EC8651 TRANSMISSION LINES AND RF SYSTEMS

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OBJECTIVES:

- To introduce the various types of transmission lines and its characteristics
- To give thorough understanding about high frequency line, power and impedance measurements
- To impart technical knowledge in impedance matching using smith chart
- To introduce passive filters and basic knowledge of active RF components
- To get acquaintance with RF system transceiver design

UNIT I TRANSMISSION LINE THEORY 9

General theory of Transmission lines - the transmission line - general solution - The infinite line - Wavelength, velocity of propagation - Waveform distortion - the distortion-less line - Loading and different methods of loading - Line not terminated in Z0 - Reflection coefficient - calculation of current, voltage, power delivered and efficiency of transmission - Input and transfer impedance - Open and short circuited lines - reflection factor and reflection loss.

UNIT II HIGH FREQUENCY TRANSMISSION LINES 9

Transmission line equations at radio frequencies - Line of Zero dissipation - Voltage and current on the dissipation-less line, Standing Waves, Nodes, Standing Wave Ratio - Input impedance of the dissipation-less line - Open and short circuited lines - Power and impedance measurement on lines - Reflection losses - Measurement of VSWR and wavelength.

UNIT III IMPEDANCE MATCHING IN HIGH FREQUENCY LINES 9

Impedance matching: Quarter wave transformer - Impedance matching by stubs - Single stub and double stub matching - Smith chart - Solutions of problems using Smith chart - Single and double stub matching using Smith chart.

UNIT IV WAVEGUIDES 9

General Wave behavior along uniform guiding structures – Transverse Electromagnetic Waves, Transverse Magnetic Waves, Transverse Electric Waves – TM and TE Waves between parallel plates. Field Equations in rectangular waveguides, TM and TE waves in rectangular waveguides, Bessel Functions, TM and TE waves in Circular waveguides.

UNIT V RF SYSTEM DESIGN CONCEPTS 9

Active RF components: Semiconductor basics in RF, bipolar junction transistors, RF field effect transistors, High electron mobility transistors Basic concepts of RF design, Mixers, Low noise amplifiers, voltage control oscillators, Power amplifiers, transducer power gain and stability considerations.

TOTAL:45 PERIODS

OUTCOMES:

Upon completion of the course, the student should be able to:

- Explain the characteristics of transmission lines and its losses
- Write about the standing wave ratio and input impedance in high frequency transmission lines
- Analyze impedance matching by stubs using smith charts
- Analyze the characteristics of TE and TM waves
- Design a RF transceiver system for wireless communication

EC8651–TRANSMISSION LINES & RF SYSTEMS

(Regulation 2017)

UNIT – 1 Transmission Line Theory

Part – A

1) Define Characteristic Impedance.
2) Find the characteristic impedance of a line at 1600 Hz if Zoc =750∠-30°Ω and Zoc=600∠-20°Ω
3) State the condition for a distortion less line?
4) Give the Campbell's formula for a uniformly loaded line?
5) Define Propagation Constant.
6) Give the relation between Reflection factor & Reflection loss?
7) Define Reflection loss & insertion loss.

Part – B

1) Derive the transmission line equation and hence obtain expressions for voltage and current on a transmission line.
2) Derive the equation of attenuation constant and phase constant of transmission line in terms of line constants R,L,C and G
3) Prove that an infinite line equal to finite line terminated its characteristic impedance
4) Explain in detail about the wave form distortion and also derive the condition for distortion less line
5) i) Explain in detail the reflection on a line not terminated by its characteristic

   Impedance Z_0.

   ii) The constants of a transmission line are R=6Ω/km, L=2.2mH/km,C=0.005µF/km & G=0.25x10^{-}3/km. Calculate the Z_0, attenuation constant & phase constant at 1000Hz.
6) A Generator of 1 V, 1000 Hz supplies power to a 100 km open wire line terminated in Z0 and having the following parameters: R = 10.4 Ohm/km, G = 0.8 x 10-6 mho/km, L= 0.00367 henry/m and C = 0.00835 µF/km. Find the characteristic impedance(Z0), propagation constant(γ), attenuation constant (α),phase shift constant(β), velocity of propagation(v) and wavelength(λ). Also find the sending end current, receiving end current and received power.

UNIT - 2 High Frequency Transmission Lines

Part – A

1) Give the minimum and maximum values of SWR and Reflection Coefficient.
2) Write the expression for standing wave ratio in terms of reflection co efficient
3) Write the equations of Voltage and Current on the dissipation less line.
4) For the line of zero dissipation, what will be the value of attenuation constant and Characteristic Impedance?
5) Write the Expression for input impedance of open and short circuited dissipation less line.
6) List the Parameters of the open wire line at high frequencies.
7) What is mean by Reflection Loss?
8) What are the assumptions to simplify the analysis of line performance at high frequencies?

Part – B

1) Derive the Transmission line equations at radio frequencies (open wire & Co axial line parameters)

2) Discuss in detail about the voltage and current on the dissipation less line

3) Derive the expression for power and find the input impedance of dissipation less line, when the load is short circuited, open circuited and for a matched line.

4) A lossless line in air having a characteristic impedance of 300Ω is terminated in unknown impedance; the first voltage minimum is located at 15cm from the load. The standing wave ratio is 3.3. Calculate the wavelength and Terminated Impedance.

5) A 50Ω loss line transmission line is connected to a load composed of 75Ω resistor in series with a capacitor of unknown capacitance. If at 10 MHz, the voltage standing wave ratio on the line was measured as '4'. Determine the capacitance 'C'?

6) A 30m long lossless transmission line with Z_0 = 50W operating at 2MHz is terminated with a load Z_L = 60+j40. If u = 0.6C on the line.

Note: (where 'C' is Velocity of light and 'u' is Phase velocity)

Find i) Reflection Co-efficient (G)

ii) Standing wave Ratio (S)

iii) The Input Impedance (Z_{in})

UNIT – 3 Impedance Matching in High Frequency Lines

Part – A

1) What is Impedance Matching and state the need for impedance matching.
2) What is Stub Matching? Name the impedance matching devices.
3) Explain the significance of smith chart.
4) List the applications of a quarter wave line.
5) What is impedance matching in stub?
6) Write down the Expression to determine the position of Stub?
7) Write down the Expression to determine the length of Stub?

Part – B

1) Explain the significance of smith chart and its applications in a transmission lines.
2) A single stub is to match a load 400Ω line to a load of 200-j100Ω. The wave length is 3m. Determine the position and length of the short circuited stub.
3) A 300Ω transmission line is connected to a load impedance of (450-j600)Ω at 10MHz. Find the position and length of short circuited stub required to match the line using smith chart.
4) Explain the technique of impedance matching by stubs and discuss the operation of quarter wave transformer
5) Explain the procedure for obtaining the smith chart using R and X circles.
6) Determine the following

   i) Standing wave Ratio (SWR)

   ii) Load Impedance (Z_L)

   iii) Distance between load and the first voltage minimum along the transmission line for a line with a characteristic impedance of 300Ω and terminated in a load of 175+j207Ω. An electrical signal of 200 MHz is transmitted along the line in free space.

UNIT – 4 Waveguides and cavity Resonators

Part – A

1) What are rectangular wave guides
2) Write the expression for cutoff wavelength of the wave which is propagated in between two parallel planes.
3) What are the dominant mode and degenerate modes in rectangular wave guides
4) Justify, why TM_{01} and TM_{10} modes in a rectangular wave guide do not exist.
5) Give the expression for wave impedance and power transmission in TE & TM waves in a rectangular wave guide
6) Calculate the cut-off frequency of a rectangular wave guide whose inner dimension are a= 2.5cm & b=1.5cm operating at TM_{10} mode
7) State the relation between the attenuation factor for TE waves and TM waves for parallel plate waveguide.

Part – B

1) Briefly explain about general wave behavior along uniform guiding structures, TE, TM & TEM waves.
2) A rectangular air filled copper waveguide with dimension 0.9 inch X 0.4 inch cross section and 12 inch length is operated at 9.2Ghz with a dominant mode. Find cut off frequency, guide wavelength, Phase velocity, characteristic impedance and the loss
3) Derive an expression for the transmission of TM waves between parallel perfectly conducting planes for the field components
4) An air filled circular waveguide having an inner radius of 1 cm is excited in dominant mode at 10Ghz. Find a) cut off frequency of the dominant mode at 10 Ghz b) Guide wave length c) wave impedance. Also find the bandwidth for operation in the dominant mode.
5) Write Bessel's differential equation & derive the TE Wave components in circular wave guides.
6) A TE_{10} wave at 10 GHz propagates in a brass $\sigma_c = 1.57 \times 10^7$ S/m rectangular wave guide with inner dimensions a= 1.5 cm & b= 0.6 cm, which is filled with $\varepsilon_r = 2.25$, $\mu_r = 1$, loss tangent = 4×10^{-4}. Determine i) Phase constant ii) Guide wavelength iii) Phase velocity iv) Wave impedance v) attenuation due to loss in the dielectric vi) attenuation due to loss in the guide walls

UNIT – 5 RF System Design Concepts

Part – A

1) What is MESFET? List the different types of FETs used in RF Circuits.
2) What is mean by Unilateral transducer gain G_{TU} & Transducer gain G_T
3) What is the main purpose of LNA & what are the problems arise due to non-linear characteristics of LNA?
4) Define Conduction angle and give its significance in power amplifiers.
5) Define unconditional stability.
6) Give the expression for noise of a two port amplifier.
7) What are multipliers based mixers and mention their types.
8) What are three operating points in MESFET?
9) What are the factors for selecting a matching network?
10) What are the advantages of RF/microwave transistors?

Part – B

1) Briefly explain about design process of RF/Microwave Circuits
2) Give the analysis of frequency response of MESFET, What are its limiting values?
3) Discuss about the basic architecture of RF System and importance of RF Circuit design.
4) Briefly explain about High Electron Mobility Transistor (HEMT) & explain the functionality of HEMT.
5) A Ga As MESFET has the following parameters: $N_D = 10^{16}$ cm^{-3}, d=0.75 µm, W= 10µm, L= 2µm, $\varepsilon_r = 12.0$, $V_d = 0.8$v and $\mu_n = 8500$ cm^2 /vs. Determine a) The Pinch off voltage b) Threshold voltage c) The Maximum Saturation Current I_{DSS}.
6) List and give constructional figures of the different types of FETs based on the way the gate is connected to the conducting channel.
7) What is the importance of RF Circuit design and design consideration steps for RF Systems.
8) i) Write short notes on sub-harmonic mixer and its characteristics

   ii) Explain the behavior of LNA topologies with its design constraints.
9) Derive the equation for power gain, available power gain and transducer power gain.
10) Give the analysis of unconditional stability in RF Amplifier.

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SUBJECT : EC8651 TRANSMISSION LINES AND RF SYSTEMS

SEM / YEAR: VI/ III Year B.E.

UNIT I - TRANSMISSION LINE THEORY

General theory of Transmission lines - the transmission line - general solution - The infinite line - Wavelength, velocity of propagation - Waveform distortion - the distortion-less line - Loading and different methods of loading - Line not terminated in Z_0 - Reflection coefficient - calculation of current, voltage, power delivered and efficiency of transmission - Input and transfer impedance - Open and short circuited lines - reflection factor and reflection loss.

PART A (2 marks)

Q.No.	Questions	BT Level	Competence
1.	Define transmission line.	BTL 1	Remembering
2.	List the conditions for distortion less line.	BTL 1	Remembering
3.	State phase distortion and frequency distortion.	BTL 1	Remembering
4.	What are primary constants and secondary constants of a transmission line?	BTL 1	Remembering
5.	How to avoid the distortion that occurs in the line?	BTL 1	Remembering
6.	Choose the properties of infinite line.	BTL 1	Remembering
7.	Write the expressions for the phase constant and velocity of propagation.	BTL 2	Understanding
8.	Discuss the effect of inductance loading of telephone cable.	BTL 2	Understanding
9.	Deduce the relationship between characteristic impedance and propagation constant.	BTL 2	Understanding
10.	Conclude the general equation for the input impedance and transfer impedance of a transmission line.	BTL 2	Understanding
11.	Build the voltage and current equations at any point on a uniform transmission line.	BTL 3	Applying
12.	Illustrate how practical lines made appear as infinite lines.	BTL 3	Applying
13.	Sketch the equivalent circuit of a unit length of transmission line.	BTL 3	Applying
14.	Analyze the expression for reflection coefficient and reflection loss.	BTL 4	Analyzing
15.	The open circuit and short circuit impedance of a transmission line at 1500 Hz are 800 $\angle$-30 Ω and 400 $\angle$-10 Ω respectively, Examine its propagation constant.	BTL 4	Analyzing
16.	Assess the expression for reflection factor.	BTL 4	Analyzing
17.	Prove that 1 neper=8.686dB.	BTL 5	Evaluating

18.	Estimate the reflection coefficient of a 50ohm transmission line when it is terminated by a load impedance of 60+j40Ω.	BTL 5	Evaluating
19.	Formulate the equation to find the relation between characteristic impedance and primary constants of a transmission line.	BTL 6	Creating
20.	Design Campbell's formula for a uniformly loaded line.	BTL 6	Creating

PART –B (13 marks)

1.		Obtain the general transmission line equation for the voltage and current at any point on a transmission line.	(13)	BTL 1	Remembering
2.	(i)	Outline the expression for the attenuation and phase constants after obtaining an expression for the characteristic impedance.	(7)	BTL 1	Remembering
	(ii)	How to solve the Campbell's equation for the loading cables?	(6)		
3.	(i)	Discuss in detail about inductance loading of telephone cables and recall the attenuation constant, phase constant and velocity of signal transmission for the uniformly loaded cable.	(7)	BTL 1	Remembering
	(ii)	Describe about the reflection on a line not terminated in its characteristic impedance.	(6)		
4.	(i)	What is a loading? Specify the types of loading of lines.	(7)	BTL 1	Remembering
	(ii)	Write a short note on reflection factor and reflection loss and give expressions.	(6)		
5.	(i)	Explain in detail about the waveform distortion and also derive the condition for distortion less line.	(7)	BTL 2	Understanding
	(ii)	Predict the expression for open and short circuited impedance.	(6)		
6.	(i)	Identify the conditions (α, β) required for a distortion less line.	(7)	BTL 2	Understanding
	(ii)	A distortion less transmission line has attenuation constant α=1.15×10-2 Np/m, and capacitance of 0.01 nF/m. the characteristic resistance L/C=50Ω. Identify the resistance, inductance and conductance per more of the line.	(6)		
7.	(i)	A telephone line has parameters of R = 6.5 Ω/km, L= 0.4 mH/km, C=0.05μF/km and G=0.5 μmho/km. Extend the calculation for finding the input impedance of the line at a frequency of 500 Hz given that the line is very long.	(7)	BTL 2	Understanding
	(ii)	A lossless transmission line with Z_0 = 75ohm and electrical length l= 0.3 λ is terminated with a complex load impedance of Z_R = 40+j20ohm. Estimate reflection	(6)		

		coefficient and VSWR of the line.			
8.	(i)	The characteristic impedance of a certain transmission line is 682.5-j195.7 ohm. The frequency of operation is 1 KHz. At this frequency, the attenuation constant in the line was observed to be 0.01 neper/km and phase constant 0.035 radians /km. Prepare the line constants R, L, G and C per km of the line.	(8)	BTL 3	Applying
	(ii)	Draw and explain the reflection loss due to mismatch between source and load impedances.	(5)		
9.	(i)	An open wire line having R=10.15 ohm/km, L = 3.93 mH/km, G=0.29 μmho/km and C=0.00797 μF/km is 100 km long and terminated in $Z_{0.}$ Solve $Z_{0,}$ α, β and γ.	(7)	BTL 3	Applying
	(ii)	Illustrate the Zo and in terms of primary constants.	(6)		
10.	(i)	Connect the value of attenuation constant 'α' as **R/2** $\sqrt{\boldsymbol{C}/\boldsymbol{L}}$ + **G/2**$\sqrt{\boldsymbol{L}/\boldsymbol{C}}$, when the series resistance R and shunt resistance G of the transmission line are small but not negligible.	(7)	BTL 4	Analyzing
	(ii)	Demonstrate the concept of attenuation and phase constant of an infinite line.	(6)		
11.	(i)	Analyze the expressions for short circuited and open circuited impedance.	(6)	BTL 4	Analyzing
	(ii)	Point out the propagation constant of continuously loaded cable.	(7)		
12.	(i)	A 2 meter long transmission line with characteristic impedance of 60+j40 is operating at $=10^6$ rad/sec has attenuation constant of 0.921 Np/m and phase shift constant of 0 rad/miff the line is terminated by a load of 20+j50,find the input impedance of this line.	(7)	BTL 4	Analyzing
	(ii)	Simplify the expression for input impedance and transfer impedance of transmission lines.	(6)		
13.	(i)	Summarize how an infinite line equal to finite line terminated in its characteristic impedance.	(6)	BTL 5	Evaluating
	(ii)	The characteristics impedance of a 805m-long transmission line is 94 angle $-23.2^0\Omega$, the attenuation constant is 74.5×10^{-6} Np/m. and the phase shift constant is 174×10^{-6} rad/m at 5KHz. Assess the line parameters R,L,G and C per meter and the phase	(7)		

		velocity on the line.			
14.		The constants of a transmission line are R= 6Ω/km, L=2.2mH/km, C=0.005×10^{-6}F/km and G=0.25×10^{-6} mho/km. calculate at the frequency of 800 HZ, Design (a) The terminating impedance for which no reflection will be setup in the line, (b) The attenuation in dB suffered by the signal after travelling a distance of 50Km when the line is properly terminated and the phase velocity with which the signal would travel.	(6) (7)	BTL 6	Creating
		PART C (15 marks)			
1.	(i)	Adapt the condition for minimum attenuation in a distortion less line	(7)	BTL 6	Creating
	(ii)	Develop and derive the relation between primary constants and secondary constants.	(8)		
2.		A 100 km long line is terminated in its characteristic impedance. A generator of internal impedance of 600 ohm and a voltage of 5 volts operating at frequency of 800 Hz is connected at the input end of the line. The characteristic impedance of the line is 550 ∠ -15^0 and the propagation constant γ = 0.045 +j0.0825 per km. Observe the parameter such as (a) Primary Constants (b)Sending end current and sending end voltage (c)Receiving end current and Receiving end Voltage (d) Sending end power and Receiving end power.	(15)	BTL 6	Creating
3.		Open circuited and short circuited measurements at a frequency of 5KHz on a line of length 200km yielded the following results: Zoc = 570 ∠ -48^0 ohm , Zsc = 720∠ 34^0 ohm EvaluateZo , α, β and primary constants given that the approximate velocity of propagation to be 1.8 *10^6 km/sec.	(15)	BTL 5	Evaluating
4.	(i)	An open wire line is 200 km long is correctly terminated. The generator at the sending end has Es = 10 V, f = 1 KHz and internal impedance of 500 ohm. At that frequency Zo=683-j138 and γ= 0.0074+j0.0356 per km. Measure the sending and receiving end voltage, current and power.	(10)	BTL 5	Evaluating
	(ii)	A cable has α = 0.01 nepers /km and β = 0.0018/km and having length of 100 km.Estimate the receiving end voltage when the line is terminated in its characteristic impedance and Es = 5V.	(5)		

UNIT II - HIGH FREQUENCY TRANSMISSION LINES

Transmission line equations at radio frequencies - Line of Zero dissipation - Voltage and current on the Dissipation less line, Standing waves, Nodes, Standing wave ratio - Input impedance of the dissipation-less line - Open and short circuited lines - Power and impedance measurement on lines - Reflection losses -

Measurement of VSWR and wavelength.			
PART A (2 marks)			
Q.No.	**Questions**	**BT Level**	**Competence**
1.	Define Skin effect.	BTL 1	Remembering
2.	Label the assumptions made for the analysis of performance of the line at radio frequency.	BTL 1	Remembering
3.	Compare the values of SWR for $Z_R = 0$ and $Z_R = Z_0$.	BTL 1	Remembering
4.	Recognize the dissipationless line with the proper condition.	BTL 1	Remembering
5.	What are nodes and antinodes on a line?	BTL 1	Remembering
6.	Find the nature and value of Z0 for the dissipation less line.	BTL 1	Remembering
7.	Give the relation between standing wave ratio and magnitude of reflection co efficient.	BTL 2	Understanding
8.	Express reflection coefficient in terms of SWR.	BTL 2	Understanding
9.	Predict the expression for input impedance of RF line.	BTL 2	Understanding
10.	Indicate the minimum values and maximum values of SWR and reflection coefficient.	BTL 2	Understanding
11.	Relate the nature of input impedance of open circuited and short circuited and matched load condition for dissipation less line.	BTL 3	Applying
12.	Solve the terminating load for a certain R.F transmission line which has the characteristic impedance of the line 1200 Ω and the reflection co-efficient was observed to be 0.2.	BTL 3	Applying
13.	Sketch standing waves on a line having open or short circuit termination.	BTL 3	Applying
14.	Deduce an expression for inductance of an open wire line and coaxial line.	BTL 4	Analyzing
15.	Analyze the line with dissipationless line and find the values of attenuation constant and characteristic impedance.	BTL 4	Analyzing
16.	Explain the concept of dissipation less line.	BTL 4	Analyzing
17.	How would you justify that the point of voltage minimum is measured rather than the voltage maximum?	BTL 5	Evaluating
18.	A lossless transmission has a shunt capacitance of 100 pF/m and a series inductance of 4μH/m. Evaluate the characteristic impedance.	BTL 5	Evaluating
19.	Can you predict the range of values of standing wave ratio?	BTL 6	Creating
20.	Adapt the condition for open and short circuited line.	BTL 6	Creating
PART B (13 marks)			

1.		Derive the expressions for voltage and current at any point on the radio frequency dissipation less line. Obtain the expressions for the same for different receiving end conditions.	(13)	BTL 1	Remembering
2.	(i)	Brief notes on Standing waves, nodes, standing wave ratio also make relation between the standing wave ratio S and the magnitude of the reflection coefficient.	(7)	BTL 1	Remembering
	(ii)	State the condition for the open wire line at high frequencies and derive the parameters.	(6)		
3.	(i)	Explain the parameters of open wire line and co axial at RF. Mention the standard assumptions made for radio frequency line. Label the Line constants for zero dissipation.	(7)	BTL 1	Remembering
	(ii)	Enumerate the voltage and current equation on dissipation less line.	(6)		
4.	(i)	Discuss the reflection coefficient of different transmission lines.	(6)	BTL 1	Remembering
	(ii)	The ratio of spacing'd' to the radius 'a' of an open wire dissipation less line is 25 and the space between the conductors has a dielectric of relative permittivity of 8. Recognize (a)Inductance (b) Capacitance (c) Characteristic impedance	(7)		
5.	(i)	Compare the features of open wire and co axial cable at high frequencies.	(7)	BTL 2	Understanding
	(ii)	Outline the variation of input impedance along open and short circuit lines with relevant graphs.	(6)		
6.	(i)	Describe the various parameters of open wire and co axial lines at radio frequency.	(7)	BTL 2	Understanding
	(ii)	Summarize the concept of Standing wave ratio.	(6)		
7.		Explain how the VSWR and wavelength of the line measured in detail.	(13)	BTL 2	Understanding
8.	(i)	Derive the line constants of a zero dissipation less line.	(7)	BTL 3	Applying
	(ii)	Sketch the voltages and currents on dissipation less line for the conditions given below. (a) Open circuit (b) Short circuit (c) Rr =R_0	(6)		
9.	(i)	A low loss transmission line of 100Ω characteristic impedance is connected to a load of 200Ω.Apply the formula to calculate the voltage reflection coefficient and the standing wave ratio.	(7)	BTL 3	Applying

	(ii)	Solve the standing wave ratio and reflection co – efficient on a line having Z_0 =300ohm and terminated in Z_R=300+j400	(6)		
10.	(i)	Draw the standing wave pattern for (a) Open circuited load (b) Short circuited load (c) matched load	(7)	BTL 4	Analyzing
	(ii)	Predict that the reflection coefficient [|Emax| - |Emin|] / [|Emax| +|Emin|].	(6)		
11.		Explain the expressions for the input impedance of the dissipation less line. Deduce the input impedance of open and short circuited dissipation less line.	(13)	BTL 4	Analyzing
12.	(i)	Examine the voltage and currents at any point on the dissipation less line along with incident and reflected voltage wave phasor diagrams which should satisfy the conditions such as open circuit, short circuit, R_R=R_0.	(7)	BTL 4	Analyzing
	(ii)	Analyze the standing waves with neat diagram.	(6)		
13.	(i)	Summarize the relation between standing wave ratio (S) and magnitude of relation co – efficient.	(7)	BTL 5	Evaluating
	(ii)	Find the reflection coefficient and voltage standing wave ratio of a line having Ro = 100 ohm, Zr = 100 – j100 ohm.	(6)		
14.		Formulate the following parameters (a) Standing waves (b) Standing wave ratio (c) Relation between SWR and K (d) Nodes and Antinodes	(13)	BTL 6	Creating
PART C (15 marks)					
1.		A Dissipation less co-axial cable has an inner copper conductor of radius 3mm and an outer copper conductor of radius 15mm. It is filled with dielectric material of relative permittivity ε_r. When it is excited at one end by an a.c. source, the phase velocity of the wave was observed to be 1.5×10^8 m/s. The other end is terminated in a load resistance Z_R = R_R which produces standing wave ratio of 3.8. What would you recommend the values for following parameters? (a) Characteristic impedance Zo = Ro		BTL 5	Evaluating

		(b) Dielectric constant (c) Load resistance $Z_R = R_R$ (d) Reflection Coefficient K	(4) (4) (3) (4)		
2.		Generalize the expressions for voltage and current at any point on the radio frequency dissipation less line. Obtain the expressions for the same for different receiving end conditions.	(15)	BTL 5	Evaluating
3.		How could you adapt the length of dissipationless line to obtain an inductance of 15µH at 60 MHz frequency with open circuit termination? Given that characteristic impedance of the line is 400 ohm.	(15)	BTL 6	Creating
4.	(i)	How would you make up the expression for maximum and minimum impedances on the line for a lossless line as R_oS and R_o/S respectively?	(8)	BTL 6	Creating
	(ii)	What way would you design the coaxial line at high frequencies? Design a graph to show the variation of R_o for a coaxial line.	(7)		

UNIT III - IMPEDANCE MATCHING IN HIGH FREQUENCY LINES

Impedance matching: Quarter wave transformer - Impedance matching by stubs - Single stub and double stub matching - Smith chart - Solutions of problems using Smith chart - Single and double stub matching using Smith chart.

PART A (2 marks)

Q.No.	Questions	BT Level	Competence
1.	What is the need for impedance matching?	BTL 1	Remembering
2.	List the requirements of a better transmission line	BTL 1	Remembering
3.	Interpret the effect of impedance mismatching.	BTL 2	Understanding
4.	Express standing wave ratio in terms of reflection coefficient.	BTL 2	Understanding
5.	Discuss about nodes and anti nodes in a transmission line.	BTL 4	Analyzing
6.	Why do standing waves exist on transmission lines?	BTL 1	Remembering
7.	Give the minimum and maximum value of SWR and reflection coefficient.	BTL 1	Remembering

8.	Calculate the standing wave ratio if the reflection co-efficient of a line is 0.3 $\angle$-66°.	BTL 3	Applying
9.	A lossless line has a characteristic impedance of 400Ω. Determine the standing wave ratio if the receiving end impedance is 800+j0Ω	BTL 5	Evaluating
10.	List the application of a quarter wave line.	BTL 1	Remembering
11.	Justify the statement - quarter wave lines are termed as impedance inverter.	BTL 5	Evaluating
12.	Analyze why Quarter wave line is called as copper insulator.	BTL 4	Analyzing
13.	A 75Ω lossless transmission line is to be matched to a resistive load impedance of Z_L =100Ω via a quarter wave section.	BTL 6	Creating
14.	Determine the characteristic impedance of the quarter wave transformer.	BTL 2	Understanding
15.	Mention the advantages of Smith Chart.	BTL 2	Understanding
16.	Illustrate the procedure to find the impedance from the given admittance using smith chart.	BTL 3	Applying
17.	Define the term Stub used in transmission line.	BTL 1	Remembering
18.	Examine why short circuited stub is preferred to open circuited stub.	BTL 3	Applying
19.	Generalize the method to determine the positon and the length of a single stub connected across the transmission line.	BTL 6	Creating
20.	Compare single stub matching and double stub matching.	BTL 3	Applying

PART –B (13 Marks)

1.	(i)	Examine the operation and application of quarter wave transformer.	(7)	BTL 2	Understanding
	(ii)	Consider a line with a load of Z_R/R_O =2.6+j, which is 28° long. Find the input impedance.	(6)		
2.	(i)	Deduce the expression for input impedance of a quarter wave transformer and mention its applications.	(8)	BTL 6	Creating
	(ii)	Design a Quarter wave transformer to match a load of 200Ω to a source resistance of 500Ω which operates at the frequency of 200 MHz.	(5)		
3.		Analyze the transmission line circle diagram by deriving the expression for constant S and constant βs circle.	(13)	BTL 4	Analyzing
4.		A lossless line with Z_O =70Ω is terminated with Z_R =115-80jΩ. Wavelength of transmission is 2.5 λ. Using smith chart evaluate the VSWR, reflection coefficient, input impedance and input admittance	(13)	BTL 5	Evaluating
5.		Describe the impedance matching technique using single stub and obtain the expression for the stub location and stub length.	(13)	BTL 1	Remembering
6.		Consider a line of R_O = 55 ohms terminated with 115+j75 ohms. If the total length of the line is 1.183λ, find the reflection coefficient, VSWR, input impedance and admittance	(13)	BTL 1	Remembering
7.		What is the procedure for double stub matching on a transmission line, explain with an example.	(13)	BTL 1	Remembering

8.		A UHF lossless transmission line working at 1 GHz is connected to an unmatched line producing a voltage reflection coefficient of 0.5(0.866+j 0.5). Calculate the length and position of the stub to match the line using corresponding equations verify the values using Smith Chart.	(13)	BTL 2	Understanding
9.		A transmission line is terminated in Z_{L}. Measurements indicate that the standing wave minima are 102 cm apart and that the last minimum is 35 cm from the load end of the line. The value of standing wave ratio is 2.4 and R_0 = 250Ω. Determine frequency, wavelength, Real and reactive components of the terminating impedance. Also Verify the results obtained from equations using the smith chart.	(13)	BTL 2	Understanding
10.		VSWR of a lossless line is found to be 5 and successive voltage minima are 40cm apart. The first voltage minima is observed to be 15cm from the load. The length of the line is 160cm and Z_o is 300 Ω. Apply the values in smith chart to find the load impedance and input impedance.	(13)	BTL 3	Applying
11.		A RF transmission line with $Z_o=300\angle 0°$ Ω is terminated in an impedance of $100\angle 45°$Ω. This load is to be matched to the transmission line by using a short circuited stub. With the help of smith chart, Determine the length and location of the stub.	(13)	BTL 4	Analyzing
12.		A 50 Ω transmission line feeds an inductive load 35+j35Ω. Analyze and design a double stub tuner to match this load to the line using smith chart. Spacing between the two stubs is λ/4.	(13)	BTL 1	Remembering
13.		Derive the expression of radius and center for constant R and X circles in Smith Chart.	(13)	BTL 4	Analyzing
14.		Examine the transmission line to provide an impedance matching using a stub. Obtain the length and location of the stub to provide an impedance match on a line of 600 ohms terminated with 200 ohms. Assuming that the stub is short circuited at one end.	(13)	BTL 3	Applying
PART – C (15 Marks)					
1.	(i)	Determine length and location of a single short circuited stub to produce an impedance match on a transmission line with characteristic impedance of 600 ohm and terminated in 1800 ohm.	(8)	BTL 5	Evaluating
	(ii)	A 300 Ω transmission line is connected to a load impedance of (450-j600) Ω at 10MHz.Evaluate the position and length of a short circuited stub required to match the line using	(7)		

		smith chart.			
2.		For a normalized load impedance of 0.8+j1.2 design a double stub tuner with the distance between them as 3λ/8. Considering the stubs are short circuited determine the length of the stubs and the position of the first stub from the load. Verify the answer using Smith Chart.	(15)	BTL 6	Creating
3.	(i)	A line having characteristic impedance of 50 Ω is terminated in load impedance [75+j75] Ω. Determine the reflection coefficient and voltage standard wave ratio.	(8)	BTL 5	Evaluating
	(ii)	Mention the significance of smith chart and its application in transmission lines.	(7)		
4.	(i)	Develop the expression for the input impedance of the dissipation less line and thus obtain the expression for the input impedance of the quarter wave line. Also discuss the application of the quarter wave line.	(10)	BTL 6	Creating
	(ii)	Design a single stub match for a load of 150+j225 Ω for a 75 Ω line a 500 MHz using smith chart.	(5)		

UNIT IV - WAVE GUIDES

General Wave behavior along uniform guiding structures – Transverse Electromagnetic Waves, Transverse Magnetic Waves, Transverse Electric Waves – TM and TE Waves between parallel plates. Field Equations in rectangular waveguides, TM and TE waves in rectangular waveguides, Bessel Functions, TM and TE waves in Circular waveguides.

PART A (2 marks)

Q.No.	Questions	BT Level	Competence
1.	What are guided Waves? Give examples for guiding structures.	BTL 1	Remembering
2.	Interpret the characteristics of E wave and H wave.	BTL 2	Understanding
3.	Write about Principal wave.	BTL 1	Remembering
4.	Express the dominant mode in the wave propagating in the waveguide.	BTL 2	Understanding
5.	Deduce the expression for cut off frequency when the wave is propagated in between two parallel plates.	BTL 4	Analyzing
6.	Examine the Characteristics of TEM waves.	BTL 3	Applying
7.	Justify, why TM_{01} and TM_{10} modes in a rectangular waveguide do not exists.	BTL 5	Evaluating
8.	Define cutoff frequency of a waveguide.	BTL 1	Remembering
9.	Illustrate the features of TE and TM mode.	BTL 2	Understanding
10.	Mention about the dominant mode of a rectangular waveguide.	BTL 1	Remembering
11.	Discuss about the dominant mode and degenerate modes in rectangular	BTL 5	Evaluating

	waveguide.		
12.	Explore the relation between group velocity, phase velocity and free space velocity.	BTL 3	Applying
13.	Why rectangular waveguides preferred over circular waveguides?	BTL 1	Remembering
14.	Exhibit the nature of the evanescent mode.	BTL 3	Applying
15.	Analyze why TM_{01} and TM_{10} modes in a rectangular waveguide do not exist	BTL 4	Analyzing
16.	How would you categorize the modes as degenerate modes in a rectangular waveguide?	BTL 4	Analyzing
17.	Consider an air filled rectangular waveguide with a cross – section of 5 cm × 3 cm. For this waveguide, deduce the cut off frequency (in MHz) of TE_{21} mode.	BTL 6	Creating
18.	List the dominant mode in circular waveguide.	BTL 1	Remembering
19.	Write Bessel's functions of first kind of order zero?	BTL 2	Understanding
20.	Formulate the size of the circular waveguide required to propagate TE_{11} mode if λc=8cm and p'_{11}=1.841.	BTL 6	Creating

PART - B (13 marks)

1.		Obtain the expression for the field components of an electromagnetic wave propagating between a pair of perfectly conducting planes?	(13)	BTL 1	Remembering
2.		Derive the expression for the field strength for TE waves between parallel plates propagating in Z direction?	(13)	BTL 6	Creating
3.	(i)	Explain about transverse electromagnetic waves between a pair of perfectly conducting planes?	(7)	BTL 1	Remembering
	(ii)	Determine the expression of wave impedance of TE, TM and TEM wave between a pair of Perfectly conducting planes.	(6)		
4.		Illustrate the transmission of TM waves between two parallel perfectly conducting planes with necessary equations and diagram.	(13)	BTL 2	Understanding
5.		A pair of perfectly conducting plates is separated by 10cm in air and carries a signal frequency of 6GHz in TE1 mode. Find Cut-off frequency, Angle of incidence on planes, Phase velocity, group velocity, Phase constant, Cut-off wavelength, characteristic wave impedance, and wavelength along guiding walls. Is it possible to propagate TE_3 mode.	(13)	BTL 3	Applying
6.	(i)	Interpret the propagation of TE waves in a rectangular waveguide with necessary expressions for the field components.	(6)	BTL 2	Understanding
	(ii)	Summarize the characteristics of TE and TM waves and also derive the cutoff frequency and phase velocity from	(7)		

		propagation constant.			
7.		Analyze the field configuration, cut off frequency and velocity of propagation for TM waves in rectangular wave guides.	(13)	BTL 4	Analyzing
8.		A rectangular air filled copper waveguide with dimension 0.9inch x 0.4inch cross section and 12inch length is propagated at 9.2GHz with a dominant mode. Find the cutoff frequency, Guide wavelength, Phase velocity, characteristic impedance and the loss.	(13)	BTL 5	Evaluating
9.		An air filled rectangular waveguide of 5cm x 2 cm cross section is operating in the TE_{10} mode at a frequency of 4GHz. Determine (a)the group velocity (b)the guide wavelength (c) the attenuation to be expected at a frequency which is 0.95 times the cut off frequency (assuming the guide walls is made of perfect conductors).	(3) (4) (6)	BTL 2	Understanding
10.		Using Bessel differential equation Obtain the TM field components in circular waveguides.	(13)		
11.		A TE_{11} wave is propagating through a circular waveguide. The diameter of the guide is 10cm and the guide is air-filled. Given p'_{11}=1.842 (a)Find the cut off frequency (b)Find the wavelength λg in the guide for a frequency of 3GHz. (c)Determine the wave impedance in the guide.	(4) (4) (5)	BTL 4	Analyzing
12.		Analyze the expressions for the transmissions of TE waves in a circular waveguide conducting planes for the field components.	(13)	BTL 4	Analyzing
13.		An air filled circular waveguide has a radius of 2 cm. Examine the cut off frequency and the phase constant for the dominant mode (p11' = 1.841 and p11 = 2.405.)	(13)	BTL 3	Applying
14.		Obtain the field distribution of transverse and longitudinal components of the electric and magnetic fields in circular waveguide with necessary equations.	(13)	BTL 1	Remembering
PART C (15 marks)					
1.	(i)	A hollow rectangular waveguide is to be used to transmit signals at a carrier frequency of 6GHz. Choose its dimensions so that the cut off frequency of the dominant TE mode is lower than the carier by 25 % and that of the next mode is atleast 25 % higher than the carrier.	(8)	BTL 6	Creating
	(ii)	Evaluate the ratio of the area of a circular waveguide to that of a rectangular one if both are to have the same cut off frequency for dominant mode.	(7)		

2.	(i)	The interior of a 20/3 cm × 20/4 cm rectangular waveguide is completely filled with a dielectric of $\varepsilon_r = 4$. Waves of free space wave – length shorter than which can be propagated in the TE_{11} mode.	(8)	BTL 5	Evaluating
	(ii)	A rectangular waveguide having TE_{10} mode as dominant mode is having a cut off frequency of 18 GHz for the TE_{30} mode. Evaluate the inner broad – wall dimension of the rectangular waveguide.	(7)		
3.		Determine the cut off frequencies of the first two propagating modes of a circular waveguide with a=0.5cm and ε_r = 2.25 the guide is 50cm in length operating at f=13GHz. Determine the cut off wavelength and propagation constant.	(15)	BTL 5	Evaluating
4.		A TE wave propagating in a dielectric filled waveguide of unknown permittivity has dimensions a=5 cm and b = 3cm.If the x – components of the electric field is given by Ex=36cos(40πx)sin(100πy)sin(2.4π×1010t−52.9πz) (V/m). Devise (a) ε_r of the material in the guide, (b) the cutoff frequency, (c) the expression for H_y.	(5) (4) (6)	BTL 6	Creating

UNIT V - RF SYSTEM DESIGN CONCEPTS

Active RF components: Semiconductor basics in RF, bipolar junction transistors, RF field effect transistors, High electron mobility transistors Basic concepts of RF design, Mixers, Low noise amplifiers, voltage control oscillators, Power amplifiers, transducer power gain and stability considerations.

