ii) $BWFN = \frac{114.6}{(L/\lambda)}$ degree

$= \frac{114.6}{\frac{7.5\lambda}{\lambda}} = 1$

(ii) For End fire array

$G_{Dmax} = 4(L/\lambda)$

$15 = 4(L/\lambda)$

$L =$

(ii) $BWFN = 114.6\sqrt{\frac{2}{L/\lambda}}$ degree

$= 114.6\sqrt{\frac{2}{\left(\frac{3.75\lambda}{\lambda}\right)}}$

$BWFN =$

Pattern Multiplication :-

The total field pattern of an array of non isotropic but similar sources are the multiplication of the individual source patterns & the pattern of array of isotropic point sources each located at the phase centre of individual sources & having the relative amplitude & phase, whereas the total phase pattern is the addition of the phase pattern of the individual sources & that of the array of isotropic point sources

Total field (E) = $\{E_i(\theta,\phi) \times E_a(\theta,\phi)\} \times$ (multiplication of field pattern) $\{E_{pi}(\theta,\phi) + E_{pa}(\theta,\phi)\}$ (Addition of phase pattern)

where

$E_i(\theta,\phi) \rightarrow$ Field pattern of individual source

$E_a(\theta,\phi) \rightarrow$ Field pattern of array of isotropic point sources.

$E_{pi}(\theta,\phi) \rightarrow$ phase pattern of individual source

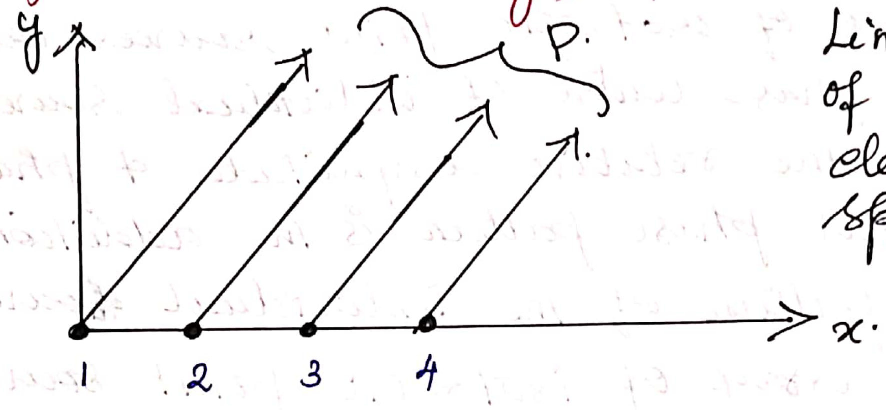
$E_{pa}(\theta,\phi) \rightarrow$ Phase pattern of array of isotropic point source.

angle $\theta \rightarrow$ vertical / polar / Elevation angle.

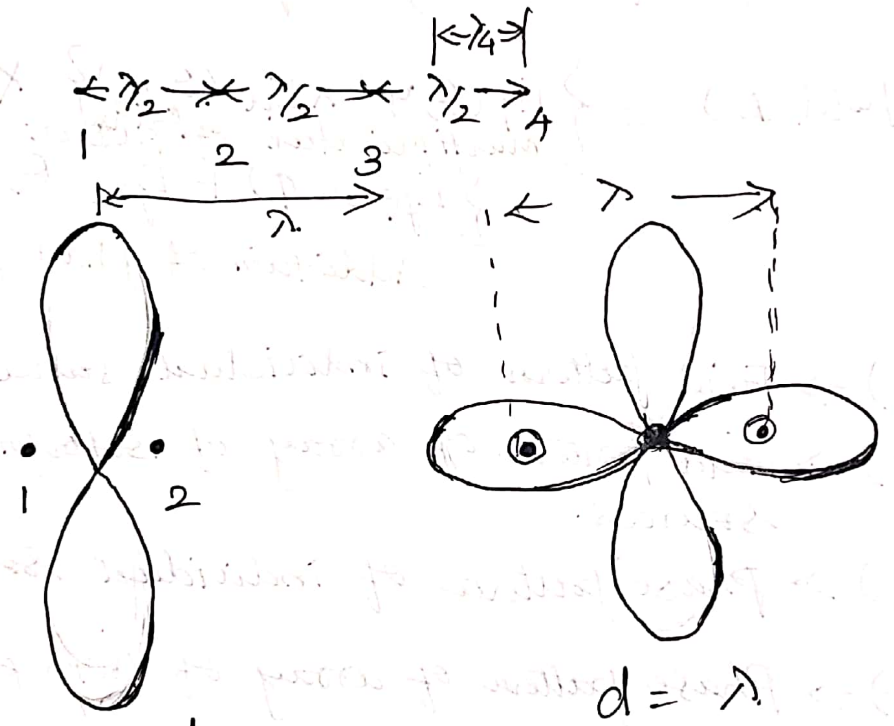
$\phi \rightarrow$ horizontal / azimuth angle.

$\left.\begin{array}{l}\text{Resultant} \\ \text{Field pattern}\end{array}\right\} = \left\{\begin{array}{l}\text{Individual} \\ \text{Source} \\ \text{Pattern}\end{array}\right\} \times \left\{\begin{array}{l}\text{Pattern of array of 2} \\ \text{point sources each} \\ \text{located at the phase} \\ \text{Centre of individual} \\ \text{source}\end{array}\right\}$

(i) Radiation pattern of 4 - isotropic Elements Fed in phase & spaced $\lambda/2$ ~~asp~~ apart. (Uniform linear array) :-



Linear Array of 4 isotropic elements spaced $\lambda/2$ apart fed in phase



$\lambda/2 = d$
$\alpha = 0$

Individual pattern. (unit pattern.) of 2 individual elements.

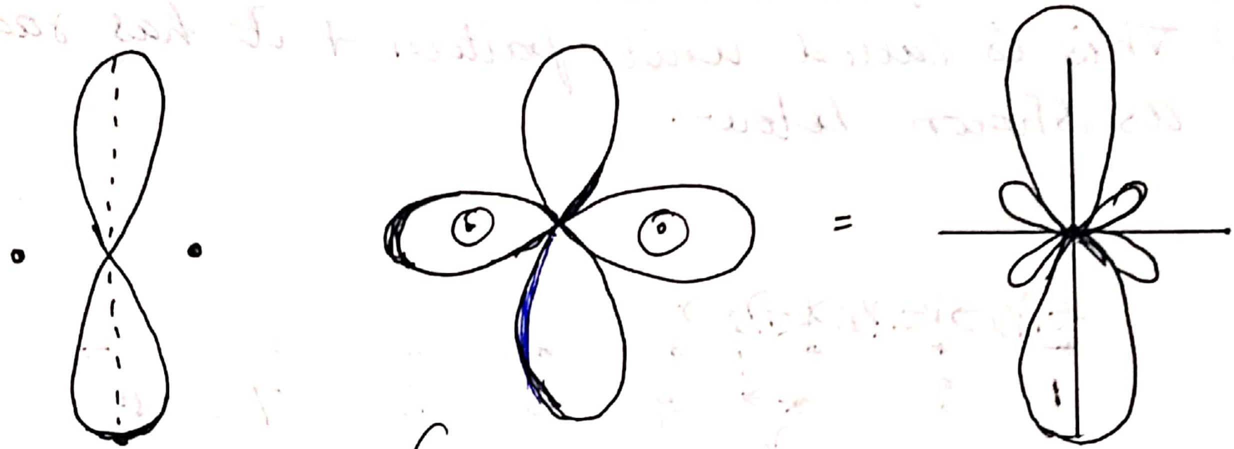
$d = \lambda$

Bidirectional pattern.

Group pattern due to array of two isotropic separated by λ.

(ie)
* The elements 1 & 2 are considered as one unit & this new unit is considered to be placed between the mid way of ① & ②. Similarly ③ & ④

* 4 elements spaced $\lambda/2$ have replaced by 2 units spaced λ, the radiation has been reduced to find out the radiation pattern of 2 antennas spaced λ.



Individual (unit pattern) pattern due

Group pattern due to array of two isotropic separated by λ.

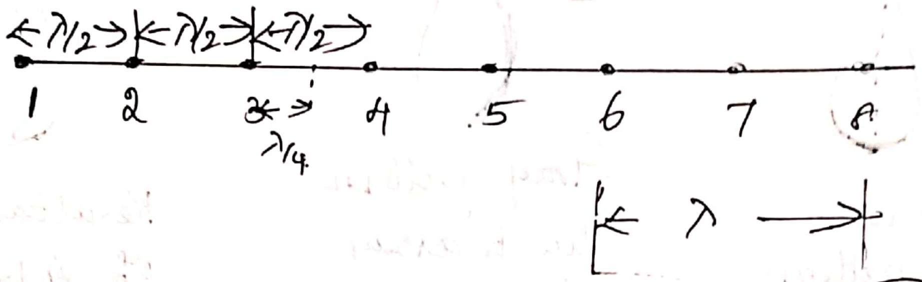
Resultant pattern of 4 isotropic elements.

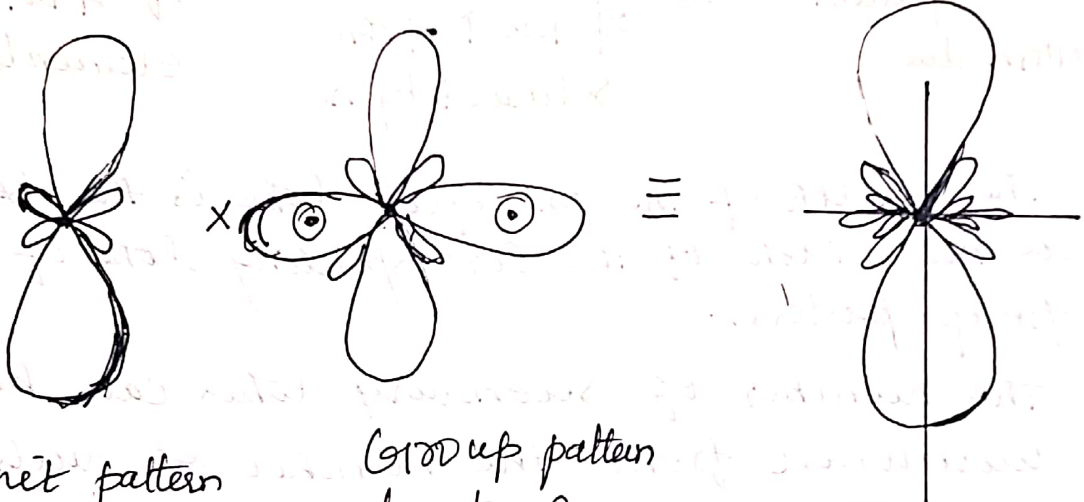
* The width of the principal lobe is the same as the width of the corresponding lobe of the group pattern.

* The number of secondary lobes can be determined from the number of nulls in the resultant pattern, which is the sum of the nulls in the unit & group patterns

(ii) Radiation pattern of 8 isotropic elements fed in phase & spaced $\lambda/2$ apart.

Here consider four elements as one unit & another four elements as another similar unit. This is called unit pattern & it has radiation as shown below.





Unit pattern due to 4 individual element

Group pattern due to 2 isotropic element spaced 2λ apart

Resultant pattern of 8 isotropic elements

Resultant radiation pattern of 8 isotropic elements by pattern multiplication

Advantages:

(i) It is the speedy method for sketching the pattern

(ii) It provides to be an useful tool in the design of antenna arrays.

Uniformly Spaced Arrays with uniform & Non uniform Excitation Amplitudes.

Disadvantages of uniform linear Array:-

* When the array length is increased to increase the directivity, but secondary or minor lobes also appear.
* This has to be reduced to a minimum desired level in comparison to principal (or) main lobes because considerable amount of power is wasted in this directions.

Example, in radar, while target finding false target may be indicated.

This could be overcome by Non uniform current excitation.

* The amplitudes of the radiating sources are arranged according to the coefficients of successive terms of the binomial series & therefore it is named as Binomial array.

Binomial Series

$$(a+b)^{n-1} = a^{n-1} + \frac{n-1}{1!} a^{n-2} \cdot b + \frac{(n-1)(n-2)}{2!} a^{n-3} b^2 + \frac{(n-1)(n-2)(n-3)}{3!} a^{n-4} b^3 + \cdots$$

where $n \rightarrow$ Numbers of radiating sources in the array.

Concept of Binomial array:-

If the array is arranged in such a way that radiating sources in the centre of the broad side array radiates more strongly than the radiating sources at the edges, thus minor lobes can be eliminated.

Conditions:-

(i) The space between the 2 consecutive radiating sources does not exceed $\lambda/2$

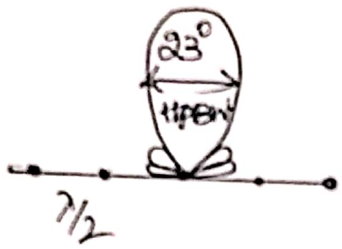
(ii) The current amplitudes in radiating sources (from outer towards centre source) are proportional to the coefficients of the successive terms of the binomial series.

These 2 conditions are satisfied in binomial arrays & the coefficients which corresponds to the amplitude of the sources are obtained by putting n = 1, 2, 3, 4, 5 ... in binomial series eqn

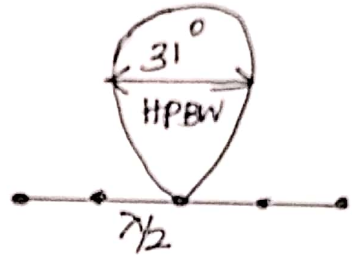
No. of sources	Relative Amplitude
n=1	1
n=2	1 1
n=3	1 2 1
n=4	1 3 3 1
n=5	1 4 6 4 1
n=6	1 5 10 10 5 1
n=7	1 6 15 20 15 6 1
n=8	1 7 21 35 35 21 7 1
n=9	1 8 28 56 70 56 28 8 1
n=10	1 9 36 84 126 126 84 36 9 1

$$(1+x)^{m-1} = 1 + \frac{(m-1)x}{1!} + \frac{(m-1)(m-2)x^2}{2!} + \cdots$$

* In binomial array, elimination of secondary lobes takes place at the cost of directivity.
* HPBW of binomial array is more than that of uniform array for the same length of the array
* For $n = 5$, $d = \lambda/2$, HPBW is 31° for binomial array.



Uniform Array



Binomial array with Amplitude ratio 1 : 4 : 6 : 4 : 1.

* Thus in uniform array secondary lobes appear but principal lobe is sharp & narrow.
* In binomial array, width of beam widens but without secondary lobes.

Disadvantages:-

(i) HPBW increases & hence the directivity decreases

(ii) For design of a large array, larger amplitude ratio of sources is required

UNIT - IV

Passive & Active Microwave Devices

Microwave Passive Components :-

A passive element is an electrical component that does not generate power, but instead dissipates, stores, &/or releases it.

Most commonly used passive devices are terminators, attenuators, phase shifters, directional couplers, power dividers, T-junctions, hybrides etc...

Attenuators :-

Attenuators are passive devices used to control power levels in a microwave system by partially absorbing the transmitted signal wave.

Types :-

1. Fixed Attenuator → Coaxial, waveguide.

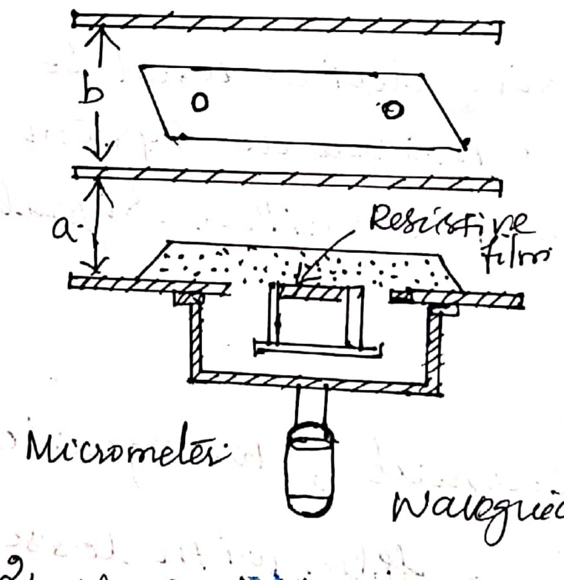
* Coaxial ~~waveguide~~ Attenuator → film with losses on the centre conductor to absorb power

* ~~Fixed~~ waveguide Attenuator :-

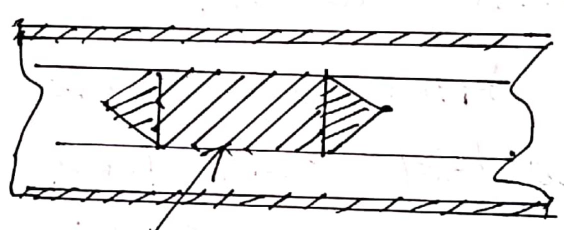
Consists of a thin dielectric strip coated with resistive film & placed at the centre of the waveguide parallel to the maximum E field.

whenever, the incident wave falls on the waveguide, current induced on the resistive film & it produces power dissipation, it leads to attenuation of microwave energy.

The dielectric strip is tapered at both ends up to a length of more than $\lambda/2$ to reduce reflections. The resistive vane is supported by two dielectric rods separated by an odd multiple of quarter wave length & is perpendicular to the electric field



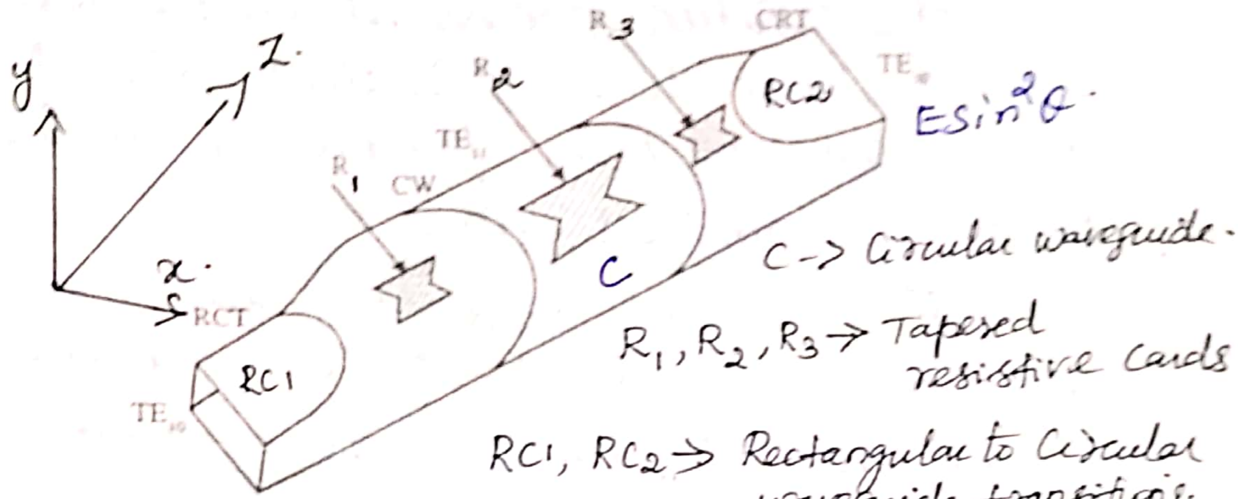
Waveguide Attenuator.



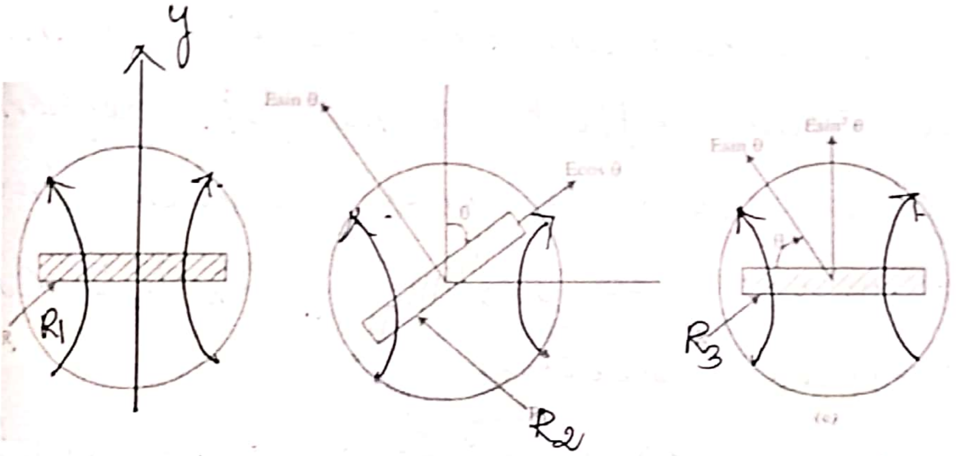
Coaxial type Attenuator.

2. Variable type Attenuator:-

It is constructed by moving the resistive vane by means of micrometer screw from one side of the narrow wall to the centre where the E field is maximum, or by changing the depth of insertion of a resistive vane at an E field maximum through a longitudinal slot at the middle of the broad wall.



Precision type variable attenuator.



A precision type variable attenuator makes use of a circular section (C) containing a very thin tapered resistive card (R_2) to both sides of which are connected axisymmetric sections of circular to rectangular waveguide tapered transitions (RC1 & RC2).

The centre circular section with the resistive card can be precisely rotated by 360° w.r. to the two fixed sections of circular to rectangular waveguide transitions.

The induced current on the resistive card R_2 due to the incident signal is dissipated as heat producing attenuation of the transmitted signal.

The incident TE_{10} dominant wave on the rectangular waveguide is converted into a dominant TE_{11} mode in the circular waveguide.

A very thin tapered resistive card is placed perpendicular to the E field at the circular end of each transition section so that it has a negligible effect on the field perpendicular to it but absorbs any component parallel to it. Therefore a pure TE_{11} mode is excited in the middle section.

If the resistive card in the centre section is kept at an angle θ relative to the E field direction of the TE_{11} mode, the component $E\cos\theta$ parallel to the card gets absorbed while the component $E\sin\theta$ is transmitted without attenuation, finally it appears as electric field component $E\sin^2\theta$ in a rectangular output guide.

Scanned with CamScanner

Therefore, the attenuation of the incident wave is

$$\alpha = \frac{E}{E\sin^2\theta} = \frac{1}{\sin^2\theta} = \frac{1}{|S_{21}|}$$

$$\alpha(dB) = -40\log(\sin\theta) = -20\log|S_{21}|$$

Attenuators are normally matched reciprocal devices, so that

$$|S_{21}| = |S_{12}|$$

$$|S_{11}| \text{ or } |S_{22}| = \frac{VSWR - 1}{VSWR + 1} << 0.1$$

The S-matrix of an ideal precision rotary attenuator is

$$[S] = \begin{bmatrix} 0 & \sin^2\theta \\ \sin^2\theta & 0 \end{bmatrix}$$

Waveguide Tees :-

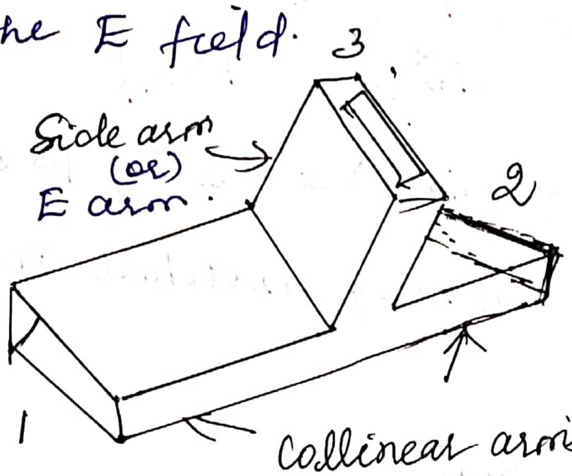
* Tees are 3 port Components.
* They are used to Connect a branch or section of the waveguide in series or parallel with the main waveguide transmission line for providing means of splitting, & also of Combining power in a waveguide system.

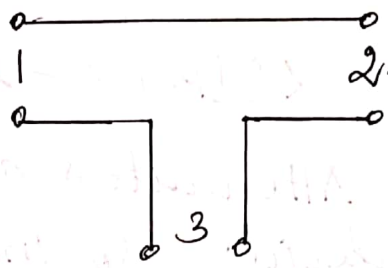
Types :-

1. Eplane Tee
2. H plane Tee
3. Hybrid Tee

1. E plane (Series) Tee :- (Voltage Junction)

The axis of the side arm is parallel to the E field.





* The ports 1 & 2 are 180° out of phase with each other.

* Side arm provides bidirectional wave propagation to form parallel port.

Let us consider 3 x 3 matrix

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \rightarrow (1)$$

Scattering coefficients S_{13} & S_{23} are out of phase by 180° with an input at port 3

$$S_{23} = -S_{13} \rightarrow (2)$$

* The port is perfectly matched to the junction

$$S_{33} = 0 \rightarrow (3)$$

* From the symmetry property

$$S_{ij} = S_{ji} \rightarrow (A)$$

$$S_{12} = S_{21} \; ; \; S_{23} = S_{32} \; ; \; S_{13} = S_{31}. \rightarrow (4)$$

Considering eqn (3) & (4)

$$\therefore [S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \rightarrow (5)$$

* From unitary property

$$[S][S]^* = [I]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

After multiplying we get

R_1C_1 : $S_{11}S_{11}^* + S_{12}S_{12}^* + S_{13}S_{13}^* = I$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \rightarrow (6)$$

R_2C_2 : $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \rightarrow (7)$

R_3C_3 : $|S_{13}|^2 + |S_{13}|^2 = 1 \rightarrow (8)$

R_3C_1 : $S_{13}S_{11}^* - S_{13}S_{12}^* = 1 \rightarrow (9)$

Equating the equations (6) & (7)

$$S_{11} = S_{22} \rightarrow (10)$$

From eqn (8)

$2|S_{13}|^2 = 1$ or $S_{13} = \frac{1}{\sqrt{2}} \rightarrow (11)$

From eqn (9)

$$S_{13}(S_{11}^* - S_{12}^*) = 1$$

$$S_{13} \neq 1$$

$$\therefore \; S_{11}^* - S_{12}^* = 1$$

$$S_{11}^* = S_{12}^* \; ; \Rightarrow \; S_{11} = S_{12} \rightarrow (12)$$

using the equations (10), (11) & (12), in eqn (6) we get

$$|S_{11}|^2 + |S_{11}|^2 + 1/2 = 1$$

$$2|S_{11}|^2 = 1 - 1/2 = 1/2$$

$$|S_{11}|^2 = \frac{1}{4}$$

$$S_{11} = 1/2 \rightarrow (13)$$

Substituting the values from the above eqn [S] matrix.

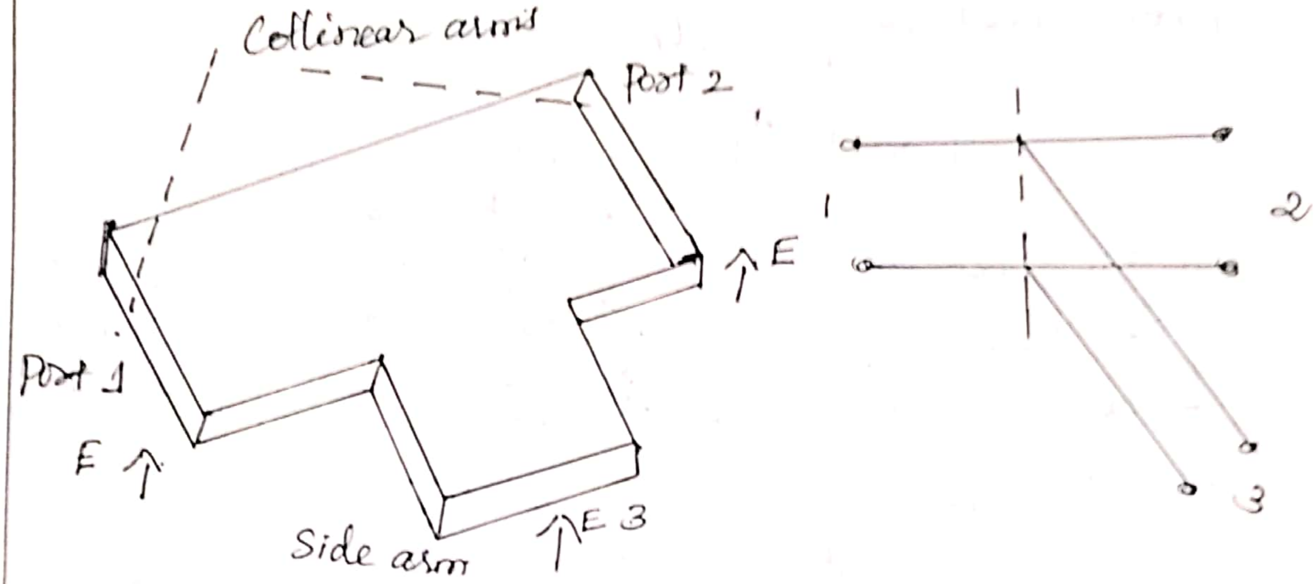
$$[S] = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}$$

$$\therefore \; b = [S][a]$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & -1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}$$

H Plane Tee :- Shunt Tee or Current junction

The axis of the side arm is parallel to the magnetic field.



The bidirectional propagation of side arm forms a serial port

Let us consider 3x3 matrix

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \rightarrow (1)$$

Scattering coefficients S_{13} & S_{23} are equal as the junction is symmetrical in plane.

From the symmetry property

$$S_{ij} = S_{ji}$$

$$S_{12} = S_{21}\ ;\ S_{23} = S_{32} = S_{13};\ S_{13} = S_{31}.$$

The port is perfectly matched $S_{33} = 0.$

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \rightarrow \text{eqn } (2)$$

From unitary property.

$$[S][S^*] = [I]$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{13}^* \\ S_{13}^* & S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Multiplying

$R_1 C_1 \Rightarrow S_{11} S_{11}^* + S_{12} S_{12}^* + S_{13} S_{13}^* = 1 \rightarrow (3)$

$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \rightarrow (3)$

$R_2 C_2 \Rightarrow |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \rightarrow (4)$

$R_3 C_3 \Rightarrow |S_{13}|^2 + |S_{13}|^2 = 1 \rightarrow (5)$

$R_3 C_1 \Rightarrow S_{13} S_{11}^* - S_{13} S_{12}^* = 0 \rightarrow (6)$

$2|S_{13}|^2 = 1$ (or) $S_{13} = 1/\sqrt{2} \rightarrow (7)$

$|S_{11}|^2 = |S_{22}|^2$

$S_{11} = S_{22} \rightarrow (8)$

From eqn (6) $S_{13}(S_{11}^* + S_{12}^*) = 0$

$S_{13} \neq 0$, $S_{11}^* + S_{12}^* = 0$, $S_{11}^* = -S_{12}^*$

Since $S_{13} \neq 0$; $S_{11}^* + S_{12}^* = 0$; $S_{11}^* = -S_{12}^*$

$S_{11} = -S_{12}$ or $S_{12} = -S_{11} \rightarrow (9)$

using these

$S_{13} \neq 0$; $S_{11}^* + S_{12}^* = 0$; or $S_{11}^* = -S_{12}^*$

$|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1$ or $2|S_{11}|^2 = \frac{1}{2}$ or $S_{11} = \frac{1}{2} \rightarrow (10)$

From (8) & (9)

$S_{12} = -\frac{1}{2} \longrightarrow (11)$

$S_{22} = \frac{1}{2} \longrightarrow (12)$

Sub

$$S = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$[b] = [S][a]$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Magic Tee :- Hybrid Tee or 3 dB Coupler

E - H plane Tee junction is formed by attaching two simple waveguides one parallel & the other series to a rectangular waveguide which is already has two ports.

Port 3 -> H arm or Sumport or parallel port

port 4 => E arm or Difference port or series port.

Characteristics of E-H-plane Tee.

* If a signal of equal phase & magnitude is sent to port 1 & port 2 then the output at port 4 is zero & the output at port 3 will be the additive of both the ports 1 & 2.

* If a signal is sent to port 4, E-arm, then the power is divided between port 1 & 2 equally but in opposite phase while there would be no output at port 3. Hence $S_{34} = 0$

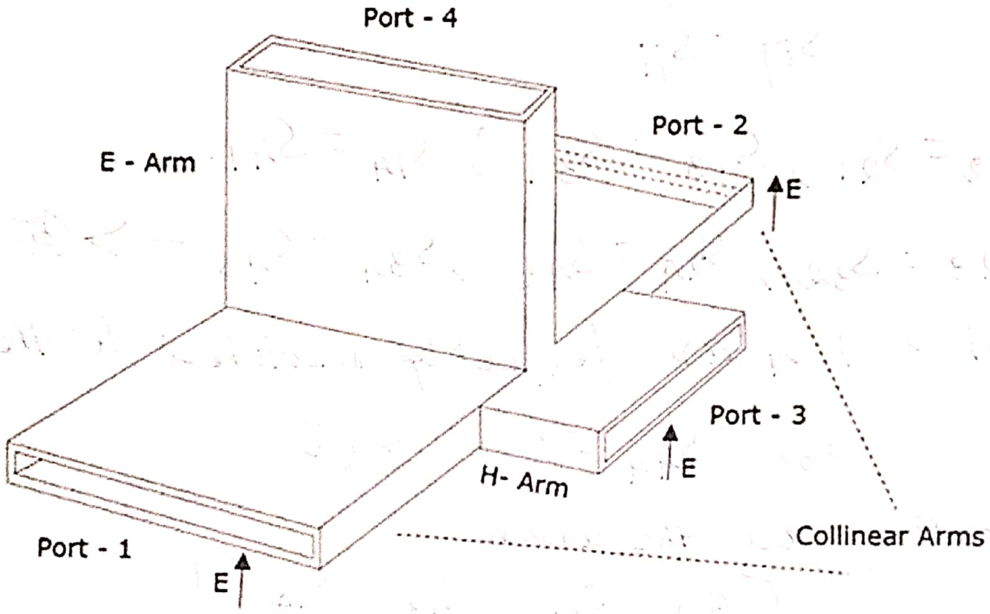
* If a signal is fed port 3, then the power is divided between port 1 & 2 equally while there would be no output at port 4. Hence $S_{43} = 0$

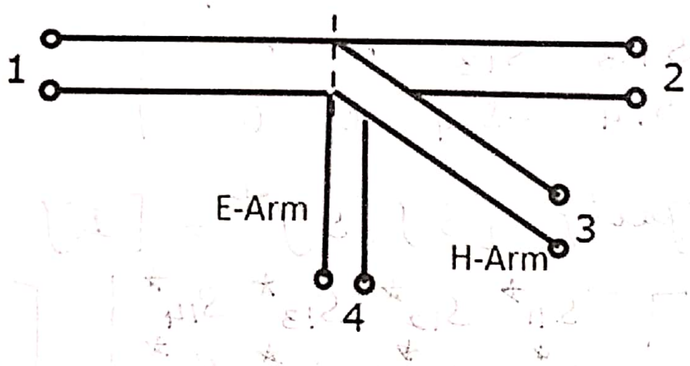
* If a signal is fed at one of the collinear ports, then there appears no output at the other collinear port as the E arm produces a phase delay & the H arm produces a phase advance

$$S_{12} = S_{21} = 0.$$

Properties of E-H plane Tee can be defined by its $[S]_{4\times4}$ matrix.