PART A (2 marks)

Q.No.	Questions	BT Level	Competence
1.	List some of the active RF components.	BTL 1	Remembering
2.	Enumerate the band gap energy for Si and Ge used for semiconductor diodes.	BTL 4	Analyzing
3.	Draw the cross section of multifinger Bipolar Junction Transistor	BTL 1	Remembering
4.	What is called as HBTs ?	BTL 1	Remembering
5.	Define reverse active mode in bipolar junction transistor.	BTL 1	Remembering
6.	Classify RF field effect transistors based on physical construction.	BTL 2	Understanding
7.	Compare the enhancement type FET with Depletion type FET.	BTL 3	Applying
8.	Outline the characteristics of modulation doped field effect transistor	BTL 1	Remembering
9.	Illustrate the generic RF amplifier design.	BTL 2	Understanding
10.	Mention the various types of mixers.	BTL 1	Remembering
11.	Summarize the basic steps in the design process of RF amplifier circuits.	BTL 5	Evaluating
12.	Distinguish between oscillator and Mixer	BTL 4	Analyzing
13.	Examine the importance of voltage controlled oscillator in RF system.	BTL 2	Understanding
14.	Interpret the basic parameters of RF amplifier.	BTL 2	Understanding

15.	Generalize the concept of unconditional stability of an amplifier.			BTL 6	Creating
16.	Analyze the techniques of efficiency boosting in RF power amplifier			BTL 4	Analyzing
17.	Evaluate the significance of negative resistance in oscillation of a circuit			BTL 5	Valuating
18.	Devise the operation of single ended and differential ended LNA.			BTL 5	Creating
19.	Deduce the transducer power gain of a RF power amplifier.			BTL 3	Applying
20.	Demonstrate typical output stability circle and input stability circle			BTL 3	Applying
		PART –B (13 Marks)			
1.		Outline the process to compute the junction capacitance and the space charge region length of a *pn* junction semiconductor device.	(13)	BTL 1	Remembering
2.	(i)	For a Si pn junction the doping concentration are given as $N_A = 10^{18}$ cm^{-3} and $N_D = 10^{15}$ cm^{-3} with an intrinsic concentration of $n_i = 1.5 \times 10^{10} cm^{-3}$. Find the barrier voltages for T = 300^o K.	(7)	BTL 5	Evaluating
	(ii)	Considering the electron concentration and hole concentration in a semiconductor as ***n*** and ***p*** respectively infer that $\boldsymbol{np} = \mathbf{n}_i^2$ where ni is the intrinsic concentration.	(6)		
3.		Elaborate the construction and the functionality of the bipolar junction transistor.	(13)	BTL 3	Applying
4.		Discuss about the different operating modes of a bipolar junction transistor with appropriate diagram.	(13)	BTL 3	Applying
5.		Derive the drain saturation voltage and maximum saturation current for a field effect transistor.	(13)	BTL 6	Creating
6.	(i)	Compare the field effect transistor with the bipolar junction transistor	(6)	BTL 4	Analyzing
	(ii)	Explain the distinct features of high electron mobility transistors.	(7)		
7.	(i)	Analyze the steps involved to design a low noise amplifier	(6)	BTL 4	Analyzing
	(ii)	Distinguish power match and noise match in a Low Noise Amplifier.	(7)		
8.	(i)	Interpret the various types of mixers with its principle of operation	(6)	BTL 2	Understanding
	(ii)	Examine the following parameters of Conversion gain, Linearity and isolation of a mixer.	(7)		
9.		Illustrate the design principles of RF amplifier and impedance matching.	(13)	BTL 2	Understanding
10.	(i)	Write about the method used to design an integer N frequency synthesizer.	(7)	BTL 1	Remembering
	(ii)	Determine the transfer function of a voltage controlled oscillator.	(6)		
11.		Obtain the expression for unilateral power gain with necessary signal flow diagram.	(13)	BTL 1	Remembering
12.		Describe the various power gain for a two port RF network	(13)	BTL 1	Remembering

		considering the stability of the amplifier involved.			
13.		Discuss about input and output stability circles in the complex Γ_L and Γ_S planes, also derive the condition for unconditional stability.	(13)	BTL 2	Understanding
14.		A MESFET operated at 5.7GHz ha the following S parameters: $S_{11}=0.5\angle-60°$, $S_{12}=0.02\angle0°$, $S_{21}=6.5\angle115°$ and $S_{22}=0.6\angle-35°$. Determine if the circuit is unconditionally stable and Find the maximum power gain under optimal choice of reflection coefficients, assuming unilateral design ($S_{12}=0$).	(13)	BTL 4	Analyzing
PART – C (15 Marks)					
1.		An abrupt pn junction made of Si has the acceptor and donor concentration of $N_A= 10^{18}$ cm^{-3} and $N_D = 5 \times 10^{15} cm^{-3}$, respectively .Assuming that the device operates at the room temperature , Formulate (a) the barrier voltage (b) the space charge width in the p- and n- type semiconductors (c) the peak electric field across the junction (d) the junction capacitance for a cross sectional area of $10^{-4}cm^2$ and a relative dielectric constant of $\varepsilon_r = 11.7$	(15)	BTL 6	
2.		An RF amplifier has the following S parameters: $S_{11}=0.3\angle-70°$, $S_{21}=3.5\angle85°$, $S_{12}=0.2\angle-10°$, $S_{22}=0.4\angle-45°$. Further Vs=5V$\angle$0°, Zs=40Ω and Z_L=73Ω. Assuming Zo=50Ω. Find GT, GTU, GA and G. Also find Power delivered to the load PL, available power from source PA and incident power to amplifier Pinc.	(15)	BTL 5	Evaluating
3.		Consider a Si bipolar junction transistor whose emitter, base , collector are uniformly doped with the following concentrations $N^E_D = 10^{21}$ cm^{-3} , $N^B_A = 2\times10^{19}cm^{-3}$, $N^C_D = 10^{19}$ cm-3. Assume that the base emitter voltage is 0.75 and the collector –emitter potential is set to 2 V. The cross sectional area of both junctions is 10-4 cm2 and the emitter, base and collector thickness are d_E = 0.8μm , d_B = 1.2μm and d_C = 2μm respectively. Assuming that the device is operated at room temperature : (a) Find the space charge region extents for both junctions. (b) Compute the base, emitter and collector currents (c) Calculate the forward and reverse current gains	(5) (5) (5)	BTL 5	Evaluating
4.	(i)	Generelize the homodyne and heterodyne architecture of RF system	(8)	BTL 6	Creating
	(ii)	Devise the various stabilization methods for a RF amplifier circuit.	(7)		

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EC8561 – Transmission Lines and RF Systems

Unit – I
TRANSMISSION LINE THEORY

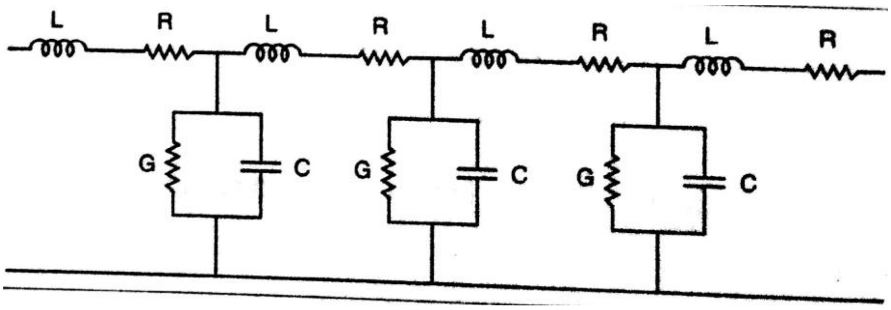
Find the reflection coefficient of a 50-ohm transmission line when it is terminated by a load impedance of 60 + j40 ohm.

$$K = \frac{Z_R - Z_O}{Z_R + Z_o}$$

$$K = \frac{50 - 60 - j40}{50 + 60 - j40}$$

K = 0.35 124.01

Equivalent circuit of a unit length of a transmission line:



Infinite line:

When S=∞, in the infinite line the travelling waves continue in one direction indefinitely and there is no source of energy or discontinuity to send back a reflected wave along the line.

Delay distortion:

For an applied voice-voltage wave the received waveform may not be identical with the input waveform at the sending end, since some frequency components will be delayed more than those of other frequencies. This phenomenon is known as delay or phase distortion.

A transmission line has $Z_o = 745 \ \ 12^0$ Ω and is terminated is Z_R = 100 Ω. Calculate the reflection loss in dB.

$$\text{:Reflection Factor } k = \left|\frac{2\sqrt{Z_R Z_0}}{Z_R + Z_0}\right| = \left|\frac{2\sqrt{745 \times 100}}{745 + 100}\right| = 0.645.$$

$$\text{Reflection Loss} = 20\log\frac{1}{|k|} = 20 \log \frac{1}{0.645} = 3.7751 \text{ dB}$$

Unit – II

HIGH FREQUENCY TRANSMISSION LINES

Input impedance of open and short - circuited dissipation less line:

Short circuited impedance

$$Z_{sc} = Z_0 \tanh \gamma l$$

The open circuited impedance

$$Z_{oc} = Z_0 \coth \gamma l$$

State the assumptions for the analysis of the performance of the radio frequency line:

1) Due to the skin effect, the currents are assumed to flow on the surface of the conductor. The internal inductance is zero. 2) The resistance R increases with f while inductance L increases with f. Hence ωL >> R. 3) The leakage conductance G is zero.

Standing wave ratio:

The ratio of the maximum to minimum magnitudes of voltage or current on a line having standing waves called standing wave ratio. S = $\frac{|E\max|}{|E\min|} = \frac{|\mathrm{Im}\,ax|}{|\mathrm{Im}\,in|}$

Input impedance of a dissipation less line:

The input impedance of a dissipation less line is given by, Zs= $\frac{Es}{Is} = Ro\frac{1+k<\phi-2\beta s}{1-k<\phi-2\beta s}$

Range of values of standing wave ratio:

The range of values of standing wave ratio is theoretically 1 to infinity.

Relation between SWR and reflection coefficient:

S = $\frac{|1+k|}{|1-k|}$, Also $|K| = \frac{s-1}{s+1}$

Unit – III

IMPEDANCE MATCHING IN HIGH FREQUENCY LINES

Use of eighth wave line:

An eighth wave line is used to transform any resistance to an impedance with a magnitude equal to Ro of the line or to obtain a magnitude match between a resistance of any value and a source of Ro internal resistance.

Input impedance of eighth wave line:

The input impedance of eighth wave line terminated in a pure resistance R_r. Is given by $Zs = (Z_R+jR_o) / (R_{o}+ j Z_R)$. From the equation it is seen that $| Zs | = Ro$.

Impedance inverter:

A quarter wave line may be considered as an impedance inverter because it can transform low impedance into high impedance and vice versa.

Copper insulator:

An application of the short -circuited quarter wave line is an insulator to support an open wire line or the center conductor of a coaxial line. This application makes some of the fact that the input impedance of a quarter wave line is very high, such lines are sometimes referred to as copper insulators.

Double stub matching is preferred over single stub matching:

Double stub matching is preferred over single stub due to following disadvantages of single stub.

1. Single stub matching is useful for a fixed frequency. So as frequency changes the location of single stub will have to be changed.
2. The single stub matching system is based on the measurement of voltage minimum; hence for coaxial line it is very difficult to get such voltage minimum, without using slotted line section.

1. Design a quarter wave transformer to match a load of 200Ω to a source resistance of 500 Ω. The operating frequency is 200 MHz.

$$Ro = \sqrt{Z_s Z_R} = \sqrt{500x200} = 316.22\ \Omega.$$

$$\lambda = C / f = 1.5 \text{ m} \qquad \lambda / 4 = 0.375 \text{ m}.$$

Unit – IV

WAVE GUIDES

TEM wave or principal wave:

TEM wave is a special type of TM wave in which an electric field E along the direction of propagation is also zero. The Tem waves are waves in which both electric and magnetic fields are transverse entirely but have no components of Ez and Hz. It is also referred to as the principal wave.

Characteristics of TEM waves:

a) It is a special type of TM wave.
b) It doesn't have either E or H component.
c) Its velocity is independent of frequency.
d) Its cot-off frequency is zero.

Dominant mode for the rectangular waveguide:

The lowest mode for TE wave is TE_{10} (m=1, n=0) whereas the lowest mode for TM wave is TM_{11} (m=1, n=1). The TE_{10} wave has the lowest cutoff frequency compared to the TM $_{11}$ mode. Hence the TE_{10} (m=1, n=0) is the dominant mode of a rectangular waveguide. Because the TE_{10} mode has the lowest attenuation of all modes in a rectangular waveguide and its electric field is definitely polarized in one direction everywhere.

A rectangular has the following dimensions l = 2.54 cm, b = 1.27 cm. Waveguide thickness = 0.127 cm. Calculate the cut off frequency for TE_{11} mode:

A rectangular waveguide has the following dimensions:
a = 2.54 cm, b = 1.27 cm, Waveguide thickness = 0.127 cm.

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)2+\left(\frac{n\pi}{b}\right)2} = 16.15 \text{ GHz.}$$

a = 2.54×10^{-2} x 0.127 = 0/02286 m b = 1.27×10^{-2} x 0.127 = 0.01016 m.

Quality factor of a resonator:

The quality factor Q is a measure of frequency selectivity of the resonator. It is defined as Q = 2 x Maximum energy stored / Energy dissipated per cycle = W / P.
Where W is the maximum stored energy, P is the average power loss.

Unit – V

RF SYSTEM DESIGN CONCEPTS

Comparison of conditional and unconditional stabilities of an amplifier:

Conditional stabilities	unconditional stabilities
Conditional stabilities refers to a network that is stable when its input and output see the intended characteristic impedance Zo	Unconditional stabilities refers to a network that can see any possible impedance on the smith chart from the center to the perimeter at any phase angle. Gamma<1 means that the real part of the impedance is positive
If there is a mismatch, there is a region of either source or load impedances that will definitely cause it to oscillate. The term potentially unstable refers to the same condition	Note that any network can oscillate if it sees a real impedance that is negative, so if your system goes outside the normal smith chart all bets on stability are off

1. Power gain of amplifier:

$$G_{TU} = \frac{\left(1-|\Gamma_L|^2\right)|S_{21}|^2\left(1-|\Gamma_S|^2\right)}{|1-\Gamma_L S_{22}|^2\,|1-S_{11}\Gamma_S|^2}$$

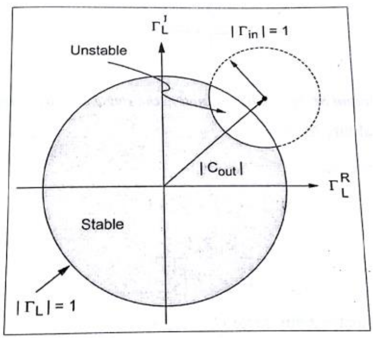
Requirement of impedance matching:

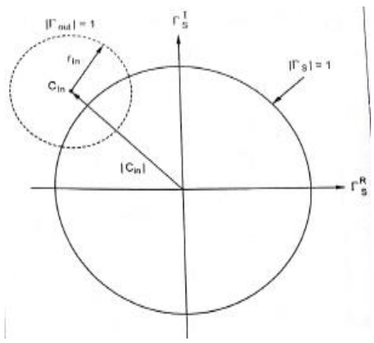
1. Minimum power loss in the feed line.
2. Maximum power transfer
3. Improving the S/N ratio of the system for sensitive receiver components

   Required Other considerations:

   1. Complexity 2. Band width requirement 3. Adjustability
   4. Implementation

Output stability circle and input stability circle:





Transducer power gain:

Transducer power gain is nothing but the gain of the amplifier when placed between the source and load.

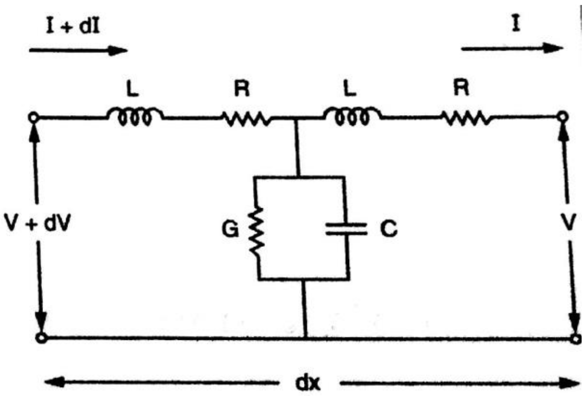
$$G_T = \frac{\text{Power delivered to the load}}{\text{Available power from the source}}$$

$$= \frac{P_L}{P_A}$$

Relation between nodal quality factor (Q_n) with loaded quality factor (Q_L):

$$Q_L = Q_n / 2$$

General Solution:



$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{ZY}$$

$$V = A e^{\gamma x} + B e^{-\gamma x}$$

$$I = C e^{\gamma x} + D e^{-\gamma x}$$

$$I = A\sqrt{\frac{Y}{Z}}\, e^{\sqrt{ZY}x} - B\sqrt{\frac{Y}{Z}}\, e^{-\sqrt{ZY}x} \qquad V = C\sqrt{\frac{Z}{Y}}\, e^{\sqrt{ZY}x} - D\sqrt{\frac{Z}{Y}}\, e^{-\sqrt{ZY}x}$$

$$I_R = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}} \qquad A = \frac{V_R}{2}\left[1 + \frac{Z_o}{Z_R}\right] \quad B = \frac{V_R}{2}\left[1 - \frac{Z_o}{Z_R}\right]$$

$$V_R = C\sqrt{\frac{Z}{Y}} - D\sqrt{\frac{Z}{Y}} \qquad C = \frac{I_R}{2}\left[1 + \frac{Z_R}{Z_o}\right] \quad D = \frac{I_R}{2}\left[1 - \frac{Z_R}{Z_o}\right]$$

$$V = \frac{V_R}{2}\left(1 + \frac{Z_o}{Z_R}\right) e^{\sqrt{ZY}x} + \frac{V_R}{2}\left(1 - \frac{Z_o}{Z_R}\right) e^{-\sqrt{ZY}x}$$

$$I = \frac{I_R}{2}\left(1 + \frac{Z_R}{Z_o}\right) e^{\sqrt{ZY}x} + \frac{I_R}{2}\left(1 - \frac{Z_R}{Z_o}\right) e^{-\sqrt{ZY}x}$$

$$V = V_R\left(\frac{e^{\sqrt{ZY}x} + e^{-\sqrt{ZY}x}}{2}\right) + I_R Z_o\left(\frac{e^{\sqrt{ZY}x} - e^{-\sqrt{ZY}x}}{2}\right) \quad [\because V_R = I_R Z_R]$$

$$I = I_R\left(\frac{e^{\sqrt{ZY}x} + e^{-\sqrt{ZY}x}}{2}\right) + \frac{V_R}{Z_o}\left(\frac{e^{\sqrt{ZY}x} - e^{-\sqrt{ZY}x}}{2}\right) \quad \left[\because I_R = \frac{V_R}{Z_R}\right]$$

$$V = V_R \cosh\sqrt{ZY}\,x + I_R Z_o \sinh\sqrt{ZY}\,x$$

$$I = I_R \cosh\sqrt{ZY}\,x + \frac{V_R}{Z_o}\sinh\sqrt{ZY}\,x$$

$$V_S = V_R\left[\cosh\sqrt{ZY}\,l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\,l\right]$$

$$I_S = I_R\left[\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right]$$

The input impedance of the transmission line is,

$$Z_S = \frac{V_S}{I_S}$$

$$Z_S = \frac{Z_0\,(Z_R\cosh\sqrt{ZY}\,l + Z_0\sinh\sqrt{ZY}\,l)}{(Z_0\cosh\sqrt{ZY}\,l + Z_R\sinh\sqrt{ZY}\,l)}$$

$$Z_S = Z_0\left[\frac{Z_R + Z_0\tanh\gamma l}{Z_0 + Z_R\tanh\gamma l}\right]$$

$$Z_S = Z_0\left[\frac{e^{\gamma l} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)e^{-\gamma l}}{e^{\gamma l} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)e^{-\gamma l}}\right]$$

$$Z_S = Z_0\left[\frac{e^{\gamma l} + K\,e^{-\gamma l}}{e^{\gamma l} - K\,e^{-\gamma l}}\right]$$

Wavelength and velocity of propagation:

The propagation constant (γ) and characteristic impedance (Z_0) are called secondary constants of a transmission line.

Propagation constant is usually a complex quantity $\gamma = \alpha + j\beta$

$$\gamma = \sqrt{ZY}$$

The characteristic impedance of the transmission line is also a complex quantity.

$$Z_0 = \sqrt{\frac{Z}{Y}}$$

$$\alpha + i\beta = \sqrt{RG - \omega^2 LC + j\omega(LG + RC)}$$

$$\alpha^2 = \beta^2 + RG - \omega^2 LC$$

$$2\,\alpha\beta = \omega\,(LG + RC)$$

$$\therefore\ \beta = \sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2\,(LG + RC)^2}}{2}}$$

$$\therefore\ \alpha = \sqrt{\frac{RG - \omega^2 LC + \sqrt{(RG - \omega^2 LC)^2 + \omega^2\,(LG + RC)^2}}{2}}$$

Velocity:

The velocity of propagation is given by,

$$v = \lambda f$$

$$= 2\pi f \frac{\lambda}{2\pi}$$

$$v = \frac{1}{\sqrt{LC}}$$

Wavelength:

The distance travelled by the wave along the line while the phase angle is changing through 2π radians is called wavelength.

$$\lambda = \frac{2\pi}{\beta} \quad \text{or} \quad \lambda = \frac{v}{f}$$

Waveform distortion:

The received waveform will not be identical with the input waveform at the sending end. This variation is known as distortion.

1. Frequency distortion
2. Delay or phase distortion

Frequency Distortion: A complex (voice) voltage transmitted on a transmission line will not be attenuated equally and the received waveform will not be identical with the input waveform at the transmitting end. This variation is known as frequency distortion.

Delay or Phase Distortion: For an applied voice-voltage wave the received waveform may not be identical with the input waveform at the sending end, since some frequency components will be delayed more than those of other frequencies. This phenomenon is known as delay or phase distortion.

The Distortion Less Line:

If a line is to have neither frequency nor delay distortion, then attenuation factor α and the velocity of propagation v cannot be functions of frequency.

$$\text{If} \qquad v = \frac{\omega}{\beta}$$

β must be a direct function of frequency

$$\beta = \sqrt{\frac{\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2}}{2}}$$

For β to be a direct function of frequency, the term $(RG - \omega^2 LC)^2 + \omega^2 (LG + CR)^2$ must be equal to $(RG + \omega^2 LC)^2$

$$\frac{R}{L} = \frac{G}{C}$$

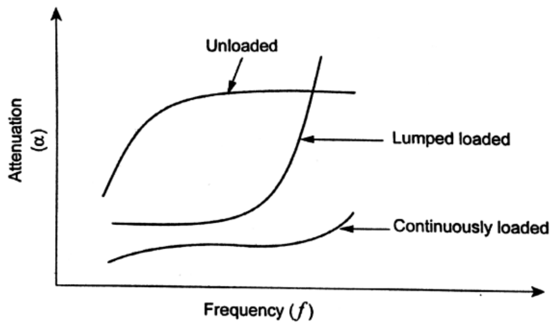
This is the condition for distortionless line.

Loading:

- To achieve distortion less condition → increase L/C ratio
- Increasing inductance by inserting inductances in series with the line is termed as loading such lines are called as loaded lines
- Lumped inductors → loading coils

Types of loading

(a) Lumped loading
(b) Continuous loading
(c) Patch loading



Comparison of loaded and unloaded cable characteristics

Inductance loading of Telephone cables:

Consider an uniformly loaded cable with G = 0. Then,

$$Z = R + j\omega L$$

$$Y = j\omega C$$

$$Z = \sqrt{R^2 + (L\omega)^2} \ \underline{\Big|\ \tan^{-1}\left(\frac{L\omega}{R}\right)}$$

Propagation constant $\gamma = \sqrt{ZY}$

$$= \omega\sqrt{LC}\ \sqrt[4]{1+\left(\frac{R}{L\omega}\right)^2}\ \angle\left(\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{L\omega}\right)$$

$$\therefore\ \gamma = \omega\sqrt{LC}\ \angle\left(\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{L\omega}\right)$$

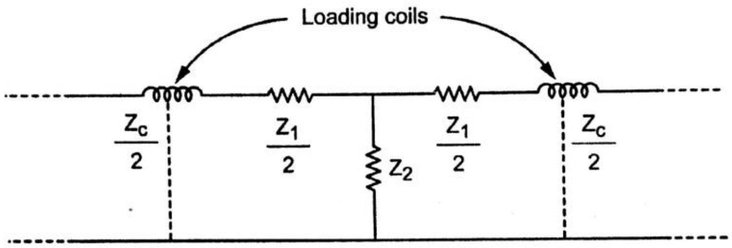
$$= \frac{R}{2}\sqrt{\frac{C}{L}} + j\omega\sqrt{LC}$$

$$\therefore \text{ Attenuation constant } \alpha = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\text{Phase-shift } \beta = \omega\sqrt{LC}$$

$$\text{Velocity of propagation } v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

Campbell's Equation



Equivalent T section for part of a line between two lumped loading coils

$$\frac{Z_1'}{2} = \frac{Z_c}{2} + \frac{Z_1}{2}$$

$$\therefore\ \frac{Z_1'}{2} = \frac{Z_c}{2} + Z_o \tanh\frac{\gamma l}{2}$$

$$\cosh\gamma' l = \frac{Z_c}{2Z_o}\sinh\gamma l + \cosh\gamma l$$

Derive the expressions for open circuited and short circuited lines:

$$V_S = V_R\left[\cosh\sqrt{ZY}\,l + \frac{Z_o}{Z_R}\sinh\sqrt{ZY}\,l\right]$$

$$I_S = I_R\left[\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_o}\sinh\sqrt{ZY}\,l\right]$$

The input impedance of a transmission line is given by

$$Z_S = \frac{V_S}{I_S}$$

$$Z_S = Z_o \left(\frac{Z_R \cosh \gamma l + Z_o \sinh \gamma l}{Z_o \cosh \gamma l + Z_R \sinh \gamma l} \right)$$

Short circuited impedance

$$Z_{sc} = Z_o \tanh \gamma l$$

The open circuited impedance

$$Z_{oc} = Z_o \coth \gamma l$$

Reflection on a line not terminated in its characteristic impedance (Z_0):

When the load impedance is not equal to the characteristic impedance of a transmission line, reflection takes place, i.e., $Z_R \neq Z_0$, reflection occurs.

If a transmission line is not terminated in Z_0, then part of the wave is reflected back. The reflection is maximum when the line is open circuit or short circuit.

From the general solution of a transmission line, the equations for voltage and current are expressed as:

$$E = \frac{E_R(Z_R + Z_0)}{2Z_R}\left[e^{\gamma s} + \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)e^{-\gamma s}\right]$$

$$I = \frac{I_R(Z_R + Z_0)}{2Z_R}\left[e^{\gamma s} - \left(\frac{Z_R - Z_0}{Z_R + Z_0}\right)e^{-\gamma s}\right]$$

Incident voltage component is given by

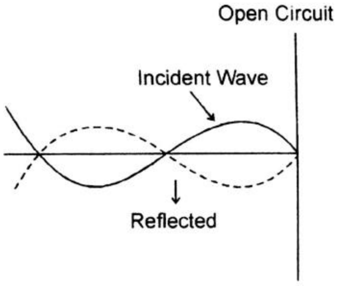
$$E_1 = \frac{E_R(Z_R + Z_0)}{2Z_R}e^{\gamma s} = \frac{E_R\left(1 + \frac{Z_0}{Z_R}\right)}{2}e^{\gamma s}$$

Reflected voltage component is given by,

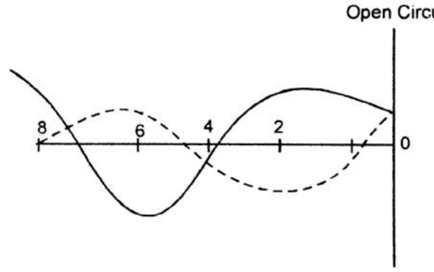
$$E_2 = \frac{E_R(Z_R - Z_0)}{2Z_R}e^{-\gamma s} = \frac{E_R\left(1 - \frac{Z_0}{Z_R}\right)}{2}e^{-\gamma s}$$

If $Z_R = \infty$ which represents an open circuited line,

At $s = 0$, both E_1 and E_2 have an amplitude of $E_R/2$. Thus at the receiving end, initial value of the reflected wave is equal to incident voltage.



(i) For time instant t = 0



(ii) For time instant t = 1/8 f

Input impedance and transfer impedance:

Input impedance :

The equations for voltage and current at the sending end of a transmission line of length 'l' are given by

$$V_S = V_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}\,l\right)$$

$$I_S = I_R\left(\cosh\sqrt{ZY}\,l + \frac{Z_R}{Z_0}\sinh\sqrt{ZY}\,l\right)$$

The input impedance of the transmission line is,

$$Z_S = \frac{V_S}{I_S}$$

$$Z_S = \frac{Z_0\,(Z_R\cosh\sqrt{ZY}\,l + Z_0\sinh\sqrt{ZY}\,l)}{(Z_0\cosh\sqrt{ZY}\,l + Z_R\sinh\sqrt{ZY}\,l)}$$

Let $\sqrt{ZY} = \gamma$

The input impedance of the line is

$$Z_S = Z_0\left[\frac{Z_R\cosh\gamma l + Z_0\sinh\gamma l}{Z_0\cosh\gamma l + Z_R\sinh\gamma l}\right]$$

or

$$Z_S = Z_0\left[\frac{Z_R + Z_0\tanh\gamma l}{Z_0 + Z_R\tanh\gamma l}\right]$$

If $K = \frac{Z_R - Z_0}{Z_R + Z_0}$, then

$$Z_S = Z_0\left[\frac{e^{\gamma l} + K\,e^{-\gamma l}}{e^{\gamma l} - K\,e^{-\gamma l}}\right]$$

Transfer impedance :

$$Z_T = \frac{V_S}{I_R}$$

$$Z_T = \frac{V_S}{I_R} = \frac{Z_R + Z_0}{2}\,(e^{\gamma l} + K\,e^{-\gamma l})$$

$$Z_T = Z_R\cosh\gamma l + Z_0\sinh\gamma l$$

UNIT – II - HIGH FREQUENCY TRANSMISSION LINES

Parameters of open-wire and coaxial lines:

The inductance of an open wire line is given by,

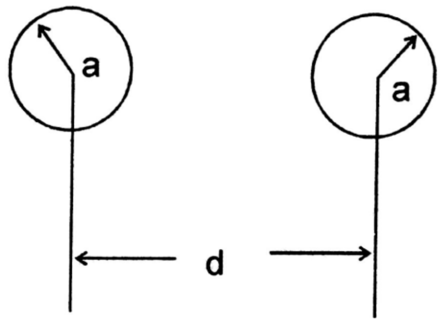
$$L = 10^{-7}\left(\frac{\mu}{\mu_r} + 4\ln\frac{d}{a}\right)$$

The first term on the right hand side of the above expression represents internal inductance of the line due to internal flux linkages in the conductors and is zero for a open wire line.

Hence the inductance of the open wire line is

$$L = 4 \times 10^{-7} \ln\frac{d}{a} \text{ henrys/m}$$

$$= 9.21 \times 10^{-7} \log_{10}\frac{d}{a} \text{ henrys/m}$$



$a \rightarrow$ radius of conductor

$d \rightarrow$ distance between conductors.

The value of capacitance of a line is not affected by skin effect or frequency and hence the capacitance of a open wire line with air dielectric is given by,

$$C = \frac{\pi\varepsilon_v\varepsilon_r}{\ln\frac{d}{a}} \text{ farads/m}$$

where ε_v = Permittivity of free space = 8.85×10^{-12} f/m,

$$\varepsilon_r = 1 \text{ for air}$$

$$C = \frac{27.7}{\ln\frac{d}{a}} \ \mu\mu\text{f/m.}$$

$$C = \frac{12.07}{\log_{10}\frac{d}{a}} \mu\mu f/m$$

The resistance of a round conductor of radius `a' meters to direct current is inversely proportional to the area as,

$$R_{dc} = \frac{k}{\pi a^2}$$

While that of a round conductor with alternating current flowing in a skin of thickness δ is,

$$R_{ac} = \frac{k}{2\pi a\delta}$$

Therefore the ratio of resistance to alternating current to resistance to direct current is given by,

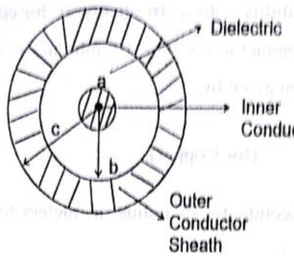
$$\frac{R_{ac}}{R_{dc}} = \frac{a\sqrt{\pi f\mu\sigma}}{2} = \frac{a}{2\delta}$$

For copper

$$\frac{R_{ac}}{R_{dc}} = 7.53a\sqrt{f}$$

PARAMETERS OF THE COAXIAL LINE AT HIGH FREQUENCIES

Because of the skin effect, the current flows on the outer surface of the inner conductor and the inner surface of the outer conductor.



For a coaxial line the inductance is given by,

$$L = 10^{-7}\left[2\ln\frac{b}{a} + \frac{2C^4 \ln\frac{c}{b}}{(C^2-b^2)^2} - \frac{C^2}{C^2-b^2}\right] H/m$$

second term and third term represents flux linkages inside the inner and outer conductors.

The skin effect eliminates flux linkages and hence the inductance of coaxial line is given by,

$$L = 2 \times 10^{-7} \ln\frac{b}{a} \text{ henrys/m}$$

$$L = 4.6 \times 10^{-7} \log_{10}\frac{b}{a} \text{ henrys/m}$$

The capacitance of the coaxial line is not affected by the frequency.

$$C = \frac{2\pi\varepsilon}{\ln \frac{b}{a}} \text{ farads/m}$$

$$C = \frac{24.14\varepsilon_r}{\log_{10} \frac{b}{a}} \ \mu\mu\text{f/m.}$$

Due to skin effect resistance increases and the resistance of coaxial copper line is

Due to skin effect resistance increases and the resistance of coaxial copper line is

$$R_{ac} = 4.16 \times 10^{-8} \sqrt{f}\left[\frac{1}{b}+\frac{1}{a}\right]\Omega/m$$

The ac resistance of the coaxial cable is derived as follows,

$$R_{ac} = \frac{1}{2\pi a\,\delta\sigma} + \frac{1}{2\pi b\,\delta\sigma} = \frac{1}{2\pi\,\delta\sigma}\left[\frac{1}{a}+\frac{1}{b}\right]$$

The ac resistance per unit length of a copper conductor is given by,

$$R_{ac} = \frac{1}{2\pi\left(\frac{0.0664}{\sqrt{f}}\right)\left(5.75\times10^{7}\right)}\left[\frac{1}{a}+\frac{1}{b}\right]$$

$$R_{ac} = 4.16 \times 10^{-8} \sqrt{f}\left[\frac{1}{a}+\frac{1}{b}\right]\Omega/m.$$

The dc resistance of a coaxial line is given by,

$$R_{dc} = \frac{1}{\pi\sigma}\left[\frac{1}{a^2}+\frac{1}{(c^2-b^2)}\right] \ \Omega/\text{m}$$

1. Line constants for zero dissipation line:

In general the line constants for a transmission line are:

$$Z = R + j\omega L$$

$$Y = G + j\omega C$$

Characteristic impedance $Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

Propagation constant $\gamma = \sqrt{ZY} = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$\gamma = \alpha + j\beta.$$

For a transmission of energy at high frequencies, $\omega L >> R$. We assume negligible losses or zero dissipation and G is also assumed to be zero. $(G = 0)$

Using the inductance and capacitance a **open wire line at high frequency**, the value of characteristic impedance of the open wire line can be found as,

$$L = 4 \times 10^{-7} \ln \frac{d}{a} \text{ h/m} \qquad C = \frac{27.7}{\ln d/a} \ \mu\mu\text{f/m}$$

$$R_0 = \sqrt{\frac{L}{C}} = 120 \ln \frac{d}{a} \text{ ohms.}$$

(or) $\quad R_0 = 276 \log_{10} d$ ohms.

The characteristic impedance of the coaxial line can be computed as,

$$L = 4.60 \times 10^{-7} \log_{10} b/a \text{ h/m} \qquad L = 2 \times 10^{-7} \ln b/a \text{ h/m}$$

$$C = \frac{24.14\varepsilon_r}{\log_{10} b/a} \ \mu\mu\text{F/m} \qquad C = \frac{55.5\varepsilon_r}{\ln b/a} \ \mu\mu\text{f/m}$$

$$R_0 = \sqrt{\frac{L}{C}} = \frac{138}{\sqrt{\varepsilon_r}} \log_{10} b/a \text{ ohms} \qquad R_0 = \sqrt{\frac{L}{C}} = \frac{60}{\sqrt{\varepsilon_r}} \ln \frac{b}{a} \text{ ohms}$$

→ **The propagation contant** g is given by,

$$\gamma = \sqrt{ZY} = \sqrt{(-j\omega L)(j\omega C)} = \sqrt{j^2\omega^2 LC} = j\omega\sqrt{LC}$$

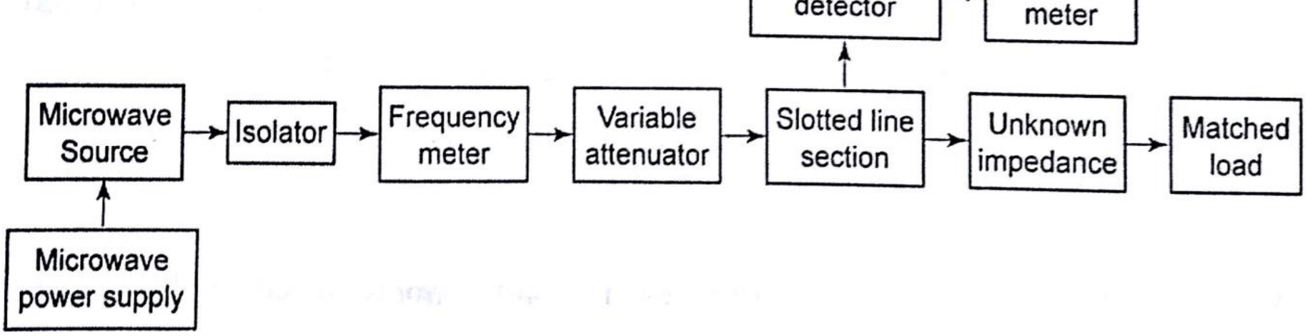
$$= \sqrt{-j\omega^2 LC}$$

The velocity of propagation can be calculated as

$$V = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}} \text{ m/sec.}$$

Measurement of VSWR and Wavelength:

- ***VSWR*** and the ***magnitude of voltage reflection*** coefficient are very important parameters which determine the ***degree of impedance matching.***
- VSWR and Γ are also used for measurement of ***load impedance*** by the slotted line method.



- When a load $Z_L \neq Z_o$ is connected to the transmission line, the standing waves are produced.
- By inserting a slotted line system in the line, ***standing waves*** can be traced by moving the carriage with a tunable probe detector along the line.
- VSWR can be measured by detecting V_{max} and V_{min} in the ***VSWR meter.***

$$\text{Standing wave ratio (S)} = \frac{V_{max}}{V_{min}} = \frac{1+\Gamma}{1-\Gamma} \qquad \dots (1)$$

$$\Gamma = \text{Reflection coefficient} = \frac{P_{reflected}}{P_{incident}} \qquad \dots (2)$$

- Here, $P_{reflected}$ is a reflected power and $P_{incident}$ is a incident power of unknown impedance. S varies from ***1 to ∞***.As Γ varies from ***0 to ∞.***

LOW VSWR (S < 20)

- Values of VSWR ***not exceeding 20*** are very easily measured ***directly on the VSWR meter*** using the experimental set-up shown in Fig.18.9 as follows,
  (1) The variable attenuator is adjusted to ***10dB.*** The microwave source is set to the required frequency. The ***1kHz modulation is adjusted*** for maximum reading on the VSWR meter in a 30dB scale.
  (2) The probe on the slotted waveguide is moved to get ***maximum*** reading on the meter (corresponding to V_{max}).
  (3) The attenuation is now adjusted to get full-scale reading. This full-scale reading is noted down. Next the probe on the slotted line is adjusted to get ***minimum*** reading on the meter (corresponding to V_{min}).
  (4) The ratio of $\frac{V_{max}}{V_{min}}$ gives the ***VSWR.***
- The experiment is repeated for other frequencies as required to obtain a set of values of *S* Vs *f*.

The Possible Sources of Error in this Measurements are:

(1) V_{max} and V_{min} ***may not be*** measured in the ***square – law region*** of the ***crystal detector.***

(2) The probe thickness and depth of the penetration may produce ***reflections*** in the line and also ***distortion*** in the field to be measured.

(3) When ***VSWR < 1.05,*** the associated VSWR of connector produces ***significant error*** in VSWR measurement. Very good ***low VSWR (<1.01)*** connectors should be used for very low VSWR measurements.

- For high power, ***double minimum method is used.*** The electromagnetic field at any point of transmission line may be considered as the sum of two traveling waves: the ***'Incident wave'*** which propagates from generator and ***'reflected wave'*** which propagates towards the generator.
- The reflected wave is set up by reflection of incident wave from a discontinuity on the line or from the load impedance.
- The magnitude and phase of reflected wave depends upon amplitude and phase of the reflecting impedance.
- The superposition of two traveling waves, gives rise to standing wave along the line.
- The maximum field strength is found where two waves are in phase and it is minimum where two waves adds in an opposite phase.
- The distance between two successive minimums (or maximums) is half the guide wavelength on the line.

Reflection Coefficient:

The ratio of electrical field strength of reflected and incident wave is called the reflection coefficient.

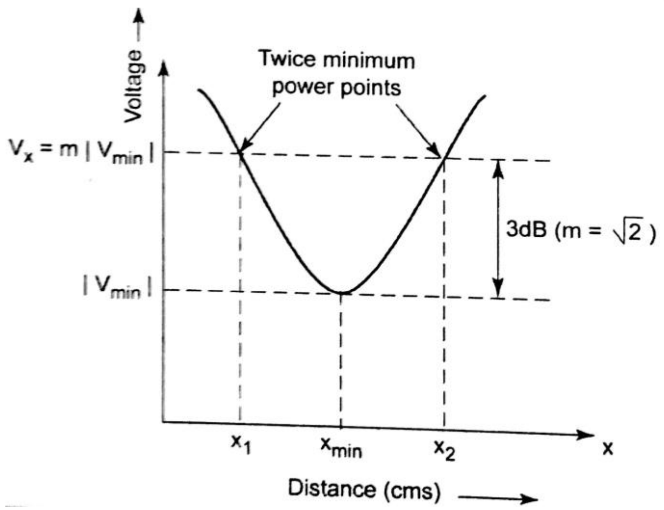
- Reflection coefficient, ρ is

$$\Gamma = \frac{E_r}{E_I} = \frac{Z - Z_0}{Z + Z_0}$$

where, Z is the impedance at a point, and

Z_0 is characteristic impedance.

- The above equation gives following equation $|\Gamma| = \dfrac{S-1}{S+1}$



Double minima method

VSWR:

- VSWR denoted by S is,

$$S = \frac{E_{max}}{E_{min}} = \frac{|E_I| + |E_r|}{|E_I| - |E_r|}$$

where,

E_I – Incident voltage, and

E_r – Reflected voltage.

- In this method, the probe is inserted to a depth where the minimum can be real without difficulty.
- The probe is then moved to a point where the power is ***twice the minimum.*** Let this position be denoted by x_1.
- The probe is then moved to ***twice the power*** point on the other side of the minimum (say x_2).

$$P_{min} \propto V_{min}^2$$

$$2 P_{min} \propto V_x^2$$

$$\frac{1}{2} = \frac{V_{min}^2}{V_x^2}$$

$$V_x^2 = 2(V_{min})^2$$

$$V_x = \sqrt{2}\, V_{min}$$

Guide Wavelength:

- By moving the probe between two successive minima, a distance equal to $\frac{\lambda_g}{2}$ is found to determine the guide wavelength λ_g.

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_C}\right)^2}}$$

Quarter wave line and Half wave line:

The input impedance of a dissipationless transmission line is

$$Z_S = R_0\left[\frac{Z_R + j\,R_0 \tan\beta x}{R_0 + j\,Z_R \tan\beta x}\right]$$

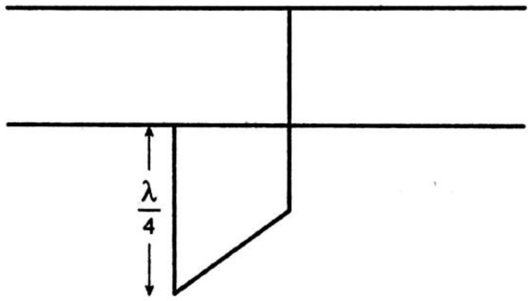
$$Z_S = R_0\left[\frac{\dfrac{Z_R}{\tan\beta x} + j\,R_0}{\dfrac{R_0}{\tan\beta x} + j\,Z_R}\right]$$

For a quarter wave line $x = \lambda/4$,

$$\beta x = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z_S = R_0 \left[\frac{\dfrac{Z_R}{\tan \pi/2} + j\, R_0}{\dfrac{R_0}{\tan \pi/2} + j\, Z_R} \right] = R_0 \left[\frac{j\, R_0}{j\, Z_R} \right]$$

$$Z_S = \frac{R_0^2}{Z_R}$$



Half-Wave Line

The input impedance of a dissipationless transmission line is

$$Z_S = R_0 \left[\frac{Z_R + j\, R_0 \tan \beta x}{R_0 + j\, Z_R \tan \beta x} \right]$$

For a half-wave line $x = \lambda/2$

$$\beta x = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

$$Z_S = R_0 \left[\frac{Z_R + j\, R_0 \tan \pi}{R_0 + j\, Z_R \tan \pi} \right]$$

$$= R_0 \frac{Z_R}{R_0}$$

$$Z_S = Z_R$$

Single stub matching:

Location and length of the stub using reflection coefficient:

The input impedance of the line is given by

$$Z_i = Z_0 \frac{1 + K e^{-2\gamma l}}{1 - K e^{-2\gamma l}}$$

For lossless line $\alpha = 0, \gamma = j\beta$ and $K = |K| e^{j\phi}$

where ϕ is the angle of reflection coefficient.

$$Z_i = Z_0 \frac{1 + |K| e^{j\phi} e^{-j 2\beta l}}{1 - |K| e^{j(\phi - 2\beta l)}}$$

$$= Z_0 \frac{1 + |K| e^{j(\phi - 2\beta l)}}{1 - |K| e^{j\phi} e^{-j 2\beta l}}$$

The input admittance is given by

$$Y_i = G_0 \frac{1 - |K| e^{j(\phi - 2\beta l)}}{1 + |K| e^{j(\phi - 2\beta l)}}$$

where the characteristic conductance is

$$G_0 = \frac{1}{Z_0} = \frac{1}{R_0} \qquad [\because Z_0 \text{ is resistive}]$$

$$Y_i = G_0 \frac{1 - |K| [\cos(\phi - 2\beta l) + j \sin(\phi - 2\beta l)]}{1 + |K| [\cos(\phi - 2\beta l) + j \sin(\phi - 2\beta l)]}$$

$$= G_0 \frac{1 - |K| [\cos(\phi - 2\beta l) - j |K| \sin(\phi - 2\beta l)]}{1 + |K| [\cos(\phi - 2\beta l) + j |K| \sin(\phi - 2\beta l)]}$$

Multiplying the numerator and denominator by

$1 + |K| [\cos(\phi - 2\beta l) - j |K| \sin(\phi - 2\beta l)$

$$Y_i = G_0 \frac{1 - |K|^2 - 2j |K| \sin(\phi - 2\beta l)}{1 + |K|^2 + 2 |K| \cos(\phi - 2\beta l)}$$

Since $Y_i = G_i + j S_i$, then

$$\frac{Y_i}{G_0} = \frac{G_i}{G_0} + \frac{j S_i}{G_0} = \frac{1 - |K|^2 - 2j |K| \sin(\phi - 2\beta l)}{1 + |K|^2 + 2 |K| \cos(\phi - 2\beta l)}$$

Equating the real parts

$$\frac{G_i}{G_0} = \frac{1 - |K|^2}{1 + |K|^2 + 2 |K| \cos(\phi - 2\beta l)}$$

Equating the imaginary parts

$$\frac{S_i}{G_0} = \frac{-2|K|\sin(\phi - 2\beta l)}{1+|K|^2+2|K|\cos(\phi - 2\beta l)}$$

At the location of stub $Z_i = Z_0$ for matching.

Since there is no reflection, $l = l_s$

$$\therefore G_i = G_0$$

$$\frac{G_i}{G_0} = 1$$

$$\frac{1-|K|^2}{1+|K|^2+2|K|\cos(\phi - 2\beta l_s)} = 1$$

$$1-|K|^2 = 1+|K|^2+2|K|\cos(\phi - 2\beta l_s)$$

$$2|K|\cos(\phi - 2\beta l_s) = -2|K|^2$$

$$\cos(\phi - 2\beta l_s) = -|K|$$

$$\phi - 2\beta l_s = \cos^{-1}(-|K|)$$

$$\text{But} \quad \cos^{-1}(-|K|) = -\pi + \cos^{-1}|K|$$

$$\therefore \quad \phi - 2\beta l_s = -\pi + \cos^{-1}|K|$$

$$2\beta l_s = \phi + \pi - \cos^{-1}|K|$$

$$l_s = \frac{\phi + \pi - \cos^{-1}|K|}{2\beta}$$

$$\text{or} \quad l_s = \frac{\lambda}{4\pi}[\phi + \pi - \cos^{-1}|K|] \qquad \left[\because \beta = \frac{2\pi}{\lambda}\right]$$

The normalized susceptance (imaginary part) equation is

$$\frac{S_i}{G_0} = \frac{-2|K|\sin(\phi - 2\beta l)}{1+|K|^2+2|K|\cos(\phi - 2\beta l)}$$

$$\text{But} \quad (\phi - 2\beta l_s) = -\pi + \cos^{-1}|K| \quad \text{and}$$

$$\cos(\phi - 2\beta l_s) = -|K|$$

$$\therefore \quad \frac{S_i}{G_0} = \frac{-2|K|\sin(-\pi + \cos^{-1}|K|)}{1+|K|^2+2|K|(-|K|)}$$

$$= \frac{2|K|\sin(\cos^{-1}|K|)}{1+|K|^2-2|K|^2}$$

Let $\cos^{-1}|K| = \theta$, then $|K| = \cos\theta$ and

$$\sin(\cos^{-1}|K|) = \sin\theta$$

$$= \sqrt{1-\cos^2\theta} = \sqrt{1-|K|^2}$$

$$\therefore \frac{S_i}{G_0} = \frac{2|K|\sqrt{1-|K|^2}}{1-|K|^2}$$

$$S_i = G_0 \frac{2|K|}{\sqrt{1-|K|^2}}$$

The susceptance of the stub is $G_0 \cot\beta l_t$

$$G_0 \cot\beta l_t = G_0 \frac{2|K|}{\sqrt{1-|K|^2}}$$

$$\frac{1}{\tan \beta l_t} = \frac{2|K|}{\sqrt{1-|K|^2}}$$

$$\tan \beta l_t = \frac{\sqrt{1-|K|^2}}{2|K|}$$

$$\beta l_t = \tan^{-1}\frac{\sqrt{1-|K|^2}}{2|K|}$$

$$l_t = \frac{1}{\beta}\tan^{-1}\frac{\sqrt{1-|K|^2}}{2|K|}$$

$$l_t = \frac{\lambda}{2\pi}\tan^{-1}\frac{\sqrt{1-|K|^2}}{2|K|}$$

The location of the stub 'l_s' and length of the stub 'l_t' can be determined, if the reflection coefficient and frequency are known.

Determine the stub length and the distance of the stub from the load. Given that a complex load ZL = 50 – j100 ohms is to be matched to a 75 ohms transmission line using a short circuited stub.

(a) Characteristic impedance (Z_0) of the line = 75Ω

Load impedance $Z_L = 50 - j100\ \Omega$

Normalized load impedance = $Z'_L = \frac{Z_L}{Z_0} = \frac{50 - j100\Omega}{75\Omega}$

$$Z'_L = 0.667 - j1.33$$

Normalized load impedance Z_L' is plotted at the intersection of constant R circle with R = 0.67 and with X = 1.33. This is point A. The impedance circle is drawn.

(b) The normalized load admittance point B is determined by drawing a line from point A through the center to the opposite side of the S circle. (i.e., Point B),

$Y = 0.3 + j0.6$ ℧.

(c) Travel along the constant S circle in the clockwise direction from load to generator to reach a point C on $g_r = 1$ circle (or) $\frac{Y}{G_0} = 1$ circle. Draw a line from O to C and extend the line to C' on the outer rim.

(d) The distance between $B'C'$ gives the distance of the stub from the load.

i.e., $s = 0.18\ \lambda - 0.09\ \lambda = 0.09\ \lambda$.

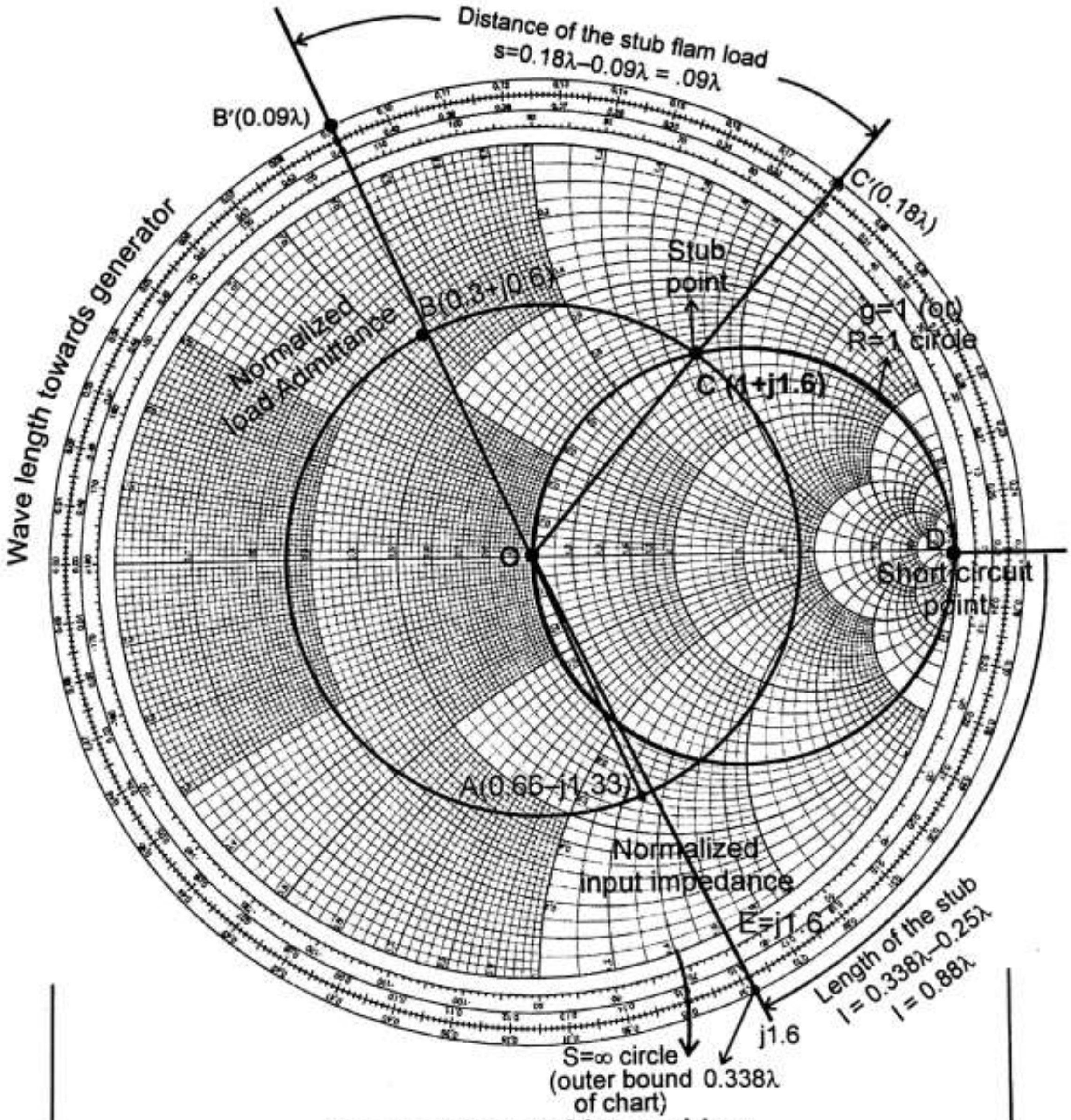
(e) At the point C, the normalized admittance value is $1 + j1.6$. This is the point at which the stub is connected. Thus the stub should provide a susceptance of – j1.6,

(f) The determine the length of the shorted stub that has opposite reactive component to the input admittance, the outside of the smith chart ($g = 0$ circle) is moved around until a point with susceptance of $-j1.6$ is reached which is point E. The point E represents a susceptance of –j1.6.

(g) The distance between D and E is the length of the sub length of the stub l,

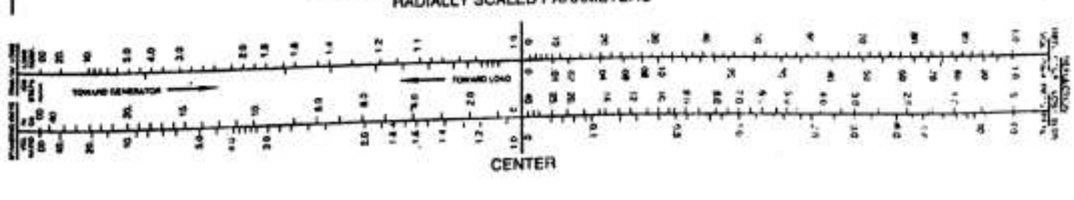
$$= 0.338\ \lambda - 0.25\ \lambda$$

$$l = .088\ \lambda.$$



Single stub matching problem

RADIALLY SCALED PARAMETERS



A 50 Ω loss less feeder line is to be matched for an antenna with Z_L = (75-j20) Ω at 100MHz using single shorted stub. Calculate the stub length and distance between the antenna and the stub using smith chart.

Given:

$$Z_0 = 50\ \Omega$$

$$Z_L = (75 - j20)\ \Omega$$

$$f = 100\ \text{MHz}$$

$$= \frac{c}{f} = \frac{3 \times 10^8}{100 \times 10^6}$$

$$= 3\ \text{m}$$

Normalized load impedance $\overline{z}_L = \frac{Z_L}{Z_0} = \frac{75 - j20}{50} = 1.5 - j0.4$

Normalized load impedance $\overline{z}_L$ is plotted at P on the Smith chart and the impedance circle with 'O' centre and OP as radius is drawn.

From the smith chart OS read as SWR = 1.7

The normalized load admittance is diametrically opposite to the normalized load impedance at Q *i.e.,* $\overline{y}_L = 0.62 + j0.17$.

$\overline{y}_L$ is moved in clockwise direction to a point A on the impedance circle where it intersects R = 1 circle *i.e.,* at $1 + j0.525$.

The distance between Q and A is the distance from the load to the location of the stub.

$$d = 0.1455\ \lambda - 0.0415\ \lambda$$

$$= 0.104\ \lambda$$

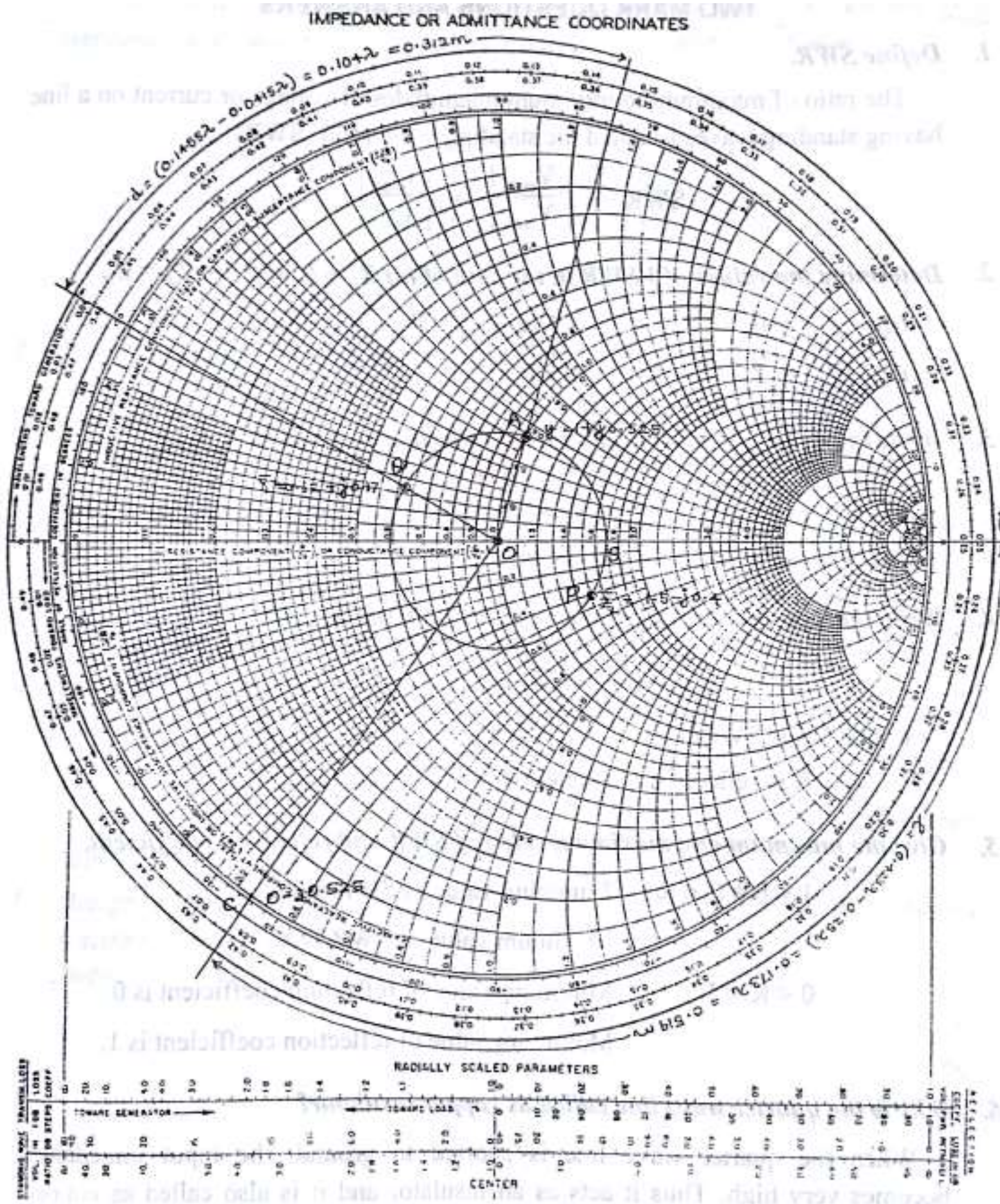
$$= 0.104 \times 3 = 0.312\ \text{m}$$

The stub must have zero resistance and susceptance that has an exactly opposite value at C *i.e.,* $y_{\text{stub}} = 0 - j0.525$.

The length of the stub is measured from the right side of the chart (X = 0) at B to the point C.

$$l = (0.423 - 0.25)\ \lambda = 0.173\ \lambda$$

$$= 0.173 \times 3 = 0.519\ \text{m}$$



A 75 Ω lossless line transmission line is to be match with a 100-j80Ω load using single stub. Calculate the stub length and its distance from the load corresponding to the frequency of 30 MHz using Smith Chart:

Given:

$$Z_0 = 75\ \Omega$$

$$Z_L = 100 - j80\ \Omega$$

$$f = 30\ \text{MHz}$$

$$\lambda = \frac{c}{f}$$

$$= \frac{3 \times 10^8}{30 \times 10^6}$$

$$= 10\ \text{m}$$

Normalized load impedance $z_L = \dfrac{Z_L}{Z_0}$

$$= \frac{100 - j80}{75} = 1.33 - j1.067$$

The normalized load impedance z_L is marked in Smith chart at P and the impedance circle with $(1 + j0)$ as centre 'O' and 'OP' as radius.

$$\text{From chart SWR} = OS = 2.5$$

The normalized load admittance is diametrically opposite to the normalized load impedance at Q is $\bar{y}_L = 0.46 + j0.36$. $\bar{y}_L$ is moved in clockwise direction to a point A on the impedance circle where it intersects R = 1 circle *i.e.,* at $1 + j0.956$.

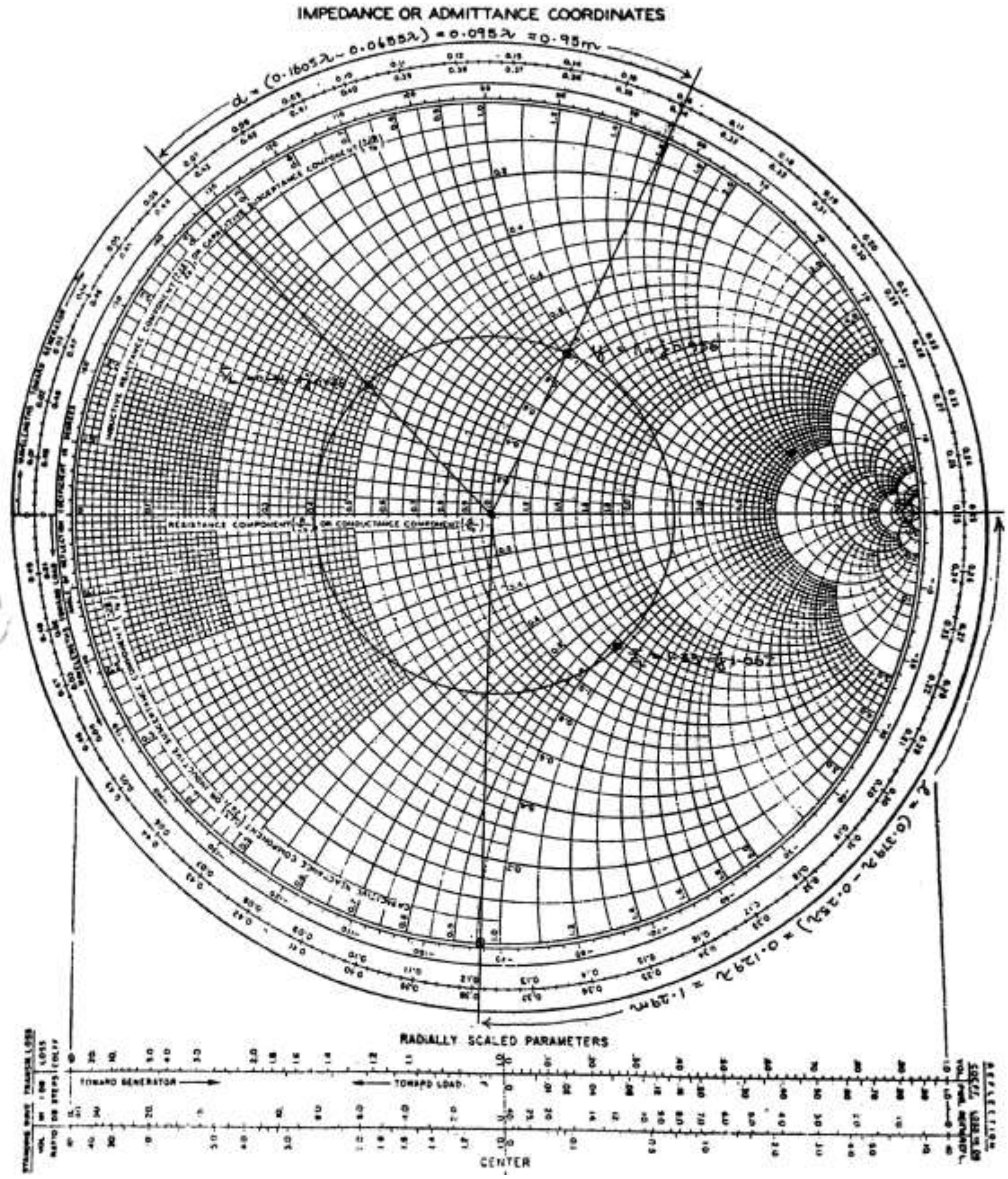
The distance between Q and A is the distance from the load to the location of the stub

$$d = 0.1605\,\lambda - 0.0655\,\lambda = 0.095\,\lambda$$
$$= 0.095 \times 10 = 0.95 \text{ m}$$

The stub must have zero resistance and susceptance that has an exactly opposite value at C *i.e.,* $y_{stub} = 0 - j0.095$.

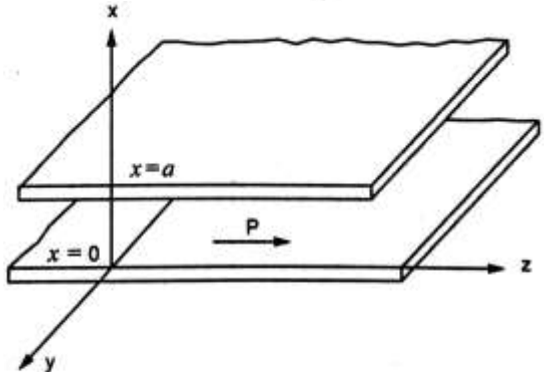
The length of the stub is measured from the right side of the chart (X = 0) at B to the point C.

$$l = (0.379 - 0.25)\,\lambda = 0.129\,\lambda$$
$$= 0.129 \times 10 = 1.29 \text{ m}$$



Electric field and magnetic field expression between the parallel plates:

Consider an electromagnetic wave propagating between a pair of parallel perfectly conducting planes of infinite extent in the y and z directions as shown in Fig



Maxwell's equations will be solved to determine the electromagnetic field configurations in the rectangular region.

Maxwell's equations for a non-conducting rectangular region are given as

$$\nabla \times \mathrm{H} = j\,\omega\,\varepsilon\,\mathrm{E}$$
$$\nabla \times \mathrm{E} = -j\,\omega\,\mu\,\mathrm{H}$$

$$\nabla \times \mathrm{H} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \mathrm{H}_x & \mathrm{H}_y & \mathrm{H}_z \end{vmatrix}$$

$$= \bar{a}_x\left(\frac{\partial \mathrm{H}_z}{\partial y} - \frac{\partial \mathrm{H}_y}{\partial z}\right) + \bar{a}_y\left(\frac{\partial \mathrm{H}_x}{\partial z} - \frac{\partial \mathrm{H}_z}{\partial x}\right) + \bar{a}_z\left(\frac{\partial \mathrm{H}_y}{\partial x} - \frac{\partial \mathrm{H}_x}{\partial y}\right)$$

$$= j\,\omega\,\varepsilon\;[\,\bar{a}_x\,\mathrm{E}_x + \bar{a}_y\,\mathrm{E}_y + \bar{a}_z\,\mathrm{E}_z\,]$$

Equating x, y and z components on both sides.

$$\left.\begin{aligned} \frac{\partial \mathrm{H}_z}{\partial y} - \frac{\partial \mathrm{H}_y}{\partial z} &= j\omega\varepsilon\mathrm{E}_x \\ \frac{\partial \mathrm{H}_x}{\partial z} - \frac{\partial \mathrm{H}_z}{\partial x} &= j\omega\varepsilon\mathrm{E}_y \\ \frac{\partial \mathrm{H}_y}{\partial x} - \frac{\partial \mathrm{H}_x}{\partial y} &= j\omega\varepsilon\mathrm{E}_z \end{aligned}\right\}$$

$$\text{Similarly,} \quad \nabla \times E = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$= \bar{a}_x\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \bar{a}_y\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + \bar{a}_z\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

$$= -j\omega\mu\,[\bar{a}_x H_x + \bar{a}_y H_y + \bar{a}_z H_z]$$

Equating x, y and z components on both sides,

$$\left.\begin{aligned} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} &= -j\omega\mu H_x \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} &= -j\omega\mu H_y \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= -j\omega\mu H_z \end{aligned}\right\}$$

The wave equation is given by

$$\nabla^2 E = \gamma^2 E$$

$$\nabla^2 H = \gamma^2 H$$

where $\gamma^2 = (\sigma + j\omega\varepsilon)\,(j\omega\mu)$

For a non-conducting medium, it becomes

$$\nabla^2 E = -\omega^2\mu\varepsilon E$$

$$\nabla^2 H = -\omega^2\mu\varepsilon H$$

$$\left.\begin{aligned} \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} &= -\omega^2\mu\varepsilon E \\ \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} &= -\omega^2\mu\varepsilon H \end{aligned}\right\}$$

It is assumed that the propagation is in the z direction and the variation of field components in this z direction may be expressed in the form $e^{-\gamma z}$,

where γ is propagation constant.

$$\gamma = \alpha + j\beta$$

If $\alpha = 0$, wave propagates without attenuation.

If α is real *i.e.*, $\beta = 0$, there is no wave motion but only an exponential decrease in amplitude.