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \rightarrow (1)$$





Magic Tee junction

As it has H-plane Tee section

$$S_{23} = S_{13} \rightarrow (2)$$

As it has E-plane Tee section

$$S_{24} = -S_{14} \rightarrow (3)$$

The E-Arm & H-arm port are so isolated that the other won't deliver an output, if an input is applied at one of them. Hence, this can be noted as

$$S_{34} = S_{43} = 0 \rightarrow (4)$$

From symmetry property

$$S_{ij} = S_{ji}$$

$$S_{12} = S_{21}, \; S_{13} = S_{31} \; ; \; S_{14} = S_{41}.$$

$$S_{23} = S_{32}, \; S_{24} = S_{42}, \; S_{34} = S_{43} \rightarrow (5)$$

If port 3 & 4 are perfectly matched to the junction

$$S_{33} = S_{44} = 0 \rightarrow (6)$$

Substituting all the above

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \rightarrow (7)$$

From unitary property $[S][S]^* = [I]$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1C_1 = |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 + |S_{14}|^2 = 1 \rightarrow (8)$$

$$R_2C_2 = |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 + |S_{14}|^2 = 1 \rightarrow (9)$$

$R_3C_3 \Rightarrow |S_{13}|^2 + |S_{13}|^2 = 1 \longrightarrow$ (10)

$R_4C_4 : |S_{14}|^2 + |S_{14}|^2 = 1 \longrightarrow$ (11)

From (10) & (11)

$$S_{13} = \frac{1}{\sqrt{2}} \longrightarrow (12)$$

$$S_{14} = \frac{1}{\sqrt{2}} \longrightarrow (13)$$

Comparing (8) & (9)

$$S_{11} = S_{22} \longrightarrow (14)$$

Using these values from the eqn

$$|S_{11}|^2 + |S_{12}|^2 + \tfrac{1}{2} + \tfrac{1}{2} = 1.$$

$$|S_{11}|^2 + |S_{12}|^2 = 0$$

$$S_{11} = S_{22} = 0 \longrightarrow (15)$$

From eqn (9) we get $S_{22} = 0$

Ports (1) & (2) are perfectly matched to the junction. As this is a 4 port junction, whenever two ports are perfectly matched, the other two ports are also perfectly matched to the junction

The junction where all the four ports are perfectly matched is called as Magic Tee junction

By substituting the eqn (12) & (16) in (7)

$$S = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

Applications:

1. E-H plane junction is used to measure the impedance.
2. E-H plane Tee is used as duplexer.
3. E-H plane Tee is used as mixer.

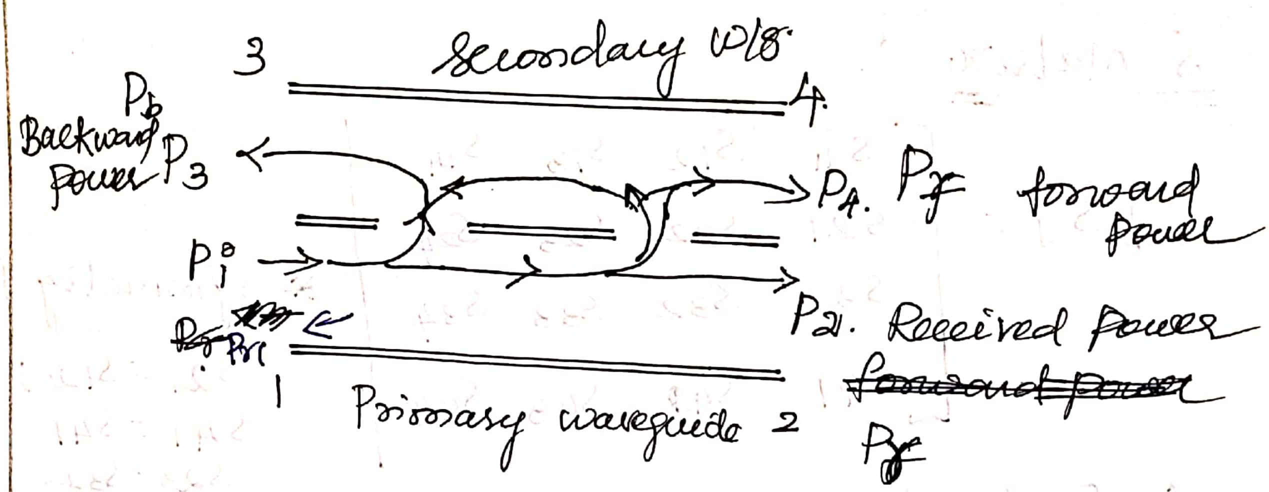
E-H plane Tee junction is also used as Microwave bridge, Microwave discriminator.

Directional Couplers :-

* A directional coupler is a four port passive device commonly used for coupling a known fraction of the microwave power to a port (coupled port) in the auxiliary line while flowing from the ~~po input~~ input port to the output port in the main line. The remaining port is an ideally isolated port & matched terminated

Types:-

1. Multiple aperture
2. Coupled coaxial or strip or microstrip line
3. Branch line couplers.



The performance of a directional coupler is measured in terms of four basic parameters.

(i) Coupling factor (C)

$$C\,(dB) = 10 \log \frac{P_i}{P_f}$$

$$= 10 \log P_1/P_4$$

(ii) Transmission loss (T) dB

$$T\,(dB) = 10 \log \frac{P_i}{P_r}$$

$$= 10 \log P_1/P_2$$

(iii) Directivity (D) dB $= 10 \log P_f/P_b$

$$= 10 \log P_4/P_3$$

(iv) Return loss (R) dB $= 10 \log P_i/P_{r1}$.

(v) Isolation (I) dB $= 10 \log \dfrac{P_i}{P_b}$

$$= 10 \log P_1/P_3.$$

S matrix

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

* symmetry ppty

$S_{21} = S_{12}$; $S_{31} = S_{13}$

$S_{41} = S_{41}$

$S_{23} = S_{32}$

* Perfectly matched ; $S_{11} = S_{22} = S_{33} = S_{44} = 0$

* Third port is perfectly matched.

$\therefore S_{13} = S_{31} = 0$; $S_{24} = S_{42} = 0$

$$[S] = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix}$$

$R_1 C_1$ $\quad |S_{12}|^2 + |S_{14}|^2 = 1 \rightarrow (1)$

$R_2 C_2$ $\quad |S_{12}|^2 + |S_{23}|^2 = 1 \rightarrow (2)$

$R_3 C_3$ $\quad |S_{23}|^2 + |S_{34}|^2 = 1 \rightarrow (3)$

$R_4 C_4$ $\quad |S_{14}|^2 + |S_{34}|^2 = 1 \rightarrow (4)$

$R_1 C_3$ $\quad S_{12} S_{23}^* + S_{14} S_{34}^* = 0 \rightarrow (5)$

from (1) & (2)

$$|S_{14}| = |S_{23}|$$

$$S_{14} = S_{23}.$$

from ② & ③

$$|S_{12}| = |S_{34}|$$

$$S_{12} = S_{34}.$$

Say $\alpha = S_{12} = S_{34}.$

from ⑤ $S_{12} S_{23}^* + S_{14} S_{34}^* = 0$

$$\therefore S_{12} = S_{34}$$

$$(S_{12} S_{23}^* + S_{14} S_{12}^*) = 0$$

$$S_{12} (S_{23}^* + S_{14}) = 0$$

$$\alpha (S_{23}^* + S_{14}) = 0$$

$\alpha \neq 0$; $S_{23}^* + S_{14} = 0$

$$S_{23} = -S_{14}$$

Let say

$$S_{23} = -S_{14} = \beta.$$

$$S_{14} = -S_{23} = \beta$$

$$\text{or } S_{23} = -\beta.$$

Thus

$$[S] = \begin{bmatrix} 0 & \alpha & 0 & \beta \\ \alpha & 0 & -\beta & 0 \\ 0 & -\beta & 0 & \alpha \\ \beta & 0 & \alpha & 0 \end{bmatrix}$$

Resonators :-

Microwave resonators are used in a variety of applications, including filters, oscillators, frequency meters, & tuned amplifiers.
Its operation is very similar to that of lumped elements of circuit theory.

Microwave resonators can be also constructed from closed sections of waveguide. Because radiation loss from an open ended waveguide can be significant waveguide resonators are usually short circuited at both ends, thus forming a closed box or cavity.

Electric & Magnetic energy is stored within the cavity enclosure, & power is dissipated in the metallic walls of the cavity as well as in the dielectric material that may fill the cavity.

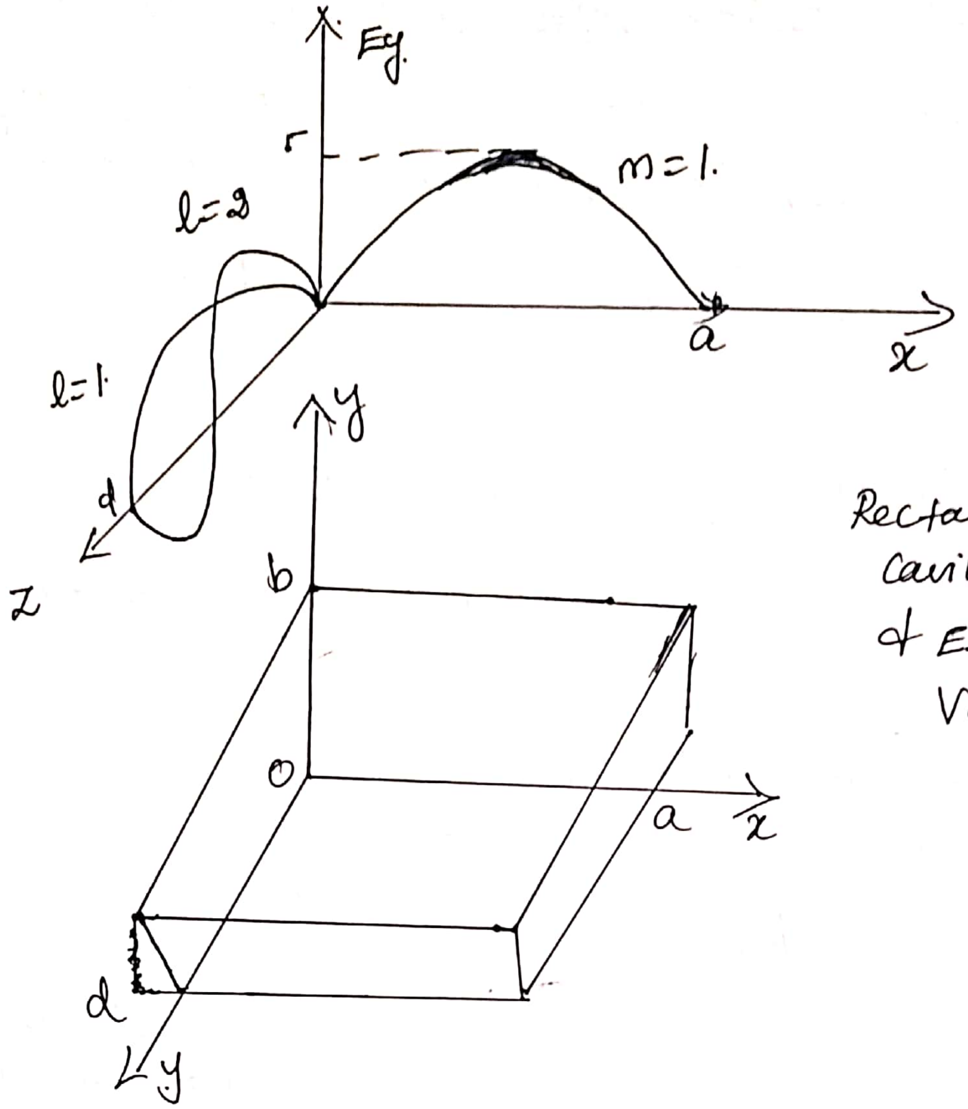
Coupling to a cavity resonator may be by a small aperture or a small probe or loop.

* Derivation of resonant frequencies of TE or TM resonant mode of R/c cavity.
* Derivation of unloaded Q of the TE_{101} mode.

Resonant frequencies :-

The rectangular cavity consists of a length 'd' of rectangular waveguide shorted at both ends ($z=0, d$).

The boundary conditions on the side walls ($x=0, a$ & $y=0, b$) of the cavity.



Rectangular cavity resonator & Electric field Variations.

The transverse electric fields (E_x, E_y of the TE_{mn} or TM_{mn} rectangular waveguide mode can be

$$\bar{E}_t(x,y,z) = \bar{e}(x,y)\left(A^+ e^{-j\beta_{mn}z} + A^- e^{j\beta_{mn}z}\right)$$

where $\bar{e}(x,y) \rightarrow$ Transverse Variation of the mode.

$A^+, A^- \rightarrow$ arbitrary amplitudes of forward & backward traveling waves.

The propagation Constant of the m, n th TE or TM mode is

$$\beta_{mn} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$ where $k = \omega\sqrt{\mu\varepsilon}$.

$\mu \rightarrow$ Permeability $\varepsilon \rightarrow$ permittivity of the material filling in the ...

(i) Single - Ended Mixer Design :.

Applying the condition $\bar{E}_t = 0$ at $z=0$, implies $A^+ = -A^-$. Then the condition that $\bar{E}_t = 0$ at $z=d$

$$\bar{E}_t(x, y, d) = -\bar{e}(x, y) A^+ 2j \sin \beta_{mn} d = 0.$$

$$A^+ \neq 0.$$

then solution $\beta_{mn} d = l\pi$, $l = 1, 2, 3, \ldots$

A resonance wave number for the rectangular cavity

$$k_{mnl} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

TE_{mnl} or TM_{mnl} are resonant mode of the cavity where the indices $m, n, l \rightarrow$ no. of variations in the standing wave pattern in x, y, z dirn.

The resonant frequency of the TE_{mnl} or TM_{mnl} mode

$$f_{mnl} = \frac{c k_{mnl}}{2\pi\sqrt{\mu_r \epsilon_r}} = \frac{c}{2\pi\sqrt{\mu_r \epsilon_r}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

If $b < a < d$, the dominant resonant mode (lowest resonant frequency) will be the TE_{101} mode.

TE$_{mnp}$ mode field :-

$$H_z = H_0 \cos(m\pi x/a)\cos(n\pi y/b)\sin(p\pi z/d);$$

$$H_y = \frac{1}{K_c^2}\frac{\partial^2 H_z}{\partial y\,\partial z}$$

$$H_y = -H_0/K_c^2 \,(p\pi/d)(n\pi/b)\cos(m\pi x/a)\sin(n\pi y/b)\cos(p\pi z/d)$$

$$H_x = \frac{1}{K_c^2}\frac{\partial^2 H_z}{\partial x\,\partial z}$$

$$= -H_0/K_c^2\,(p\pi/d)(n\pi/b)\sin(m\pi x/a)\cos(n\pi y/b)\cos(p\pi z/d).$$

$$E_z = 0$$

$$E_y = j\frac{\omega\mu H_0}{K_c^2}\frac{\partial H_z}{\partial x}$$

$$= -j\frac{\omega\mu H_0}{K_c^2}\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{d}\right)$$

$$E_x = -j\frac{\omega\mu H_0}{K_c^2}\frac{\partial H_z}{\partial y}$$

$$= j\frac{\omega\mu H_0}{K_c^2}\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{d}\right)$$

where $m = 0, 1, 2, \ldots$ $n = 0, 1, 2, 3 \ldots$

$p = 1, 2, 3, 4 \ldots$

$m, n \rightarrow$ mode Excitations

$$K_c^2 = (m\pi/a)^2 + (n\pi/b)^2$$

$K_c \rightarrow$ cut off wave number.

TMmnp mode field :-

$$E_z = E_0 \sin(m\pi x/a) \sin(n\pi y/b) \cos(p\pi z/d)$$

$$E_y = \frac{E_0}{k_c^2} \cdot \frac{\partial^2 H_z}{\partial y \partial z}$$

$$= \frac{-E_0}{k_c^2} \left(\frac{n\pi}{b}\right)\left(\frac{p\pi}{d}\right) \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right)$$

$$E_x = E_0 / k_c^2 \cdot \frac{\partial^2 E_z}{\partial x \partial z}$$

$$= \frac{-E_0}{k_c^2} \left(\frac{m\pi}{a}\right)\left(\frac{p\pi}{d}\right) \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right)$$

$$H_z = 0.$$

$$H_y = \frac{-j\omega\varepsilon_r E_0}{k_c^2} \frac{\partial E_z}{\partial x}$$

$$= \frac{-j\omega\varepsilon_r E_0}{k_c^2} \left(\frac{m\pi}{a}\right) \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \cos\left(\frac{p\pi z}{d}\right).$$

$$H_x = \frac{j\omega\varepsilon_r E_0}{k_c^2} \partial E_z / \partial y.$$

$$= \frac{j\omega\varepsilon_r E_0}{k_c^2} \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \cos\left(\frac{p\pi z}{d}\right)$$

$m = 1, 2, 3$, $n = 1, 2, 3$, $p = 0, 1, 2$; $m \neq 0$, $n \neq 0$

For either TEmnp or TMmnp mode, the resonant frequency is

$$f_r = \frac{1}{2\sqrt{\mu\varepsilon_r}} \sqrt{(m/a)^2 + (n/b)^2 + (p/d)^2} \text{ Hz}$$

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Circular Cavity:

Electromagnetic field analysis shows that due to φ - symmetric structure of circular cylindrical cavity, field solutions possess harmonic solutions in φ, & standing waves in the radial & z dirns. The field components inside the cavity are described in terms of TE_{nmp} & TM_{nmp} modes

TE_{nmp} mode field:

$$H_z = H_0 J_n(x'_{nm} \rho/a) \cos n\varphi \sin(p\pi z/d).$$

$$H\varphi = -H_0 \left(\frac{p\pi}{d}\right)\left(\frac{n}{\rho}\right)\left(\frac{a}{x'_{nm}}\right)^2 J_n(x'_{nm} \rho/a) \sin n\varphi \cos(p\pi z/d)$$

$$H\rho = H_0 \frac{p\pi}{d} \left(\frac{a}{x'_{nm}}\right) J'_n (x'_{nm} \rho/a) \cos n\varphi \cos(p\pi z/d)$$

$$E_z = 0.$$

$$E\varphi = jH_0 \omega\mu (a/x'_{mn}) J_n' (x'_{nm} \rho/a) \cos n\varphi \sin(p\pi z/d)$$

$$E_z = 0$$

$$E\varphi = jH_0 \omega\mu (a/x'_{mn}) J'_n (x'_{nm} \rho/a) \cos n\varphi \sin(p\pi z/d)$$

$$E\rho = jH_0 \omega\mu (n/\rho)(a/x'_{mn})^2 J_n (x'_{nm} \rho/a) \sin n\varphi \sin(p\pi z/d)$$

where $n = 0, 1, 2 \ldots$

$m = 1, 2, 3 \ldots$

TM_{nmp} mode field:

$$E_z = E_0 J_n (x_{nm} \rho/a) \cos n\varphi \cos(p\pi z/d)$$

$$E\varphi = E_0 \frac{p\pi}{d} (n/\rho)(a/x_{nm})^2 J_n (x_{nm} \rho/a) \sin n\varphi \sin n\varphi \sin(p\pi z/d)$$

$$E_\rho = -E_0 \frac{p\pi}{d} (a/x_{nm}) J_n' (x_{nm}\rho/a) \cos n\varphi \sin(p\pi z/d)$$

$H_z = 0$

$$H_\varphi = -jE_0 \omega \varepsilon_r (a/x_{nm}) J_n' (x_{nm} \rho/a) \cos n\varphi \cos(p\pi z/d)$$

$$H_\rho = -j\omega \varepsilon E_0 (a/x_{nm})^2 J_n (x_{nm} \rho/a) \sin n\varphi \cos(p\pi z/d)$$

The resonant frequencies are

TE_{nmp};

$$f_r = \frac{1}{2\pi\sqrt{(\mu\varepsilon_r)}} \sqrt{\left[(x'_{nm}/a)^2 + (p\pi/d)^2\right]} \; ; Hz$$

TM_{nmp};

$$f_r = \frac{1}{2\pi\sqrt{(\mu\varepsilon_r)}} \cdot \sqrt{\left[(x_{nm}/a)^2 + (p\pi/d)^2\right]} \; ; Hz$$

The smallest root out of x_{01} & x'_{11} generates the dominant mode.

$$F = \frac{f_r(TM_{011})}{f_r(TE_{111})} = \frac{x_{01}}{\sqrt{[x_{11}'^2 + (\pi a/d)^2]}}$$

Principles of Microwave Semiconductor Devices:

The PN junction diode is not very suitable for high frequency applications because of the high junction capacitance. These diodes formed by a metal semiconductor contact possess smaller junction capacitances & consequently reach higher frequency limits

* Example:-

Schottky diodes, PIN diodes, IMPATT diodes Gunn diodes

Schottky Barrier Diodes:-

Schottky diodes are used in RF ~~diode~~ detectors, mixers, attenuators, oscillators & amplifiers.