Let $H_y = H_y^0\, e^{-\gamma z}$

$$\frac{\partial H_y}{\partial z} = -\gamma\, H_y^0\, e^{-\gamma z} = -\gamma\, H_y$$

Similarly, $\frac{\partial H_x}{\partial z} = -\gamma\, H_x$

Let
$$E_y = E_y^0 \, e^{-\gamma z}$$
$$\frac{\partial E_y}{\partial z} = -\gamma \, E_y$$
Similarly
$$\frac{\partial E_x}{\partial z} = -\gamma \, E_x$$
There is no variation in the y direction *i.e.*, derivative of y is zero
$$\left.\begin{aligned} \gamma H_y &= j\omega\varepsilon E_x \\ -\gamma H_x - \frac{\partial H_z}{\partial x} &= j\omega\varepsilon E_y \\ \frac{\partial H_y}{\partial x} &= j\omega\varepsilon E_z \end{aligned}\right\}$$
$$\left.\begin{aligned} \gamma E_y &= -j\omega\mu H_x \\ -\gamma E_x - \frac{\partial E_z}{\partial x} &= -j\omega\mu H_y \\ \frac{\partial E_y}{\partial x} &= -j\omega\mu H_z \end{aligned}\right\}$$
$$\left.\begin{aligned} \frac{\partial^2 E}{\partial x^2} + \gamma^2 E &= -\omega^2\mu\varepsilon E \\ \frac{\partial^2 H}{\partial x^2} + \gamma^2 H &= -\omega^2\mu\varepsilon H \end{aligned}\right\}$$
$$\text{where} \quad \frac{\partial^2 E}{\partial z^2} = \gamma^2 E \ \text{ and } \ \frac{\partial^2 H}{\partial z^2} = \gamma^2 H$$
To solve H_x,
$$-\gamma \, H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$
$$\gamma E_y = -j\omega\mu H_x$$
From the above equations,
$$H_x = \frac{-\gamma \, E_y}{j\omega\mu}$$
$$E_y = -\frac{1}{j\omega\varepsilon}\left[\gamma H_x + \frac{\partial H_z}{\partial x}\right]$$
Substituting the value of E_y in the above equation,
$$H_x = \frac{-\gamma}{j\omega\mu}\left[-\frac{1}{j\omega\varepsilon}\left(\gamma H_x + \frac{\partial H_z}{\partial x}\right)\right]$$
$$H_x = \frac{-\gamma}{\omega^2\mu\varepsilon}\left[\gamma H_x + \frac{\partial H_z}{\partial x}\right]$$
$$H_x\left[1 + \frac{\gamma^2}{\omega^2\mu\varepsilon}\right] = \frac{-\gamma}{\omega^2\mu\varepsilon}\left[\frac{\partial H_z}{\partial x}\right]$$
$$H_x\,[\,\omega^2\mu\varepsilon + \gamma^2\,] = -\gamma \, \frac{\partial H_z}{\partial x}$$

$$\boxed{H_x = \frac{-\gamma}{\omega^2\mu\varepsilon + \gamma^2}\frac{\partial H_z}{\partial x}}$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x}$$

where $h^2 = \gamma^2 + \omega^2\mu\varepsilon$

To solve H_y,

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y$$

$$\gamma H_y = j\omega\varepsilon E_x$$

From the above equations,

$$H_y = \frac{j\omega\varepsilon}{\gamma} E_x$$

$$E_x = \frac{1}{\gamma}\left[j\omega\mu H_y - \frac{\partial E_z}{\partial x}\right]$$

Substituting the value of E_x in the above equation,

$$H_y = \frac{j\omega\varepsilon}{\gamma}\cdot\frac{1}{\gamma}\left[j\omega\mu H_y - \frac{\partial E_z}{\partial x}\right]$$

$$H_y = \frac{-\omega^2\mu\varepsilon}{\gamma^2} H_y - \frac{j\omega\varepsilon}{\gamma^2}\frac{\partial E_z}{\partial x}$$

$$H_y\left(1 + \frac{\omega^2\mu\varepsilon}{\gamma^2}\right) = -\frac{j\omega\varepsilon}{\gamma^2}\frac{\partial E_z}{\partial x}$$

$$H_y = \frac{-j\omega\varepsilon}{(\gamma^2 + \omega^2\mu\varepsilon)}\frac{\partial E_z}{\partial x}$$

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon$$

$$\boxed{H_y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}}$$

To solve E_x,

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y$$

$$H_y = \frac{j\omega\varepsilon}{\gamma} E_x$$

Substituting the value of H_y in the above equation,

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu\left[\frac{j\omega\varepsilon}{\gamma} E_x\right]$$

$$= \frac{-\omega^2\mu\varepsilon}{\gamma} E_x$$

$$\gamma E_x + \frac{\omega^2\mu\varepsilon}{\gamma} E_x = -\frac{\partial E_z}{\partial x}$$

$$E_x\left[\gamma + \frac{\omega^2\mu\varepsilon}{\gamma}\right] = -\frac{\partial E_z}{\partial x}$$

$$E_x [\gamma^2 + \omega^2 \mu\varepsilon] = -\gamma \frac{\partial E_z}{\partial x}$$

$$E_x = \frac{-\gamma}{h^2} \frac{\partial E_z}{\partial x}$$

To solve E_y,

$$\gamma H_x + \frac{\partial H_z}{\partial x} = -j\omega\varepsilon E_y$$

$$H_x = -\frac{\gamma E_y}{j\omega\mu}$$

Substituting the value of H_x in the above equation,

$$\frac{-\gamma^2 E_y}{j\omega\mu} + \frac{\partial H_z}{\partial x} = -j\omega\varepsilon E_y$$

$$E_y \left[\frac{-\gamma^2}{j\omega\mu} + j\omega\varepsilon \right] = -\frac{\partial H_z}{\partial x}$$

$$E_y [\gamma^2 + \omega^2 \mu\varepsilon] = j\omega\mu \frac{\partial H_z}{\partial x}$$

$$E_y = \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial x}$$

where $$h^2 = \gamma^2 + \omega^2 \mu\varepsilon$$

Electric field and magnetic field expressions for TE waves between parallel plates:

Transverse electric (TE) waves are waves in which the electric field strength E is entirely transverse. It has a magnetic field strength H_z in the direction of propagation and no component of electric field E_z in the same direction. ($E_z = 0$).

Substituting the value of $E_z = 0$ in the following equations.

$$E_x = -\frac{\gamma}{h^2} \frac{\partial E_z}{\partial x} \text{ and } H_y = \frac{-j\omega\varepsilon}{h^2} \frac{\partial E_z}{\partial x}$$

Then $E_x = 0$ and $H_y = 0$

The wave equation for the component E_y

$$\frac{\partial^2 E_y}{\partial x^2} + \gamma^2 E_y = -\omega^2 \mu\varepsilon E_y$$

$$\frac{\partial^2 E_y}{\partial x^2} = -\omega^2 \mu\varepsilon E_y - \gamma^2 E_y$$

But $$h^2 = \gamma^2 + \omega^2 \mu\varepsilon$$

$$\boxed{\frac{\partial^2 E_y}{\partial x^2} + h^2 E_y = 0}$$

This is a differential equation of simple harmonic motion. The solution of this equation is given by

$$E_y = C_1 \sin hx + C_2 \cos hx$$

where C_1 and C_2 are arbitrary constants.

If E_y is expressed in time and direction ($E_y = E_y^0 \, e^{-\gamma z}$), then the solution becomes,

$$E_y = (C_1 \sin hx + C_2 \cos hx)\, e^{-\gamma z}$$

The arbitrary constants C_1 and C_2 are determined from the boundary conditions.

The tangential component of E is zero at the surface of conductors for all values of z.

$$E_y = 0 \text{ at } x = 0$$

$$E_y = 0 \text{ at } x = a$$

Applying the first boundary condition ($x = 0$)

$$0 = 0 + C_2$$

$$C_2 = 0$$

Then $$E_y = C_1 \sin hx \; e^{-\gamma z}$$

Applying the second boundary condition ($x = a$)

$$\sin ha = 0$$

$$h = \frac{m\pi}{a}$$

where $$m = 1, 2, 3, \ldots\ldots\ldots$$

Therefore, $$E_y = C_1 \sin\left(\frac{m\pi}{a} x\right) e^{-\gamma z}$$

$$\therefore \quad \frac{\partial E_y}{\partial x} = \frac{m\pi}{a} C_1 \cos\left(\frac{m\pi}{a} x\right) e^{-\gamma z}$$

$$\gamma E_y = -j\omega\mu H_x$$

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z$$

From the first equation, $$H_x = \frac{-\gamma E_y}{j\omega\mu}$$

Substituting the value of E_y in the above equation

$$H_x = \frac{-\gamma}{j\omega\mu} C_1 \sin\left(\frac{m\pi}{a} x\right) e^{-\gamma x}$$

From the second equation, $$H_z = -\frac{1}{j\omega\mu} \frac{\partial E_y}{\partial x}$$

Substituting the value of E_y in the above equation

$$H_z = \frac{-m\pi}{j\omega\mu a} C_1 \cos\left(\frac{m\pi}{a} x\right) e^{-\gamma z}$$

$$H_z = \frac{jm\pi}{\omega\mu a} C_1 \cos\left(\frac{m\pi}{a} x\right) e^{-\gamma z}$$

The field strengths for TE waves between parallel planes are

$$\left.\begin{aligned}
E_y &= C_1 \sin\left(\frac{m\pi}{a}x\right)e^{-\gamma z} \\
H_x &= \frac{-\gamma}{j\omega\mu} C_1 \sin\left(\frac{m\pi}{a}x\right)e^{-\gamma z} \\
H_z &= \frac{-m\pi}{j\omega\mu a} C_1 \cos\left(\frac{m\pi}{a}x\right)e^{-\gamma z}
\end{aligned}\right\}$$

$$\gamma = j\beta$$

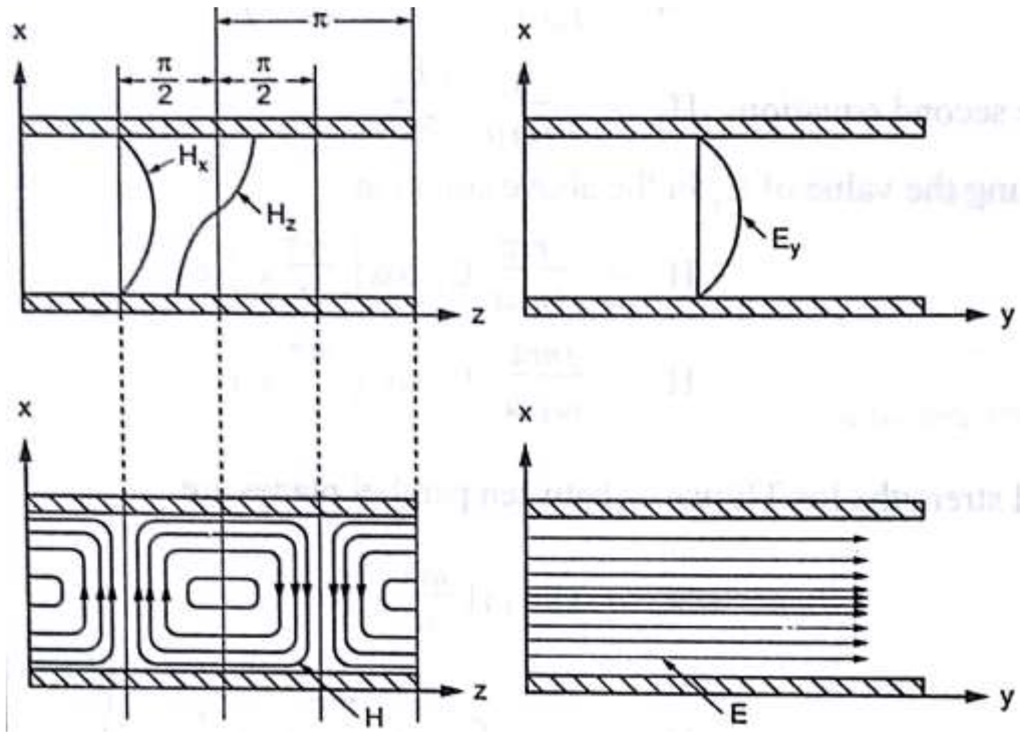
Then the field strengths for TE waves are

$$E_y = C_1 \sin\left(\frac{m\pi}{a}x\right)e^{-j\beta z}$$

$$H_x = \frac{-\beta}{j\omega\mu} C_1 \sin\left(\frac{m\pi}{a}x\right)e^{-j\beta z}$$

$$H_z = \frac{jm\pi}{\omega\mu a} C_1 \cos\left(\frac{m\pi}{a}x\right)e^{-j\beta z}$$

The field distributions for TE_{10} mode between parallel planes are shown in Fig.



Electric field and magnetic field expressions for TE waves between rectangular waveguides:

The wave equation in a rectangular waveguide is given by

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z = -\omega^2 \mu\varepsilon H_z$$

The solution of the equation is

$$H_z(x, y, z) = H_z^\circ (x, y)\, e^{-\gamma z}$$

Let $$H_z^\circ (x, y) = XY$$

where X is the function of x only.

Y is the function of y only.

Substituting the value of H_z in the wave equation,

$$Y\frac{d^2X}{dx^2} + X\frac{d^2Y}{dy^2} + \gamma^2 XY = -\omega^2 \mu\varepsilon XY$$

$$Y\frac{d^2X}{dx^2} + X\frac{d^2Y}{dy^2} + h^2 XY = 0$$

where $h^2 = \gamma^2 + \omega^2\mu\varepsilon$.

Dividing by XY,

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + h^2 = 0$$

$$\frac{1}{X}\frac{d^2X}{dx^2} + h^2 = \frac{-1}{Y}\frac{d^2Y}{dy^2}$$

The expression relates a function of x alone to a function of y alone and this can be equated to a constant.

$$\frac{1}{X}\frac{d^2X}{dx^2} + h^2 = A^2$$

$$\frac{1}{X}\frac{d^2X}{dx^2} + h^2 - A^2 = 0$$

Let $$B^2 = h^2 - A^2$$

$$\frac{1}{X}\frac{d^2X}{dx^2} + B^2 = 0$$

The solution of this equation is

$$X = C_1 \cos Bx + C_2 \sin Bx$$

Similarly, $$-\frac{1}{Y}\frac{d^2Y}{dy^2} = A^2$$

$$\frac{1}{Y}\frac{d^2Y}{dy^2} + A^2 = 0$$

The solution of this equation is $Y = C_3 \cos Ay + C_4 \sin Ay$

But $H_z^\circ = XY$

$$= (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay)$$

$$= C_1 C_3 \cos Ay \cos Bx + C_2 C_3 \cos Ay \sin Bx + C_1 C_4 \cos Bx \sin Ay + C_2 C_4 \sin Ay \sin Bx$$

It is known that

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

For TE waves $E_z = 0$.

$$E_x = -\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$= -\frac{j\omega\mu}{h^2}[-C_1 C_3 A \sin Ay \cos Bx - C_2 C_3 A \sin Ay \sin Bx + C_1 C_4 A \cos Bx \cos Ay + C_2 C_4 A \cos Ay \sin Bx]$$

Applying boundary conditions, $E_z = 0$ when $y = 0$, $y = b$.

If $y = 0$, the general solution is

$$E_x = -\frac{j\omega\mu}{h^2}[C_1 C_4 A \cos Bx + C_2 C_4 A \sin Bx] = 0$$

For $E_x = 0$, $C_4 = 0$. (C_4 is common)

Then the general solution is

$$E_x = \frac{-j\omega\mu}{h^2}[-C_1 C_3 A \sin Ay \sin Bx - C_2 C_3 A \sin Ay \sin Bx]$$

If $y = b$, $E_x = 0$.

For $E_x = 0$, it is possible either $B = 0$ or $A = \frac{n\pi}{b}$. If $B = 0$, the above solution is lentically zero. So it is better to select $A = \frac{n\pi}{b}$.

The general solution is

$$E_x^\circ = \frac{j\omega\mu}{h^2}[C_1 C_3 A \sin Ay \cos Bx + C_2 C_3 A \sin Ay \sin Bx]$$

Similarly for E_y,

$$E_y = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial y} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$= \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \qquad [\because E_z = 0]$$

$$= \frac{j\omega\mu}{h^2}[-C_1 C_3 B \cos Ay \sin Bx + C_2 C_3 B \cos Ay \cos Bx - C_1 C_4 B \sin Bx \sin Ay + C_2 C_4 B \sin Ay \cos Bx]$$

Applying boundary conditions

$$E_y = 0; \quad x = 0 \text{ and } x = a$$

If $x = 0$,

$$E_y^\circ = \frac{j\omega\mu}{h^2}[C_2 C_3 B \cos Ay + C_2 C_4 B \sin Ay]$$

For $E_y^\circ = 0$, $C_2 = 0$.

Then the general expression is

$$E_y^\circ = \frac{j\omega\mu}{h^2}[-C_1 C_3 B \cos Ay \sin Bx - C_1 C_4 B \sin Bx \sin Ay]$$

If $x = a$, then $E_y^\circ = 0$.

$$E_y^\circ = \frac{-j\omega\mu}{h^2}[C_1 C_3 B \sin B a \cos A y + C_1 C_4 \sin B a \sin A y]$$

For $E_y^\circ = 0$, $B = \frac{m\pi}{a}$.

$$E_y^\circ = -\frac{j\omega\mu}{h^2}[C_1 C_3 B \sin Bx \cos Ay + C_1 C_4 B \sin Bx \sin Ay]$$

$$E_x^\circ = \frac{j\omega\mu}{h^2}[C_1 C_3 A \sin Ay \cos Bx + C_2 C_3 A \sin Ay \sin Bx]$$

Substituting the value $C_2 = C_4 = 0$.

$$E_x^\circ = \frac{j\omega\mu}{h^2} C_1 C_3 A \cos Bx \sin Ay$$

$$= \frac{j\omega\mu}{h^2} C_1 C_3 A \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y$$

$$E_y^\circ = -\frac{j\omega\mu}{h^2} C_1 C_3 B \sin Bx \cos Ay$$

$$= -\frac{j\omega\mu}{h^2} C_1 C_3 B \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y$$

Let $C = C_1 C_3$

$$E_x^\circ = \frac{j\omega\mu}{h^2} C A \sin Ay \cos Bx$$

$$E_y^\circ = -\frac{j\omega\mu}{h^2} C B \sin Bx \cos Ay$$

where $A = \frac{n\pi}{b}$ and $B = \frac{m\pi}{a}$

Similarly for H_x°,

$$H_x^\circ = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} + \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y} = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} \qquad [\because E_z = 0]$$

For propagation, $\gamma = j\beta$, $[\because \alpha = 0]$

$$H_x^\circ = -\frac{j\beta}{h^2}\frac{\partial H_z}{\partial x}$$

But $$E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$\frac{\partial H_z}{\partial x} = \frac{h^2}{j\omega\mu} \cdot E_y$$

Substituting the value of $\frac{\partial H_z}{\partial x}$ in the above $\overset{\circ}{H_x}$ equation

$$\overset{\circ}{H_x} = \frac{-j\beta}{h^2} \cdot \frac{h^2}{j\omega\mu} \overset{\circ}{E_y}$$

$$= \frac{-\beta}{\omega\mu} \overset{\circ}{E_y}$$

Substituting the value of $\overset{\circ}{E_y}$ in the above $\overset{\circ}{H_x}$ equation

$$\overset{\circ}{H_x} = \frac{-\beta}{\omega\mu}\left[\frac{-j\omega\mu}{h^2} \text{ C B} \sin \text{B}x \cos \text{A}y\right]$$

$$\overset{\circ}{H_x} = \frac{j\beta}{h^2} \text{ CB} \sin \text{B}x \cos \text{A}y$$

$$\overset{\circ}{H_x} = \frac{j\beta}{h^2} \text{ CB} \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y$$

Similarly for $\overset{\circ}{H_y}$,

$$\overset{\circ}{H_y} = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

$$= \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y} \qquad [\because E_z = 0]$$

For propagation, $\gamma = j\beta$. $\qquad [\because \alpha = 0]$

$$\overset{\circ}{H_y} = \frac{-j\beta}{h^2}\frac{\partial H_z}{\partial y}$$

But $E_x = \frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$

$$\frac{\partial H_z}{\partial y} = \frac{-h^2}{j\omega\mu} E_x$$

Substituting this value of $\frac{\partial H_z}{\partial y}$ in the above $\overset{\circ}{H_y}$ equation

$$\overset{\circ}{H_y} = \frac{-j\beta}{h^2}\frac{(-h^2)}{j\omega\mu}\overset{\circ}{E_x} = \frac{\beta}{\omega\mu}\overset{\circ}{E_x}$$

Substituting the value of E_x in the above $\overset{\circ}{H_y}$ equation

$$\overset{\circ}{H_y} = \frac{\beta}{\omega\mu}\left[\frac{j\omega\mu}{h^2} \text{ C A} \sin \text{A}y \cos \text{B}x\right]$$

$$\overset{\circ}{H_y} = \frac{j\beta}{h^2} \text{ C A} \cos \text{B}\,x \sin \text{A}\,y$$

$$\overset{\circ}{H_y} = \frac{j\beta}{h^2} \text{ C A} \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y$$

$$
\begin{aligned}
H_z^{\circ} &= XY \\
&= C_1 C_3 \cos Ay \cos Bx + C_2 C_3 \cos Ay \sin Bx \\
&\qquad + C_1 C_4 \cos Bx \sin Ay + C_2 C_4 \sin Ay \sin Bx
\end{aligned}
$$

But $C_2 = C_4 = 0$

$$
\begin{aligned}
H_z^{\circ} &= C_1 C_3 \cos Ay \cos Bx \\
C &= C_1 C_3 \\
H_z^{\circ} &= C \cos Ay \cos Bx \\
H_z^{\circ} &= C \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y
\end{aligned}
$$

The field equations for TE waves are as follows :

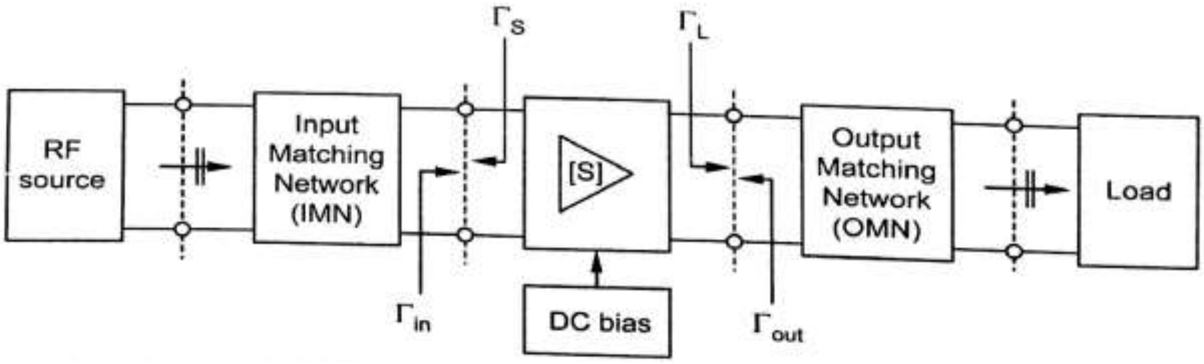
$$
\begin{aligned}
H_x^{\circ} &= \frac{j\beta}{h^2} CB \sin Bx \cos Ay \\
H_y^{\circ} &= \frac{j\beta}{h^2} CA \cos Bx \sin Ay \\
H_z^{\circ} &= C \cos Ay \cos Bx \\
E_x^{\circ} &= \frac{j\omega\mu}{h^2} CA \cos Bx \sin Ay \\
E_y^{\circ} &= \frac{-j\omega\mu}{h^2} CB \sin Bx \cos Ay
\end{aligned}
$$

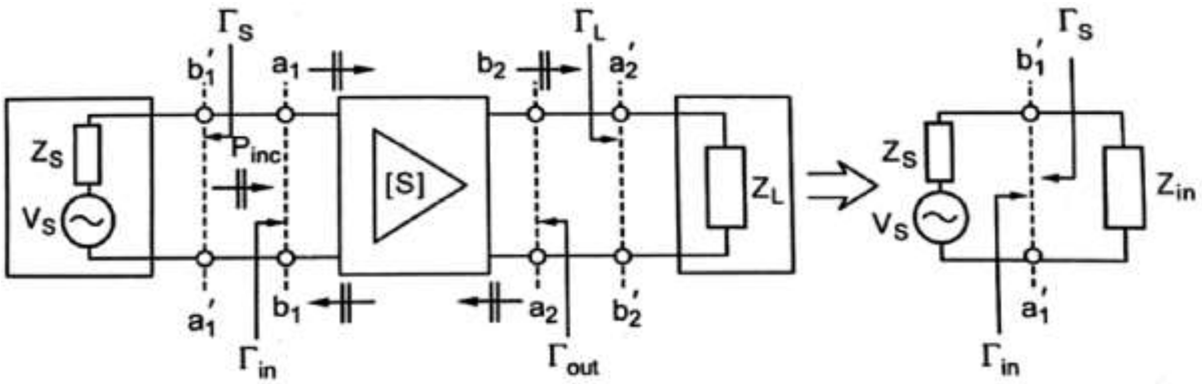
where $A = \frac{n\pi}{b}$ and $B = \frac{m\pi}{a}$

UNIT – V - RF SYSTEM DESIGN CONCEPTS

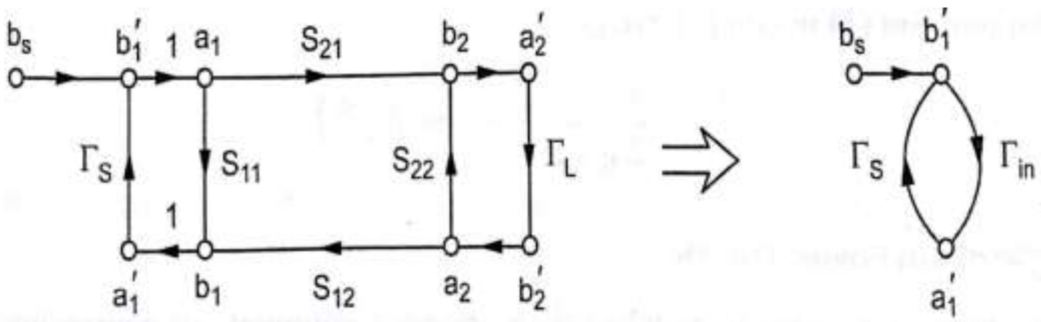
Amplifier Power Relation:

Generic single stage amplifier configuration with input and output matching networks is shown in fig.





(a) Simplified schematics of a single-stage amplifier



(b) Signal flow graph

RF source:

Incident Wave power:

The incident wave power at node b_1' is given by,

$$P_{inc} = \frac{|b_1'|^2}{2}$$

$$= \frac{1}{2} \frac{|b_S|^2}{|1-\Gamma_{in}\Gamma_S|^2} \qquad \rightarrow 1$$

Where, Source node $b_s = \frac{\sqrt{Z_o}}{Z_S+Z_o} V_s$

Input power:

$$P_{in} = P_{inc}\left(1-|\Gamma_{in}|^2\right) \qquad P_{in} = \frac{1}{2}\frac{|b_S|^2}{|1-\Gamma_{in}\Gamma_S|^2}\left(1-|\Gamma_{in}|^2\right)$$

Transducer power gain

$$G_T = \frac{\left(1-|\Gamma_L|^2\right)|S_{21}|^2\left(1-|\Gamma_S|^2\right)}{|(1-S_{11}\Gamma_S)(1-S_{22}\Gamma_L)-S_{21}S_{12}\Gamma_L\Gamma_S|^2}$$

Unilateral power gain(G_{TU}):

$$G_{TU} = \frac{\left(1-|\Gamma_L|^2\right)|S_{21}|^2\left(1-|\Gamma_S|^2\right)}{|1-\Gamma_L S_{22}|^2|1-S_{11}\Gamma_S|^2}$$

Additional power relations

Available Power Gain (G_A) at Load:

$$G_A = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)}{\left(1-|\Gamma_{Out}|^2\right)|1-S_{11}\Gamma_S|^2}$$

Power Gain (Operating Power Gain):

The operating power gain is defined as "the ratio of the power delivered to the load to the power supplied to the amplifier".

$$G = \frac{Power\ delivered\ to\ the\ load}{Power\ supplied\ to\ the\ amplifier}$$

Stability considerations and Stabilization Methods:

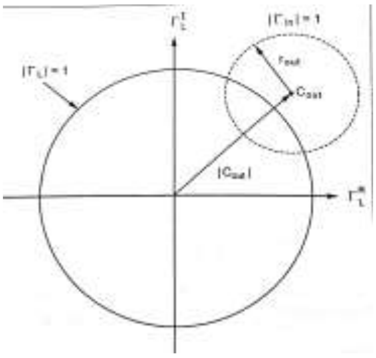
- An amplifier circuit must be stable over the entire frequency range
- The RF circuits (amplifier) tend to oscillate depending on operating frequency and termination

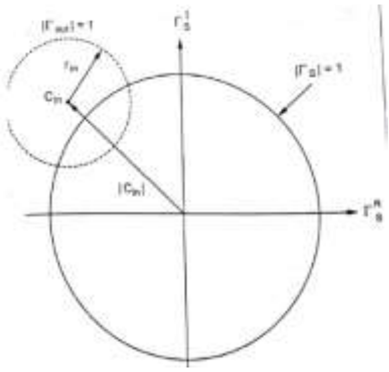
(i) If $|\Gamma| > 1$, then the magnitude of the return voltage wave increases called positive feedback, which causes instability (oscillator)

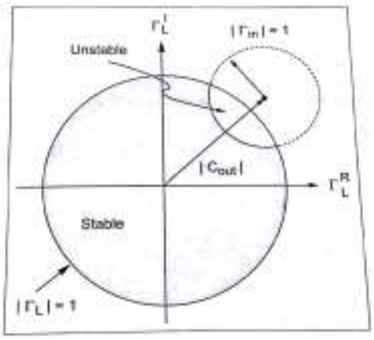
(ii) If $|\Gamma| < 1$, the return voltage wave is totally avoided (amplifier). Its called as negative feedback

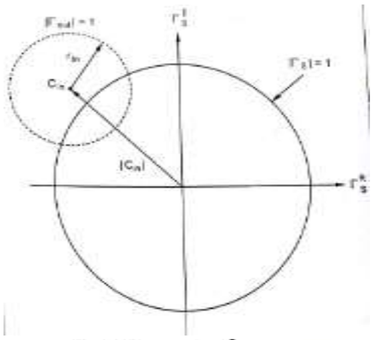
Two port network amplifier is characterized by its S-parameters
The amplifier is stable, when the magnitudes of reflection coefficients are less than unity

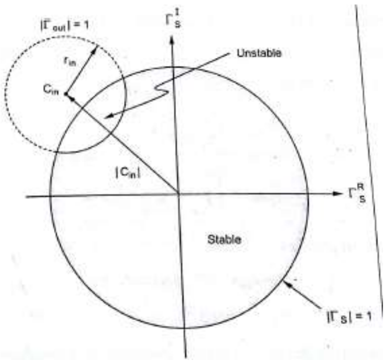
$$|\Gamma_L| < 1 \text{ and}$$
$$|\Gamma_S| < 1$$

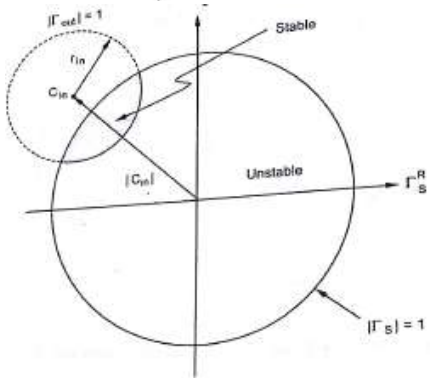












Stabilization Methods:

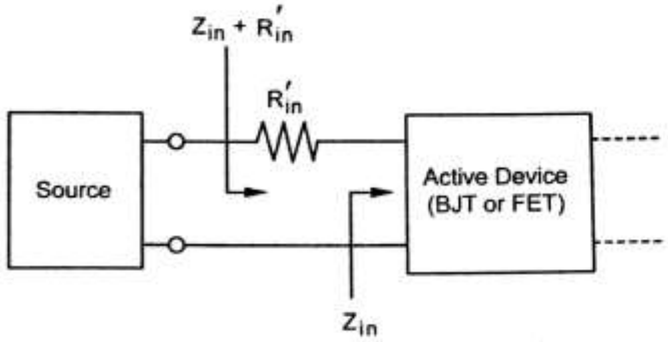
If the operation of a FET or BJT is unstable, we take steps to make them stable

The instability conditions $|\Gamma_{in}| > 1$ and $|\Gamma_{out}| > 1$ can be written in terms of the input and output impedances.

To stabilize the active devices, a series resistance or a conductance will be added to the port.

Configuration at input port:

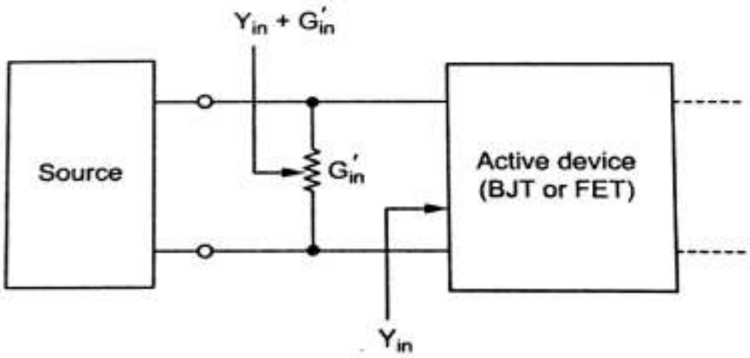
In the input port, the addition of $R_e(Z_S)$ must compensate the negative contribution of $R_e(Z_{in})$



Stabilization of input port through series resistance

$$\mathrm{Re}(Z_{in} + R'_{in} + Z_S) > 0$$

Stabilization of input port through addition of shunt conductance.



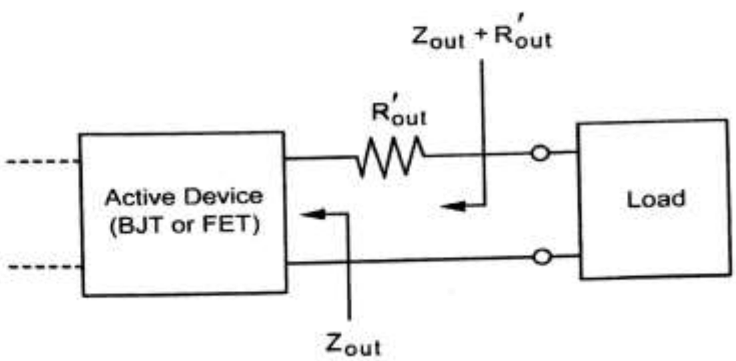
Stabilization of input port through shunt conductance

$$\mathrm{Re}(Y_{in} + G'_{in} + Y_S) > 0$$

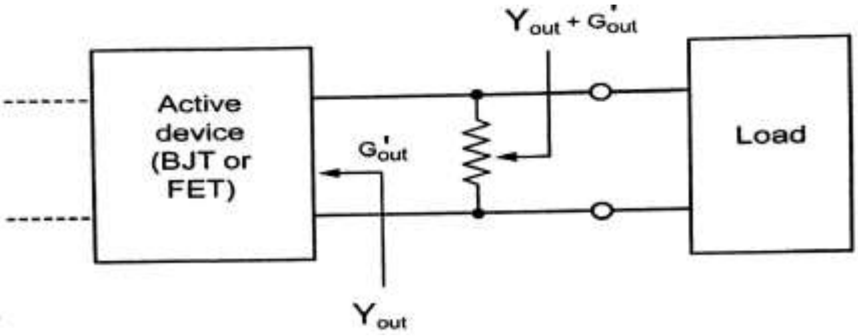
Configuration at output port:

In the output port, the addition of $R_e(Z_L)$ must compensate the negative contribution of $R_e(Z_{out})$

$$\mathrm{Re}(Z_{out} + R'_{out} + Z_L) > 0$$



Stabilization of output port through series resistance



Stabilization of output port through shunt conductance

$$\mathrm{Re}(Y_{out} + G'_{out} + Y_L) > 0$$

EC 8651 - Transmission lines & RF systems :.

Topics :. Unit - 1 :.

1.1. General theory of transmission lines - The transmission line. General solution.

1.2 The infinite line - wavelength, velocity of propagation.

1.3 Wave form distortion - The distortion less line - Loading and different methods of loading.

1.4 Line not terminated in Zo - Reflection co-efficient - Calculation of current, Voltage, power delivered and efficiency of transmission.

1.5 Input and transfer impedance - open and short circuited lines - Reflection factor & reflection loss.

Objectives :.

(i) Introduction to transmission line theory.

(ii) Definitions & General solution of transmission line.

(iii) Concept of Infinite line

(iv) Wave form distortion & Loading Techniques.

(v) Definition of Reflection co-efficient, Reflection factor & reflection loss.

(vi) Derivation of Input & transfer impedance of an open and short circuited lines.

* The study of 'Transmission lines' provides an introduction on the important concepts such as propagation, reflection & interference of signals.

* This unit explains the general theory of transmission lines, general solution, characteristics of distortion and distortion less line, different types of loading, impedances, open and short circuited lines.

⇒ In this unit, we study the various line characteristics & the effects on line. Also deals with the techniques that prevents the wave form distortion.

What is transmission line? Range of Radio Spectrum → 3 KHz — 300 GHz.

A pair of electrical conductors used to transmit the electrical energy from one point to another point at radio frequencies is known as transmission line.

* Transmission line is the cable (or) medium that transmits the radio frequency signal over long distances.
* Used to connect the radio Txs & Rxs.

Different types of Transmission lines?

1) co-axial line
2) parallel plane transmission line.
3) Two-wire parallel transmission line.
4) Microstrip lines.
5) Strip lines.

Ex: Telephone system. Computer data in LAN.

RF ⇒ KHz to MHz.

frequency $= \frac{1}{T}$ (Number of cycles/second.

1) co-axial line:-

* A co-axial cable is an arrangement of two concentric cylindrical conductors that are separated by a dielectric (air (or) polythene).

1000 Hz – 1 KHz
1 million Hz → MHz.
1 billion Hz → GHz.

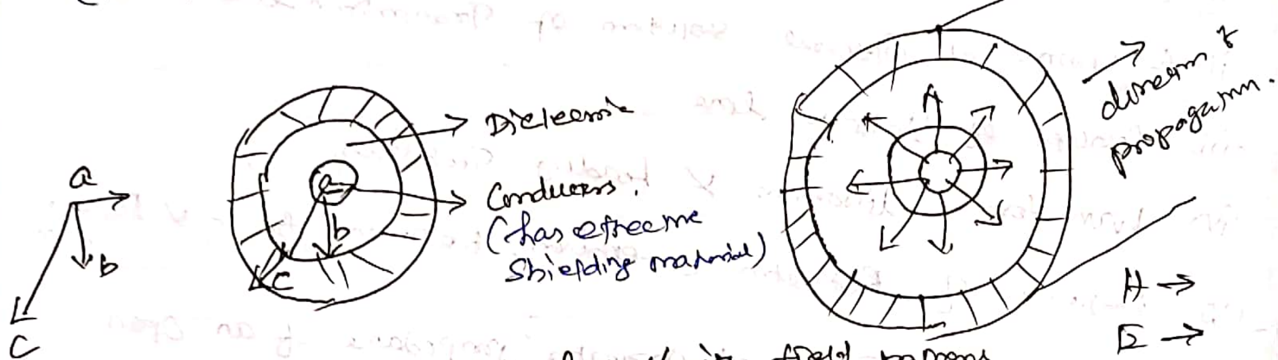
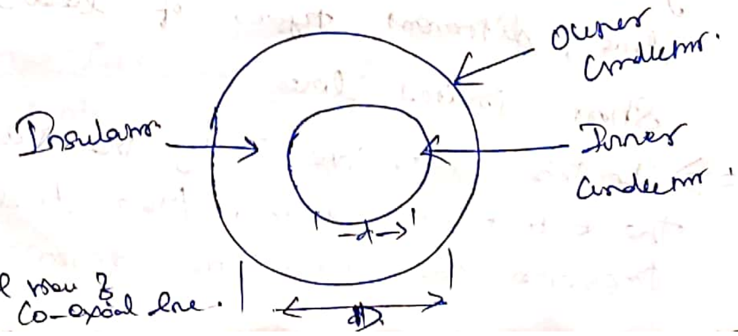


Fig: co-axial line & its field patterns.

→ The geometry of a co-axial line with its field patterns is as shown above fig.

* The co-axial transmission lines can be used as input cables in oscilloscopes, spectrum analyzer and network analyzers.

* These cables can also be employed as antenna feed lines.



Cross sectional view of Co-axial line.

Advantages of co-axial cable?

(i) It allows high rates of data transmission.

(ii) These are cost effective when compared to optical fibre cables.

(iii) Flexible due to copper braiding of outer conductor.

(iv) In co-axial cable, outer conductor has effective shielding material. It behaves as a dielectric, attenuates the external electro-magnetic signals.

(v). Electric & magnetic fields are bounded around inner conductor because of closed field configuration in co-axial cable. This completely stops the radiation in to free space.

Disadvantages of co-axial cable:

(i) Signals can be tapped easily. These are less secured signals.

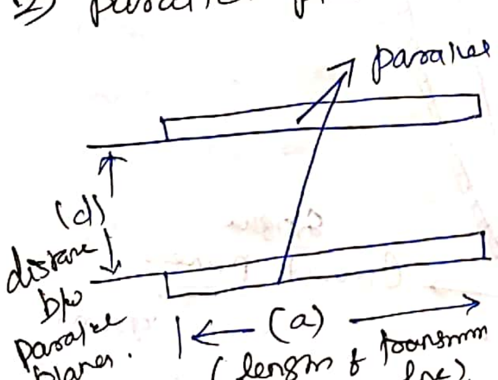
(ii) It is prone to electrical hazards.

(iii) Heavier due to the presence of two cylindrical conductors.

(iv) Transmission of signals using co-axial cable is limited only upto 1 GHz, greater efficiency losses are observed at frequencies beyond 1 GHz.

(v) External shielding is required, for reducing RF interference, which makes them costlier (or) expensive.

2) parallel plane Transmission line:



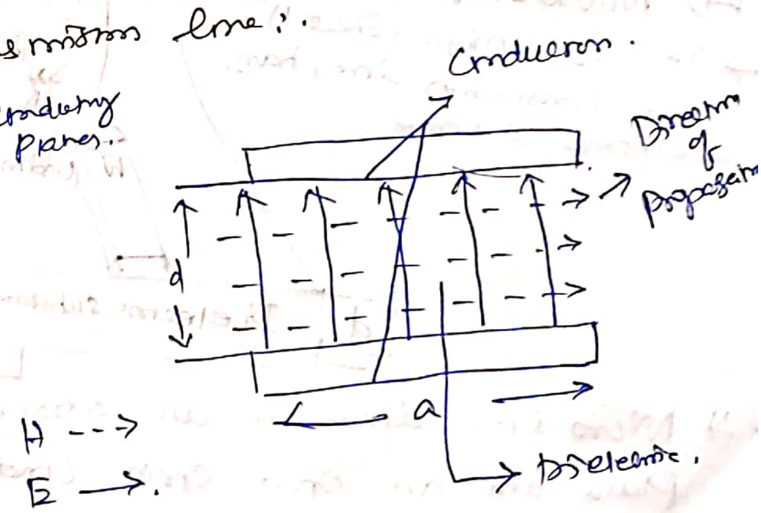


Fig: parallel plane Transmission line.

MW

* Parallel plane transmission lines consists of two (or) more metallic conducting planes arranged in parallel and separated by a dielectric material.
* The cross section of a parallel plane transmission line and its field pattern are as shown in fig.

3) Two wire parallel transmission line:

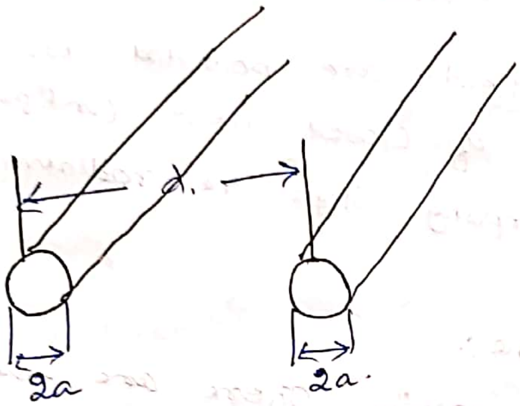


Fig: Two wire parallel Transmission line.

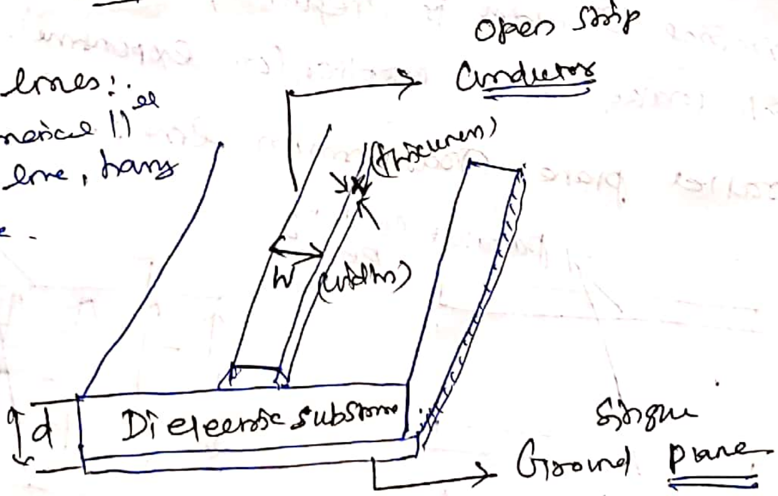
* A Two wire parallel transmission line consists of two wires of radius 'a' arranged in parallel and separated by a distance 'd' as shown in fig.

→ In this line, transmission of current takes place from one wire to the source of other.

→ The most commonly used two-wire transmission lines are power lines & Telephone lines.

4) Micro strip lines:

→ It is an unsymmetrical parallel plane transmission line, having dielectric substrate.



* Micro strip line is an arrangement of a single ground plane and an open strip conductor having thickness 't' and width 'w'.

→ The plane and conductor are separated by a dielectric substrate as shown in above fig.

* Micro strip lines are mainly used in printed circuit Boards (PCB) for component interconnects.

(5) Strip lines :-

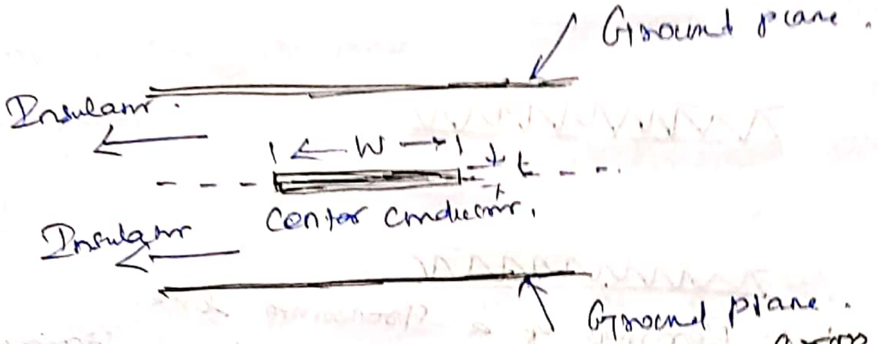


Fig: Strip line transmission line (cross sectional view).

* A strip line consists of a central thin conducting strip of width 'W' which is greater than its thickness 't'.
* It is placed inside the low loss dielectric (ϵ_r) substrate of thickness b/2 between two wide ground planes.
→ Strip lines consists of same arrangement as that of micro strip lines except that the conductor is closed in strip lines.

Parameters of Transmission line:-

* The two parameters describes the characteristics of a transmission line are → called as distributed elements.
   1) Primary constants. (RLCG)
   2) Secondary constants. (are derived from primary constants). (Z_0, γ) $(\gamma = \alpha + j\beta)$ Attenuation constant, phase constant $\left(Z_0 = \frac{E_0}{I_0}\right)$

1) Primary constants:- (R, L, C, G)
* The primary constants (or) electric parameters of a transmission line given as,
   (i) Resistance (R)
   (ii) Inductance (L)
   (iii) Capacitance (C)
   (iv) Conductance (G)

(i) Resistance (R).

→ When current flows through the conductors of a transmission line, there exists an opposition towards it (current)

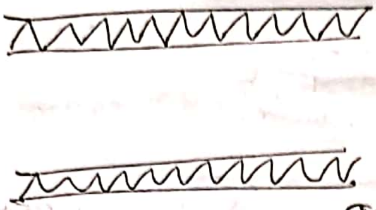


Fig: Resistance of a Transmission line.

→ Resistance is a distributed constant. As it uniformly distributed along the length of the line.

→ It is denoted by R.

→ It is expressed in ohms per unit length.

(ii) Inductance (L)

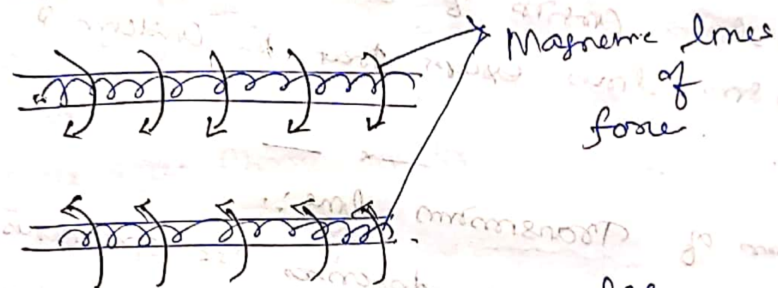


Fig: Inductance of a transmission line.

→ When current flows through the conductors of a transmission line, magnetic lines of force are generated around them as shown in fig.

→ Since the variation in the flow of current proportionally varies the magnetic field, there exists a certain amount of inductance.

→ It is also a distributed constant & denoted by L.

→ It is expressed in micro henry per unit length.

(iii) Capacitance (C) :

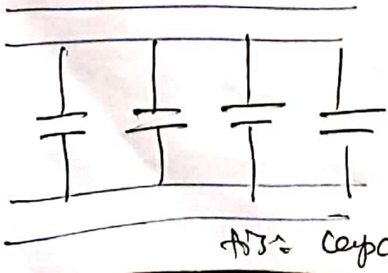


Fig: Capacitance of a transmission line.

* When two parallel plates are separated by a dielectric material, there exist capacitance b/w them. (7)

→ This is due to the presence of electric field between the conductors (or) lines.

→ Fig. shows the existence of capacitance b/w the conductors of a transmission line.

* Capacitance is also a distributed constant and is denoted by 'C'.

⇒ The capacitance between lines is generally expressed in picofarads per unit length.

iv) Conductance :-

* When current flows through the two ||el plates, a leakage current flows between them due to imperfect insulation of dielectric.

→ This can be represented by a shunt conductance as shown in fig.

→ This is denoted by 'G'.

⇒ It is reciprocal of resistance.

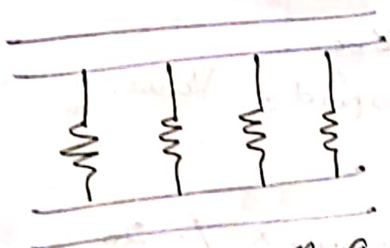


fig :- Conductance of a transmission line.

2) Secondary Constants :-

* Secondary constants are derived from primary constants. These includes (i) Characteristic impedance (Z_0).

(ii) Propagation constant (γ)

(a) Attenuation constant (α)

(b) phase constant (β).

i) Characteristic impedance (Z_0) :-

→ It is defined as the input impedance of a uniform transmission line whose length is infinite.

→ It is defined as the steady state vector ratio of voltage to current at the input of a finite line.

Expression for Charac. Impedance of a Transmission line is given by $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$ (8)

* It is denoted by Z_0.
* If E_0 is a Source Voltage and I_0 is a line current, Then characteristic impedance Z_0 is given by

$Z = R+j\omega L$
$Y = G+j\omega C$

$$\boxed{Z_0 = \frac{E_0}{I_0}}$$

Where E_0 - Source Voltage
I_0 : Line current.

$Z_0 = \sqrt{\frac{Z}{Y}}$
Z - Series imp.
Y - Shunt imped.

Prob: Find the characteristic impedance of a line @ 1600 Hz. If $Z_{oc} = 750 \angle -30^\circ \Omega$ &
$Z_{sc} = 600 \angle -20^\circ \Omega$.

$$\boxed{Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}}$$

R - Conductor Resistance (Ω)
L = Inductance (H)
G - Dielectric Conductance (℧)
C - Capacitance (F)
ω - operating frequency (radian/sec)

Ans: Given, Frequency $f = 1600$ Hz.
Open circuit impedance $Z_{oc} = 750 \angle -30^\circ \Omega$
Short circuit impedance $Z_{sc} = 600 \angle -20^\circ \Omega$.
Characteristic impedance $Z_0 = ?$

* The relation between Z_{oc} and Z_{sc} is given as.

$$Z_0^2 = Z_{sc} \cdot Z_{oc}.$$

$$\therefore \quad Z_0 = \sqrt{Z_{sc} \cdot Z_{oc}}.$$

By substituting the corresponding values in above eqn. result

$$Z_0 = \sqrt{750 \times \angle -30 \times 600 \times \angle -20}$$

$$= 670.82 \angle \frac{-30 - 20}{2}$$

$$= 670.82 \angle -25$$

$$\therefore \boxed{Z_0 = 607.96 - j\,283.5\,\Omega}$$

(ii) Propagation Constant:

* Propagation Constant is a complex quantity and is denoted by 'γ'.

Mathematically $\boxed{\gamma = \alpha + j\beta.}$

Where
α : Attenuation constant
β : Phase Constant.

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* It is defined as a measure of change in the amplitude and phase of the electro magnetic wave when propagating in a particular direction.

(a) Attenuation Constant (α):

* It is the natural logarithm of magnitude of ratio of the transmitted current $\underline{I_t}$ to the Received current $\underline{I_r}$.

ie $$\boxed{\alpha = \ln \left| \frac{I_t}{I_r} \right|}$$

* units = Neper/meter.

→ Attenuation Constant of a line is defined as the reduction in Voltage (or) current along the transmission line.

(b) Phase Constant (β):

* It is defined as the phase difference between the transmitted current & received current.

ie $$\boxed{\beta = \frac{\angle I_t}{\angle I_r}}$$ units: radians/meter.

* The propagation constant, in terms of primary constants is given by $$\boxed{\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}}$$

Where R - Resistance / unit length
L = Inductance / unit length
G = Conductance / unit length
C = Capacitance / unit length.
ω = angular frequency.

Prob 2. A transmission line is having the following parameters at 1 KHz. $R = 10.4\,\Omega/m$.

$L = 0.00367\ H/m$

$G = 0.8 \times 10^{-4}\ \mho/m$

$C = 0.00835\ \mu F/m$.

Calculate Z_0 & γ.

Sol:

The expression for characteristic impedance of line is given by

$$\boxed{Z_0 = \sqrt{\frac{Z}{Y}}} \quad \text{--- (1)}$$

where Z = series impedance.
Y = shunt ~~impedance~~ admittance.

where Series impedance $\boxed{Z = R + j\omega L}$

Shunt impedance $\boxed{Y = G + j\omega C.}$

$\therefore$ Series impedance $Z = R + j\omega L$ $\quad [\because \omega = 2\pi f]$

$= 10.4 + j\,2\pi f L$

$= 10.4 + j\,2 \times 3.14 \times (1.0 \times 10^3)(0.003$

$= 10.4 + j\,23.04.$

$$\boxed{Z = 25.27 \angle 65.70^\circ\, \Omega.}$$

Shunt admittance, $Y = G + j\omega C$

$Y = 0.8 \times 10^{-4} + j\,2\pi f.C.$

$= 0.8 \times 10^{-4} + j2 \times 3.14 \times (1.0 \times 10^3)(0.00835 \times 10^{-6}$

$= 0.8 \times 10^{-4} + j\,6.28 \times 10^{-3} \times 0.00835.$

$= 0.8 \times 10^{-4} + j\,0.0524 \times 10^{-3}.$

$$\therefore \boxed{Y = 9.563 \times 10^{-5} \angle 33.22\, \Omega.}$$

Substituting the values of Z and Y in eqn (1)

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{25.27 \angle 65.70^\circ}{9.563 \times 10^{-5} \angle 33.22^\circ}}$$

$$= 514.05 \angle \frac{65.70 - 33.22}{2}.$$

$$= 514.05 \angle 16.24\, \Omega$$

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$= 514.05 (\cos(16.24) + j \sin(16.24))$

$= 514.05 (0.96 + j0.27)$.

$\therefore \boxed{Z_0 = 493.4 + j138.7\,\Omega.}$ //

② Propagation Constant :- (γ).

* The expression for propagation constant is given by

$$\gamma = \sqrt{ZY}. \quad \text{——} ②.$$

Substituting the values of Z and Y in eqn ②, we get.

$$\gamma = \sqrt{(25.27 \angle 65.70^\circ)(9.563 \times 10^{-5} \angle 33.22^\circ)}$$

$$= 0.049 \angle \frac{65.70^\circ + 33.22^\circ}{2}.$$

$$= 0.049 \angle 49.46^\circ$$

$$= 0.049 (\cos 49.46 + j \sin 49.46)$$

$$= 0.049 (0.649 + j\, 0.759).$$

$\therefore \boxed{\gamma = 0.031 + j\, 0.037.}$ //

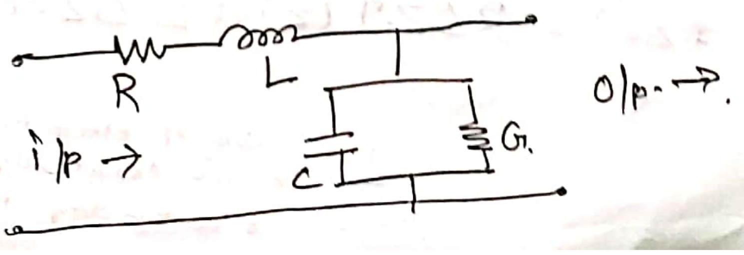
1.1. Derivation of General transmission line equations for
(a) Voltage & current at any point on a line. (X)

→ General solution of a transmission line :-

Transmission line equations :-

→ The second order differential equations of Voltage & Current at a random point on a uniform line are known as transmission line equations.

→ Equivalent circuit of a transmission line :-



A Two Conductor transmission line of small section of length Δz is considered in below fig.

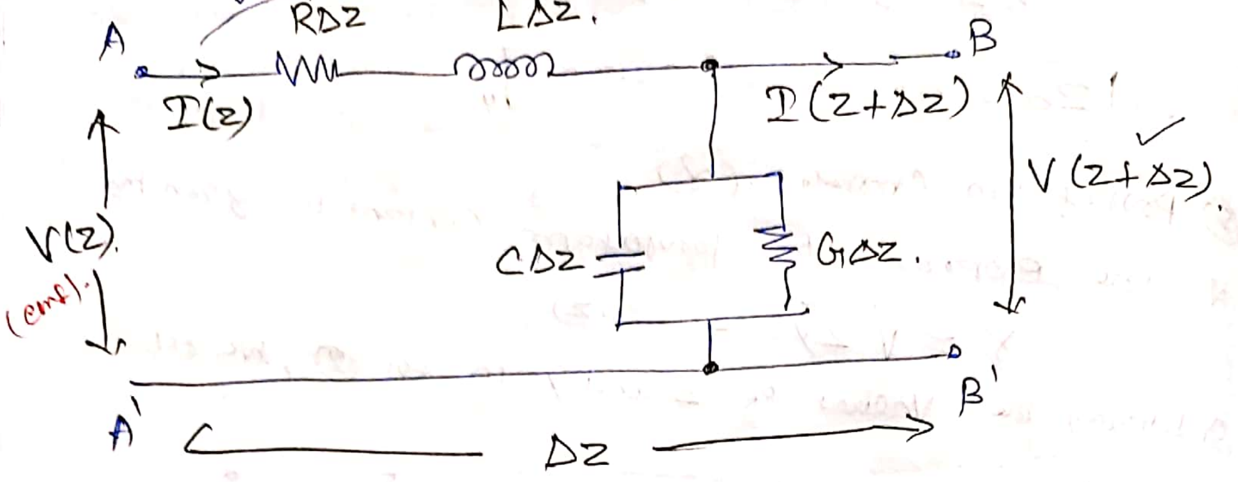


Fig : Voltage & current on Infinitesimal section of Transmission line.

(i) The line has the resistance, inductance, conductance & Capacitance of $R\Delta z, L\Delta z, G\Delta z$ & $C\Delta z$ respectively.

(ii) The input to the transmission line is a sinusoidal Voltage of frequency 'ω' applied between AA'.

* This causes current 'I' to flow into the terminal 'A'.
* The R and L elements connected in series causes a Voltage drop at BB', whereas the parallel arrangement of C and G changes the Value of output current at 'B'.

(iii) Consider $I(z+\Delta z)$ & $V(z+\Delta z)$ be the current & Voltage at BB' respectively.

→ On applying Kirchoff's Voltage law to the outer loop of the circuit shown, we get.

$$\boxed{V(z) = R\Delta z \cdot I(z) + L\Delta z \cdot \frac{\partial I(z)}{\partial t} + V(z+\Delta z)}$$

(∵ By KVL).

$$\Rightarrow V(z) - V(z+\Delta z) = R\Delta z \cdot I(z) + L \cdot \Delta z \cdot \frac{\partial I(z)}{\partial t}$$

KVL = Sum of electro motive force (emf) in any circuit loop is equal with the sum of Voltage drops in the same loop.

On dividing the above equation by Δz, we get.

$$-\frac{V(z+\Delta z)-V(z)}{\Delta z} = RI(z) + L.\frac{\partial I(z)}{\partial t}.$$

Taking. $V(z+\Delta z) - V(z)$ as $\Delta V(z)$ &

applying the limit as $\Delta z \to 0$ in the above eqn, we get

$$-\lim_{\Delta z \to 0} \frac{\Delta V(z)}{\Delta (z)} = RI(z) + L\frac{\partial I(z)}{\partial t}.$$

⇒ Above eqn can be written as.

$$\boxed{-\frac{\partial V(z)}{\partial z} = RI(z) + L.\partial I(z) \longrightarrow (1)}$$

→ On applying Kirchoff's current law to the principal node of figure, we get.

KCL: Sum of current enters a junction is equal to sum of currents that leave the junction

$$\boxed{I(z) = I(z+\Delta z) + G\Delta z.V(z+\Delta z) + C\Delta z\frac{\partial V(z+\Delta z)}{\partial t}.}$$

[∵ By KCL]

On dividing the above equation by Δz, we get,

$$-\left|\frac{I(z+\Delta z)-I(z)}{\Delta z}\right| = GV(z+\Delta z) + C\frac{\partial v(z+\Delta z)}{\partial t}.$$

Taking $I(z+\Delta z) - I(z)$ as $\Delta I(z)$ &

applying the limit $\Delta z \to 0$ in the above eqn, we get.

$$-\lim_{\Delta z \to 0} \frac{\Delta I(z)}{\Delta z} = GV(z+\Delta z) + C\frac{\partial V(z+\Delta z)}{\partial t}$$

⇒ Above eqn can be written an.

$$\boxed{-\frac{\partial I(z)}{\partial z} = GV(z) + C\frac{\partial V(z)}{\partial t} \longrightarrow (2).}$$

Consider the differential length is dz, then the series impedance becomes $(R+j\omega L)dz$ and shunt impedance becomes $(G+j\omega C)dz$.

→ Consider V, I, V+dV & I+dI are the voltages and currents at the transmitting & receiving ends of the differential length, such that: (14)

$$V-(V+dV) = I(R+j\omega L)\,dz.$$

$$I-(I+dI) = V(G+j\omega C)\,dz.$$

$$\Rightarrow \frac{dV}{dz} = -(R+j\omega L)\cdot I. \longrightarrow (3)$$

$$\frac{dI}{dz} = -(G+j\omega C)V. \longrightarrow (4)$$

(3) & (4) are the first order differential eqns of Voltage & current and are referred to as telegrapher's equation.

[∵ pair of linear partial eqns that describe V & I on transmission line with distance & time]

→ on differentiating (3) with respect to 'z', we get,

(i)

$$\frac{d^2V}{dz^2} = -(R+j\omega L)\cdot\frac{dI}{dz} \longrightarrow (5)$$

→ on substituting eqn (4) in (5), we get:

$$\frac{d^2V}{dz^2} = -(R+j\omega L)\left(-(G+j\omega C)V\right).$$

$$\Rightarrow \frac{d^2V}{dz^2} = (R+j\omega L)(G+j\omega C)V \longrightarrow (6).$$

→ on differentiating eqn (4) with respect to 'z' we get.

(ii)

$$\frac{d^2I}{dz^2} = -(G+j\omega C)\cdot\frac{dV}{dz} \longrightarrow (7).$$

on substituting (3) in (7) we get.

$$\frac{d^2 I}{dz^2} = (R + j\omega L)\,(G + j\omega C) I \longrightarrow (8).$$

(6) and (8) can be written as.

$$\frac{d^2 V}{dz^2} = \gamma^2 . V \longrightarrow (9).$$

$$\frac{d^2 I}{dz^2} = \gamma^2 . I \longrightarrow (10)$$

Where $\gamma^2 = (R + j\omega L).(G + j\omega C)$

γ = propagation constant.

∴ (9) & (10). are the second order differential eqns of Voltage & current of transmission line.

Since, γ is constant, the homogenous equation with constant co-efficients can be expressed as.

$$V = V^+ e^{-\gamma z} + V^- e^{\gamma z} \text{ ——— } (11)$$

$$\& \quad I = I^+ e^{-\gamma z} + I^- e^{\gamma z} \text{ ——— } (12).$$

Where, V^+, V^-, I^+, I^- are the constants representing the forward Voltage, Reverse Voltage, forward current & reverse current respectively. //

1.1 (B). Derive the equation of attenuation constant 'α' and phase constant (β) of transmission lines in terms of line constants R, L, C and 'G'. (15)

Ans:

The expression for attenuation constant 'α' and phase constant 'β' are obtained from the propagation constant 'γ' as follows,

→ The expression for 'γ' is given as,

$$\boxed{\gamma = \alpha + j\beta}$$

$$\boxed{\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}} \longrightarrow (1).$$

(i) Taking the magnitude of (1), we get,

$$|\gamma| = \underbrace{\sqrt{|R+j\omega L|\,|G+j\omega C|}}_{LHS} = \underbrace{|\alpha+j\beta|}_{RHS}.$$

$$|\gamma| = \sqrt{\underbrace{\sqrt{R^2+\omega^2L^2}\angle\tan^{-1}\left(\frac{\omega L}{R}\right)}_{LHS}\sqrt{G^2+\omega^2C^2}\angle\tan^{-1}\left(\frac{\omega C}{G}\right)}$$

$$= \underbrace{\sqrt{\alpha^2+\beta^2}\angle\tan^{-1}\left(\frac{\beta}{\alpha}\right)}_{RHS}$$

$$\underbrace{= \sqrt[4]{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\left[\tan^{-1}\left(\frac{\omega L}{R}\right)+\tan^{-1}\left(\frac{\omega C}{G}\right)\right]}_{LHS} = \underbrace{\sqrt{\alpha^2+\beta^2}\angle\tan^{-1}\frac{\beta}{\alpha}}_{RHS} \rightarrow (2)$$

(ii) Upon equating magnitude of LHS & RHS we get.

$$\sqrt{\alpha^2+\beta^2} = \sqrt[4]{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$\Rightarrow \alpha^2+\beta^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} \longrightarrow (3)$$

From eq (1), we can write.

$$\gamma = \alpha+j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}.$$

(ii) Squaring on both sides, we get.

$$\gamma^2 = (\alpha+j\beta)^2 = (R+j\omega L)(G+j\omega C)$$

$$\Rightarrow \alpha^2-\beta^2+2j\alpha\beta = RG+j\omega RC+j\omega LG-\omega^2 LC \rightarrow (4)$$

upon Comparing LHS & RHS of above eqn. we get.

$$\alpha^2-\beta^2 = RG-\omega^2 LC \longrightarrow (5)$$

Adding (3) and (4) we get.

$$2\alpha^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + RG-\omega^2LC$$

$$\alpha^2 = \frac{1}{2}\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} + RG-\omega^2LC$$

$$\boxed{\therefore \alpha = \sqrt{\frac{1}{2}(R^2+\omega^2L^2)(G^2+\omega^2C^2) + RG-\omega^2LC}}$$

$\Rightarrow$ This is the expression for attenuation constant 'α'.

Subtracting eqn (5) from eqn (4), we get.

$$2\beta^2 = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} - (RG-\omega^2LC)$$

$$\beta^2 = \frac{1}{2}\left[\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} - RG-\omega^2LC\right]$$

$$\boxed{\therefore \beta = \sqrt{\frac{1}{2}\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)} - RG+\omega^2LC}}$$

$\Rightarrow$ This is the expression for phase constant 'β'.

Problems: The constant of transmission line are $R = 6\,\Omega/km$, $L = 2.2\,mH/km$

(Q44) $C = 0.005\,\mu F/km$ & $G = 0.25\times10^{-3}$ mho/km. @ 1000 Hz.

Calculate the Z_0, γ, (attenuation constant α & Phase constant β).

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}, \qquad \gamma = \alpha+j\beta$$

Phase Constant

$\gamma = \sqrt{ZY}$, Ans. $Z_0 = ?$

$Z = 6+j13.816 \rightarrow = 15.06\angle 66.53^\circ \Rightarrow \boxed{Z_0 = 244.74\angle 29.686^\circ}$

$\gamma = 0.4913 + j0.0368$, $Y = 2.51\times10^{-4}\angle 7.158^\circ$

Prob 3:

A Telephone line has $R = 30\,\Omega/km$, $L = 0.1\,H/km$, $C = 20\,\mu F/m$ and $G = 0$. At $f = 10\,KHz$, find the secondary constants & phase velocity.

Ans: Given that, for a telephone line,

Resistance $R = 30\,\Omega/km$, Inductance $L = 0.1\,H/km$.

Capacitance $C = 20\,\mu F/m$, Conductance $G = 0$.

Operating frequency $f = 10\,KHz$.

Secondary constants = ?

Phase velocity V_p = ?

Secondary constants of a transmission line are characteristic impedance, Z_0 and propagation constant γ.

(i) Characteristic impedance Z_0 :

→ The expression for the characteristic impedance of the line is given by,

$$\boxed{Z_0 = \sqrt{\frac{Z}{Y}}} \quad \text{--- (1)}$$

Where Z = series impedance $\underline{Z = R + j\omega L}$.

Y = Shunt ~~impedance~~ admittance, $\underline{Y = G + j\omega C}$.

(i) Series impedance $Z = R + j\omega L$

$= 30\,\Omega + j\,2\pi f L$.

$= 30 + j\,2\pi\,(10 \times 10^3)\,(0.1)$

$= 30 + j\,6.283 \times 10^3$

$Z = 6283.07 \angle 89.72^\circ\,\Omega/km$.

(ii) Shunt admittance $Y = G + j\omega C$.

$= G + j\,2\pi f C$.

$= 0 + j\,2\pi\,(10 \times 10^3)\,(20 \times 10^{-3})$

$= 0 + j\,1.256 \times 10^3$

$\therefore Y = 1256 \angle 90^\circ$ ℧/km

Substituting the values of Z and Y in (1), we get,

$$Z_0 = \sqrt{\frac{6283.07 \angle 89.72^\circ}{1256 \angle 90^\circ}}$$

$$= 2.236 \angle \frac{89.72^\circ - 90^\circ}{2}$$

$$= 2.236 \angle -0.14^\circ \,\Omega.$$

$$= 2.236 - j\,0.0055 \,\Omega$$

$$\therefore \boxed{Z_0 = 2.236 - j(0.0055)\,\Omega}$$

(ii) propagation constant (γ):

The expression for propagation constant is given by.

$$\boxed{\gamma = \sqrt{ZY}}$$

Substituting Z and Y values in above eqn, we get.

$$\gamma = \sqrt{(6283.07 \angle 89.72^\circ)(1256 \angle 90^\circ)}$$

$$= 2809 \angle \frac{89.72^\circ + 90^\circ}{2}$$

$$= 2809 \angle 89.86^\circ.$$

$$= 6.863 + j\,2809.09.$$

$$\boxed{\therefore \gamma = \alpha + j\beta = 6.863 + j\,2809.09.}$$

From the above expression,

Attenuation constant $\alpha = 6.863$ Np/m.

Phase constant $\beta = 2809.09$ rad/m.

(iii) Phase Velocity (V_p).

* The expression for phase velocity, V_p is given by

$$V_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta}.$$

* Phase velocity is defined as the rate of change in phase of the propagating wave.

$\Rightarrow V_p = \lambda g / t.$

$V_p = \lambda . f.$

$$\Rightarrow V_p = \frac{2\pi(10\times10^3)}{2809.09}$$

$$= 22.367 \text{ m/sec}$$

$$\boxed{\therefore V_p = 22.367 \text{ m/sec}}$$

1.2 The Infinite line – wavelength, velocity of propagation

Explain the concept of Infinite line:.

Infinite line:. A transmission line whose length is infinite is known as infinite line. An infinite length transmission line is as shown in fig.

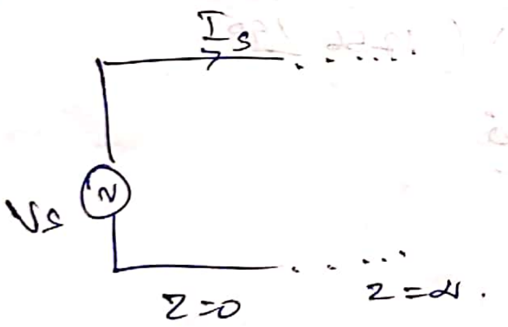
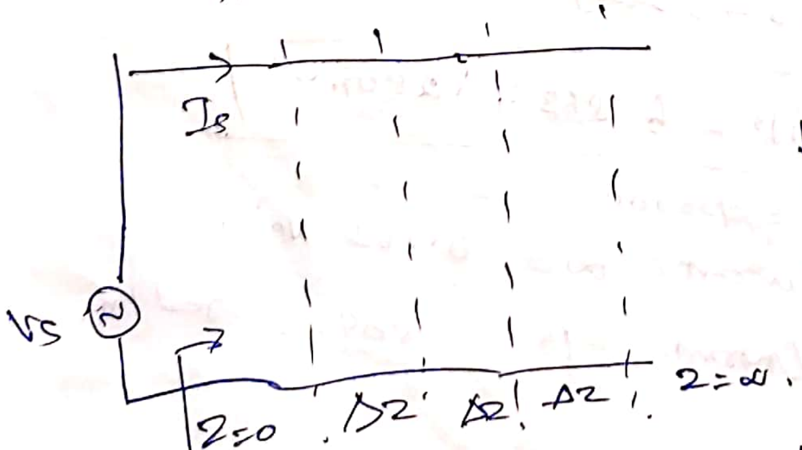


fig: Infinite line.

Basically, an infinite line is formed by cascading a number of symmetrical sections as shown below



$$Z_{in} = Z_0 = \frac{V_s}{I_s}.$$

A finite line ~~A Infinite~~ line behaves as an infinite line when $Z_{in} = Z_0$,

$\Rightarrow$ The input impedance (Z_{in}) of the first n/w is the characteristic impedance (Z_0) which is obtained when the symmetrical section is terminated by Z_0.

The expression for the characteristic impedance
(i) of the parallel wire line is

$$Z_0 = \frac{276}{\sqrt{K}} \log_{10}\left(\frac{d}{a}\right) \Omega.$$

Where K = dielectric constant, d - Distance b/w the wires.
a - Diameter of inner conductor.

(ii) The expression for characteristic impedance of co-axial cable line is

$$Z_0 = \frac{138}{\sqrt{K}} \log_{10}\left(\frac{b}{a}\right) \Omega.$$

K - dielectric constant.
b - Diameter of outer conductor.
a - Diameter of inner conductor.

Wavelength: (λ)

* Wavelength of a transmission line is defined as.

$$\therefore \boxed{\lambda = \frac{2\pi}{\beta}}$$

λ in terms of primary constant is given as

$$\lambda = \frac{2\pi}{\sqrt{\frac{1}{2}\left[(\omega^2 LC - RG) - \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]}}$$

→ Distance b/w two points along the line at which current (or) voltage undergoes a phase shift of 2π radians.

Velocity of propagation (v):

* Velocity of propagation of a transmission line is defined as.

$$\boxed{V = \frac{\omega}{\beta}}$$

Where β: phase constant.
ω = Angular frequency.

* β in terms of primary constants is expressed as

$$\beta = \sqrt{\frac{1}{2}\left[(\omega^2 LC - RG) - \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]}$$

Substituting this in eqn (B)

$$V = \frac{\omega}{\sqrt{\frac{1}{2}\left[(\omega^2 LC - RG) - \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]}}$$

Wave form distortion - Distortion less line - loading & different methods of loading. (22)

Name the types of distortions in the transmission line & Explain:

* Distortions in the signal are said to be occurred when the output produced at the receiver is not the replica of the input signal.

* The distortions in transmission lines are categorized depending on the variation of different parameters as.

(1) Variation in characteristic impedance (Z_0) with frequency.

(2) Variation in attenuation constant (α) with frequency (frequency distortion).

(3) Variation in phase constant (β) with frequency (phase ~~distort~~ distortion).

① Variation in Z_0 with frequency:-

The change in frequency causes variation in Z_0.

* The transmission line gets terminated at a point where impedance is independent of change in frequency, which causes distortions.

∴ The power gets absorbed at particular range of frequencies while the same is reflected for another range of frequencies.

→ The expression for Z_0 is $Z_0 = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}$.

$$\boxed{Z_0 = \sqrt{\dfrac{R\left(1+j\omega \frac{L}{R}\right)}{G\left(1+j\omega \frac{C}{G}\right)}}}$$

Remedy:-

when $LG = CR$

$\Rightarrow \frac{L}{R} = \frac{C}{G}$

$\therefore Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$

* Since, Z_0 is independent of frequency, It eliminates distortion.

(2) Frequency distortion:

* Frequency distortion is caused due to the variation in attenuation constant with frequency.

→ The expression for attenuation constant (α) is.

$$\alpha = \sqrt{\frac{1}{2}\left(RG - \omega^2 LC\right) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}$$

⇒ from the above eqn, it is observed that the attenuation constant (α) is dependent of frequency.

→ Hence the attenuation of the transmitted signal of different input frequencies occur in uneven manner.

Thus, the output produced is not replica of the input resulting in distortion called "frequency distortion".

Remedy

* This type of ~~signal~~ distortion is prominant in audio signals compared to video signals which can be terminated using equalizers by making resultant frequency response uniform.

(3) Phase distortion:

Phase distortion occurs due to the variation in phase constant (β) with frequency.