Schottky barrier diode has a different reverse saturation current mechanism, which is determined by the thermionic emission of the majority carriers across the potential barrier. This current is order of magnitude larger than the diffusion - driven minority carriers constituting the reverse saturation current of the ideal PN junction diode.

The Schottky diode has a typical reverse saturation current density on the order of 10^{-6} A/cm^2 compared with 10^{-11} A/cm^2 of a conventional Si based PN junction diode.

The metal electrode (aluminium, gold etc) is in contact with a weakly doped n - semiconductor layer epitaxially grown on a highly doped n^+ substrate.

The dielectric is assumed to be ideal, i.e the conductance is zero. The current voltage characteristic is described by

$$I = I_S \, e^{(V_A - IR_s)} - 1$$

where $I_S \rightarrow$ reverse saturation current is given by

$$I_S = A \left(R^* T^2 \exp\left[\frac{-q V_b}{KT} \right] \right)$$

$R^* \rightarrow$ Richardson Constant.

The junction resistance R_j is dependent on the basis current, just as the diode series resistance, which is composed of epitaxial & substrate resistances $R_S = R_{epi} + R_{sub}$. as shown in circuit model

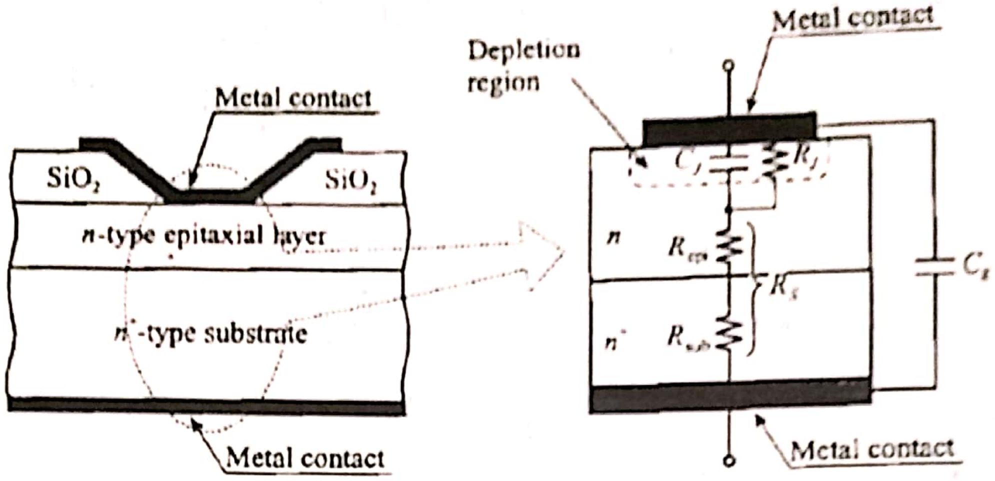
For certain applications, the series resistance may form a feedback loop, which means the resistance is multiplied by a gain factor of potentially large magnitude

In circuit realizations of high frequency Schottky diodes, the planar configuration gives rise to relatively large parasitic capacitances for very small metal contacts of typically 10 μm diameter & less.

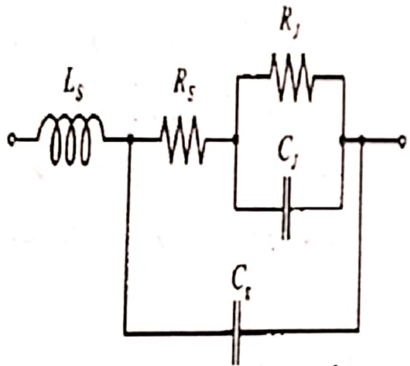
The stray capacitances can be somewhat minimized through the addition of an isolation ring.

The small signal junction capacitance & junction resistance around the quiescent or operating point V_Q.

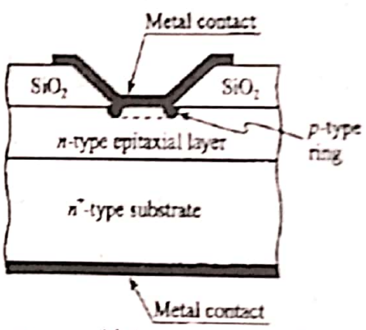
A small AC signal carrier frequency component V_d



Cross Sectional View of Si Schottky Barrier Diode.



Circuit Model of typical Schottky diode



Schottky Diode with additional isolation ring suitable for very high frequencies applications

$V = V_Q + V_d$.

The negligible IR_S

$$I = I_S\left(e^{V/V_T} - 1\right) = I_S\left(e^{V_Q/V_T}\, e^{V_d/V_T} - 1\right)$$

Expanding this equation in Taylor Series about Q point & retaining the first two terms

$$I(V) \simeq I_Q + \left.\frac{dI}{dV}\right|_{V_Q} v_d = I_Q + \frac{I_S v_d}{V_T} e^{V_Q/V_T}$$

$$= I_Q + (I_Q + I_S)\frac{v_d}{V_T}$$

$$= I_Q + \frac{v_d}{R_f}.$$

The Junction resistance $R_f(v_Q)$ is identified as

$$\boxed{R_f(v_Q) = \frac{V_T}{I_Q + I_S}}$$

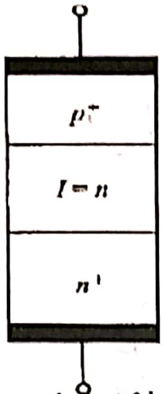
PIN Diode :-

PIN diodes find applications as high frequency switches. They Contain an additional layer of an intrinsic (I layer) or lightly doped semiconductor sanwiched between highly doped p^+ & n^+ layers.

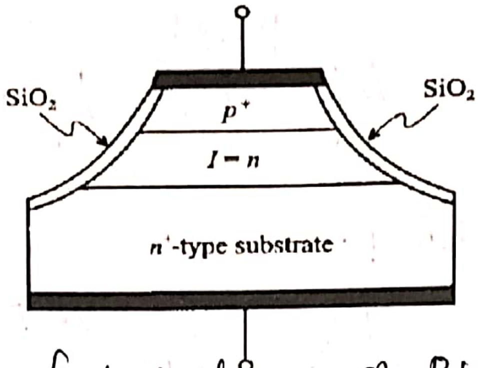
Depending upon applications & frequency range, the thickness of the middle layer ranges from 1 to 100 μm.

In forward direction, the diode behaves as if it possesses a variable resistance Controlled by the applied current. However, in reverse direction the lightly doped inner layer creates space charges whose extent reaches the highly doped outer layers.

This effect even takes place for small reverse voltages & remains constant up to high voltages with the ~~consequence~~ consequence that the diode behaves similar to a dual plate capacitor.

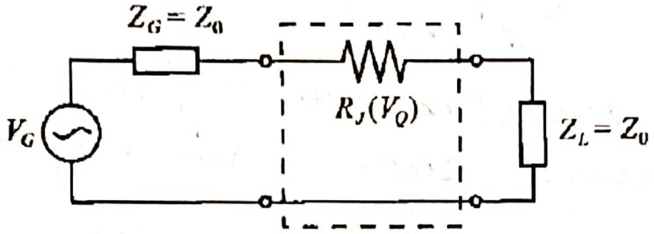


PIN diode

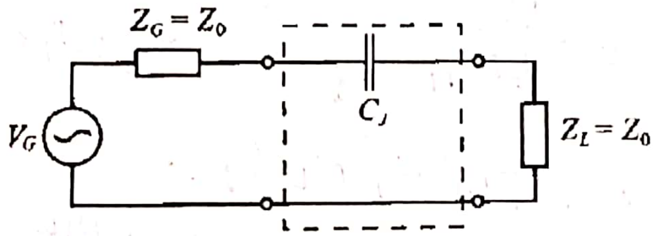


fabrication of PIN diode

PIN Diode Construction



(a) Forward bias



(b) Reverse bias (isolation)

PIN diode in Series Connection

The mathematical representation of I-V characteristic depends on the level & direction of current flow.

To keep things simple,

In forward direction & for a weakly doped n-type intrinsic layer the current through the diode

$$I = A\left[\frac{q n_i^2 W}{N_D \tau_p}\right]\left(e^{V_A/(2V_T)} - 1\right)$$

where $W \rightarrow$ width of the intrinsic layer

$\tau_p \rightarrow$ Excess minority carrier lifetime.

$N_D \rightarrow$ doping concentration.

$$I = A\left(\frac{q n_i W}{\tau_p}\right)\left(e^{V_A/2V_T} - 1\right)$$

$$Q = I\tau_p.$$

$$C_d = \frac{dQ}{dV_A} = \tau_p\left(\frac{dI}{dV_A}\right) = \frac{I\tau_p}{2V_T}$$

$$C_J = \varepsilon_I (A/W)$$

The dynamic resistance of a PIN diode can be found through Taylor series expansion around the Q point.

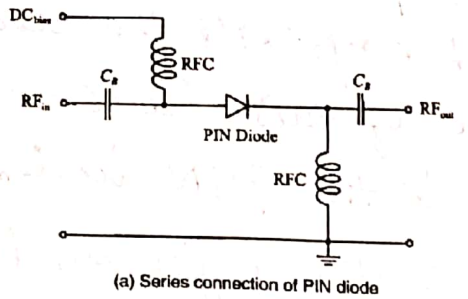
$$R_j(V_Q) = \frac{dV}{dI}\bigg|_{i=I_Q} = \frac{2V_T}{I_Q + I_{p0}}$$

$$I_{p0} = A\left(q n_i^2 W / N_D \tau_p\right).$$

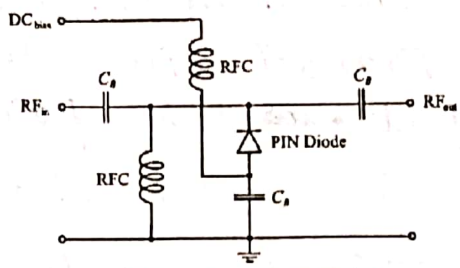
Based on the PIN diode's resistive behavior under forward bias (switch on) & capacitive behaviour under reverse bias (switch off or isolation)

The bias point setting required to operate the PIN diode has to be provided through a DC circuit that must be separated from the RF signal path. The DC isolation is achieved by a radio frequency coil (RFC), representing a short circuit at DC & an open circuit at

PIN diode in Series Connection



(a) Series connection of PIN diode



(b) Shunt connection of PIN diode

DC and an open circuit at high frequency. Blocking Capacitors (C_B) represent at open circuit DC + a short circuit at RF.

A low frequency AC bias can also be employed. The diode consists of two components such as

$$I = (dQ/dt) + Q/\tau_p$$

For positive DC bias voltage, the series connected PIN diode represents a low resistance to the RF signal to appear at the output port.

The shunt connection acts like a high attenuation device with high insertion loss. The situation is reversed for negative bias condition where the series connected PIN diode, behaves like a capacitor with high impedance or high insertion loss, where the shunt connected diode with a high shunt impedance does not affect the RF signal appreciably.

The transducer loss TL

$$TL = -20 \log |S_{21}| = -20 \log \left| \frac{2V_2}{V_G} \right|$$

IMPATT Diode:

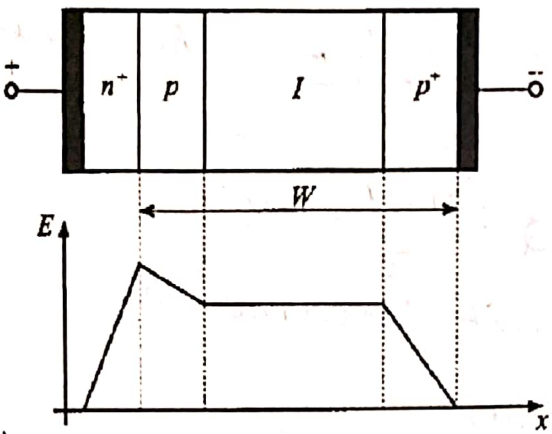
The IMPATT stands for IMPact Avalanche & Transit Time diode & exploits the avalanche effect. The principle of this diode construction which is very similar to the PIN diode.

The key difference is the high electric field strength that is generated at the interface between the n^+ & p layer resulting in an avalanche of carriers through impact ionization.

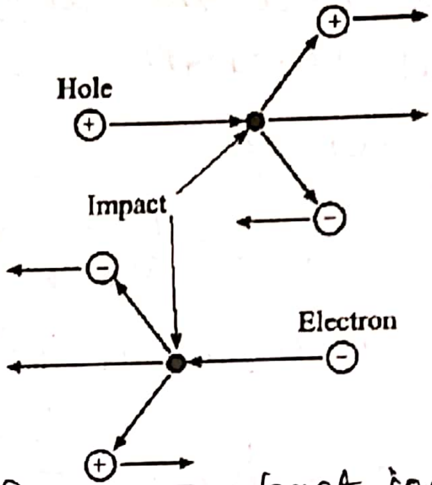
The additional ionization current I_{ion} that is generated when the applied RF voltage V_A produces an electric field that exceeds the critical threshold level.

The current slowly decreases during the negative voltage cycle as the excess carriers are removed. The phase shift between this ionization current & the applied voltage can be tailored so as to reach 90°.

The total diode current suffers an additional delay since the excess carriers have to travel through the intrinsic layer to the p^+ layer. The time constant is dependent on the length & drift velocity. Choosing the intrinsic layer layer length approximately in conjunction with a suitable doping concentration can create an additional time delay of 90°

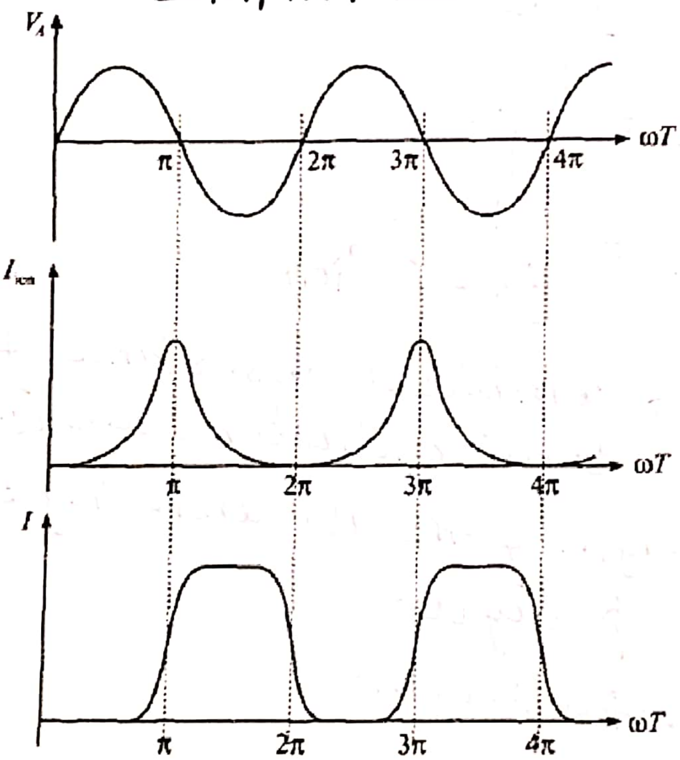


Layer Structure & Electric field profile



Impact ionization

IMPATT diode behavior.



Applied Voltage, ionization Current & total current of an IMPATT diode.

The total resistance is positive for $f < f_0$ & becomes negative $f > f_0$

The resonance frequency is determined based on the operating current I_Q, dielectric constant, saturation drift velocity v_{dmax}, & the differential change in the ionization coefficient α w.r.to the differential change in electric field strength $\alpha' = \partial\alpha/\partial E$. The resonant frequency

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{2 I_Q \, V_{dmax}}{\varepsilon}\alpha'}$$

The additional circuit parameters

$$R = R_L + \frac{V'_{dmax}}{2\pi^2 f_0^2 C_L W\left[1 - (f/f_0)^2\right]}$$

$$C_L = \frac{\varepsilon A}{W}$$

$$C_{ion} = \frac{\varepsilon A}{d}$$

$$L_{ion} = \frac{1}{(2\pi f_0)^2 C_{ion}}$$

where $R_L \rightarrow$ combined resistance of semiconductor layers.

$d \rightarrow$ length of the avalanche region of the p layer.

$W \rightarrow$ total length.