* The expression for phase constant 'β' is.

$$\beta = \sqrt{\frac{1}{2}\left[\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2L^2)} - (RG - \omega^2 LC)\right]}$$

Also $\beta = \frac{\omega}{v} = \frac{2\pi f}{v}$

where v - velocity of propagation, f - frequency.

→ Velocity of propagation and phase constant both are dependent on frequency.

Thus, the waves with high velocity travel fast and those with less velocity are delayed resulting in the distortion of output wave. Hence it is also called "delay distortion".

Remedy: This type of distortion is prominent in video signal and picture transmission. Usage of co-axial cables for picture & video signal transmission eliminates the distortion. Thus, faithful output is produced at the receiver.

Explain in detail about the waveform distortion & also derive the condition for distortionless line. (Or) Derive the condition for minimum attenuation in a distortionless line.

Distortion: Distortion in the signal are said to be occurred when the output produced at the receiver is not the replica of the input signal transmitted.

Distortionless line: A transmission line is said to be distortionless if it satisfies the following two conditions,

1. The attenuation constant (α) is independent of frequency.
2. The phase constant (β) is linearly dependent on the frequency.

Mathematically, a transmission line is said to be distortionless if $\frac{R}{L} = \frac{G}{C}$.

The expression for the propagation constant (γ) is given by,

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \sqrt{R\left(1+\frac{j\omega L}{R}\right)G\left(1+\frac{j\omega C}{G}\right)}$$

$$= \sqrt{RG\left(1+\frac{j\omega L}{R}\right)\left(1+\frac{j\omega C}{G}\right)}$$

$$= \sqrt{RG\left(1+\frac{j\omega C}{G}\right)\left(1+\frac{j\omega C}{G}\right)}$$

$$= \sqrt{RG\left(1+\frac{j\omega C}{G}\right)^2}$$

$$\gamma = \sqrt{RG}\left(1+\frac{j\omega C}{G}\right)$$

Since $\gamma = \alpha + j\beta$ $\therefore$ Attenuation constant $\alpha = \sqrt{RG}$ — (1)

Phase constant $\beta = \omega\sqrt{LC}$ — (2)

From (1) & (2) It is proved that, α is frequency independent and β is linear function of frequency.

→ The expression for characteristic impedance is given by,

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$= \sqrt{\frac{R(1+j\omega L/R)}{G(1+j\omega C/G)}} \quad \left[\because \frac{R}{L} = \frac{G}{C}\right]$$

$$= \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} = R_0 + jX_0$$

Where $R_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$ and $X_0 = 0$.

$$\therefore \boxed{Z_0 = R_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}}$$

The phase velocity is given by

$$V_P = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}.$$

$$\therefore \boxed{V_P = \frac{1}{\sqrt{LC}}}$$

Q: Calculation of I, V, P delivered & efficiency of transmission.

(or)

Prove that an infinite line equal to finite line terminated in its characteristic impedance.

"A finite line terminated in its characteristic impedance behaves as an infinite line".

Proof :-

Consider a transmission line of an infinite length with input impedance Z_0, as represented below.

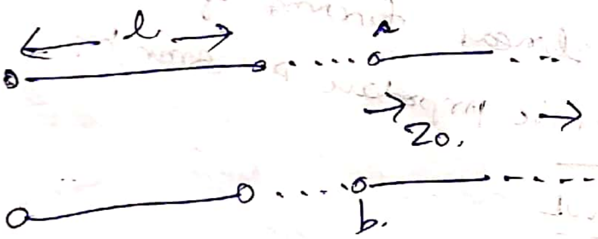
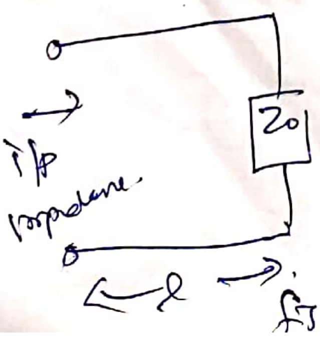


fig: infinite line.

* A small section of length 'l' is considered from the line shown.

→ Since the line is of infinite length, its length remains same and the impedance across 'a' and 'b' is Z_0.

The finite line terminated in Z_0 is shown below.



* Assume V_R & I_R are the voltage and current at receiving end ~~respectively~~ respectively.

fig: Finite line terminated in Z_0.

The characteristic impedance Z_0 is given by (27)

$$\boxed{Z_0 = \frac{V_R}{I_R}} \rightarrow (*)$$

* The general solution for current & voltage equations at any point on the transmission lines are defined by.

$$\boxed{\begin{aligned} V &= V_R \cosh px + I_R Z_0 \sinh px \rightarrow (1) \\ I &= I_R \cosh px + \frac{V_R}{Z_0} \sinh px \rightarrow (2) \end{aligned}}$$

* The voltages and currents of finite line at the sending end (ie at $x=0$) and receiving end (ie at $x=l$) are obtained as follows:

At $\boxed{x=0,\ V=V_s \text{ and } I = I_s}$

Hence $V_s = V_R(1) + I_R Z_0 (0) = V_R$

$I_s = I_R(1) + \frac{V_R}{Z_0}(0) = I_R$

From eqn (1) and (2) we have.

$$\boxed{\begin{aligned} V &= V_s \cosh px + I_s Z_0 \sinh px \rightarrow (3) \\ I &= I_s \cosh px + \frac{V_s}{Z_0} \sinh px \rightarrow (4) \end{aligned}}$$

Ill^{ly}, $\boxed{\text{at } x=l,\ V = V_R \text{ and } I = I_R}$, hence the equations (3) & (4) imply,

$V_R = V_s \cosh pl + I_s Z_0 \sinh pl$.

$I_R = I_s \cosh pl + \frac{V_s}{Z_0} \sinh pl$.

Substituting eqns (3) & (4) in Z_0, we have.

$$Z_0 = \frac{V_R}{I_R} = \frac{V_s \cosh pl + I_s Z_0 \sinh pl}{I_s Z_0 \cosh pl + \frac{V_s}{Z_0} \sinh pl}.$$

$$1 = \frac{V_s \cosh \beta l + I_s Z_0 \sinh \beta l}{I_s Z_0 \cosh \beta l + V_s \sinh \beta l}$$

$$\Rightarrow I_s Z_0 \cosh \beta l + V_s \sinh \beta l = V_s \cosh \beta l + I_s Z_0 \sinh \beta l.$$

$$\Rightarrow V_s (\sinh \beta l - \cosh \beta l). = I_s . Z_0 (\sinh \beta l - \cosh \beta l)$$

$$\boxed{Z_0 = \frac{V_s}{I_s}}$$

N the ratio of $\frac{V_s}{I_s}$ represents the input impedance (or the sending end impedance of the line.

Here $Z_0 = Z_{in}$.

Since the "input impedances of both finite and infinite line are identical, the finite line terminated in its characteristic impedance behaves as a infinite line"

Prob: A generator of 1V, 1KHz supplies power to a 100km open wire line terminated in Z_0 and having follows the line parameters are $R = 10.4\,\Omega/Km$, $L = 3.8\,mH/Km$ (10^{-3}), $C = 0.085\,\mu F/Km$ & $G = 0.8 \times 10^{-6}$ mho/km. Calculate Z_0, α, β, λ, V; Also find the received power?

Sol. ① $Z_0 = \sqrt{\frac{Z}{Y}}$ $\quad Z = R + j\omega L$ (series impedance) $\quad Y = G + j\omega C$ (shunt admittance)

② Attenuation constant (α) : [$\gamma = \alpha + j\beta$]

$$\boxed{\gamma = \sqrt{ZY}}$$

③ Wavelength (λ) $= \frac{2\pi}{\beta}$

(4) propagation velocity (v)

$$v = \frac{\omega}{\beta} = \frac{2\pi f}{\beta}$$

(5) Receiving power (P_R)

$$P_R = E_R I_R \cos\theta$$

where
E_R = Received Voltage
I_R = Receiving current

$$\boxed{I_R = I_s . e^{-\gamma l}}$$ 100 km

$$\boxed{I_s = \frac{E_s}{Z_0}}$$

from $Z_s = Z_0$. $\therefore I_s = \frac{E_s}{Z_0}$

[Given Generator Voltage = 1V]

$$\boxed{E_R = I_R Z_0}$$

Sol: (1) $Z_0 = \sqrt{\frac{Z}{Y}} \rightarrow (1)$

S.T $Z = R + j\omega L$

$= 10.4 + j2\pi f L$

$= 10.4 + j2 \times 3.14 \times 1000 \times 3.8 \times 10^{-3}$

$= 10.4 + j\,23.864$

$\therefore Z = 26.03 \angle 66.45^\circ \ \Omega/km$

Sh: Ad. $Y = G + j\omega C$

$= 0.8 \times 10^{-6} + j\,2\pi f C$

$= 0.8 \times 10^{-6} + j\,2 \times 3.14 \times 1000 \times 0.0085 \times 10^{-6}$

$= 0.8 \times 10^{-6} + j\,53.38 \times 10^{-6}$

$\therefore Y = 53.38 \times 10^{-6} \angle 89.14^\circ \ \mho/km$

Substituting the value of Z and Y in equation (1) we get

$$Z_0 = \sqrt{\frac{26.03 \angle 66.45^\circ}{53.38 \times 10^{-6} \angle 89.14^\circ}}$$

$$= 698.30 \angle \left(\frac{66.45^\circ - 89.14^\circ}{2}\right)$$

$$Z_0 = 698.30 \angle -11.345^\circ \ \Omega$$

$Z_0 = 698.30\,(\cos(-11.345) - j\sin(-11.345))$

$\therefore \boxed{Z_0 = 684.95 - j\,137.36.}$

Attenuation Constant (α) -

The expression for propagation constant is given by.

$\gamma = \alpha + j\beta$ where, α = Attenuation Constant

β = phase Constant

Substituting the value of Z and γ in $\boxed{\gamma = \sqrt{ZY}} \rightarrow (3)$ eq (2) we get.

$\gamma = \alpha + j\beta \rightarrow (2)$

$= \sqrt{ZY}$. Sub Z, Y values.

$\gamma = \sqrt{26.03\,\angle 66.45^\circ)\,(53.38\times10^{-6}\,\angle 89.14^\circ}$

$= 0.0372\,\angle \dfrac{66.45^\circ + 89.14^\circ}{2}$

$= 0.0372\,\angle 77.795^\circ$

$= 0.0372\,(\cos(77.795^\circ) + j\sin(77.795^\circ))$

$\therefore \boxed{\gamma = 0.007864 + j\,0.03635} \longrightarrow (4)$

from (3) & (4) we get.

Attenuation Constant $\rightarrow \boxed{\alpha = 0.007864 \text{ nepers/km.}}$

Phase constant (β).

Since $\gamma = \alpha + j\beta$.

from (4), β is obtained as.

$\boxed{\therefore \text{phase Constant } \beta = 0.03635 \text{ radian/km}}$

Wavelength $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\times3.14}{0.03635}$

$\boxed{\lambda = 172.764\,\text{km.}}$

propagation Velocity (v):

$$v = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2 \times 3.14 \times 10^3}{0.03635}$$

$$= 172764.788 \times 10^3$$

$$\boxed{\therefore \text{propagation Velocity } v = 172764 \text{ km/sec}}$$

Received power (P_R), of the power received @ Receiving end of the transmission line is defined as

$P = E \times I$

$P_R = E_R I_R$ Case. $\rightarrow$ (5)

Where E_R - Received Voltage I_R - Received current

of the received current I_R is given by

$$I_R = I_s e^{-\gamma l}.$$

Here the source current I_s is obtained as.

$$I_s = \frac{E_s}{Z_s}.$$

Since the line is terminated by Z_0,

E: Generator Voltage = 1V

$$Z_s = Z_0.$$

$$I_s = \frac{E_s}{Z_0} = \frac{1 \angle 0^\circ}{698.30 \angle -11.345^\circ}$$

$$\boxed{I_s = 1.43 \angle 11.345^\circ \text{ mA.}}$$

Hence, $I_R = I_s . e^{-\gamma l}$

$$= 1.43 \times 10^{-3} \angle 11.345^\circ e^{-(0.007864 + j0.03635) \times 100}$$

$$= 1.43 \times 10^{-3} \angle 11.345^\circ e^{-0.7864} e^{-j3.635}.$$

Here, $e^{-j3.635}$ represents the phase of -3.635 radians

Since 1 radian $= \frac{180}{\pi}$ degrees.

$$\therefore 3.635 \text{ radians} = \frac{180}{\pi} \times 3.635$$

$$= 208.375^\circ$$

Hence, I_R = ~~0.000651~~

$= 0.000651 \angle 11.245^\circ - 8.375^\circ$.

$I_R = 0.000651 \angle -197.03^\circ$ A.

→ The received Voltage E_R is given by,

$E_R = I_R \cdot Z_0$,

$= 0.000651 \angle -197.03^\circ \times 698.30 \angle -11.71^\circ$

$= 0.454 \angle -208.74^\circ$

From (A) we get.

$P_R = Re(0.454 \times 0.000651 \angle -208.74^\circ - 197.03^\circ)$

$= 295.5 \times 10^{-6} \cos(405.54)$

$P_R = 206.9 \times 10^{-6}$ watts

$\boxed{\therefore \text{Received power, } P_R = 206.9\ \mu W}$

Prob: A distortionless transmission line has the following parameters $Z_0 = 60\Omega$, $\alpha = 20$ Np/m and $V = 21 \times 10^7$ m/sec. Find R, L, G and C.

Ans: Given data,

For a distortionless transmission line,

Characteristic impedance $Z_0 = 60\Omega$

Attenuation constant $\alpha = 20$ Np/m.

Velocity of propagation $v = 21 \times 10^7$ m/sec.

$R, L, G, C = ?$

Resistance (R) : The condition for a distortionless line is

(1) $\frac{R}{L} = \frac{G}{C} \rightarrow (1)$.

* For a distortionless line, the attenuation constant is given by

$$\alpha = \sqrt{RG}.$$

$$= \sqrt{R \cdot \frac{CR}{L}} \qquad \left[\because G = \frac{CR}{L}\right]$$

$$= \sqrt{\frac{R^2C}{L}}$$

$$= R\sqrt{\frac{C}{L}} \Rightarrow \alpha = \frac{R}{Z_0}.$$

$$\therefore \boxed{R = \alpha \cdot Z_0} \rightarrow (2)$$

Substitute the values in (2), we get.

$$R = (20\times10^{-3})\times 60.$$

$$\boxed{R = 1.2\,m\Omega/m.}$$

(2) Inductance (L) = The expression for characteristic impedance is

$$Z_0 = \sqrt{\frac{L}{C}} \qquad \left[\because v = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{Lv^2}\right]$$

$$= \sqrt{\frac{L}{(1/Lv^2)}}$$

$$= \sqrt{L^2v^2}$$

$$\Rightarrow Z_0 = Lv.$$

$$\therefore L = \frac{Z_0}{v} \rightarrow (3)$$

$$L = \frac{60}{21\times10^{7}} = 2.8571\times10^{-7}$$

$$\therefore \boxed{L = 285.71\,nH/m.}$$

③ Capacitance (C) :

The expression for characteristic impedance is.

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$= \sqrt{\frac{1}{v^2 c^2}} \qquad \left[\because v = \frac{1}{\sqrt{LC}} \quad L = \frac{1}{v^2 c} \right]$$

$$C = \frac{1}{vc} \rightarrow ④$$

$$\therefore C = \frac{1}{60 \times 21 \times 10^{7}} = 7.936 \times 10^{-11}$$

$$\boxed{C = 79.36\ pF/m}$$

④ Conductance (G).: from eq ① $\frac{R}{L} = \frac{G}{C}$

$$G = \frac{CR}{L}$$

$$G = \frac{\alpha^2}{R} \longrightarrow ⑤$$

$$\therefore G = \frac{(20 \times 10^{-3})^2}{1.20}$$

$$= 3.333 \times 10^{-4}.$$

$$\boxed{G = 333.3\ \mu\ \text{mhos}/m.}$$

Define loading: The process of increasing the inductance of line by incorporating lumped inductors along the line at equal intervals is known as "loading"

Types:

1) Continuous loading :→ by inserting an iron core (or) magnetic tape along the conductors of transmission line.

2) patch loading :→ by using continuously loaded cables separated with unloaded cables along the transmission line.

3) Lumped loading :→ by inserting lumped inductors along the line at uniform intervals.

Discuss in detail about lumped loading & derive the Campbell's equation. 35

Lumped loading:-

* In lumped loading, the inductance of line can be increased by inserting lumped inductors along the line at uniform intervals. This phenomenon is known as lumped loading.

fig: Lumped loading.

(i) The lumped loaded lines behave as a low pass filter and this method of loading is more convenient than the continuous loading which provides a limited frequency range upto 'fc'.

(ii) The attenuation frequency characteristics of the line is shown below.

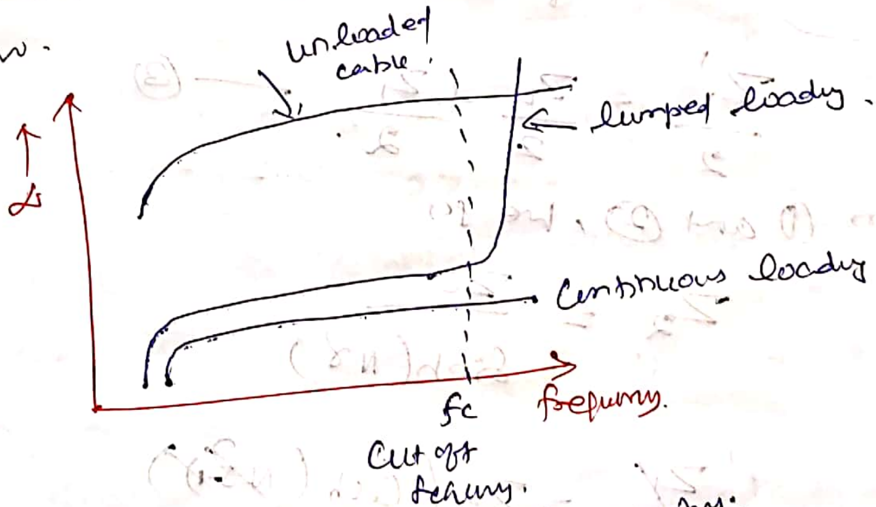


fig: Attenuation frequency characteristics.

(iii) Derivation of Campbell's equation:-

* The performance of lumped loading is analyzed by considering section of line which is symmetrically loaded with coils and the impedance of coil loaded be 'Zc'.

* This section of line is then replaced with an equivalent T-circuit of line with lumped parameters as shown below.

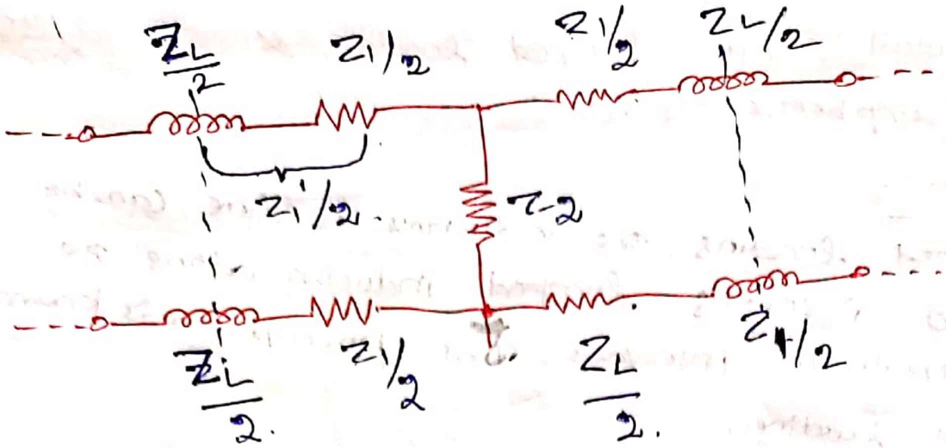


fig: Equivalent T-circuit.

* Consider a symmetric T-section of line. If the separation between series arms is 'N' then.

$$\sinh(N\gamma) = \frac{Z_0}{Z_2} \quad \text{——} ①$$

$$\cosh(N\gamma) = 1 + \frac{Z_1}{2Z_2} \quad \text{——} ②$$

* The overall impedance of the series arm of the loaded section of line is given as

$$\frac{Z_1'}{2} = \frac{Z_L}{2} + \frac{Z_1}{2} \quad \text{——} ③$$

From ① and ②, we get.

$$Z_2 = \frac{Z_0}{\sinh(N\gamma)}$$

$$\frac{Z_1}{2} = Z_2 \left(\cosh\left(\cancel{N\gamma - 1}\right)\right)$$

$$\frac{Z_1}{2} = Z_2\left(\cosh(N\gamma) - 1\right)$$

$$= \frac{Z_0}{\sinh(N\gamma)}\left(\cosh(N\gamma) - 1\right)$$

Substituting the above eqn in (3), we get.

$$\frac{Z_1'}{2} = \frac{Z_L}{2} + \frac{Z_0}{\sinh(N\gamma)}\left(\cosh(N\gamma)-1\right) \longrightarrow (4).$$

* Assuming that, the propagation constant of a loaded section is 'γ'' then,

$$\cosh(N\gamma') = 1 + \frac{Z_1'}{2Z_2}$$

From (4) and (5), we get,

$$\cosh(N\gamma') = 1 + \frac{1}{Z_2}\left[\frac{Z_L}{2} + \frac{Z_0}{\sinh(N\gamma)}\left(\cosh(N\gamma)-1\right)\right]$$

$$= 1 + \frac{Z_L}{2Z_2} + \cosh(N\gamma) - 1 \qquad \left[\because Z_2 = \frac{Z_0}{\sinh(N\gamma)}\right]$$

Hence.

$$\boxed{\cosh(N\gamma') = \cosh(N\gamma) + \frac{Z_L}{2Z_0}\sinh(N\gamma)} \longrightarrow (6).$$

* Equation (6) is known as 'campbell's equation. This equation allows to find value of γ' for section of line, which comprises lumped and distributed elements ~~[illegible]~~ partially.
* It also permits to measure the reduced attenuation and distortion caused over the line due to loading coils in the line section.

Define reflection factor (k). Give the relation between reflection factor and reflection loss. (3A)

Ans: Reflection factor (k)

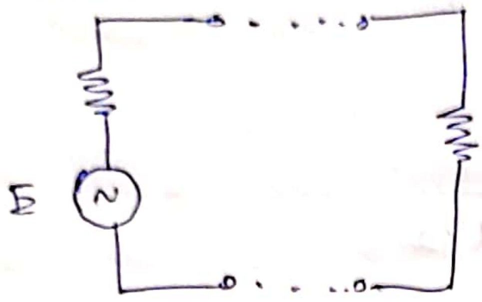


fig:: Generator of Impedance Z_0 Connected to Load Z_L.

* In fig, a transmission line having characteristic impedance Z_0, is terminated by an impedance Z_L.
* If $Z_L \neq Z_0$, it results in mismatch and the transmitted energy is reflected back.

* Due to this reflection, the transmitted energy is not fully delivered to the load. i.e. power dissipation takes place. This loss in energy is known as "Reflection loss".

* The Magnitude of reflection loss is expressed as,

$$|\Gamma_L| = \left|\frac{I_{load}}{I_{source}}\right|_{at\ z=z_0}$$

where I_{Load} = current flow in load.

I_{source} = Current flow in source

* Impedance matching is achieved by using an ideal transformer along with a lossless phase shifter between load and the source; Thus, the current ratio is expressed as,

$$\frac{I_{source}}{I_{load}} = \sqrt{\frac{Z_L}{Z_0}}$$

* Under impedance matching conditions, the current flow through the source is given as

$$I_{source} = \frac{E}{2Z_0}$$

* Then the current I_L' is expressed as.

$$I_L' = \frac{E}{2Z_0} \cdot \sqrt{\frac{Z_0}{Z_L}} = \frac{E}{2\sqrt{Z_L Z_0}}$$

* where I_L' is the current flowing in boundary transformer (or) load.

(i) When $Z_0 \neq Z_L$, the magnitude of current I_L is given as,

$$|I_L| = \frac{|E|}{|Z_0+Z_L|}$$

∴ the magnitude of current ration $\left|\frac{I_L}{I_L'}\right|$ at $Z_0 = Z_L$.

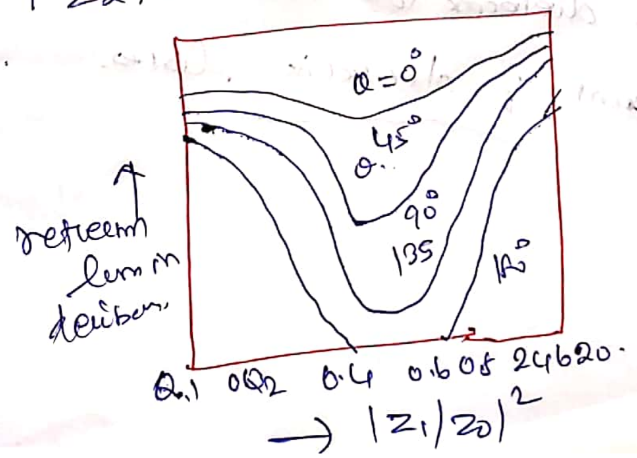
$$\left|\frac{I_L}{I_L'}\right| = \frac{\frac{|E|}{|Z_0+Z_L|}}{\frac{|E|}{|2\sqrt{Z_0Z_L}|}} = \frac{|2\sqrt{Z_0Z_L}|}{|Z_0+Z_L|}$$ //.

* This ratio represents the change in I_L due to reflection at the mis-matched junction. This is known as reflection factor 'k'.

Reflection loss:

* The below fig, represents reflection loss due to mismatch of load and source impedance.

* when $\left|\frac{Z_1}{Z_2}\right| = 1$, the curves are symmetric in logarithmic scale.



Explain the various types of losses in transmission lines. (40)

(1) Conductor heating loss

(2) Radiation loss.

(3) Dielectric heating loss.

1) Conductor heating loss:-

An electric current flowing through the transmission line having finite resistance causes a significant amount of power. This loss is known as conductor loss (or) power dissipation. This loss is known as conductor loss (or) Conductor heating loss.

→ This loss can be reduced by decreasing the length of transmission line (or) increasing the diameter of conductor.

2) Radiation loss:-

In a transmission line, if the distance of separation b/w the conductors is approximately equal to the wavelength (λ, $\lambda/2$ (or) $\lambda/4$) of the signal, then it radiates energy which results in radiation loss.

→ Usage of properly shielded cables can reduce radiation loss in transmission lines.

3) Dielectric Heating loss:-

It occurs due to generation of heat in dielectric material. This is because of the difference in voltage b/w the conductors of a transmission line.

→ Usage of free space dielectric (or) air dielectric lines have negligible amount of dielectric losses.

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Unit - 1 :

Important Questions

Part A:

1. Define characteristic impedance.
2. Find the characteristic impedance of a line at 1600 Hz if $Z_{oc} = 750 \angle -30^\circ \Omega$ and $Z_{sc} = 600 \angle -20^\circ \Omega$
3. Define propagation constant
4. State the condition for a distortionless line.
5. Give Campbell's formula for a uniformly loaded line.
6. Define reflection loss & insertion loss.

Part B

1. Derive the general transmission line equations for voltage and current at any point on a line.
2. Prove that an infinite line equal to finite line terminated in its characteristic impedance.
3. Derive the equation of attenuation constant & phase constant of transmission lines in terms of line constants R, L, C, & G
4. Explain in detail about the wave form distortion and also derive the condition for distortionless line.
5. A communication line has $L = 3.67$ mH/km, $G = 0.08 \times 10^{-6}$ mhos/km, $C = 0.0083$ μF/km and $R = 10.4$ ohms/km. Determine the characteristic impedance, propagation constant. Phase constant, velocity of propagation, sending and receiving end current for given frequency = 1000 Hz. sending end voltage is 1 volt and transmission line length is 100 km.

Unit-2. High Frequency transmission lines:

→ Transmission line equations @ Radio frequencies.
→ Line of Zero dissipation (LOZ)
→ V & I on the dissipation less line. [If the resistance of the line is negligible compare to other parameter of the line]
→ Standing wave, Standing wave Ratio, Nodes,
→ I/P Impedance of the dissipation less line.
→ Open & Short circuited lines.
→ Power & Impedance Measurement on lines.
→ Reflection loss.
→ Measurement of VSWR & wavelength.

① Write the equations of Voltage and current on the dissipation less line.

$$E = E_R \cos\left[\left(\frac{2\pi}{\lambda}\right)l\right] + jI_R R_0 \sin\left[\left(\frac{2\pi}{\lambda}\right)l\right]$$

$$I = I_R \cos\left[\left(\frac{2\pi}{\lambda}\right)l\right] + j\frac{E_R}{R_0} \sin\left[\left(\frac{2\pi}{\lambda}\right)l\right]$$

② Transmission line equation at Radio frequencies - Line of Zero dissipation - V & I on the dissipationless line.

③ Explain the parameters of Open wire and co-axial lines at radio frequencies.

① Parameters of open wire at Radio frequency:
* At radio frequencies, the skin effect is dominant and hence the current flows essentially on the surface of the conductors.
* Thus, the internal flux and internal inductance get reduced to zero.

* The inductance 'L' of an open wire is given by,

$$L = 4 \times 10^{-7} \ln\left(\frac{d}{a}\right) \text{ H/m.}$$

$$L = 9.21 \times 10^{-7} \log_{10}\left(\frac{d}{a}\right) \text{ H/m} \longrightarrow ①$$

* The capacitance 'C' of an open wire is given by.

$$C = \frac{\pi \varepsilon}{\ln(d/a)} \text{ F/m}$$

$$C = \frac{12.07}{\ln(d/a)} \mu\text{F/m.} \longrightarrow ②$$

* The effective cross section of conductor is given by

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} \text{ m.} \longrightarrow ③$$

Where μ = conductor permeability.

$\mu = 4\pi \times 10^{-7}$ H/m.

σ = conductivity of conductor.

$\sigma = 5.75 \times 10^{7}$ ℧/m.

* Copper effective thickness is given by

$$\delta = \frac{0.0664}{\sqrt{f}} \text{ meter} \longrightarrow ④$$

(i) For DC, the resistance of a round conductor with radius 'a' is given by

$$R_{dc} = \frac{K}{\pi a^2} \ \Omega/\text{m.} \quad \text{where } K = \frac{1}{\sigma} \longrightarrow ⑤$$

(ii) For AC.,

$$R_{ac} = \frac{K}{2\pi a \delta} \longrightarrow ⑥$$

Thus

$$\frac{R_{ac}}{R_{dc}} = \frac{a\sqrt{\pi f \mu \sigma}}{2.} \longrightarrow ⑦$$

For Copper,

$$\frac{R_{ac}}{R_{dc}} = 7.53 (a\sqrt{f}) \rightarrow (8)$$

From (8), it is clear that for the large radius conductors, increase in the resistance with increase in frequency is considerably large as compared to that in conductors of small radius.

(2) parameters of co-axial lines at radio frequencies.

(i) The inductance of L - m coaxial line is given by.

$$L = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right) H/m.$$

$$\therefore \boxed{L = 2\times10^{-7} \log\left(\frac{b}{a}\right) H/m.} \quad [\because \mu = 4\pi\times10^{-7} H/m]$$

Where a = outer radius of inner conductor.

b = Inner radius of outer conductor

(ii) The capacitance 'c' of a co-axial line is given by,

$$C = \frac{2\pi \epsilon}{\ln\left(\frac{b}{a}\right)} F/m.$$

$$C = \frac{55.5\epsilon}{\log(b/a)}$$

$$\left[\because \epsilon = \epsilon_0 \epsilon_r, \quad \epsilon_0 = 8.854\times10^{-12} F/m\right]$$

$$\therefore \boxed{C = \frac{24.14 \epsilon_r}{\ln\left(\frac{b}{a}\right)} pF/m.}$$

(iii) Rac of coaxial line is given by,

$$R_{ac} = 4.16 \times 10^{-8} \sqrt{f\left(\frac{1}{b}+\frac{1}{a}\right)}\ \Omega/m.$$

* If the air is used as dielectric in coaxial line, then there are no shunt loss.

* If other than air as dielectric is used such as Quartz, polyethelene etc) then conductance losses are incurred.

→ power factor of the insulating material when used as dielectric determines its quality and it can be obtained by the susceptance triangle as shown below.

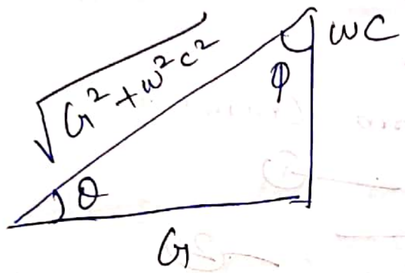

fig: Susceptance triangle.

* from the above figure, shunt susceptance is given as,

$$Y = G + jwC.$$

where

G = Conductance of insulating material.

C = Capacitance of insulating material.

→ Then, power factor is expressed as,

$$PF = \frac{G}{\sqrt{G^2+w^2C^2}}$$

⇒ For a good insulating material, conductance is very less such that $G << wC$.

$$\therefore \boxed{PF = \frac{G}{wC}}$$

$$\Rightarrow \boxed{G = wC \times PF}$$

(2) Desire the line constants of a zero dissipationless line. (5)

Ans.

Lossless transmission line :- A transmission line is said to be lossless if it satisfies the following conditions,

1) The conductors of the transmission line should be perfect ie

$$\boxed{\sigma_c = \infty.}$$

2) The dielectric medium between the transmission lines should be zero. ie $\boxed{\sigma_d = 0.}$

* The condition for lossless line is, $\boxed{R = G = 0}$.

Characterization of lossless Transmission line :-

1) Propagation constant (γ) :-

The expression for propagation constant is

$$\gamma = \alpha + j\beta. \longrightarrow (1)$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} \longrightarrow (2)$$

The condition for lossless line is $R = G = 0. \longrightarrow (3)$

On substituting eqn (3) in (2), we get,

$$\gamma = \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{(LC)}$$

$$\therefore \boxed{\gamma = j\omega\sqrt{(LC)}} \longrightarrow (4)$$

2) Attenuation constant (α) and phase constant (β) :-

* On comparing eqn (1) with (4) we get.

$$\alpha = 0.$$

$$\& \quad \beta = \omega\sqrt{LC}.$$

3) Phase velocity (V_p) :-

* The expression for phase velocity is,

$$V_p = \frac{\omega}{\beta} \longrightarrow (5)$$

(i) Substituting the value of β in (5), we get.

$$V_p = \frac{\omega}{\omega\sqrt{LC}}$$

$$\boxed{V_p = \frac{1}{\sqrt{LC}}}$$

4) Characteristic Impedance (Z_0)

→ The expression for characteristic impedance is

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \qquad [\because R = G = 0]$$

$$= \sqrt{\frac{j\omega L}{j\omega C}}$$

$$= \sqrt{\frac{L}{C}}$$

$$\therefore \boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

Prob1:- A line with zero dissipation has $R = 0.006\ \Omega/m$, $C = 4.45\ pF/m$. $L = 2.5\ \mu H/m$. find $R_0, \alpha, \beta, \lambda, V_p$. If the line is operated at 10 MHz.

Sol:

For a lossless line,

i) Characteristic impedance R_0 is expressed as $R_0 = \sqrt{\frac{L}{C}}$.

$$R_0 = \sqrt{\frac{2.5\times10^{-6}}{4.45\times10^{-12}}} \qquad \therefore \boxed{R_0 = 749.53\ \Omega}$$

(ii) For a lossless line, attenuation is zero.

Attenuation Constant $\boxed{\alpha = 0\ NP/m.}$ //

β given by

→ The Expression for Phase Constant

$$\beta = \omega\sqrt{LC}.$$

$$\beta = 2\pi f\sqrt{LC}$$

$$= 2\pi\times10\times10^{6}\sqrt{2.5\times10^{-6}\times4.45\times10^{-12}}$$

$$\therefore \beta = 0.2095$$

$$\Rightarrow \boxed{\beta = 0.21\ rad/m} \ //$$

∴ The relation between β and 'λ' is expressed as.

$$\beta = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.21} = 29.9. \quad \therefore \boxed{\lambda = 30m}$$

∴ The expression for Velocity of propagation is given by

$$v = \frac{\omega}{\beta} \text{ (or) } v = \frac{1}{\sqrt{LC}}.$$

$$v = \frac{1}{\sqrt{2.5\times10^{-6} \times 4.45\times10^{-12}}}$$

$$\boxed{V_p = 299810 \text{ km/sec}}$$

Prob2: A 50Ω coaxial cable feeds a $75+j20\Omega$ dipole antenna. Find reflection coefficient and standing wave Ratio.

Sol: Given data, $Z_0 = 50\Omega$.

Receiving end impedance $Z_R = 75+j20\Omega$

1) Reflection coefficient $\Gamma = ?$

2) Standing wave ratio, $\rho = ?$

1) Reflection coefficient, $\Gamma = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{75+j20-50}{75+j20+50}$.

$$= \frac{25+j20}{125+j20}.$$

$$\therefore \boxed{\Gamma = 0.253 \angle 29.57^\circ}$$

2) Standing wave ratio, $\rho = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.253}{1-0.253} = 1.677$

$$\therefore \boxed{\rho = 1.677}$$

Write the equations of Voltage and current on the dissipationless line. (5)

* The Voltage & current equation of dissipationless line are expressed as. (4).

$$E = E_R \cos\left[\left(\frac{2\pi}{\lambda}\right)l\right] + j I_R R_0 \sin\left[\left(\frac{2\pi}{\lambda}\right)l\right]$$

$$I = I_R \cos\left[\left(\frac{2\pi}{\lambda}\right)l\right] + j \frac{E_R}{R_0} \sin\left[\left(\frac{2\pi}{\lambda}\right)l\right].$$

Write the expressions for input impedance of $\lambda/2$, $\lambda/4$ & $\lambda/8$ lines.

* The expression for input impedance of,

1) $\lambda/2$ line, $Z_{in} = Z_L$.

2) $\lambda/4$ line, $Z_{in} = \frac{Z_0^2}{Z_L}$

3) $\lambda/8$ line, $Z_{in} = Z_0$.

Write the expression for power of a dissipationless line?

* The general expression for power of a dissipationless line is given as,

$$P = \frac{|E_{max}||E_{min}|}{R_0} \quad (or) \quad P = |I_{max}||I_{min}| \cdot R_0.$$

Prob) Calculate Standing wave Ratio and Reflection Coefficient on a line having the characteristic impedance $Z_0 = 300\Omega$ and terminating impedance in $Z_R = 300 + j400\Omega$.

1) Reflection Coefficient $\Gamma = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{300 + j400 - 300}{300 + j400 + 300} = \frac{j400}{600 + j400}$

$$= \frac{400\angle 90}{721.11\angle 33.69} = 0.55\angle 56.31^\circ \quad \therefore \boxed{\Gamma = 0.55\angle 56.31}$$

2) SWR: $\rho = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.55}{1 - 0.55} = 3.44.$

$$\therefore \boxed{\rho = 3.44}$$

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Prob: A 50Ω lossless transmission line is connected to a load composed of a 75Ω resistor in series with a capacitor of unknown capacitance. If at 10 MHz the voltage standing wave ratio on the line was measured as 3, determine the capacitance 'C'? (9)

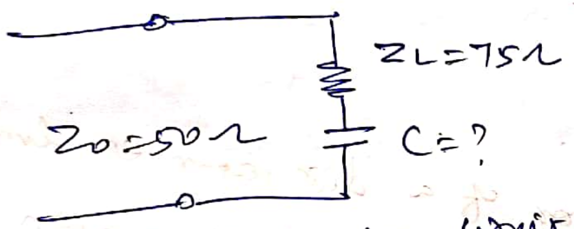
Given that, A 50Ω lossless transmission line is connected to a load of 75Ω resistor in series with a capacitor of unknown capacitance, i.e. characteristic impedance $Z_0 = 50\Omega$.

Load impedance $Z_L = 75\Omega$

Operating frequency $f = 10$ MHz. VSWR of line $S = 3$.

Capacitance = ?

→ The circuit of the given transmission line will be as



* The capacitance of the circuit can be obtained from load impedance Z_L, i.e. where $X_C = \frac{1}{\omega C}$ → (1) ⇒ $\boxed{C = \frac{1}{\omega X_C}}$ → (2)

* The reflection co-efficient of the transmission line is obtained as

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = \frac{2}{4} = 0.5$$

* Γ in terms of impedance is expressed as:

$$|\Gamma| = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma^2 = \left[\left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)\left(\frac{Z_L^* - Z_0}{Z_L^* + Z_0}\right)\right]$$

$$\Gamma^2 = \frac{Z_L Z_L^* + Z_0^2 - Z_0(Z_L + Z_L^*)}{Z_L Z_L^* + Z_0^2 + Z_0(Z_L + Z_L^*)} \rightarrow (3)$$

From equation (1)

$$Z_L = R_L - jX_C.$$

$$\Rightarrow |Z_L| = \sqrt{R_L - jX_C}$$

$$|Z_L^2| = Z_L Z_L^* = R_L^2 + X_C^2 \rightarrow (4)$$

$$Z_0(Z_L + Z_L^*) = Z_0(R_L - jX_C + R_L + jX_C)$$

$$\Rightarrow Z_0(Z_L + Z_L^*) = 2Z_0 R_L \rightarrow (5)$$

On substituting eqn (4) and (5) in (2), we get.

$$|\Gamma^2| = \frac{R_L^2 + X_C^2 + Z_0^2 - 2Z_0 R_L}{R_L^2 + X_C^2 + Z_0^2 + 2Z_0 R_L} \rightarrow (6)$$

substituting $\Gamma = 0.5$, $Z_L = 75\Omega$ and $Z_0 = 50\Omega$ in above eqn, we get.

$$|0.5|^2 = \frac{75^2 + X_C^2 + 50^2 - (2 \times 50 \times 75)}{75^2 + X_C^2 + 50^2 + (2 \times 50 \times 75)}$$

$$0.25 = \frac{625 + X_C^2}{15625 + X_C^2}.$$

$$\Rightarrow (X_C^2 + 15625)0.25 = 625 + X_C^2$$

$$\Rightarrow 0.25X_C^2 + 3906.25 = 625 + X_C^2$$

$$\Rightarrow 3906.25 - 625 = X_C^2 - 0.25X_C^2$$

$$\Rightarrow 0.75X_C^2 = 3281.25$$

$$X_C^2 = \frac{3281.25}{0.75} = 4375.$$

$$\therefore X_C = \sqrt{4375} = 66.14 \quad \therefore \boxed{X_C = 66.1\Omega}$$

From eq(2), the Capacitance is expressed as

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi f \times X_C}$$

$$= \frac{1}{2\pi \times 10^7 \times 66.1}$$

$$= 2.41 \times 10^{-10}$$

$$\therefore \boxed{C = 241\ pF}$$ //.

Discuss in detail about the Voltage and currents on the dissipationless line.

Voltage and currents on dissipationless line;

Equation for Voltage at any point at a length of 'l' units from receiving end of transmission line is

$$E = \frac{E_R}{2Z_R}(Z_R+Z_0)\left(e^{\gamma l} + \Gamma e^{-\gamma l}\right)$$

$$E = \frac{I_R}{2}(Z_R+Z_0)\left(e^{\gamma l} + \Gamma e^{-\gamma l}\right) \quad \left[\because \frac{E_R}{Z_R} = I_R\right]$$

For zero dissipation line,

$$Z_0 = R_0, \quad \gamma = j\beta, \quad \alpha = 0.$$

$$\therefore E = \frac{I_R}{2}(Z_R+Z_0)\left(e^{-j\beta l} + \Gamma e^{-j\beta l}\right) \longrightarrow (1)$$

$$\left[\because \Gamma = \frac{Z_R - Z_0}{Z_R + Z_0}\right]$$

$$= \frac{I_R}{2}(Z_R+R_0)e^{j\beta l} + \frac{I_R}{2}(Z_R - R_0)e^{-j\beta l}$$

$$= \frac{I_R}{2}\left[Z_R e^{j\beta l} + R_0 e^{j\beta l} + Z_R e^{-j\beta l} - R_0 e^{-j\beta l}\right]$$

$$= \frac{I_R}{2}\left\{Z_R\left[e^{j\beta l} + e^{-j\beta l}\right) + R_0\left(e^{j\beta l} - e^{-j\beta l}\right)\right\}$$

$$= I_R \left[Z_R \left(\frac{e^{j\beta l} + e^{-j\beta l}}{2} \right) \right] + j R_o \left[\frac{e^{j\beta l} - e^{-j\beta l}}{2j} \right] \quad (1)$$

$$= I_R Z_R \cos(\beta l) + j I_R R_o \sin(\beta l)$$

$$\boxed{\therefore E = E_R \cos(\beta l) + j I_R R_o \sin(\beta l)} \longrightarrow (2)$$

III^{ly}, equation for current at any point from receiving end of transmission line is given as,

$$I = \frac{I_R}{2Z_o}(Z_R + Z_o)(e^{\gamma l} - \Gamma e^{-\gamma l})$$

For zero dissipation line, $Z_o = R_o$, $\gamma = j\beta$, $\alpha = 0$.

$$I = \frac{I_R}{2R_o}(Z_R + R_o)\left[e^{j\beta l} - \left[\frac{Z_R - R_o}{Z_R + R_o} \right] e^{-j\beta l} \right]$$

$$= \frac{I_R}{2R_o}(Z_R + R_o) e^{j\beta l} - \frac{I_R}{2R_o}(Z_R - R_o) e^{-j\beta l}$$

$$= \frac{I_R}{2R_o}\left[Z_R e^{j\beta l} + R_o e^{j\beta l} - Z_R e^{-j\beta l} + R_o e^{-j\beta l} \right]$$

$$= \frac{I_R}{2R_o}\left[Z_R (e^{j\beta l} - e^{-j\beta l}) + R_o (e^{j\beta l} + e^{-j\beta l}) \right]$$

$$= \frac{I_R}{R_o}\left[R_o \left(\frac{e^{j\beta l} + e^{-j\beta l}}{2} \right) + j Z_R \left(\frac{e^{j\beta l} - e^{-j\beta l}}{2j} \right) \right]$$

$$\boxed{\therefore I = I_R \cos(\beta l) + j \frac{E_R}{R_o} \sin(\beta l)} \longrightarrow (3)$$

Since $\beta = \frac{2\pi}{\lambda}$, eqns (2) and (3) can be written as,

$$E = E_R \cos\left(\frac{2\pi}{\lambda} l\right) + j I_R R_0 \sin\left(\frac{2\pi}{\lambda} l\right)$$

$$I = I_R \cos\left(\frac{2\pi}{\lambda} l\right) + j \frac{E_R}{R_0} \sin\left(\frac{2\pi}{\lambda} l\right)$$

Case 1: If the dissipation less line is open circuited, then $I_R = 0$ and equation (4) & (5) are rewritten as

$$\boxed{E_{oc} = E_R \cos\left(\frac{2\pi}{\lambda} l\right) \qquad I_{oc} = j \frac{E_R}{R_0} \sin\left(\frac{2\pi}{\lambda} l\right)}$$

Case 2: If the dissipation less line is short circuited, then $E_R = 0$ and equation for E and I are expressed as

$$\boxed{E_{sc} = j I_R \cdot R_0 \sin\left(\frac{2\pi}{\lambda} l\right) \qquad I_{sc} = I_R \cos\left(\frac{2\pi}{\lambda} l\right)}$$

Derive an expression for the input impedance of a dissipationless line and also find the input impedance is maximum and minimum at a distance 's'.

* Write about impedance of a dissipationless line part.

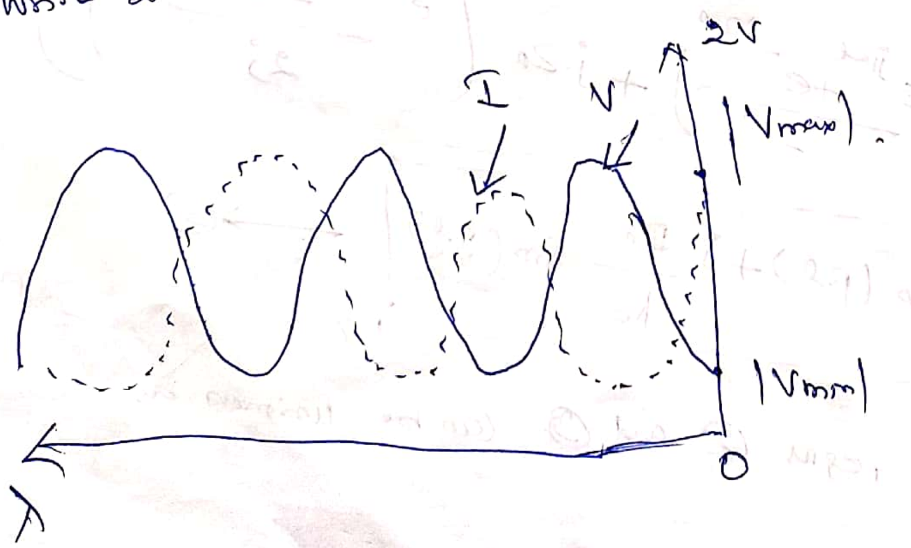


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Figure shows the standing wave due to complex load Z_L. (17)

* Impedance at a voltage minimum and voltage maximum:

i) For a lossless line, the impedances at a voltage minimum and at a voltage maximum are purely real with real characteristic impedance Z_0.

ii) At a voltage maxima (or current minima.

$$Z_{in} = Z_{max}.$$

$$= \frac{V_{max}}{I_{min}} = Z_0 \left[\frac{1+|\Gamma_L|}{1-|\Gamma_L|} \right]$$

$$\boxed{Z_{max.} = Z_0 \cdot \rho}$$

Where ρ - standing wave ratio.

iii) At a voltage minimum (or current maximum,

$$Z_{in} = Z_{max}.$$

$$= \frac{V_{min}}{I_{max}}.$$

$$= Z_0 \left[\frac{1-|\Gamma_L|}{1+|\Gamma_L|} \right] = \frac{Z_0}{\rho}.$$

$$\therefore \boxed{Z_{min} = \frac{Z_0}{\rho}}$$

Derive the expression for impedance measurement on a line.

* The load impedance Z_L of a transmission line can be easily calculated with the use of standing wave ratio value on a slotted line (or) open wire.

* The load impedance Z_L in terms of standing wave ratio is calculated as follows,

* The input impedance of a transmission line is given as,

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]$$

$$= Z_0 \left[\frac{Z_L + jZ_0 \tan\left(\frac{2\pi l}{\lambda}\right)}{Z_0 + jZ_L \tan\left(\frac{2\pi l}{\lambda}\right)} \right] \rightarrow (1) \quad \left[\because \beta = \frac{2\pi}{\lambda} \right]$$

(i) At voltage minimum point, the input impedance is given by as,

$$Z_{in} = \frac{Z_0}{\rho} \rightarrow (2)$$

Where ρ – standing wave ratio

Equating equations (1) and (2) we get,

$$\frac{Z_0}{\rho} = Z_0 \left[\frac{Z_L + jZ_0 \tan\left(\frac{2\pi l}{\lambda}\right)}{Z_0 + jZ_L \tan\left(\frac{2\pi l}{\lambda}\right)} \right]$$

(ii) At voltage minimum, the distance is l' from the load, then the above equation becomes.

$$\frac{1}{\rho} = \left[\frac{Z_L + jZ_0 \tan\left(\frac{2\pi l'}{\lambda}\right)}{Z_0 + jZ_L \tan\left(\frac{2\pi l'}{\lambda}\right)} \right]$$

$$\Rightarrow Z_0 + jZ_L \tan\left(\frac{2\pi l'}{\lambda}\right) = \rho \left[Z_L + jZ_0 \tan\left(\frac{2\pi l'}{\lambda}\right) \right]$$

$$\Rightarrow Z_0 - j\rho Z_0 \tan\left(\frac{2\pi l'}{\lambda}\right) = \rho Z_L - jZ_L \tan\left(\frac{2\pi l'}{\lambda}\right)$$

$$Z_0 \left[1 - j\rho \tan\left(\frac{2\pi l'}{\lambda}\right) \right] = Z_L \left[\rho - j\tan\left(\frac{2\pi l'}{\lambda}\right) \right]$$

$$\therefore \boxed{Z_L = Z_0 \left[\frac{1 - j\rho \tan\left(\frac{2\pi l'}{\lambda}\right)}{\rho - j\tan\left(\frac{2\pi l'}{\lambda}\right)} \right]}$$

Derive an expression for power and find the input impedance of dissipation less line, when the load is short circuited, open circuited and for a matched line. (16)

Ans: Expression for Average i/p power
i) The Voltage equation for the dissipation less line is

$$E = \frac{E_R}{2Z_R}(Z_R+Z_0)\left(e^{j\beta l}+\Gamma e^{-j\beta l}\right)$$

where R_0 - characteristic impedance

$$E = \frac{I_R}{2}(Z_R+R_0)\left(e^{j\beta l}+\Gamma e^{-j\beta l}\right) \longrightarrow (1)$$

Eq (1) can also be written as,

$$E = \frac{I_R}{2}\left[Z_R+R_0\right]\left(\left(1+|\Gamma|\angle\phi-2\beta l\right)\right) \longrightarrow (2)$$

III^{ly}

ii) Current equation for dissipation less line is

$$I = \frac{I_R}{2R_0}(Z_R+Z_0)\left(1-|\Gamma|\angle\phi-2\beta l\right) \longrightarrow (3)$$

iii) The maximum voltage and current equations are given by

$$E_{max} = \frac{I_R}{2}(Z_R+Z_0)\left(1+|\Gamma|\right)$$

$$I_{max} = \frac{I_R}{2R_0}(Z_R+Z_0)\left(1+|\Gamma|\right)$$

* The impedance R_0, is obtained on dividing E_{max} and I_{max} as,

$$R_0 = \frac{E_{max}}{I_{max}} \longrightarrow (4)$$

* The minimum voltage and current values are obtained when the reflected wave is phase reversed. Hence equation for minimum voltage and current are given as,

$$E_{min} = \frac{I_R}{2}(Z_R + R_0)(1 - |\Gamma|)$$

$$I_{min} = \frac{I_R}{2R_0}(Z_R + R_0)(1 - |\Gamma|)$$

* The impedance R_0 is obtained on dividing E_{min} and I_{min} as

$$R_0 = \frac{E_{max}}{I_{min}} \longrightarrow (5)$$

$$\Rightarrow \frac{E_{max}}{I_{min}} = \frac{\frac{I_R}{2}(Z_R + R_0)(1 + |\Gamma|)}{\frac{I_R}{2R_0}(Z_R + R_0)(1 - |\Gamma|)}$$

(1)

$$= R_0 \frac{(1 + |\Gamma|)}{(1 - |\Gamma|)}$$

$$\therefore R_{max} = R_0 \rho \rightarrow (6) \left[\because \frac{E_{max}}{I_{min}} = R_{max}\right]$$

|||ly

(2)

$$\frac{E_{min}}{I_{max}} = \frac{\frac{I_R}{2}(Z_R + R_0)(1 - |\Gamma|)}{\frac{I_R}{2R_0}(Z_R + R_0)(1 + |\Gamma|)}$$

$$= R_0 \frac{(1 - |\Gamma|)}{(1 + |\Gamma|)}$$

$$\therefore R_{min} = \frac{R_0}{\rho} \rightarrow (8) \left[\because \frac{E_{min}}{I_{max}} = R_{min}\right]$$

(a) At maximum Voltage, power flowing through Voltage loop D

$$P = \frac{E^2_{max}}{R_{max}}.$$

(b) At minimum Voltage, Power flowing through Current loop B

$$P = \frac{E^2_{min}}{R_{min}}.$$

Then $$P^2 = \frac{E^2_{max}}{R_{max}} \times \frac{E^2_{min}}{R_{min}}.$$

$$\therefore P = \sqrt{\frac{E^2_{max} \times E^2_{min}}{R_{max} \times R_{min}}} \longrightarrow (7)$$

Substituting eqns (6) and (7) in above eqn, we get

$$P = \frac{E_{max} \times E_{min}}{\sqrt{R_0 \cdot \rho \times \frac{R_0}{\rho}}}$$

$$\boxed{P = \frac{|E_{max}|\,|E_{min}|}{R_0}} \longrightarrow (8).$$

Eq (8) can also be expressed as

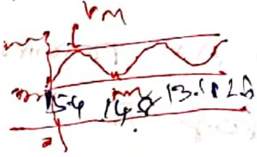
$$\boxed{P = |I_{max}|\,|I_{min}| \cdot R_0} \longrightarrow (9)$$

$\therefore$ (8) and (9) are the expressions that permit easy measurement of power flow on a line of negligible losses.

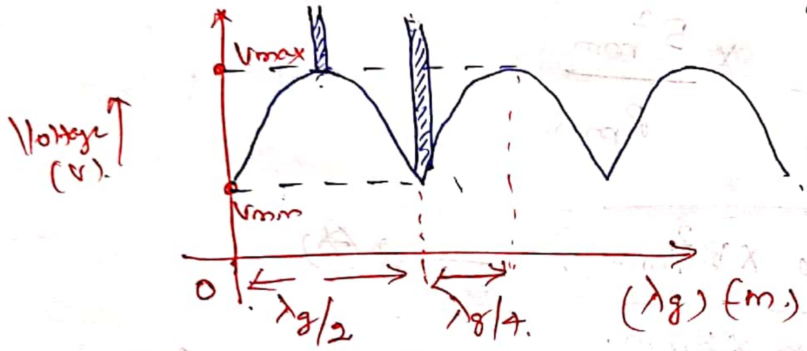
Describe an experimental setup for the determination of VSWR and length of RF Transmission.

Ans.

Voltage Standing wave Ratio :- Voltage standing wave ratio is defined as the ratio of maximum to minimum voltage on a line having standing waves.



ie $S = VSWR = \left| \frac{V_{max}}{V_{min}} \right|$.



* figure shows the standing waves along the length of the line, which are due to the mismatch of load at the termination.

* The expression for VSWR is given by,

$$S = \left| \frac{V_{max}}{V_{min}} \right| = \left[\frac{1+|\Gamma|}{1-|\Gamma|} \right]$$

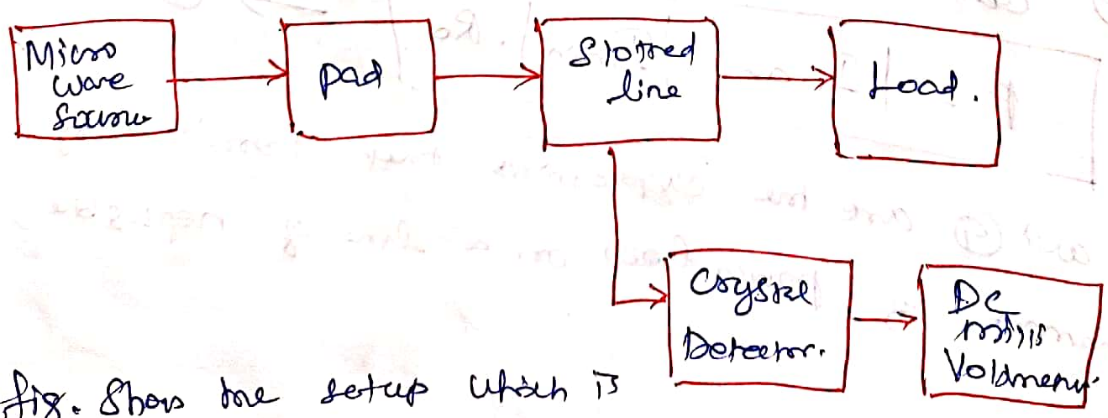
where Γ = Reflection coefficient, $= \frac{V_r}{V_i}$

When $\Gamma = 0$, $S = 1$

When $\Gamma = 1$ $S = \infty$

When $\Gamma = \infty$, $S = 1$.

(1) Low VSWR Measurement ($S < 10$) :

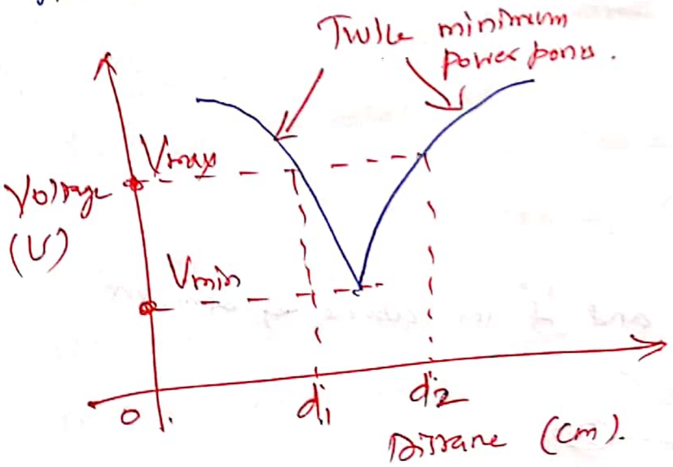


* fig. shows the setup which is used to measure low VSWR, ie less than and the readings are directly taken from the VSWR meter.

* In this method of measurement, an adequate reading on DC Millimeter is taken by simply adjusting the attenuator. (207)
* The maximum reading on the meter can be obtained by moving the probe on slotted wave guide. ie V_{max}.
* Next, the minimum reading on the meter ie V_{min} is obtained by adjusting the probe on the slotted line.
→ Thus, the ratio of first reading (V_{max}) to the second reading ie (V_{min}) gives VSWR.

(3) High SWR measurement ($S > 10$):.

* The method which is used to measure high VSWR ie greater than 10 is called "double minimum method".
* In this method, the minimum deflection on the VSWR meter can be read by inserting the probe to the required depth.
* Assuming d_1 is the position where the voltage is twice the minimum, which is obtained by probe.
* The probe is again moved in such a way to obtain the voltage that is twice the minimum and the distance is noted as d_2 shown below.



* Then VSWR can be calculated using the formula,

$$S = \frac{\lambda_g}{\pi(d_2 - d_1)}$$

where λ_g = guided wavelength

$$\lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

Cut off wavelength $\lambda_c = 2a$ (for TE_{10} mode).

Operating wavelength $\lambda_0 = \frac{c}{f}$.

Measurement of wavelength.

* Lecher measurements are used to calculate the wavelength of the dissipation less line (or) open wire
* This measurement gives the half wavelength which is equal to the distance b/w the successive maxima (or) minima of current (or) voltages.
* The setup used for this measurement is same as the VSWR Measurement.

Prob: At a frequency of 80 MHz, a lossless transmission line has a characteristic impedance of 300 Ω and a wavelength of 2.5 m. Find L and 'C'

Sol: Given that, For a lossless transmission line.

Characteristic impedance $Z_0 = 300\,\Omega$, wavelength $\lambda = 2.5\,m$.

$f = 80\,MHz$ Inductance $L = ?$ capacitance, $c = ?$

Characteristic impedance, $Z_0 = \sqrt{\frac{L}{C}} = 300\,\Omega$.

$$\sqrt{\frac{L}{c}} = 300\,\Omega.$$

$$\frac{L}{c} = 90{,}000 \quad \text{—— (1)}$$

wavelength $\lambda = \frac{2\pi}{\beta} =$

$$\lambda = \frac{1}{f\sqrt{LC}}$$

$$\sqrt{LC} = \frac{1}{f\lambda}$$

Substituting the value of λ and 'f' in above equation, we get,

$$\sqrt{LC} = 5 \times 10^{-9} \quad \text{—— (2)}$$

Solving eq (1) and (2) we get $L = 1.5\,\mu H/m$.

$c = 0.016\,nF/m$.

Prob: A loss line has a standing wave ratio of 4, the R_0 is 150Ω and the maximum voltage measured in the line is 135 V. Find the power delivered to the load.

Sol: Given that, For a lossless line.

Standing wave ratio, $\rho = 4$.

Characteristic impedance $R_0 = 150\Omega$.

maximum voltage $V_{max} = 135$ V.

power delivered to load $P = ?$

* power delivered to load for a lossless line is given by

$$P = \frac{V^2_{max}}{R_{max}}$$

where R_{max} = maximum impedance.

$R_{max} = \rho \cdot R_0$

$= 4 \times 150$

$\boxed{R_{max} = 600\Omega}$

$\therefore P = \frac{135 \times 135}{600}$

$\boxed{P = 30.375\ W}$ //

$\therefore$ power delivered to load = 30.4 watts.

Prob: A radio frequency line with $Z_0 = 70\Omega$ is terminated by $Z_L = 115 - j80\Omega$ at $\lambda = 2.5$ m. Find the VSWR and the maximum and minimum line impedances.

Ans: Given that, For a transmission line,

Characteristic impedance $Z_0 = 70\Omega$

Load impedance $Z_L = 115 - j80\Omega$

Wavelength $\lambda = 2.5$ m. Voltage standing wave ratio, $S = ?$

maximum line impedance $Z_{max} = ?$

minimum line impedance, $Z_{min} = ?$

Reflection co-efficient (Γ) is given by,

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

(2)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}.$$

$$= \frac{115 - j80 - 70}{115 - j80 + 70}$$

$$= \frac{91.7877 \angle -60.64^\circ}{201.5564 \angle -23.38^\circ}$$

$$\therefore \Gamma = 0.4553 \angle -37.26^\circ \quad \text{—— (1)}$$

(1) Voltage Standing wave Ratio (S):

VSWR is given by the expression $S = \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$

Substituting the value of Γ from (1), we get:

$$S = \frac{1 + 0.4553}{1 - 0.4553} \qquad \boxed{S = 2.6717}$$

(2) Maximum line impedance (Z_{max}):

Maximum input impedance is given by.

$$Z_{max} = Z_0 \times S.$$

$$\therefore Z_{max} = 70 \times 2.6717$$

$$\boxed{Z_{max} = 187.02\,\Omega}$$

(3) Maximum Input impedance (Z_{min}):

$$Z_{min} = \frac{Z_0}{S} = \frac{70}{2.6717} = 26.20\,\Omega$$

$$\boxed{Z_{min} = 26.20\,\Omega}$$

Prob: A 30 m long lossless transmission line with $Z_0 = 50\Omega$ operating at 2 MHz is terminated with a load $Z_L = 60 + j40$. If $u = 0.6c$ ('c' is velocity of light, u is phase velocity) on the line, find (a) the reflection coefficient Γ.
(b) the standing wave ratio 's'
(c) the input impedance.

26

Ans: Here 'u' is referred as V_p.

Given that, For a lossless transmission line of length of 30 m.

characteristic impedance $Z_0 = 50\Omega$

Load impedance $Z_L = 60 + j40$.

Operating frequency $f = 2$ MHz. and $V_p = u = 0.6c$.

where $c = 3 \times 10^8$ m/s.

Then (a) Reflection co-efficient, $\Gamma = ?$

(b) Standing wave ratio $s = ?$

(c) input impedance $Z_{in} = ?$

(a) Reflection co-efficient (Γ): in terms of Z_0 and Z_L

The reflection co-efficient of a line is given by,

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{60 + j40 - 50}{60 + j40 + 50} = \frac{10 + j40}{110 + j40}$$

$$= \frac{1 + j4}{11 + j4} = \frac{\sqrt{17}\,\angle 75.96^\circ}{\sqrt{137}\,\angle 19.98^\circ}$$

$$\therefore \boxed{\Gamma = 0.3522\angle 55.85^\circ}$$

(b) Standing wave Ratio (s):

$$s = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.3522}{1 - 0.3522} = \frac{1.3522}{0.6478}$$

$$\therefore \boxed{s = 2.09}$$

(c) Input impedance: (Z_{in})

* The expression for input impedance of a lossless line is defined as

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh L} \right]$$

* For a lossless line, $\gamma = j\beta$, hence

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]$$

Here $\beta = \frac{\omega}{V_p} = \frac{2\pi f}{0.6c}$

$$= \frac{2\pi \times 2 \times 10^6}{0.6 \times 3 \times 10^8}$$

$$\boxed{\therefore \beta = 0.07 \text{ rad/m}}$$

Hence,

$$Z_{in} = 50 \left[\frac{60 + j40 + j50 \tan(0.07 \times 30)}{50 + j(60 + j40) \tan(0.07 \times 30)} \right]$$

$$= 50 \left[\frac{60 + j40 + j1.833}{50 + j2.2 - 1.467} \right]$$

$$= 50 \left[\frac{60 + j41.833}{48.533 + j2.2} \right]$$

$$= 76.11 \quad 34.88 - 2.59$$

$$\boxed{\therefore Z_{in} = 76.11 \angle 32.28^\circ}$$

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Prob: A lossless line in air having a characteristic impedance of 300 Ω is terminated in unknown impedance. The first voltage minimum is located at 15cm from the load. The standing wave ratio is 3.3. Calculate the wavelength & terminated impedance. (26)

Sol: Given that, For a lossless line,

* Characteristic impedance $Z_0 = 300\,\Omega$
* First voltage minimum occurs at $y_{min} = 15\text{cm}$ from the load.
* Standing wave ratio, $\rho = 3.3$

Wavelength $\lambda = ?$

Terminated impedance, $Z_R = ?$

i) The expression for voltage minimum is given as,

$$2\beta y_{min} - \theta = \pi$$

$$2\beta y_{min} = \pi \quad \left[\because \theta = 0, \text{ no phase angle for reflection Co-efficient}\right]$$

$$y_{min} = \frac{\pi}{2\beta}$$

$$= \frac{\pi}{2 \times \frac{2\pi}{\lambda}}$$

$$\Rightarrow y_{min} = \frac{\lambda}{4}$$

$$0.15 = \frac{\lambda}{4}$$

$$\therefore \boxed{\lambda = 0.6\,m}$$

ii) The expression for terminated impedance is given as,

$$Z_R = Z_0\left[\frac{1+\Gamma}{1-\Gamma}\right]$$

Where Γ – reflection Co-efficient

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Γ is expressed as,

$$\Gamma = \frac{\rho - 1}{\rho + 1} = \frac{3.3 - 1}{3.3 + 1}$$

$$\boxed{\Gamma = 0.534}$$

Substituting the corresponding values in Z_R, we get,

$$Z_R = 300 \left[\frac{1 + 0.534}{1 - 0.534} \right]$$

$$\therefore \boxed{Z_R = 987.55\ \Omega} \text{ Ans}$$

2m

① What are the assumptions to simplify the analysis of line performance at high frequencies?

→ The following are assumptions made to simplify the analysis of line performance at high frequency.

① The skin effect should be significant, such that the current flows only on the surface of conductor and internal inductance is zero.

② $\omega L \gg R$ while measuring impedance.

③ Conductance of line is zero.

② List parameters of the open wire line at high frequency?

→ The following are the parameters of the open wire line at high frequency.

1) Inductance $'L' = \mu_0 \ln\left(\frac{d}{a}\right)$ H/m.

2) Capacitance $'C' = \frac{\pi \epsilon}{\ln(d/a)}$ μF/m.

③ Effective cross section of conductor $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$ m.

4) Resistance of a round Conductor for direct current $R_{dc} = \frac{K}{\pi a^2} \ \Omega/m$.

5) Resistance of a round Conductor for alternating current $R_{ac} = \frac{K}{2\pi a \delta} \ \Omega/m$.

③ Give the minimum and maximum value of SWR and reflection co-efficient.

* The minimum and maximum value of SWR and reflection coefficient are as follows.

i) For $SWR_{(min)} = 1$, $\Gamma_{min} = 0$ and.

ii) For $SWR_{(max)} = \infty$, $\Gamma_{max} = 1$.

④ Write the expression for standing wave ratio in terms of reflection co-efficient.

* The expression for standing wave in terms of reflection co-efficient is given by

$$\rho = \frac{1+|\Gamma|}{1-|\Gamma|}$$

where ρ – standing wave ratio.
Γ – Reflection co-efficient

⑤ What are nodes and antinodes on a line?

Node: A point on a wave at which, amplitude is zero is referred as 'node'.

Anti node: A point on a wave at which, amplitude is maximum is referred as 'antinode'.

⑥ Define Standing wave Ratio?

SWR: It is defined as the ratio of magnitude of maximum voltage (or current) to magnitudes of minimum voltage (or current). It is abbreviated as 'SWR', and denoted by 'ρ'.

$$\boxed{\therefore \rho = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}}}$$

⑦ What is mean by Standing wave ?

* Consider two waves travelling on a transmission line with same frequency but in opposite direction. When these waves collide with each other, a standing wave pattern of voltage and current is generated i.e. the incident & reflected waves together form a "standing wave".

⑧ What is mean by Reflection loss.

* Consider a line having characteristic impedance (Z_0), this line is terminated by an impedance Z_L.

* If $Z_L \neq Z_0$, it results in mismatch and incident wave is reflected back.

→ Due to this reflection, the transmitted power is not fully delivered to the load.

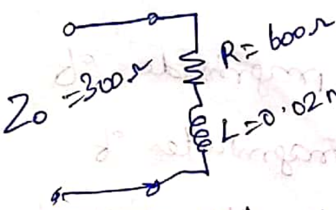
i.e. power dissipation takes place.

∴ This loss in power is termed as "Reflection loss".

⑨ What is dissipation less line?

A line for which the effect of resistance R is completely neglected is called dissipation less line.

⑩ A 300Ω lossless air transmission line is connected to a complex load composed of a resistor in series with an inductor at 5 MHz. Determine ⓐ Γ ? ⓑ S ?



ⓐ (i) $Z_L = R + j\omega L$

$= 600 + j2\pi \times 5 \times 10^6 \times 2 \times 10^{-5}$

$\boxed{Z_L = 600 + j628\,\Omega}$

(ii) $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{600 + j628 - 300}{600 + j628 + 300}$

$\boxed{\Gamma = \dfrac{300 + j628}{900 + j628} = 0.63\, e^{j29.6^\circ}}$

ⓑ $S = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = \dfrac{1 + 0.63}{1 - 0.63}$

$\boxed{S = 4.405}$

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Unit-2 High frequency transmission lines :- (30)

2 marks :-

(1) Give the minimum and maximum values of SWR and Reflection Co-efficient. (P-28)

(2) Write the expression for standing wave ratio in terms of reflection co-efficient.

(3) What do the Nodes and antinodes on a standing wave represent?

(4) Write the equations of voltage and current on the dissipationless line. (P-1)

(5) Write the expression for the input impedance of open and short circuited dissipationless line.

(6) Write the expression for power of a dissipationless line (P-8)

(7) For the line of zero dissipation, what will be the value of attenuation constant and characteristic impedance [P-5]

(8) List the parameters of the open wire line at high frequencies. (P-27)

(9) What is mean by Reflection loss?

(10) What is mean by standing wave?

12 marks :-

(1) Derive the transmission line equations at radio frequencies. (openwire & co-axial line parameters) (P-1)

(2) Discuss in detail about the voltage and currents on the dissipationless line. (P-11)

(3) Derive the expression for power and find the input impedance of dissipationless line, when the load is short circuited, open circuited and for a matched line. (P-16)

(4) (i) Derive the expression for impedance measurement on a line.
(ii) Describe the experimental setup for the determination of VSWR & length of RF Transmission.

(5) A 30m long lossless transmission line, with $Z_0 = 50\Omega$ operating at 2 MHz is terminated with a load $Z_L = 60 + j40$.
If $u = 0.6c$ (c is velocity of light on the line, u is phase velocity)

Find (a) Reflection co-efficient (Γ)

(b) Standing wave Ratio (S)

(c) The Input impedance (Z_{in})

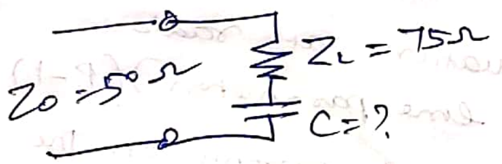
(P-24)

(6) A lossless line in air having a characteristic impedance of 300Ω is terminated in unknown impedance, the first voltage minimum is located at 15cm from the load. The standing wave ratio is 3.3. Calculate the wave length & Terminated impedance.

(P-26)

(7) A 50Ω lossless transmission line is connected to a load composed of 75Ω resistor in series with a capacitor of unknown capacitance. If at 10 MHz, the voltage standing wave ratio on the line was measured as '3'. Determine the capacitance 'C'?

(P-9)



(8) (i) At a frequency of 80 MHz, a lossless transmission line has a characteristic impedance of 300Ω and a wavelength of 25m. Find 'L' & 'C'.

(ii) A loss line has a standing wave ratio of '4'. The R_0 is 150Ω and the maximum voltage measured in the line is 135V. Find the power delivered to the load.

(P-21-22)

Unit-3 ::

Impedance matching in High frequency lines ::

(i) Impedance matching :. Quarter wave transformer - Impedance matching by stubs.

(ii) Single stub & Double stub matching.

(iii) Smith chart - Solution of problems using Smith chart.

(iv) Single & double stub matching using Smith chart

Q. Explain the technique of impedance matching by stubs & discuss the operation of QWT.

Impedance matching is defined as the process of designing the input impedance and output impedance of an electrical load to minimize the signal reflection (or) maximize the power transfer of the load. (∴ By making load impedance equal to the source impedance)

Smith chart impedance matching ::

* It is a graphical calculator used for solving complex problems of transmission lines and matching circuits.