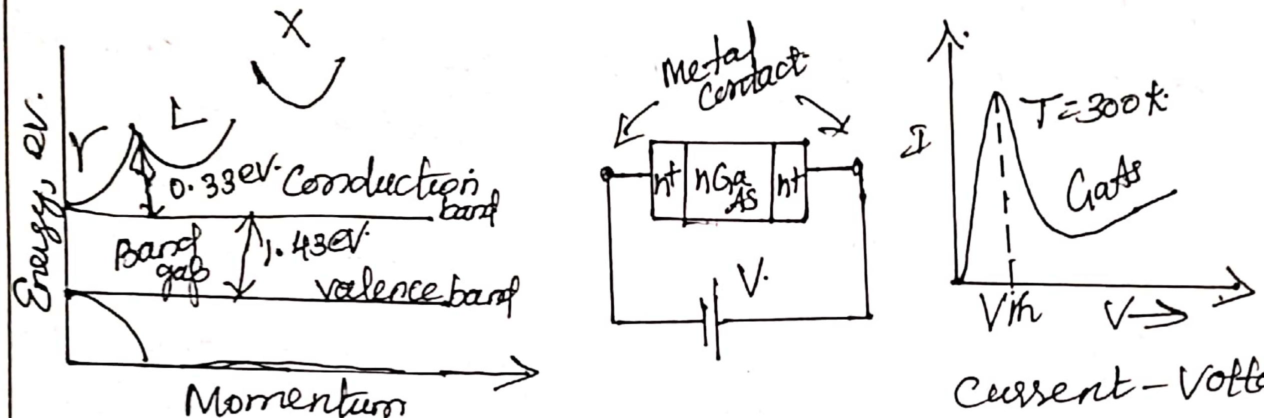
Gunn Diode:-
(Transferred Electron Devices) TED

Gunn diodes are negative resistance devices which are normally used as low power oscillator at microwave frequencies in transmitter & also local oscillator in receiver front ends.

Ex. GaAs, InP, CdTe. → These are semiconductors having a closely spaced energy valley in the conduction band.

When a dc voltage is applied across the material an electric field is established across it. At low E field in the material, most of the electrons will be located in the lower energy central valley Γ. At higher E field, most of the electrons will be transferred into high energy satellite L & X valleys where the effective electrons will be transferred into the high energy satellite L & X valleys where the effective electrons mass is larger & hence electron mobility is lower than that in the low energy Γ valley. Since the conductivity is directly proportional to the mobility, the conductivity & hence the current decreases with an increase in E field or voltage in an intermediate range, beyond a threshold value V_{th}.

This is called the transferred electron effect & the device is also called Transfer Electron Device (TED) or Gunn diode. Thus the material behaves as a negative resistance device over a range of applied voltages & can be used in microwave oscillators



Multi valley Conduction band Energies of GaAs.

Current-Voltage characteristics of GaAs.

The Gunn diode consists of n-type GaAs semiconductor with regions of high doping (n^+)

No junction → called a diode with reference to the positive end (anode) & negative end cathode of the dc voltage applied across the device.

If the voltage or an electric field at low level is applied to the GaAs, initially the current will increase with a rise in the voltage. When the diode voltage exceeds a certain threshold value V_{th}, a high electric field (3.2 kV/m for GaAs) is produced across the active region & electrons are excited from their initial lower valley to higher valley, where they become virtually immobile.

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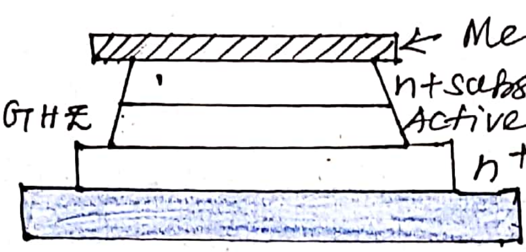
If the rate at which electrons are transferred is very high, the current will decrease with increase in voltage, resulting in equivalent negative resistance effect.

Since GaAs is a poor conductor, considerable heat is generated in the diode. The diode should be well bonded into a heat sink ~~Esses~~



Modes of oscillation :- operation :

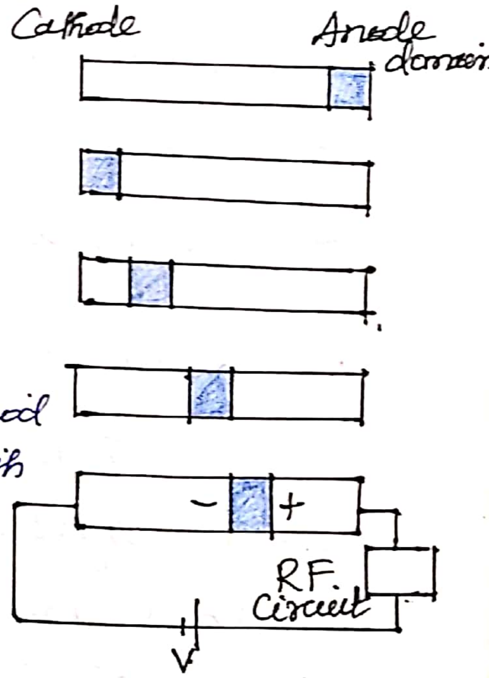
These are two principal mode of operation that result in microwave oscillations in a Gunn diode.

1. Gunn mode or Transit Time mode
2. Limited space charge (LSA) mode.
3. Quenched domain } special.
4. Delayed mode

1. Gunn or Transit Time mode :-

When the voltage applied across $n^+ n n^+$ GaAs crystal exceeds a threshold level, electrons are transferred from the low energy high mobility conduction band to a higher energy, lower or nearly zero mobility sub conduction band, where these heavier electrons bunch together to form an electric field dipole domain near the cathode. Since the applied voltage remains constant, the electric field across the domain is greater than the average field. The consequent electric field remains below the threshold level across the rest of the crystal. This prevents the formation of further domains.

All the conduction band electrons drift across the crystal at the same velocity & the less mobile bunched electrons drift across the crystal at the same velocity & the less mobile bunched electrons have reduced velocity. The current in the presence of the domain also decreases. After the high field domain had travelled into the end contact, the current returns to its higher level & a high field domain is again formed.

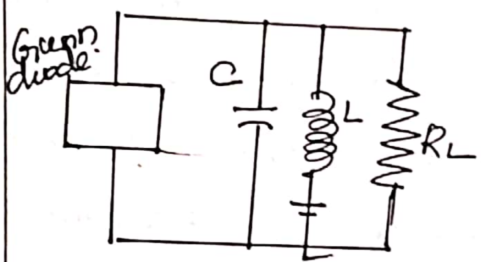


High field domain movement

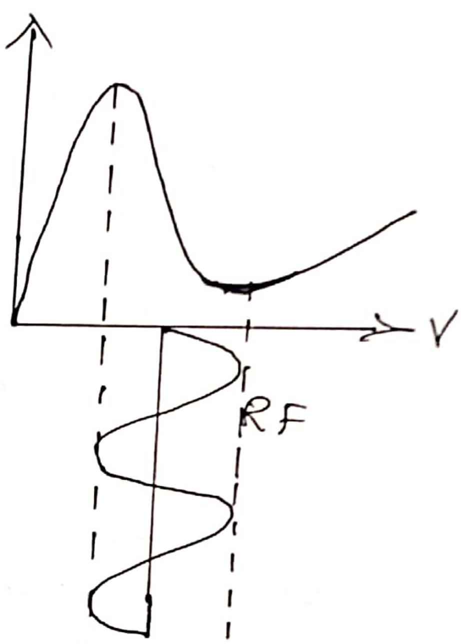
Each domain results in a pulse of current at the output. These current fluctuations occur at microwave frequencies to produce output signal at the low impedance RF circuit with a period equal to the transit time. The high field domain is quenched before it reaches the anode. Therefore the transit time is shortened & the frequency is increased. This mode of oscillation has a low efficiency of power generation & the frequency cannot be controlled by the external circuit

2. LSA mode: $\eta_{min} = 20\%$. Output power: 1 W. at 10 GHz
mW at 100 GHz.

The resonant circuit is tuned to a frequency several times greater than that of the TT mode so that dipole domains do not have sufficient time to form & the circuit operates as negative resistance oscillator when the dc voltage is adjusted to a value greater than the threshold voltage & nearly at mid point of the negative resistance region.



Gunn oscillator operating in LSA mode & RF oscillating voltage →



The resistance load RL is adjusted to a value of about 20% greater than the maximum negative resistance value of the device to enable oscillations to start & stay steady.

The amplitude of the oscillations builds up & becomes steady when the average negative resistance of the gunn diode becomes equal to the load resistance RL. The peak to peak amplitude of the microwave oscillations is approximately equal to the voltage range in the negative region.

3. Quenched domain Mode :-

If the resonant circuit is tuned to a value slightly above that of the TT mode, the dipole domain will be quenched before it arrives at the anode by the negative swing of the oscillation voltage but the Gunn diode will operate mostly like gunn mode. This mode of operation is called a quenched domain mode.

4. Delayed Mode :-

If the resonator is tuned below that of the Gunn mode, the dipole domain will arrive at the anode well in time but the formation of a new dipole domain will be delayed until the oscillation voltage ~~de~~ increases above the threshold value. This type of mode is called delayed mode.

Gunn diode oscillator:-

Gunn diode oscillators are commonly used in radars as local oscillators & also as signal source in the laboratory. A Gunn diode oscillator can be designed by mounting the diode inside a waveguide cavity formed by a short circuit termination at one end & by an iris at other end.

The diode is mounted at the center perpendicular to the broad wall where the electric field component is maximum under the dominant TE_{10} mode. The intrinsic frequency f_0 of oscillation depends on the electron drift velocity V_d due to high field domain through the effective length l

$$f_0 = V_d / l.$$

Microwave Tubes:-

Klystrons:-

* A klystron is a vacuum tube that can be used as generator (oscillator) or as an amplifier of power at microwave frequencies operated by the principles of velocity & current modulation.

Types:-

1. Two cavity or Multicavity klystron.
   → It is used as low power microwave amplifier.

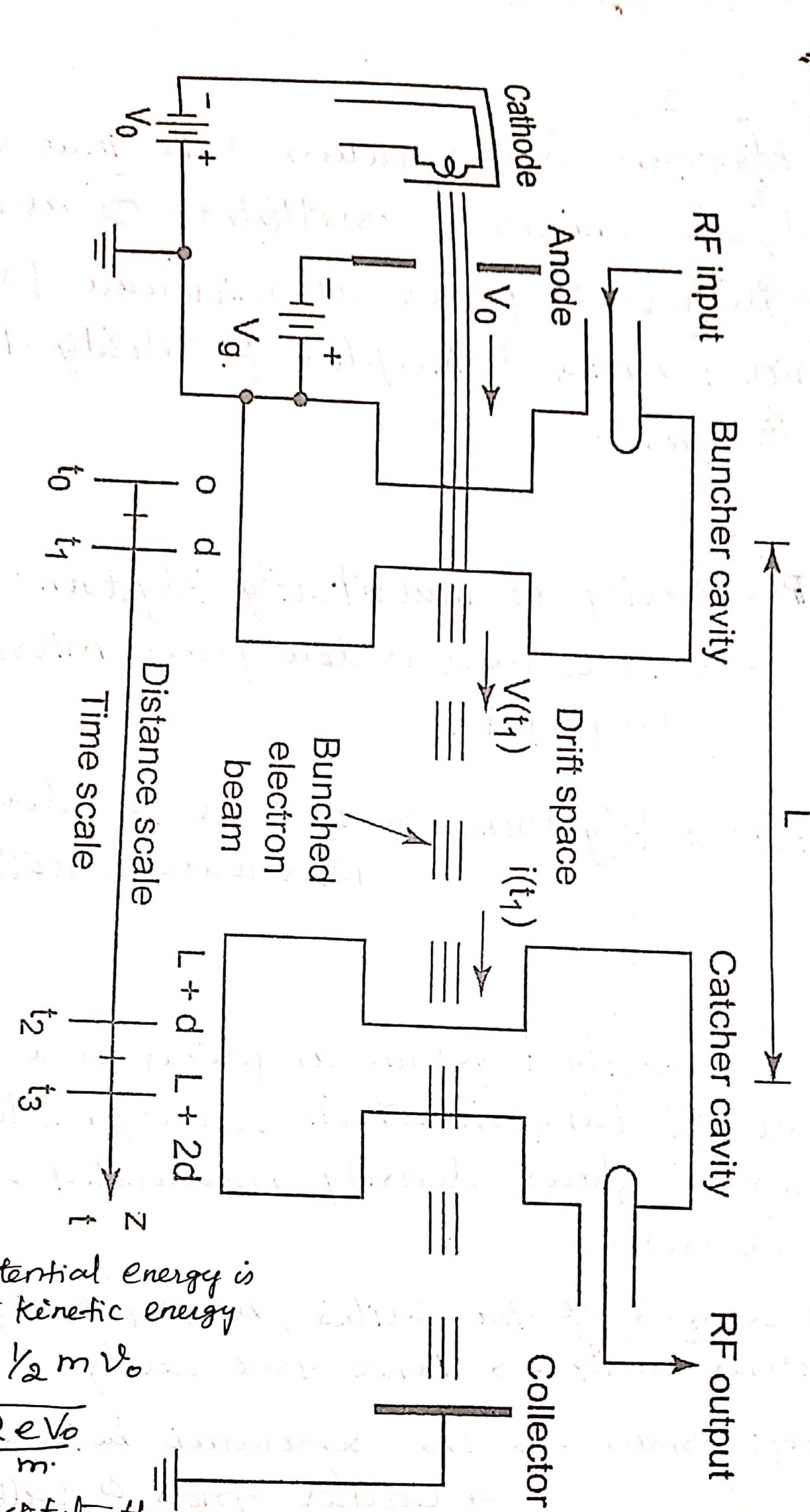
2. Reflex klystron → It is used as low power Microwave oscillator.

Two cavity klystron:-

* A two cavity klystron amplifier is a velocity modulated tube in which velocity modulation process produces density modulated stream of electrons.

* It consists of two cavities, buncher cavity & catcher cavity → hence two cavity.

* Drift space → The separation between buncher & catcher grids is called as drift space.



when potential energy is equal to kinetic energy

$$eV_0 = \frac{1}{2} m v_0$$

$$v_0 = \sqrt{\frac{2eV_0}{m}}$$

If substitute the values for e & m

$$v_0 = 0.593 \times 10^6 \sqrt{V_0} \text{ m/s}$$

Operation :-

* All electrons injected from the cathod arrive at the first cavity with uniform velocity
* The electrons beam passing the first cavity gap at zeros of the gap voltage (or) signal passes through with unchanged velocity.
* The electrons beam passing through the positive half cycles of the gap voltage undergo an increase in velocity, those passing through the negative, swings of the gap voltage undergo a decrease in velocity. As the result of these actions, the electrons gradually bunch together as they travel down the drift space.
* Velocity Modulation.

The variation in electron velocity in the drift space is known as velocity modulation.

→ first cavity → bunches & velocity modules the beam

* The density of the electrons in the second cavity gap varies cyclically with time.
* The second cavity is thus excited by the ac signal impressed on the beam in the form of a velocity modulated with resultant production of an a.c current.
* The ac current on the beam is such that the level of excitation of the second cavity is much greater than that in the buncher cavity & hence amplification takes place.

* If desired, a portion of the amplified output can be fed back to the buncher cavity in the regenerative manner to obtain self sustained oscillations.

* The maximum bunching should occur approximately midway between the second cavity grids during its retarding phase, thus the kinetic energy is transferred from the electrons to the field of the second cavity.

* The electrons then emerge from the second cavity with reduced velocity & terminate at the collector.

Catcher cavity:

The output cavity catches from the bunched electron beam. Therefore, it is also called as Catcher cavity.

Velocity Modulation Process:

When electrons are first accelerated by the high dc voltage V_0 before entering the buncher grids, their velocity is uniform.

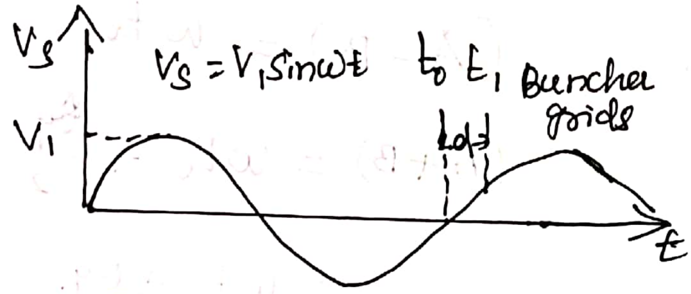
$$v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^6 \sqrt{V_0} \text{ m/s} \rightarrow (1)$$

When a microwave signal is applied to the i/p terminal, the gap voltage become the buncher grid can be

$$V_s = V_1 \sin(\omega t) \rightarrow (2)$$

where $V_1 \rightarrow$ amplitude of the signal.

$V_1 << V_0$ is assumed.



* The average transit time through buncher gap distance d is

$$\tau \simeq d/v_0 = t_1 - t_0 \rightarrow (3)$$

* The average gap transit angle

$$\theta_g = \omega\tau = \omega(t_1 - t_0) = \frac{\omega d}{v_0} \rightarrow (4)$$

* The average microwave voltage in the buncher gap

$$|V_s| = \frac{1}{\tau}\int_{t_0}^{t_1} V_1 \sin(\omega t)\, dt.$$

$$= -\frac{V_1}{\omega\tau} \cdot \left[\cos(\omega t_1) - \cos(\omega t_0)\right] \rightarrow (5)$$

from eqn (4)

$$\omega(t_1 - t_0) = \frac{\omega d}{v_0}$$

$$\omega t_1 - \omega t_0 = \omega d / v_0$$

$$\omega t_1 = \frac{\omega d}{v_0} + \omega t_0 \rightarrow (6)$$

Sub (6) in (5)

$$|V_s| = \frac{V_1}{\omega\tau}\left[\cos(\omega t_0) - \cos\left(\omega t_0 + \omega d / v_0\right)\right]$$

Let $\omega t_0 + \frac{\omega d}{2v_0} = \omega t_0 + \frac{\theta_g}{2} = A$

$$\frac{\omega d}{2v_0} = \theta g/_2 = B.$$

$$(A-B) = \omega t_0.$$

$$(A+B) = \omega t_0 + \frac{\theta g}{2} + \frac{\theta g}{2}.$$

$$= \omega t_0 + \theta g.$$

$$\boxed{(A+B) = \omega t_0 + \frac{\omega d}{v_0}.}$$

By using trigonometric relation

$$\cos(A-B) - \cos(A+B) = 2 \sin A \sin B$$

$$|V_s| = \frac{V_1}{\omega \tau} 2 \sin\left(\frac{\omega d}{2v_0}\right) \sin\left(\omega t_0 + \frac{\omega d}{2v_0}\right)$$

Sub $\tau = d/v_0$

$$|V_s| = \frac{V_1 \sin\left(\frac{\omega d}{2V_0}\right)}{\frac{\omega d}{2v_0}} \sin\left(\omega t_0 + \frac{\omega d}{2V_0}\right)$$

$$\boxed{|V_s| = V_1 \frac{\sin \theta g/2}{\theta g/2} \sin\left(\omega t_0 + \frac{\theta g}{2}\right)}$$

$$|V_s| = V_1 B_1 \sin\left(\omega t_0 + \frac{\theta g}{2}\right)$$

$$\therefore \frac{\sin\left(\frac{\omega d}{2v_0}\right)}{\omega d/2v_0} = \frac{\sin\left(\frac{\theta g}{2}\right)}{\theta g/2} = B_i.$$

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where β_i → buncher cavity beam coupling coefficient of the input cavity gap.

* Increasing the gap transit angle θ_g decreases the coupling the electron beam & the buncher cavity ie the velocity modulation of the beam for a given microwave signal is decreased.

* After velocity modulation, the exit velocity from the buncher gap is

$$v_o(t_1) = \sqrt{\frac{2e}{m}\left[V_0 + \beta_i V_1 \sin\left(\omega t_0 + \theta_g/2\right)\right]}$$

$$v_1(t_1) = \sqrt{\frac{2e}{m} V_0 \left[1 + \frac{\beta_i V_1}{V_0} \sin\left(\omega t_0 + \theta_g/2\right)\right]}$$

$\frac{\beta_i V_1}{V_0}$ → depth of velocity modulation

If $\beta_i V_1 << V_0$

$$v(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]$$

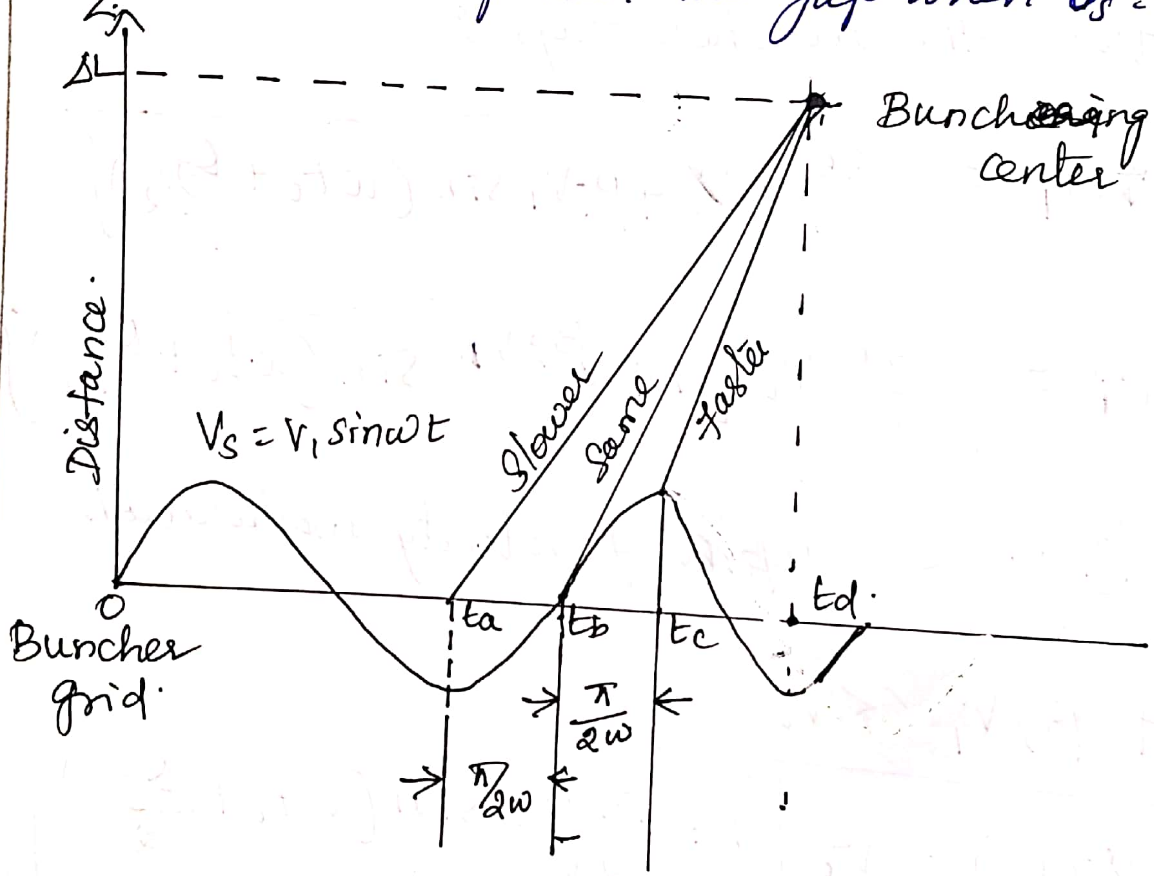
eqn of velocity modulation

(or)

$$v(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_0} \sin\left(\omega t_1 + \theta_g/2\right)\right]$$

Bunching process :-

* The effect of velocity modulation produces bunching of the electron beam or current modulation.
* The electrons that pass the buncher at $V_s = 0$ travel through with unchanged velocity v_0.
* Those electrons that pass the buncher cavity during the positive half cycles of microwave input voltage V_s travel faster than the electrons that passed the gap when $V_s = 0$



* The electron beams that pass the buncher cavity during the negative half cycles of the voltage V_s travel slower than the electrons that passed the gap when $V_s = 0$

* The distance from the buncher grid to the

location of dense electron bunching for the electron at t_b is

$$\Delta L = v_0 (t_d - t_b)$$

where
$$t_c = t_b + \frac{\pi}{2\omega}$$
$$t_b = t_a + \pi/2\omega$$
$$t_a = t_b - \pi/2\omega.$$

|||ly the distances for the electrons at t_a & t_c are

$$\Delta L = v_{min} (t_d - t_a) = v_{min} (t_d - t_b + \pi/2\omega)$$

$$\Delta L = v_{max} (t_d - t_c) = v_{min} (t_d - t_b - \pi/2\omega)$$

from velocity of modulation equation, maximum & minimum velocities are

$$v_{min} = v_0 \left(1 - \frac{\beta_i V_1}{2V_0}\right)$$

$$v_{max} = v_0 \left(1 + \frac{\beta_i V_1}{2V_0}\right)$$

Sub these into ΔL

$$\Delta L = v_{min} (t_d - t_b + \pi/2\omega)$$
$$= v_0 \left(1 - \frac{\beta_i V_1}{2V_0}\right)(t_d - t_b + \pi/2\omega)$$
$$= v_0 (t_d - t_b) + v_0 \frac{\pi}{2\omega} - \beta_i V_1 \frac{v_0}{2V_0}(t_d - t_b)$$
$$- \left(\frac{v_0 \beta_i V_1}{2V_0}\right)\left(\frac{\pi}{2\omega}\right)$$

$$\Delta L = V_0(t_d - t_b) + \left[V_0 \frac{\pi}{2\omega} - \frac{V_0 \beta_i V_1}{2V_0}(t_d - t_b) - \frac{V_0 \beta_i V_1}{2V_0} \cdot \frac{\pi}{2\omega} \right]$$

& $\Delta L = V_{max}\left(t_d - t_b - \frac{\pi}{2\omega}\right)$

$$= V_0\left[1 + \frac{\beta_i V_1}{2V_0}\right]\left[t_d - t_b - \frac{\pi}{2\omega}\right]$$

$$= \left(V_0 + \frac{V_0 \beta_i V_1}{2V_0}\right)\left(t_d - t_b - \frac{\pi}{2\omega}\right)$$

$$= V_0(t_d - t_b) - \frac{V_0 \pi}{2\omega} - \frac{V_0 \beta_i V_1}{2V_0}(t_d - t_b) - \frac{V_0 \beta_i V_1}{2V_0} \frac{\pi}{2\omega}$$

$$\Delta L = V_0(t_d - t_b) + \left[-\frac{V_0 \pi}{2\omega} + \frac{V_0 \beta_i V_1}{2V_0}(t_d - t_b) - \frac{V_0 \beta_i V_1}{2V_0} \frac{\pi}{2\omega} \right]$$

The necessary conditions for those electrons at t_a, t_b & t_c to meet at the same distance ΔL is

$$\frac{V_0 \pi}{2\omega} - \frac{V_0 \beta_i V_1}{2V_0}(t_d - t_b) - \frac{V_0 \beta_i V_1}{2V_0} \frac{\pi}{2\omega} = 0$$

& $-\dfrac{V_0 \pi}{2\omega} + \dfrac{V_0 \beta_i V_1}{2V_0}(t_d - t_b) - \dfrac{V_0 \beta_i V_1}{2V_0} \dfrac{\pi}{2\omega} =$

Equate these two

$$\frac{V_0\pi}{2\omega} - \frac{V_0\beta_i V_1}{2V_0}(t_d - t_b) - \frac{V_0\beta_i V_1}{2V_0}\frac{\pi}{2\omega} = -\frac{V_0\pi}{2\omega} + \frac{V_0\beta_i V_1}{2V_0}(t_d - t_b) - \frac{V_0\beta_i V_1}{2V_0}\,\pi/2\omega.$$

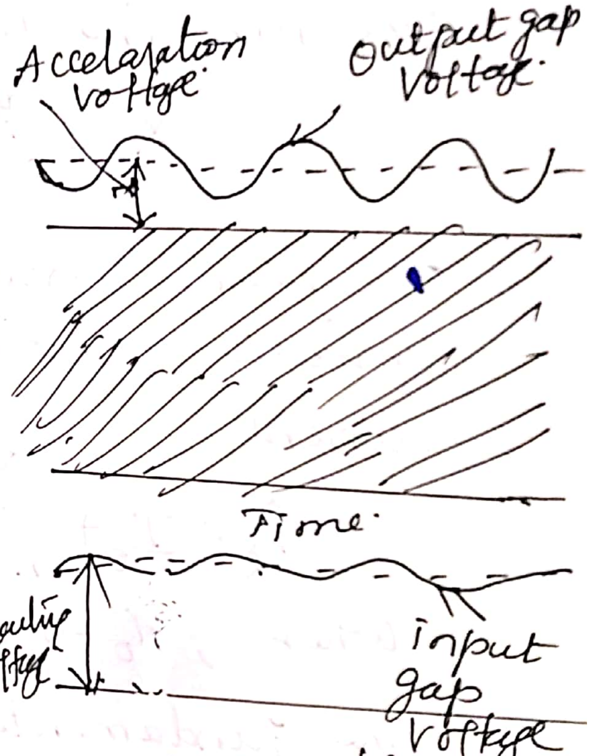
$$\frac{V_0\pi}{2\omega} + \frac{V_0\pi}{2\omega} = \frac{V_0\beta_i V_1}{2V_0}(t_d - t_b) + \frac{V_0\beta_i V_1}{2V_0}(t_d - t_b)$$

$$\frac{V_0\pi}{\omega} = \frac{V_0\beta_i V_1}{V_0}(t_d - t_b).$$

$$t_d - t_b = \frac{\pi V_0}{\omega\beta_i V_1}$$

& $\Delta L = V_0(t_d - t_b)$

$$\boxed{\Delta L = V_0\frac{\pi V_0}{\omega\beta_i V_1}}$$



$V_s = V_1\sin\omega t$

Applegate diagram.

* The transit time for an electron to travel a distance of L

$$T = t_2 - t_1 = \frac{L}{v(t)}$$

Sub $v(t)$ here. → velocity modulation

$$T = \frac{L}{v_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \theta_g/2\right)\right]}$$

$$T = T_0\left[1 - \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \theta_g/2\right)\right]$$

where $T_0 = L/v_0$ → dc transit time.

$$\omega T = \omega t_2 - \omega t_1$$

$$\omega T = \left[\omega T_0 - \left(\frac{\omega T_0 \beta_i V_1}{2V_0}\right)\sin\left(\omega t_1 - \theta_g/2\right)\right]$$

$$\omega T = \theta_0 - X \sin\left(\omega t_1 - \theta_g/2\right).$$

where $\theta_0 = \frac{\omega L}{V_0} = 2\pi N.$

→ dc transit angle between cavities

N → No. of election transit cycles in the drift space

The bunching parameter of a klystron

$$X = \frac{\beta_i V_1}{2V_0}\theta_0.$$

The beam current at the catcher cavity is a periodic waveform of period $2\pi/\omega$ about dc current

$$i_2 = I_0 + \sum_{n=1}^{\infty} 2I_0 J_n(nX)\cos\left[n\omega(t_2 - \tau - T_0)\right]$$

where, I_0 → dc current

The fundamental component of the beam ~~current~~ current at the catcher cavity has a magnitude.

$$I_f = 2I_0 J_1(X)$$

I_f has maximum amplitude at

$$X = 1.841.$$

W.K.T $X = \frac{\beta_i V_1}{2V_0}\theta_0.$

$$X = \frac{\beta_i V_1 \omega L}{2 V_0 v_0}$$

$L = L_{opt}$ when $X = 1.841$.

$$L_{opt} = \frac{2X V_0 v_0}{\omega \beta_i V_1} = \frac{2 \times 1.841 \times V_0 v_0}{\omega \beta_i V_1}$$

$$\boxed{L_{opt} = \frac{3.682 V_0 v_0}{\omega \beta_i V_1}}$$

Output power & Beam Loading :-

* The maximum bunching should occur approximately midway between the catcher grids.

* When the electrons emerge from the catcher grids, they have reduced velocity & are finally collected by the collector.

$$I_{2md} = \beta_0 I_2 = \beta_0 2 I_0 J_1(X)$$

$$P_{out} = \frac{(\beta_0 I_2)^2}{2} R_{sh} = \frac{\beta_0 I_2 V_2}{2}$$

$$\text{Efficiency} = \frac{P_{out}}{P_{in}} = \frac{\beta_0 I_2 V_2}{2 I_0 V_0}$$

Mutual conductance

$$|G_m| = \frac{I_{2md}}{V_1} = \frac{2\beta_0 I_0 J_1(X)}{V_1}$$

The input voltage V_1 can be expressed in terms of bunching parameter.

$$V_1 = \frac{2V_0}{\beta_0 \theta_0} X.$$

$$|G_m| = \frac{\beta_0^2 \theta_0 I_0 J_1(X)}{V_0 X} = \frac{\beta_0^2 \theta_0 G_0 J_1(X)}{X}.$$

$$\frac{|G_m|}{G_0} = \beta_0^2 \theta_0 \frac{J_1(X)}{X}.$$

where $G_0 = \frac{I_0}{V_0} \rightarrow$ dc beam conductance.

if $X = 1.841$

$$\boxed{\frac{|G_m|}{G_0} = 0.316 \beta_0^2 \theta_0}$$

Voltage gain

$$A_V = \left|\frac{V_2}{V_1}\right| = \frac{\beta_0 I_2 R_{sh}}{V_1}$$

$$= \frac{\beta_0 2 I_0 J_1(X)}{\frac{2 V_0 X}{\beta_0 \theta_0}} R_{sh}.$$

$$= \frac{\beta_0^2 \theta_0 I_0 J_1(X)}{V_0 (X)} R_{sh}.$$

$$\boxed{A_V = \frac{\beta_0^2 \theta_0 J_1(X)}{R_0 X} R_{sh}}$$

where $R_0 = \frac{V_0}{I_0}$ → dc beam resistance

$$A_V = G_m R_{sh}$$

Beam loading:

The difference between the average exit energy & the entrance energy must be supplied by the buncher cavity to bunch the electron beam.

Thus the electron beam is loaded by the cavity energy. This is called Beam loading.

The ratio of power required to produce bunching action

$$\frac{P_B}{P_0} = \frac{V_1^2}{2V_0^2}\left[\frac{1}{2}\beta_i^2 - \frac{1}{2}\beta_i \cos\left(\frac{\theta_g}{2}\right)\right]$$

$$= \frac{V_1^2}{2V_0^2} F(\theta_g)$$

where $F(\theta_g) \rightarrow \frac{1}{2}\left[\beta_i^2 - \beta_i \cos(\theta_g/2)\right]$

The power delivered by the electron beam to the catcher cavity

$$\frac{V_2^2}{2R_{sh}} = \frac{V_2^2}{2R_{sho}} + \frac{V_2^2}{2R_B} + \frac{V_2^2}{2R_L}$$

Effective impedance of catcher cavity

$$\frac{1}{R_{sh}} = \frac{1}{R_{sho}} + \frac{1}{R_B} + \frac{1}{R_L}$$

The loaded quality factor:

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_B} + \frac{1}{Q_{ext}}$$

Reflex klystron :-

* The reflex klystron is an oscillator with a built in feedback mechanism. It uses the same cavity for bunching & for the output cavity.

* The repeller electrode is a negative potential & sends the bunched electron beam back to the resonator cavity. This provides a positive feedback mechanism which supports oscillations.

* Due to dc voltage in the cavity circuit, RF noise is generated in the cavity. This electromagnetic noise field in the cavity becomes pronounced at cavity resonant frequency.

* When the oscillation frequency is varied, the resonant frequency of cavity & the feedback path phase shift must be redjusted for a positive feedback.

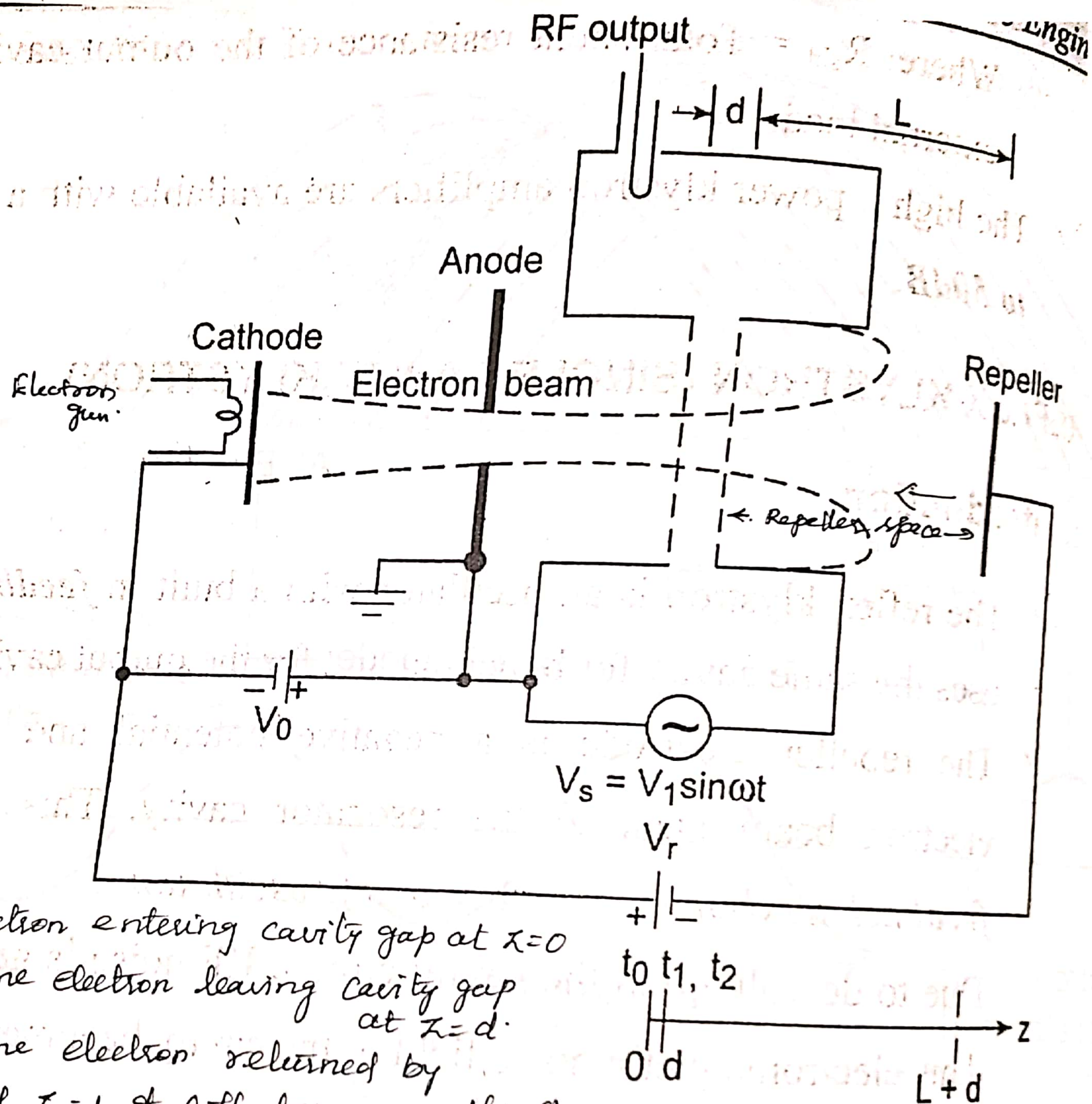
Mechanism of oscillation :-

* The electron beam injected from the cathod is first velocity modulated by cavity gap voltage.
* Some electrons accelerated by accelerating field enter the repeller space with greater velocity than those with unchanged velocity

* Some electrons decelerated by the retarding field enter the repeller region with less velocity.
* All electrons turned around by the repeller voltage then pass through the cavity gap in bunches that occur once per cycle.
* On their return journey the bunched electrons pass through the gap during the retarding phase of the alternating field & give up their kinetic energy to the electromagnetic energy of the field in the cavity.
* Oscillator output energy is then taken from the cavity.
* The electrons are finally collected by the walls of the cavity or other grounded metals parts of the tube.

Applegate Diagram:

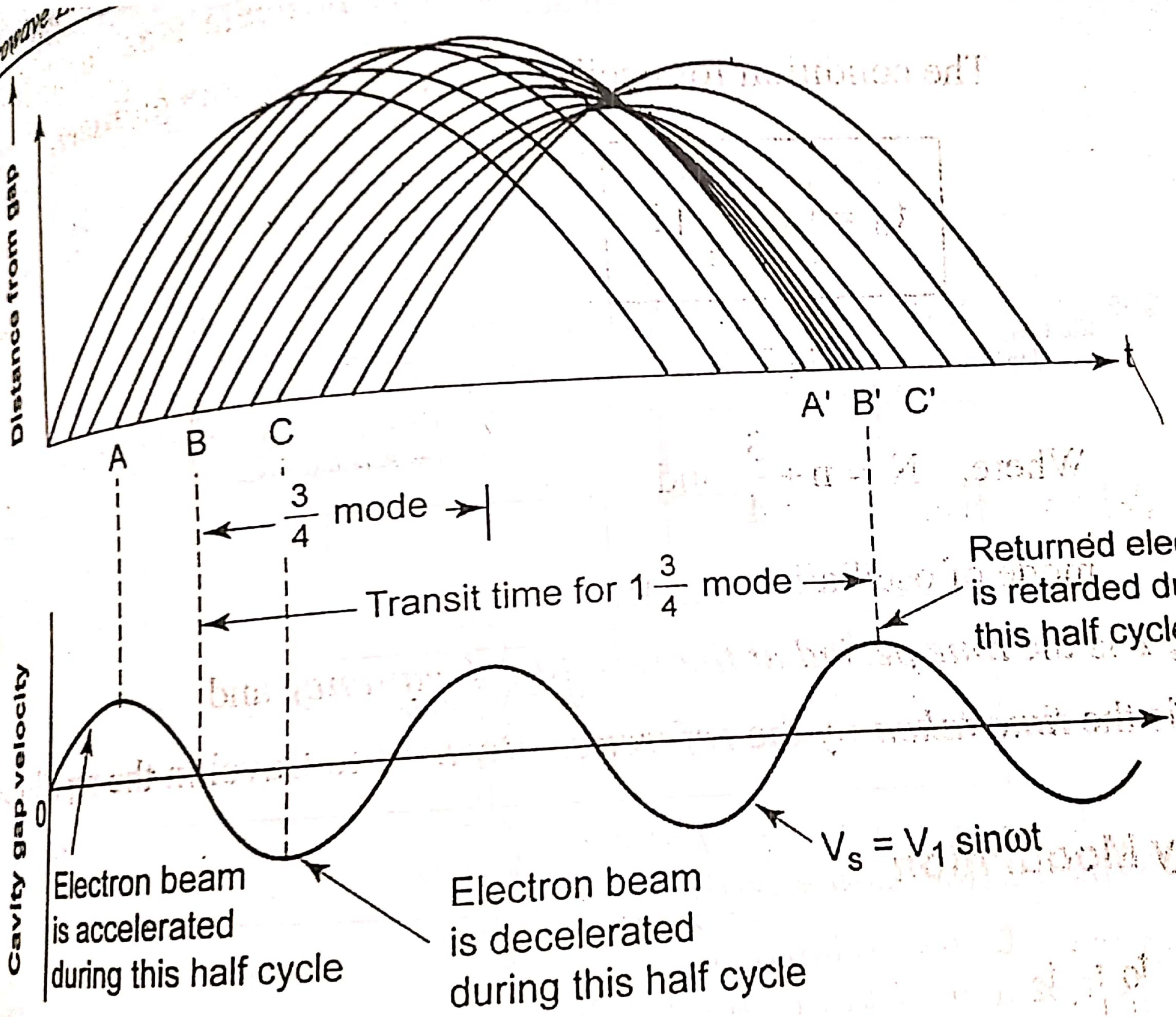
The electrons passing through the buncher grid are accelerated / retarded / passed through with unchanged initial dc velocity depending upon when they encounter the RF signal field at the buncher cavity gap at positive / negative / zero crossing phase of the cycle, respectively as shown by distance - time plot. This is called the applegate diagram.



t_0 – Time for electron entering cavity gap at $z=0$

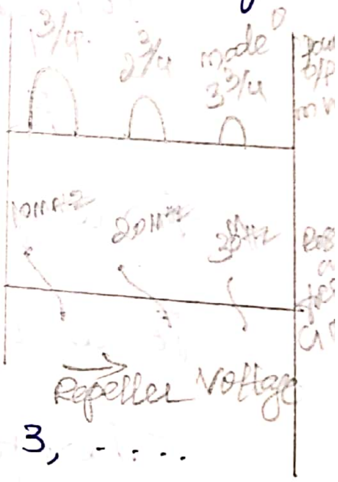
t_1 – Time for same electron leaving cavity gap at $z=d$.

t_2 – Time for same electron returned by retarding field $z=d$ & collected on walls of cavity.



Distance from gap
t
A
B
C
A'
B'
C'
3/4 mode
Transit time for 1 3/4 mode
Returned electron beam is retarded during this half cycle
Cavity gap velocity
0
t
Electron beam is accelerated during this half cycle
Electron beam is decelerated during this half cycle
$V_s = V_1 \sin\omega t$

* When the gap voltage is at positive peak, electron passing at this moment is called early electron. This electron is accelerated towards repeller & travels a distance, which is longer comparatively.
* The electron at neutral zero of gap voltage is called reference electron
* When the gap voltage is at negative peak the corresponding electron is called late electron. This electron decelerated & travels less distance. These electrons have different velocities cover different distances forms bunch at cavity gap.
* Modes of oscillation



The condition for oscillation is

$$t_0 = (n + 3/4)T = NT$$

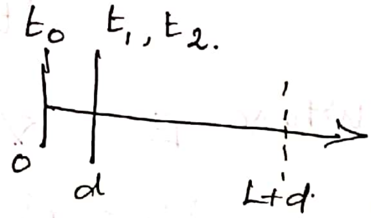
where $N = n + 3/4$.

mode of oscillation, $n = 0, 1, 2, 3, \ldots$

where $T \rightarrow$ Time period at the resonant frequency

$t_0 \rightarrow$ time taken by the reference electron to travel in the repeller space.

Velocity Modulation :-



* The electron entering the cavity gap from the cathode at $z=0$ & time t_0 is assumed to have uniform velocity

$$v_0 = 0.593 \times 10^6 \sqrt{V_0}.$$

The same electron leaves the cavity gap at $z = d$ at time t_1 with velocity

$$v(t_1) = v_0\left[1 + \frac{\beta_g V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

* The same electron is forced back to the cavity $z = d$ & time t_2 by the retarding electric field

$$E = \frac{V_r + V_0 + V_1 \sin(\omega t)}{L}.$$

* This retarding field E is assumed to be constant in z-direction. The force equation for one electron in the repeller region.

$$E = \frac{V_r + V_0}{L} \quad \text{where } |V_1 \sin\omega t| << (V_r + V_0)$$

Force of electron $= -eE = -e\left(\frac{V_r + V_0}{L}\right)$

Force of electrons = mass x acceleration $= m\frac{d^2z}{dt^2}$

Therefore, $m\frac{d^2z}{dt^2} = -eE$

$$= -e\frac{V_r + V_0}{L}$$

$$\frac{d^2z}{dt^2} = -e\left[\frac{V_r + V_0}{mL}\right]$$

where $E = -\nabla V \rightarrow$ used in z direction only

V_r : is the magnitude of the repeller Voltage.

Integrating the above eqn.

$$\frac{dz}{dt} = -e\frac{(V_r+V_0)}{mL}\int_{t_1}^{t} dt$$

$$= -e\frac{(V_r+V_0)}{mL}(t-t_1)+k$$

At $t=t_1$, $\frac{dz}{dt} = v(t_1) = k_1$; then

$$z = -e\left[\frac{V_r+V_0}{mL}\right]\int_{t_1}^{t}(t-t_1)\,dt + v(t_1)\int_{t_1}^{t} dt$$

$$= -e\left[\frac{V_r+V_0}{2mL}\right](t-t_1)^2 + v(t_1)(t-t_1)+k_2.$$

At $t=t_1$, $z=d=k_2$.

$$z = \frac{-e(V_r+V_0)}{2mL}(t-t_1)^2 + v(t_1)(t-t_1)+d.$$

the electron leaves the cavity gap at $z=d$ & time t_1 with a velocity of $v(t_1)$ & returns to the gap at $z=d$ & time t_2, then at $t=t_2$; $z=d$.

$$0 = \frac{-e(V_r+V_0)}{2mL}(t_2-t_1)^2 + v(t_1)(t_2-t_1)$$

Transit time:-

The round-trip transit time in the repeller region is given by

$$T' = \frac{2\ \text{velocity}}{\text{Acceleration}} = \frac{2v(t_1)}{d^2z/dt^2}$$

$$\therefore T' = t_2-t_1 = \frac{2mL}{e(V_r+V_0)}v(t_1)$$

The negative sign is not taken as electron bunch travels in the reverse direction

Thus

$$T' = T_0' \left[1 + \frac{\beta_i V_1}{2 V_0} \sin\left(\omega t_1 - \theta_g/2\right) \right]$$

where

$$T_0' = \frac{2 m L v_0}{e (V_r + V_0)}$$ → round trip dc transit time of the center of the bunch electron.

* Multiply the above eqn by a radiation frequency

$$\omega(t_2 - t_1) = \omega T_0' + \omega T_0' \frac{\beta_i V_1}{2 V_0} \sin(\omega t_1 - \theta_g/2)$$

$$= \theta_0' + X' \sin(\omega t_1 - \theta_g/2)$$

where $\theta_0' = \omega T_0'$ → round trip dc transit angle of the center of the bunch electron

$$X' = \frac{\beta_i V_1}{2 V_0} \theta_0'$$ → bunching parameter.

Power output & Efficiency:-

* A maximum amount of kinetic energy can be transferred from the returning electrons to the cavity walls
* For a maximum energy transfer, the round-trip angle referring to the centre of the bunch,

$$\omega(t_2 - t_1) = \omega T_0'$$

$$= (n - 1/4) 2\pi$$

$$= N 2\pi$$

$= 2\pi n - \pi/2.$

where $V_1 << V_0$.

$n \rightarrow$ any +ve integer for cycle number.

$N = n - 1/4$. no. of modes.

* The beam current injected into the cavity gap from the repeller region flows in the negative Z-direction.

The beam current of a reflex klystron oscillator

$$i_{2t} = -I_0 - \sum_{n=1}^{\infty} 2 I_0 J_n(nX') \cos[n(\omega t_2 - \theta_0' - \theta_g)]$$

where $I_0 \rightarrow$ dc beam current

* The fundamental component of the current induced in the cavity by the modulated electron beam is given $(\theta_g << \theta_0)$

$$i_2 = -\beta_i I_2 = 2 I_0 \beta_i J_1(X') \cos(\omega t_2 - \theta_0')$$

* The magnitude of fundamental component

$$I_2 = 2 I_0 \beta_i J_1(X')$$

* dc power supplied by the beam voltage V_0

$$P_{dc} = V_0 I_0$$

* The ac power delivered to the load

$$P_{ac} = \frac{V_1 I_1}{2} = V_1 I_0 \beta_i J_1(X')$$

$X' \rightarrow$ bunching parameter of reflex klystron.

* The bunching parameter

$$X' = \beta_i V \frac{\theta_0}{2V_0}$$

where $\theta_0' = \omega T_0' = 2\pi n - \pi/2$.

$$2V_0 X' = \beta_i V_1 (2\pi n - \pi/2)$$

$$\frac{V_1}{V_0} = \frac{2X'}{\beta_i (2\pi n - \pi/2)}$$

$$V_1 = \frac{2X' V_0}{\beta_i (2\pi n - \pi/2)}$$

$$\frac{V_1}{V_0} = \frac{2X'}{\beta_i (2\pi n - \pi/2)}$$

Sub V_1.

$$P_{ac} = \frac{2X' V_0 I_0 \beta_i J_1(X')}{\beta_i (2\pi n - \pi/2)}$$

$$\boxed{P_{ac} = \frac{2 V_0 I_0 X' J_1(X')}{2\pi n - \pi/2}}$$

Efficiency :-

The electronic efficiency of a reflex klystron oscillator

$$\text{Efficiency} = \frac{P_{ac}}{P_{dc}} = \eta$$

Sub

$$\therefore \eta = \frac{2V_0 I_0 x' J_1(x')}{(2\pi n - \pi/2)V_0 I_0}$$

$$\boxed{\eta = \frac{2x' J_1(x')}{2\pi n - \pi/2}}$$

The factor $x' J_1(x')$ reaches a maximum value of 1.25 at

$$x' = 2.408$$

$$J_1(x') = 0.52$$

when $n = 2$ or $1\frac{3}{4}$.

$$\therefore \eta_{max} = \frac{2(2.408) J_1(2.408)}{2\pi(2) - \pi/2}$$

$$\boxed{\eta_{max} = 22.78\%}$$

Electronic Admittance :-

* The electronic admittance of the reflex klystron is defined by the ratio of induced bunch beam current & cavity gap voltage.
* The induced bunch beam current

$$i_2 = 2I_0 \beta_i J_1(x') \cos(\omega t_2 - \theta_0')$$

Phasor form

$$i_2 = 2I_0 \beta_i J_1(x') e^{-j\theta_0'}.$$

* The voltage across the gap at time t_2 can be written in phasor form.

$$V_2 = V_1 e^{-j\pi/2}.$$

W.K.T

$$V_1 = \frac{2x' V_0}{\beta_i (2\pi n - \pi/2)}$$

Therefore

$$\boxed{V_2 = \frac{2V_0 x' e^{-j\pi/2}}{\beta_i (2\pi n - \pi/2)}}$$

Electronic admittance

$$Y_e = \frac{i_2}{V_2} = \frac{2I_0 \beta_i J_1(x') e^{-j\theta_0'} \times \beta_i (2\pi n - \pi/2)}{2V_0 x' e^{-j\pi/2}}$$

where $\theta_0' = 2\pi n - \pi/2$.

$$Y_e = \frac{I_0}{V_0} \cdot \frac{\beta_i^2 \theta_0'}{2} \cdot \frac{2J_1(x')}{x'} e^{j(\pi/2 - \theta_0')}$$

$$\therefore \boxed{Y_e = G_e + jB_e}$$

* The amplitude of the phasor admittance indicates that the electronic admittance is a function of the dc beam admittance the dc transit angle & the second transit of the electron beam through the cavity gap

Travelling - Wave Tube Amplifier:

In Klystron & Magnetrons, the microwave circuits consists of a resonant structure which limits the bandwidth (or the operating frequency range) of the tube.

Klystrons are essentially narrow band devices as they utilise cavity resonators to velocity modulate the electron beam over a narrow gap.

A travelling wave tube amplifier (TWTA) circuit uses a helix slow wave non resonant microwave guiding structure & thus a broad band microwave amplifier.

TWT consists are

(i) An electron beam

(ii) A structure supporting a slow electromagnetic wave.

* In the case of the TWT, the microwave circuit is non resonant & the wave propagate with the same speed as the electrons in the beam.

* The initial effect on the beam is a small amount velocity modulation caused by the weak electric fields associated with the travelling wave. This modulation translates to current modulation which then induces an RF current in the circuit causing amplification.

Major differences between the TWT & Klystron.

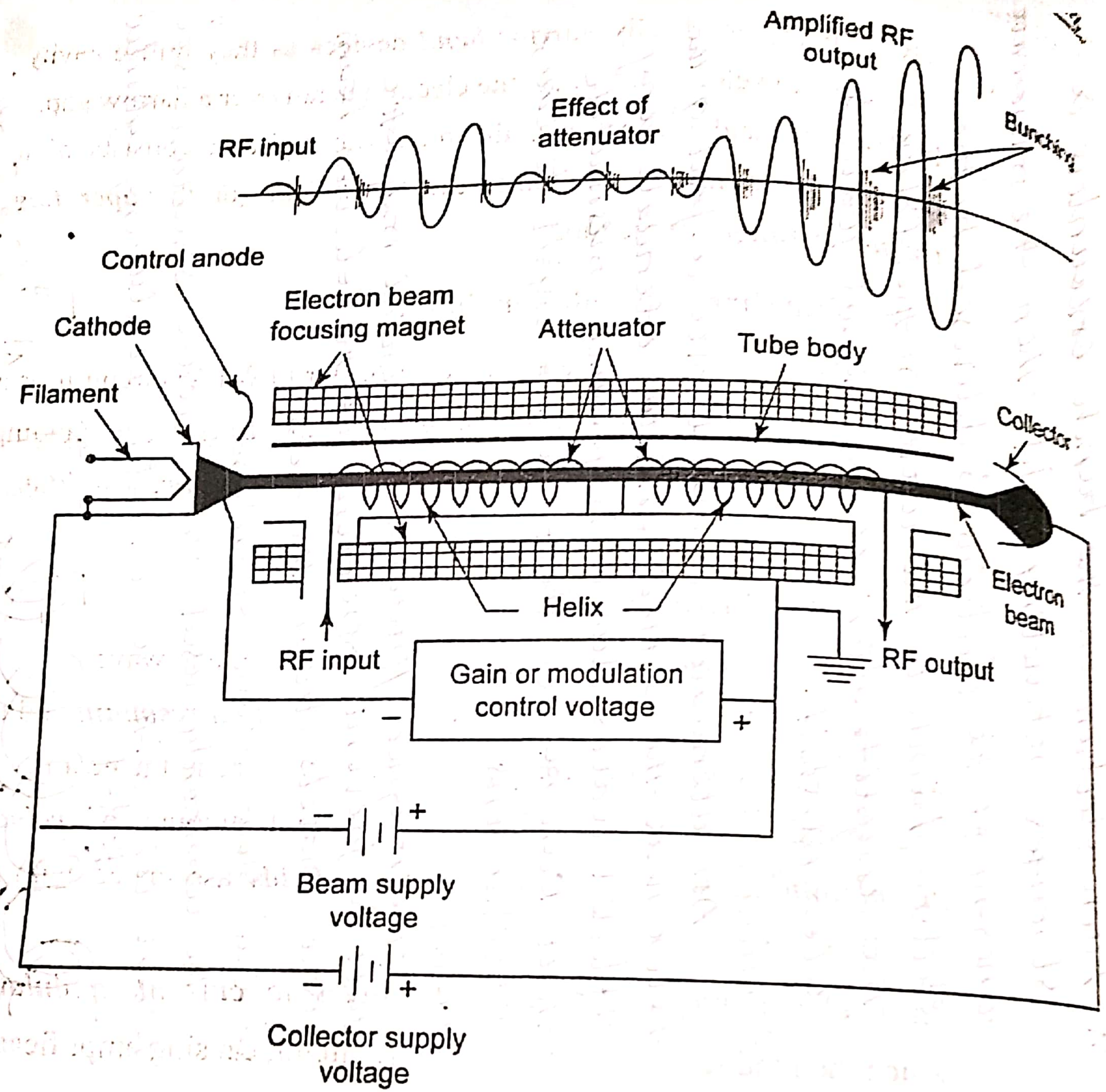
(i) The interaction of electron beam & RF field in the TWT is continuous over the entire length of the circuit but the interaction in klystron occurs only at the gaps of a few resonant cavities.

(ii) The wave in the TWT is a propagating wave, the wave in the Klystron is not propagating.

(iii) In the coupled - cavity TWT there is a coupling effect between the cavities, where as each cavity in the Klystron operates independently.

Operation :-

* The electron beam is focussed by a constant magnetic field along the electron beam & the slow wave structure. This is termed as O - type traveling wave tube.
* The slow wave structure is either the helical type or folded - back line.
* The applied signal propagates around the turns of the helix & produces an electric field at the center of the helix, directed along the helix axis.
* The axial electric field progresses with a velocity that is very close to the velocity of light multiplied the ratio of helix pitch to helix circumference.
* When the electrons enter the helix tube, an interaction takes place between the moving axial electric field & the moving electrons. The electrons transfer energy to the wave on the helix.
* The interaction causes the signal wave on the helix to become larger.
* The electrons entering the helix at zero field are not affected by the signal wave, those electrons entering the helix at the accelerating field are accelerated, & those at the retarding field are decelerated.


Amplified RF output
Effect of attenuator
RF input
Bunching
Control anode
Electron beam focusing magnet
Cathode
Attenuator
Tube body
Filament
Collector
Helix
Electron beam
RF input
Gain or modulation control voltage
RF output
Beam supply voltage
Collector supply voltage

* As the electrons travel further along the helix, they bunch at the collector end. The bunching shifts the phase by $\pi/2$.
* Each electron in the bunch encounters a stronger retarding field. Then the microwave energy of the electrons is delivered by the electron bunch to the wave on the helix. The amplification of the signal wave is accomplished.

Attenuator :-

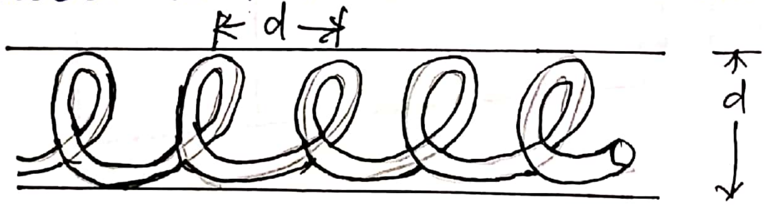
An attenuator is placed over a part of the helix near the output end to attenuate any reflected waves due to impedance mismatch that can be fed back to the input to cause oscillations

Magnet :-

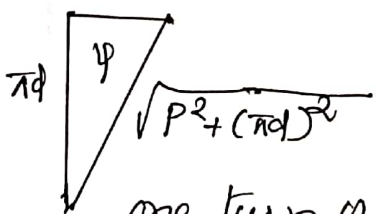
The magnet produces an axial magnetic field to prevent spreading of the electron beam as it travels down the tube.

Helical slow-wave structures :-

The commonly used slow-wave structure is a helical coil with a concentric conducting cylinder.



Helical coil.



one turn of helix

The ratio of the phase velocity v_p along the pitch to the phase velocity along the coil is given by

$$\frac{v_p}{c} = \frac{P}{\sqrt{P^2 + (\pi d)^2}} = \sin\varphi.$$

where $c \rightarrow 3 \times 10^8$ m/s.

$d \rightarrow$ helix diameter in meters

$P \rightarrow$ Helix pitch in meters

$\varphi \rightarrow$ pitch angle.

The helical coil may be within a dielectric-filled cylinder.

* The phase velocity in the axial direction

$$v_p = \frac{P}{\sqrt{\mu\epsilon\left[P^2 + (\pi d)^2\right]}}$$

* For a very small pitch angle, the phase velocity along the coil in free space is

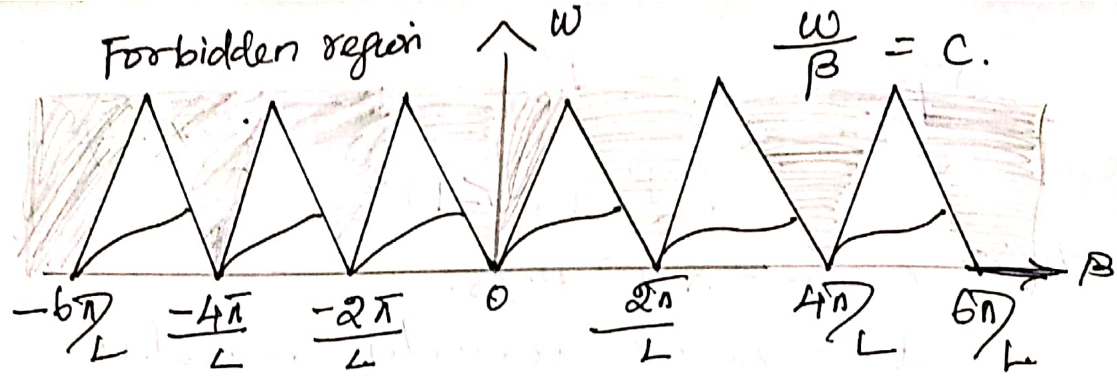
$$v_p = \frac{cP}{\sqrt{P^2 + (\pi d)^2}} \simeq \frac{cP}{\pi d} = \frac{\omega}{\beta}.$$

where, β_0 is the phase constant of average electron velocity when $P << \pi d$.

In order for a circuit to be a slow wave structure, it must have the property of periodicity in the axial direction.

$\omega - \beta$ (Brillouin) diagram

The second quadrant of the $\omega - \beta$ diagram indicates the negative phase velocity that corresponds to the negative n.



This means that the electron beam moves in the positive z direction while the beam velocity coincides with negative spatial harmonic's phase velocity. This type of tub is called a backward-wave oscillator.

* The shaded areas are the forbidden regions for propagation.

Amplification process:-

The phase shift per period of the fundamental wave on the structure is given by

$$\theta_1 = \beta_0 L.$$

where $\beta_0 = \omega / v_0 \rightarrow$ phase constant.

$L \rightarrow$ period or pitch.

* The dc transit time of an electron is

$$T_0 = \frac{L}{v_0} \rightarrow (1)$$

* The phase constant of the n^{th} space harmonic is

$$\beta_n = \omega / v_0 = \frac{\theta_1 + 2\pi n}{v_0 T_0} \rightarrow (2)$$

Sub (1) in (2).

$$\beta_n = \frac{\theta_1 + 2\pi n}{L} = \frac{\theta_1}{L} + \frac{2\pi n}{L}$$

$\therefore \beta_n = \beta_0 + \frac{2\pi n}{L}$ The axial space-harmonic phase velocity is assumed to synchronized with beam velocity for possible interactions.

Magnetron :

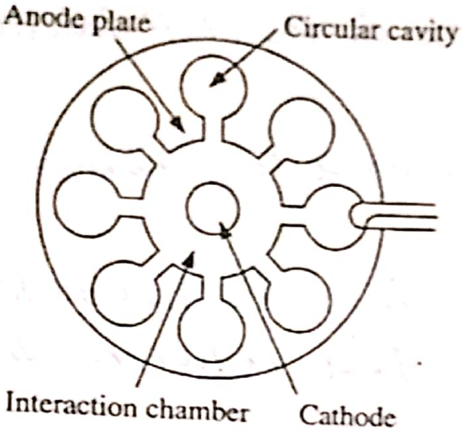
M-type devices or crossed field tubes in which the dc magnetic field & dc electric field are perpendicular to each other.

* In all crossed-field tubes the dc magnetic field plays a direct role in the RF interaction process.
* A Magnetron oscillator is used to generate high microwave power.
* All magnetrons operated in a dc magnetic field normal to a dc electric field between the cathode & anode.
* The electrons emitted from the cathode are moved in curved paths due to crossed field between the cathode & anode
* If the dc magnetic field is strong enough, the electrons will not arrive in the anode but return instead to the cathode. Consequently, the anode current is cut off

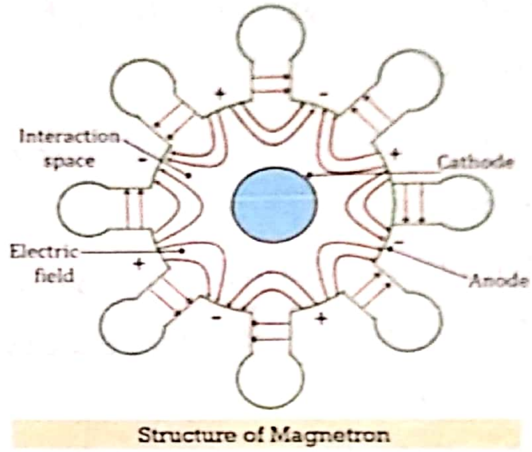
Cylindrical Magnetron :

* This type of Magnetron is called a conventional magnetron.
* In a cylindrical magnetron, several reentrant cavities are connected to the gaps.
* The dc voltage V_0 is applied between the cathode & the anode. The magnetic flux density B_0 is in the positive Z direction
* The electrons emitted from the cathode try to travel to anode, but with the influence of crossed fields E & H in the space between anode & cathode, the electrons take curved path.

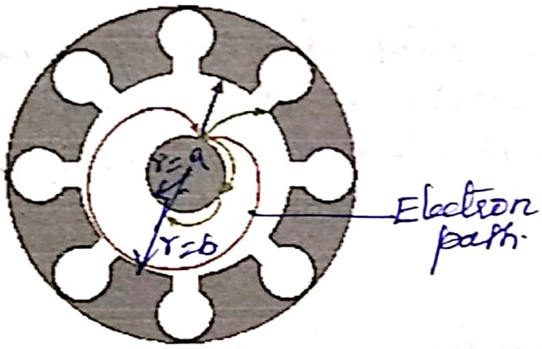
* When the dc voltage & the magnetic flux are adjusted properly, the electrons will follow ~~cylind~~ cycloidal paths in the cathode-anode space under the combined force of both electric & magnetic field.

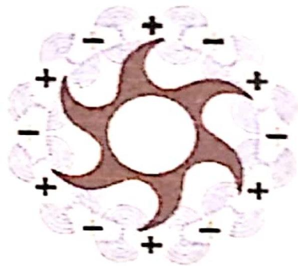


Magnetron construction



Structure of Magnetron





* The accelerated electrons in the curved trajectory, when retarded by the RF field, transfer energy from the electron to the cavities to grow RF oscillations till the system RF losses balances the RF oscillations for stability

(i) Equations of Electron Motion or Hull cut off Voltage Equation :- ⟶ Eqns of Electron Trajectory:

After emergence from the cathode with zero velocity, the electrons will acquire a velocity V having a tangential as well as a radial velocity components due to force F exerted by the crossed fields E & H

$$F = -eE - e(V \times H)$$

At zero magnetic field, the electrons, take the straight path a', by the influence of electric field, ~~the electrons take the straight path~~ & collected by the anode.

For a given V_0 if the magnetic field is increased, the electrons take the curved path b' due to the above force F to reach the anode.

At a critical value of magnetic field B_c the electrons just graze the anode surface at radius, b & take the path c' to return to the cathode for a given voltage V_0. This value B_c is called the cut off magnetic flux density.

If the magnetic field is greater than B_c, all the electrons return to the cathode as shown by a typical path d' without reaching the anode.

Therefore, when the magnetic field increases from zero to maximum, the anode current decreases from a maximum value to zero.

The average velocity of the electron in the Z-direction is constant

$$V_Z = E_0 / B_0$$

where $E_0 = V_0 / (b-a)$ is dc electric field

B_0 = dc magnetic field

At equilibrium.

the centrifugal force & velocity v experiences by the 3 forces

$$\frac{mv^2}{r} + eE = evB$$

where the electric field is in radial direction only

$$E(r) = \frac{-V_0}{r \ln b/a}$$

In the absence of an electric field, the electrons move in a circular path & return to the cathode, when

$$mv^2/r = evB$$

$$v/r = eB/m = \omega \rightarrow \text{cyclotron angular frequency}$$

The eqns of motion for electrons in crossed electric & magnetic fields

$$m(dv/dt) = F = -eE - ev \times B$$

Since the electron is emitted from the cathode in the direction opposite to $\bar{E}$, the eqn of motion for electrons in cylindrical coordinates

$$\frac{d^2r}{dt^2} - r(d\varphi/dt)^2 = +(e/m)\left[E_r - rB_z \frac{d\varphi}{dt}\right]$$

$$\& \ \frac{1}{r}\frac{d}{dt}\left(r^2 \frac{d\varphi}{dt}\right) = \frac{eB_z}{m} \, dr/dt$$

where $B_0 = B_z$ is assumed in the z direction

Cut off Magnetic field & voltage :-

$$\frac{d}{dt}\left(r^2 d\varphi/dt\right) = \frac{eB_z}{m}\frac{r\,dr}{dt} = \frac{\omega}{2}\frac{d}{dt}(r^2)$$

(or)

$$r^2 d\varphi/dt = \omega r^2/2 + K.$$

At $r = a$, $d\varphi/dt = 0$ & $K = -\omega a^2/2$.
Therefore the angular velocity of the electrons are

$$\frac{d\varphi}{dt} = \omega/2 \cdot (1 - a^2/r^2)$$

Since the electrons move in direction perpendicular to the magnetic field, the kinetic energy of the electrons is given by the electric field only.

$$eV = 1/2 \cdot m\left[(dr/dt)^2 + (r\,d\varphi/dt)^2\right]$$

At $r = b$, $V = V_0$ & $dr/dt = 0$ for the electrons to just graze the anode

$$(b\,d\varphi/dt)^2 = 2(e/m)V_0 \quad \text{or} \quad b^2\left[\omega/2(1 - a^2/b^2)\right]^2 = (2e/m)V_0$$

Sub. $\omega = eB_c/m$ at grazing.

$$b^2\left[eB_c/2m(1 - a^2/b^2)\right]^2 = 2eV_0/m \quad \text{or}$$

$$B_c = \frac{(8V_0 m/e)^{1/2}}{b(1 - a^2/b^2)}$$

Thus if $B_0 > B_c$ for a given V_0, the electrons will not reach the anode. For a given B_0 the cut off voltage is $V_c = \frac{e}{8m} b^2 (1 - a^2/b^2)^2 B_0^2$

If $V_0 < V_c$ for a given B_0, the electrons will not reach the anode. Thus B_c & V_c are called the Hull cut off magnetic & voltage eqns respectively.