* This method is also used to display the behavior of the RF parameters at one (or) more frequencies.

* Smith chart used to display parameters like impedances, admittances, reflection co-efficient, noise figure circles, scattering parameters, and mechanical vibrations etc.

→ There are 3 types of Smith charts.

(i) Impedance Smith charts (Z charts).

(ii) Admittance Smith charts (Y charts)

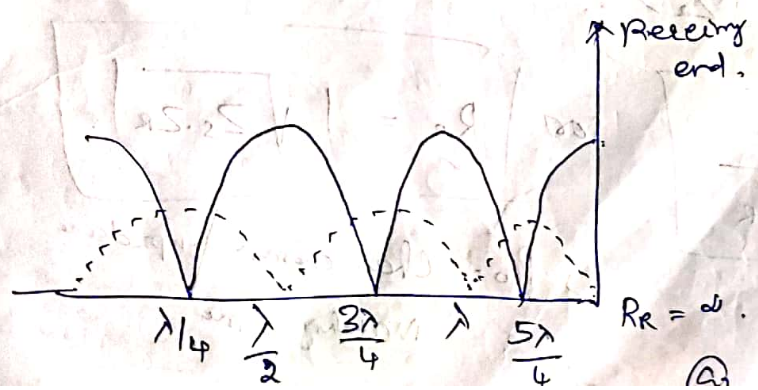
(iii) Immitance Smith charts (YZ charts).

* Name of the circles on Smith chart.

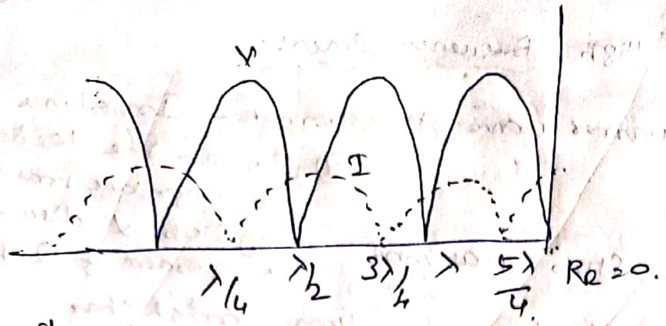
(i) Constant - R circles.

(ii) Constant - X circles

Voltages & current on dissipation less line ::



(a) Open circuit.



(b) short circuit.

Quarter wave line & Impedance matching:

i) The input impedance of a distortionless transmission line is

$$Z_s = R_0 \left[\frac{Z_R + jR_0 \tan\beta x}{R_0 + jZ_R \tan\beta x} \right].$$

Z_s = Source Impedance

Z_R = Load Impedance

$$Z_s = R_0 \left[\frac{\frac{Z_R}{\tan\beta x} + jR_0}{\frac{R_0}{\tan\beta x} + jZ_R} \right].$$

for a Quarter wave line $x = \lambda/4$.

$$\beta x = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}$$

$$= \frac{\pi}{2}$$

$$Z_s = R_0 \left[\frac{\frac{Z_R}{\tan \pi/2} + jR_0}{\frac{R_0}{\tan \pi/2} + jZ_R} \right]$$

$$= R_0 \left[\frac{jR_0}{jZ_R} \right].$$

$$\therefore \boxed{Z_s = \frac{R_0^2}{Z_R}} \quad (\text{or}) \quad \boxed{R_0 = \sqrt{Z_s \cdot Z_R}}.$$

R_0: Characteristic impedance of Matching line

* A quarter wave section of line may be considered as a transformer to match a load of Z_R to a source of Z_s.

* A quarter wave transformer may be considered as an impedance inverter in that it can transform a low impedance into a high impedance and vice versa.

→ An important application of the quarter wave matching section is to couple a transmission line to a resistive load such as an antenna.

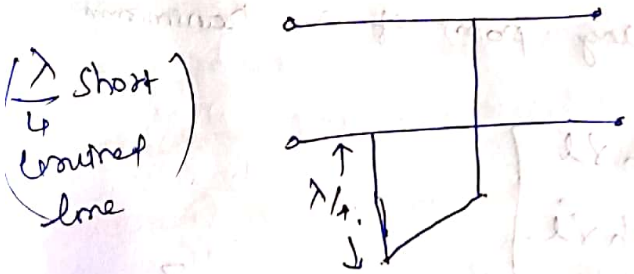


fig: Quarter wave transformer.

* To ensure maximum power is being transferred to load, the impedance of quarter wave line is matched with source & load.

What is stub matching:

If the load impedance is not equal to the complex conjugate of the input impedance, the maximum power transfer will not take place. This is known as impedance mismatching.

* So, it is necessary to introduce some form of an "impedance transforming section" between source and load to achieve impedance matching. Such a section is called as impedance matching device (Ex: quarter wave transformer).

→ Another means of accomplishing impedance matching is the use of an "open (or) short circuited line of suitable length" at a designated distance from the load. Called as stub. This is called "stub matching".

→ There are two types of stub matching. They are.

(i) Single stub matching (ii) Double stub matching.

Single stub matching: It has one stub to match the transmission line impedance.

Double stub matching: It requires two stubs for impedance matching.

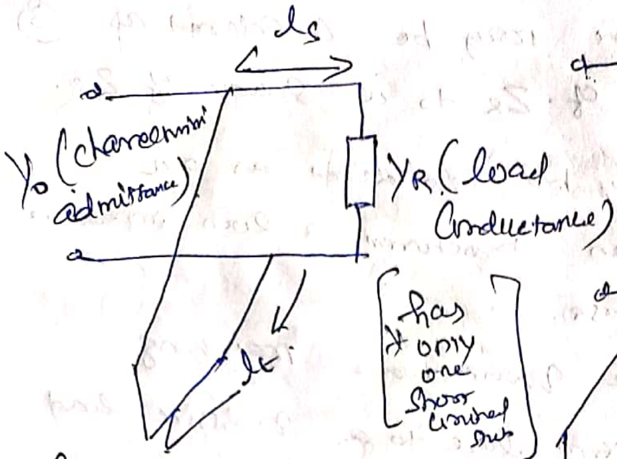


fig. Single Stub Matching.

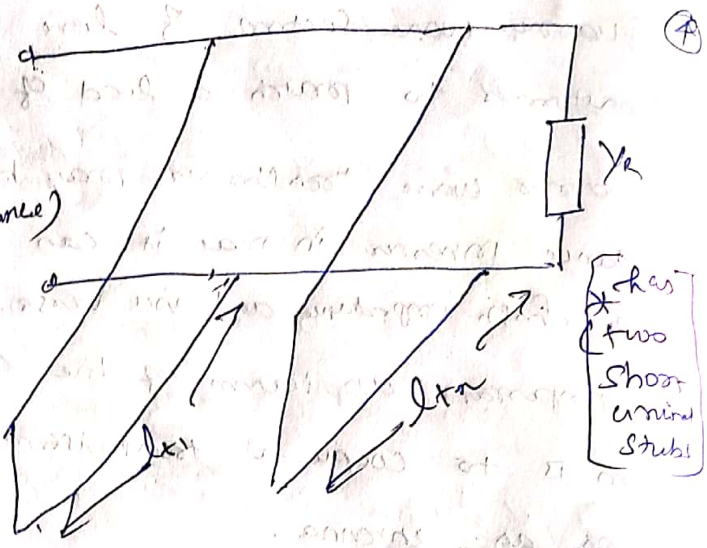


fig: Double Stub matching.

→ The input impedance at any point of a transmission line is given by

$$Z_s = Z_0\left[\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l}\right]$$

∴ The input admittance is $Y_s = Y_0\left[\frac{Y_R + Y_0 \tanh \gamma l}{Y_0 + Y_R \tanh \gamma l}\right]$.

W.K.T $\gamma = j\beta$ ($\alpha = 0$)

$$\therefore Y_s = Y_0\left[\frac{Y_R + jY_0 \tan \beta l}{Y_0 + jY_R \tan \beta l}\right]$$

→ For perfect matching

$$Y_s = Y_0 \Rightarrow \frac{Y_s}{Y_0} = 1.$$

$$\therefore \boxed{Y_{in} = 1.}$$

(i) The location of a stub l_s is given by,

$$\boxed{l_s = \frac{\lambda}{2\pi} \tan^{-1}\sqrt{\frac{Z_R}{Z_0}}}\qquad \left[\because Z_R = \frac{1}{Y_R};\ Z_0 = \frac{1}{Y_0}\right]$$

(ii) The length of the stub is given by.

$$l_t = \frac{\lambda}{2\pi} \tan^{-1}\left[\frac{\sqrt{Z_R Z_0}}{Z_R - Z_0}\right]$$

Explain the significance of Smith chart and its application to a transmission lines. (5)

Apr-May-18

Significance of Smith chart:

1) The complex computations and numbers are made easier using Smith chart due to its geometrical structure.

2) High frequency circuit parameters can be calculated accurately

3) Stub length and its position which are used for impedance matching between the line and load can be calculated easily.

4) The magnitude and phase of a polar reflection co-efficient are known directly from Smith chart.

Applications of Smith chart:

I) Smith chart used as a Admittance diagram:

→ The Smith chart can be used as admittance diagram.

* The normal input admittance (Y) is given as.

$$y = g - jb \longrightarrow (1)$$

where
g - conductance which is reciprocal of resistance.
b - susceptance which is reciprocal of reactance.

* Since the admittance is the reciprocal of the impedance

$$y = \frac{1}{z} \quad -(2)$$

Substituting eqn (1) in (2) we get,

$$g - jb = \frac{1}{z}$$

$$\Rightarrow g - jb = \frac{1}{r + jx}.$$

* Rationalizing the above equation, we get.

$$g - jb = \frac{1}{r + jx} \times \frac{r - jx}{r - jx}$$

$\Rightarrow g - jb = \dfrac{r - jx}{(r+jx)(r-jx)}$.

$g - jb = \dfrac{r - jx}{r^2 + x^2}$

$g - jb = \dfrac{r}{r^2 + x^2} - j\dfrac{x}{r^2 + x^2}$ (or).

$$\boxed{g - jb = \frac{1-K}{1+K}}$$

∴ The Smith chart is used for the admittance diagram.

(2) Smith chart is used for converting impedance into admittance:

* In general the product of the i/p impedance and the load impedance is equal to the square of the characteristic impedance Z_0 in a lossless $\lambda/4$ length.

$Z_{in} Z_R = Z_0^2$

Where Z_{in} = i/p impedance, Z_R = Load impedance

$\Rightarrow \dfrac{Z_{in}}{Z_0} = \dfrac{Z_0}{Z_R}$ (or) $Z_{in} = \dfrac{1}{Z_R}$.

$$\boxed{\therefore Z_{in} Z_R = 1}$$

→ The admittance of an impedance, point to a distance $\lambda/4$ (or 0.25λ) represents half cycle in the Smith chart. Since the admittance will be the reciprocal of the impedance.

(3) Determining the input impedance:

* The input impedance at any length of a line can be determined by considering a load impedance which is represented by a point 'p' with 'O' as center and

'OP' as radius of a circle 'S' produce a 'OP' to cut wavelength scale at 'Q' and generate the Smith chart in a clock wise direction.

→ So, the point 'T' on the wavelength scale is obtained at a distance of $1/\lambda$ from the circle 'S'.

4) Determining the load impedance:

* Smith chart can also be used to determine the load impedance if standing wave ratio (SWR) and the distance of a voltage is given.

* The point of voltage always lies in the left hand side of the horizontal axis at a distance of $\frac{1}{S}$ from the center 'O' of the chart.

* Locate V_{min} on the chart and move the given distance to V_{min} from a point 'A' and also locate the point 'Q' on the wavelength chart.

* Then 'Q_0' is joined to cut the circle at 'P'. Such that the co-ordinate of point 'P' gives a normalized load impedance.

5) Determination of Input impedance and admittance of short circuited line.

* Each point of a circle is associated with a particular value of βl which is marked in wavelength.

* The value on this circle gives the input impedance of a given short circuit line.

* The starting point on this circle will begin at $x=0$ and the phase shift of a short circuit line is $\pi/2$ rad in βl.

(6) Determination of Input impedance and Admittance of

* Determining the input impedance and admittance of an open circuit line is same as short circuit line but the angle of making a chart is shifted by $\pi/2$ radians.

* There exists a phase difference of $\pi/2$ radians between impedance or admittance of the short circuit lines and also to the open circuited lines.

Prob 1) A single stub is to match a load 400 Ω line to a load of $200 - j100\,\Omega$. The wavelength is 3m. Determine the position and length of the short circuited stub. A/m-19

Given that, for a transmission line using single stub matching.

(i) Load impedance $Z_L = 200 - j100\,\Omega$.

(ii) Characteristic impedance $Z_0 = 400\,\Omega$.

(iii) Wavelength $\lambda = 3\,m$.

position of the stub, $d = ?$

length of the stub, $l_s = ?$

⇒ The expression for location of stub in terms of reflection co-efficient (Γ) is

$$d = (\phi + \pi - \cos^{-1}(|\Gamma|) \longrightarrow (1)$$

where $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

* On substituting the values of Z_L and Z_0 in Γ, we get

$$\Gamma = \frac{200 - j100 - 400}{200 - j100 + 400}$$

$$= \frac{-200 - j100}{600 + j100}$$

$$= -\frac{(2+j)}{(6+j)}$$

$$= \frac{-2.54 \angle 26.57^\circ}{6.08 \angle 9.46^\circ}$$

$$= -0.3684 \angle 26.57^\circ - 9.46^\circ$$

$$\boxed{\Gamma = -0.3684 \angle 17.11^\circ}$$

where $|\Gamma| = |-0.3684|$

$$\boxed{\Gamma = 0.3684}$$

and,

$$\phi = 17.11^\circ \left(\frac{\pi}{180^\circ}\right)$$

$$\boxed{\phi = 0.095\pi}$$

→ By substituting the value of Γ in equation (1), we get.

$$d = \frac{\lambda}{2\pi}\left[0.095\pi + \pi - \cos^{-1}(0.364)\right]$$

$$\left[\because \cos^{-1}(0.3684) = 68.38 \times \frac{\pi}{180}\right]$$

$$d = \frac{\lambda}{2\pi}\left[0.095\pi + \pi + 0.379\pi\right) m$$

$$\therefore \boxed{d = 0.7374\lambda m}$$

→ The expression for length of the single stub in terms of reflection coefficient is given by,

$$l_s = \frac{\lambda}{2\pi} \tan^{-1}\left[\frac{\sqrt{1-|\Gamma|^2}}{2|\Gamma|}\right] \rightarrow (2)$$

By substituting the values in (2),

$$= \frac{\lambda}{2\pi} \tan^{-1}\left[\frac{\sqrt{1-(0.3684)^2}}{2 \times 0.3684}\right]$$

$$= \frac{\lambda}{2\pi}\left[51.60 \times \frac{\pi}{180^\circ}\right]$$

$$= \frac{\lambda}{2\pi} \times 0.286\,\pi$$

$$= 0.143\lambda.$$

$$\therefore \boxed{l_s = 0.143\lambda}$$

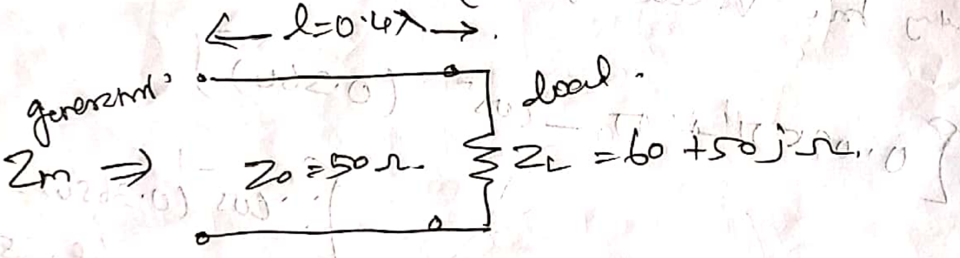
Basic Smithchart problem:

(1) use the smithchart to find the following quantities for the transmission line circuit shown in the fig.

(i) VSWR (ii) Reflection coefficient (Γ_L) (iii) Load admittance (Y_L)

(iv) Input impedance (Z_{in}) (v) distance from the load to the first voltage minimum (d_{Vmin}).

(vi) The distance from the load to the first voltage maximum (d_{Vmax}) (wavelength towards generator).



(a) VSWR:- 2.4.

(b) $\Gamma_L = |\Gamma_L| \angle\theta$

$\Gamma_L = 0.42 \angle 54^\circ$

(c) Load admittance (Y_L) =

from smith chart $\bar{Y}_L = \frac{0.5 - 0.42j}{Z_0} = \frac{\bar{Y}_L}{Z_0} = \frac{0.5 - 0.42j}{50}$

formula:- $\boxed{Y_L = \frac{\bar{Y}_L}{Z_0}}$

$\therefore Y_L = 0.01 - 8.4 \times 10^{-3} j$

a) Input impedance Zin

(1) What is impedance matching? State need for it?

* It is defined as process of matching the load impedance to characteristic impedance.

* This can be done by connecting a matching device between source and load, as shown in fig.

* It helps in transmitting maximum power from source to load.
* It reduces the reflections and formation of standing waves in transmission lines.
* Also improves the signal to noise ratio of the system.

(2) List the applications of Quarter wave line.

i) Used as an impedance inverter as it inverts low impedance to high impedance, vice versa.
ii) Act as an insulator and used to assist an open wire line.
iii) Used as a transformer for impedance matching of load with the source.
iv) Used as an antenna to combine the transmission line with resistive load.

3) What is impedance matching in stub?

* A stub is a short circuit transmission line of appropriate length.
* It is connected either in series (or) parallel to the transmission line. Short circuited (or) open circuited sections of a transmission line.

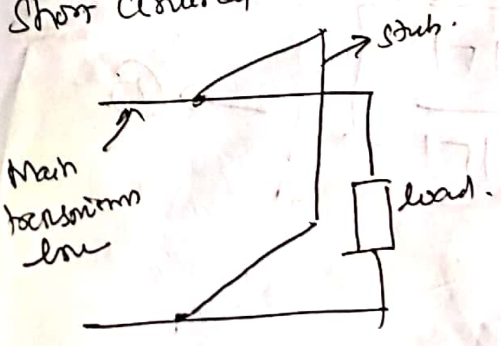


fig: Stub arrangement

* In this case, impedance of main transmission line is equal to characteristic impedance.
* The impedance of the stub is chosen such that it matches the load to transmission line with resistive impedances. → This is known as "impedance matching in stub"

Prob2 A load impedance of 90 - j50 Ω is to be matched to a line of 50 Ω using single stub matching. Find the length and position of the stub. (1)

* The expression for the location of stub in terms of reflection coefficient (Γ) is,

$$\boxed{d = \frac{\lambda}{2\pi}\left(\varphi + \pi - \cos^{-1}|\Gamma|\right)} \longrightarrow (1)$$

where

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{90 - j50 - 50}{90 - j50 + 50} = \frac{40 - j50}{140 - j50}$$

$$= \frac{4 - j5}{14 - j5} = \frac{6.403 \angle -51.34^\circ}{14.866 \angle -19.66^\circ}$$

$$\boxed{\Gamma = 0.4307 \angle -31.68^\circ}$$

where

$$|\Gamma| = 0.4307 \quad \& \quad \varphi = -31.68^\circ \left(\frac{\pi}{180^\circ}\right)$$

$$\boxed{\varphi = -0.176\pi}$$

* By substituting the value of Γ in (1),

$$d = \frac{\lambda}{2\pi}\left(-0.176\pi + \pi - \cos^{-1}(0.4307)\right)$$

$$= \frac{\lambda}{2\pi}\left(-0.176\pi + \pi - 0.3582\pi\right)$$

$$\boxed{d = 0.2329\lambda m}$$

* The expression for length of the single stub in terms of reflection coefficient is

$$\boxed{l_s = \frac{\lambda}{2\pi}\tan^{-1}\left[\frac{\sqrt{1-|\Gamma|^2}}{2|\Gamma|}\right]} \longrightarrow (2)$$

Substitute the values in (2)

$$l_s = \frac{\lambda}{2\pi} \tan^{-1}\left[\frac{\sqrt{1-(0.4307)^2}}{2 \times 0.4307}\right].$$

$$= \frac{\lambda}{2\pi} \times \left[46.315 \times \frac{\pi}{180°}\right]$$

$$= \frac{\lambda}{2\pi} \times 0.2573\pi.$$

$$\therefore \boxed{l_s = 0.1286\lambda.}$$ //

Prob3 Determine length and location of a single short circuited stub to produce an impedance match on a transmission line with characteristic impedance of 600Ω and terminated in 1800Ω

i) $d = \frac{\lambda}{2\pi}(\phi + \pi - \cos^{-1}|\Gamma|) \rightarrow (1)$ where $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$.

$$\therefore \Gamma = \frac{1800-600}{1800+600} = 0.5. \quad \text{where } |\Gamma| = 0.5 \text{ and } \phi = 0.$$

Substitution the values in (1)

$$d = \frac{\lambda}{2\pi}(0 + \pi - \cos^{-1}(0.5))$$

$$= \frac{\lambda}{2\pi}\left(\pi - \frac{\pi}{3}\right) = 0.33\lambda.$$

$$\therefore \boxed{d = 0.33\lambda m},$$

ii) $l_s = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{\sqrt{1-|\Gamma|^2}}{2|\Gamma|}\right) \rightarrow (2).$

$$\therefore l_s = \tan^{-1}\left(\frac{\sqrt{1-(0.5)^2}}{2(0.5)}\right)$$

$$l_s = \tan^{-1}(0.866)$$

$$= 0.137\lambda$$

$$\boxed{\therefore l_s = 0.137\lambda m}$$ //

Prob-5 A $300\,\Omega$ transmission line is connected to a load impedance of $450 - j600\,\Omega$ at $10\,MHz$. Find the position and length of a short circuited stub required to match the line using smith chart.

O/p May

Sol Given that, for a series RC combination,

$Z_L = 450 - j600\,\Omega$, $f = 10\,MHz$, $Z_0 = 300\,\Omega$ $d = ?$ $l_s = ?$

(1) The general expression for calculating wavelength (λ) is given as

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{10\times10^6} = 30 \quad \therefore \lambda = 30\,m.$$

Step 1: The normalised load impedance is obtained as,

$$Z_n = r + jx = \frac{Z_L}{Z_0} = \frac{450 - j600}{300} = 1.5 - j2.$$

$$\therefore \boxed{r = 1.5 \;\&\; x = -2}$$

Step 2: The point of intersection of r and x is indicated by point 'P' in the smith chart.

Step 3: A circle with radius 'OP' is drawn as graph. It cuts the centre line at 4.6 $\therefore \boxed{SWR = 4.6}$

Step 4: The line OP is extended to 'Q' as shown in smith chart.

\# The point 'Q' represents the normalized admittance

→ from chart, the normalised admittance is obtained as,

$$Y_n = 0.24 + j0.32$$

~~Step 5 The line 'OA' is extended to~~

Step 5: The circle of radius 'OA' cuts $r=1$ circle at the point 'R'.

→ From the chart, the value of 'R' is obtained as.

$$R = 1 + j1.7$$

Step 6: The distance from points G and 'H' on the rim of the chart is equal to the distance of stub from the ie,

Stub distance from load = Distance from G to H.

$\therefore$ Stub distance, $d = 0.181 - 0.051$

$= 0.13\lambda$.

$= 0.13\ (30)$ $[\because \lambda = 30]$.

$\boxed{\therefore \text{Stub distance } d = 3.9\,m.}$

Step 7: The susceptance of the load $= +j1.7$.

The susceptance of the stub $= -j1.7$.

Stub length $l_s = 0.335 - 0.250$.

$= 0.085\lambda$.

$= 0.085\ (30)$ $[\because \lambda = 30]$.

$\boxed{\therefore \text{Stub length } l_s = 2.55\,m.}$

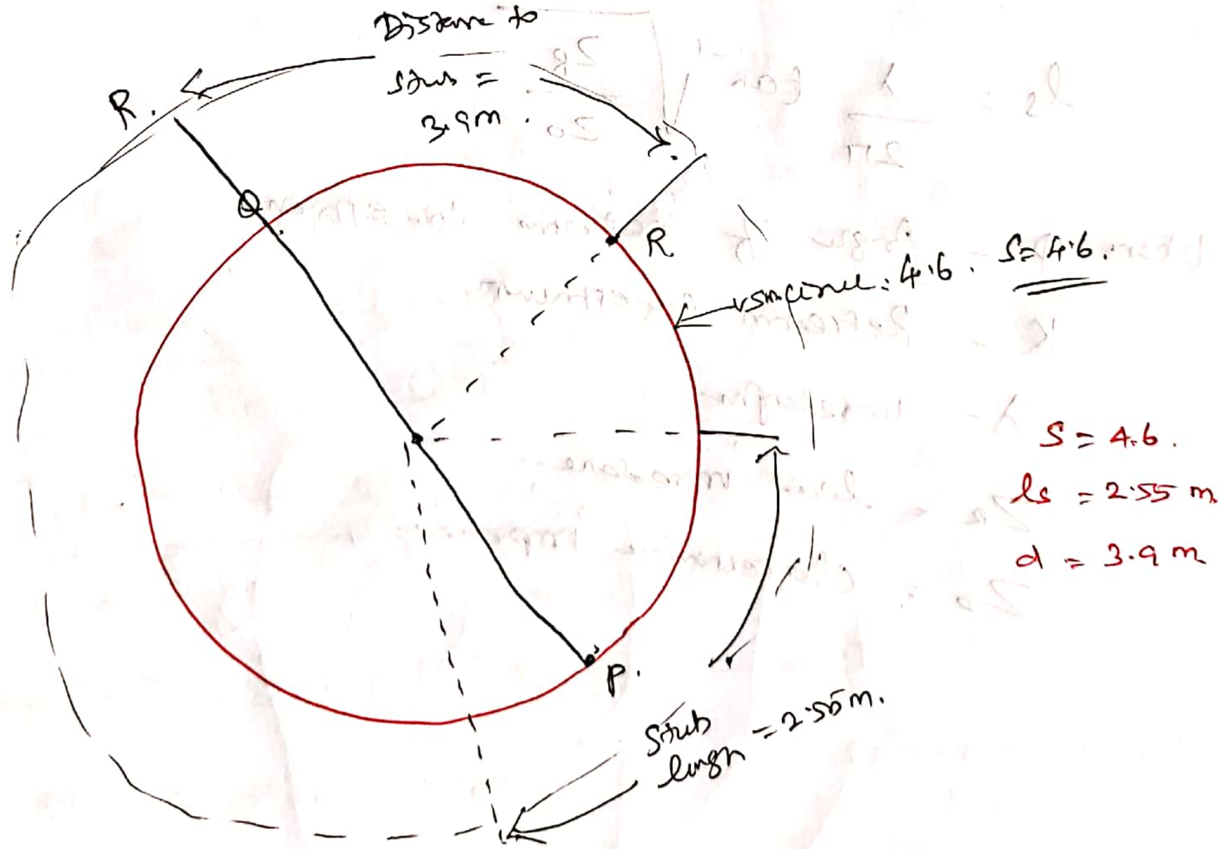


fig: Short circuited stub.

① Write down the expression to determine the length of the stub.

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \frac{1 - |K|^2}{2|K|}$$

(or)

$$l_t = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{Z_R Z_0}}{(Z_R - Z_0)}$$

Where

K = Reflection coefficient

λ = wavelength.

Z_R = Load impedance

Z_0 = characteristic impedance

② Write down the expression to determine the position of the stub:

$$l_s = \frac{\lambda}{2\pi} \left[\phi + \pi - \cos^{-1} |K| \right]$$

Eqn.

$$l_s = \frac{\lambda}{2\pi} \tan^{-1} \sqrt{\frac{Z_R}{Z_0}}$$

Where ϕ - Angle of reflection co-efficient

K - Reflection coefficient.

λ = wavelength

Z_R = load impedance

Z_0 = characteristic impedance

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1) Explain the procedure for obtaining the Smith chart (4M). using R and X circles. N/o 18.

A Smith chart is a simple graphical tool which consists of locus of constant resistance values as circles on real axis and locus of reactance values as circles on imaginary axis.

It basically converts reflection co-efficient to corresponding impedance (or admittance) and vice versa.

The expression for reflection coefficient is.

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \rightarrow (1) = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1} = \frac{Z_n - 1}{Z_n + 1}.$$

where Z_n = Normalized impedance. & $Z_n = \frac{Z_L}{Z_0} = r + jx$.

(1) can be written as,

$$\therefore \Gamma = \frac{r + jx - 1}{r + jx + 1}.$$

$$= \left[\frac{(r-1) + jx}{(r+1) + jx}\right] \times \left[\frac{(r+1) - jx}{(r+1) - jx}\right].$$

$$= \frac{(r-1)(r+1) - jx(r-1) + jx(r+1) - (jx)^2}{(r+1)^2 - (jx)^2}$$

$$= \frac{(r^2 - 1 + x^2) + j(2x)}{(r+1)^2 + x^2}$$

$$= Re[\Gamma] + Im[\Gamma] = A + jB. \rightarrow (2).$$

On equating real and imaginary parts of equation (2), we get.

$$Re[\Gamma] = A = \frac{r^2 - 1 + x^2}{(r+1)^2 + x^2} \rightarrow (3)$$

$$Im[\Gamma] = B = \frac{2x}{(r+1)^2 + x^2} \rightarrow (4)$$

Smith chart consists of two circles:

(i) Constant - r circle.

(ii) Constant x - circle.

(i) Constant r circle:

Constant r circles are obtained by eliminating x from equations (3) & (4)

i.e

$$\left(A-\frac{r}{r+1}\right)^2+B^2=\left(\frac{1}{r+1}\right)^2 \rightarrow (5)$$

→ The equation of the circle is expressed as

$$\boxed{(x-h)^2+(y-k)^2=a^2 \rightarrow (6)}$$

where a = radius, (h,k) = center at (x,y)

On comparing (5) and (6)

Radius $=\frac{1}{r+1}$, center at $(A,B)=\left(\frac{r}{r+1},0\right)$

* The r circle for different values of normalized resistance is shown below

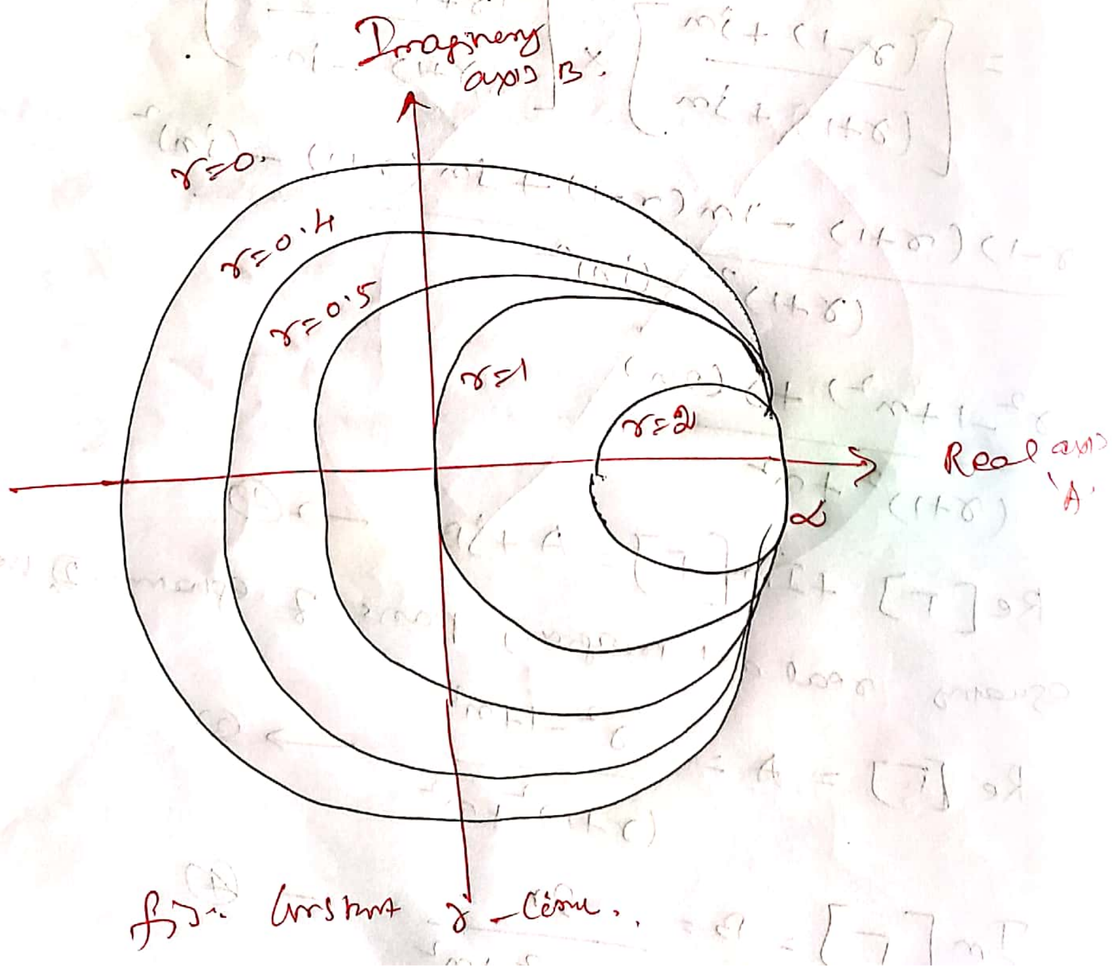


fig:- Constant r - circle.

Features of Constant 'r' circles.,

(1) In constant r circles, the centre lies on horizontal axis.

(2) The value of r is $\pm\alpha$ at point $A=1$, $B=0$.

(3) The circle pass through the point $(1,0)$.

(4) The circle compress when 'r' increases from 0 to α.

(5) When radius $r=0$ center is $(0,0)$ the circle is the outermost circle in Smith chart.

(6) When radius $=\alpha$, the circle has a point at $(1,0)$.

(2) Constant 'x' circle :

* Constant 'x' circles are obtained by eliminating 'r' from equations (3) and (4)

ie $$\boxed{(A-1)^2+\left(B-\frac{1}{x}\right)^2=\left(\frac{1}{x}\right)^2} \rightarrow (7)$$

→ on comparing (7) and (6), we get

Radius $=\frac{1}{x}$, center at $(A,B)=\left(1,\frac{1}{x}\right)$

* The circle 'x' for different values of reactance is shown below

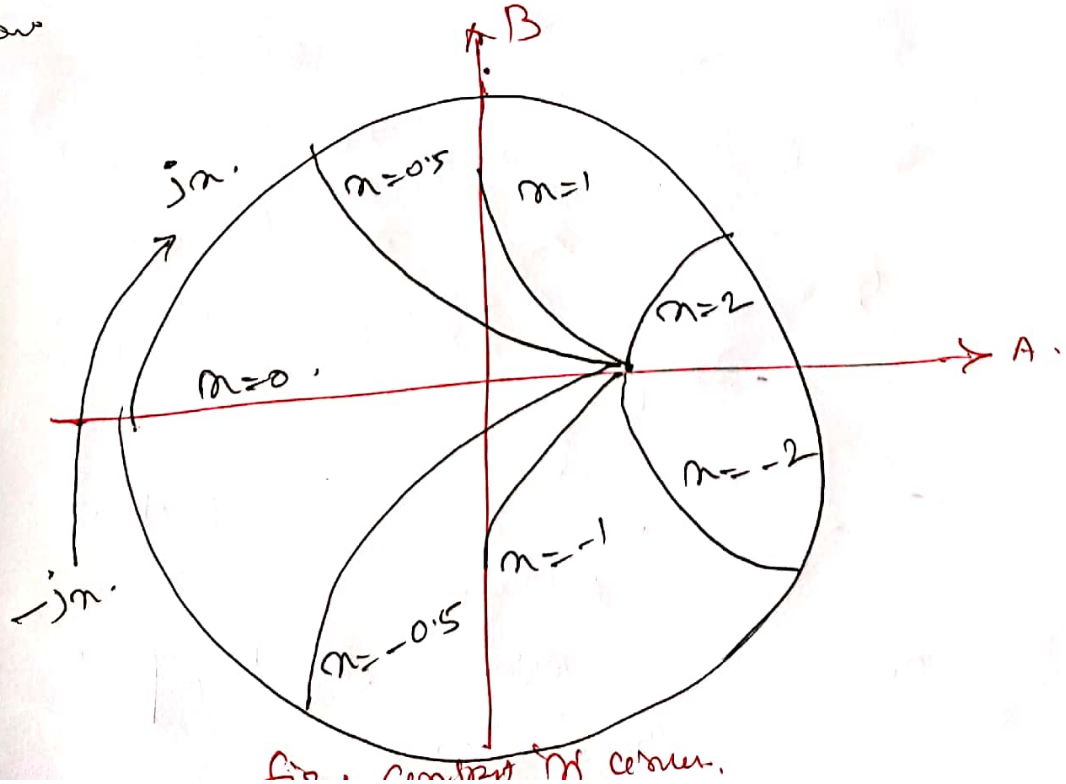


Fig. Constant 'x' circles.

Features of Constant 'm' circles:

(1) In constant 'm' circles, the centre lies on $A = 1$.

(2) When $n > 0$ (inductive reactance), the circles lies above the A-axis.

When $n < 0$ (capacitive reactance), the circle is lies below the A-axis.

(3) The circle compress when $|n|$ increases from $n = 0$ to $n = \infty$.

(4) The value of n is $\pm \infty$ at point $A = 1$, $B = 0$.

(5) The circles are centered parallel to imaginary axis.

UNIT 4

WAVEGUIDE

4.1 INTRODUCTION

A wave is defined as a means of transporting energy from source to destination. A wave is a function of space and time. In some of applications, it is necessary to confine and guide the wave energy by guided structures. In such applications the fields are confined or restricted by boundaries of materials different from those of transmission path and the waves are said to be guided by these materials. Hence it is necessary to apply Maxwell's equations, certain mathematical restrictions or boundary conditions in order to fit the general solution to the particular problem. After applying boundary conditions , it may be possible to obtain a solution showing the form and type of wave transmission that can occur in the confined region. Typical examples of such structures used in guiding waves are coaxial lines, open wire lines and wave guides.

4.2 GENERAL WAVE BEHAVIOURS ALONG UNIFORM GUIDING STRUCTURES

Consider an electromagnetic wave propagating between a pair of parallel perfectly conducting planes of infinite extent in y and z directions as shown in figure 4.1. The conducting planes are placed at $x = 0$ and $x = a$. To study the behaviour of electromagnetic field between two conducting planes Maxwell's equations are solved subject to the appropriate boundary conditions.

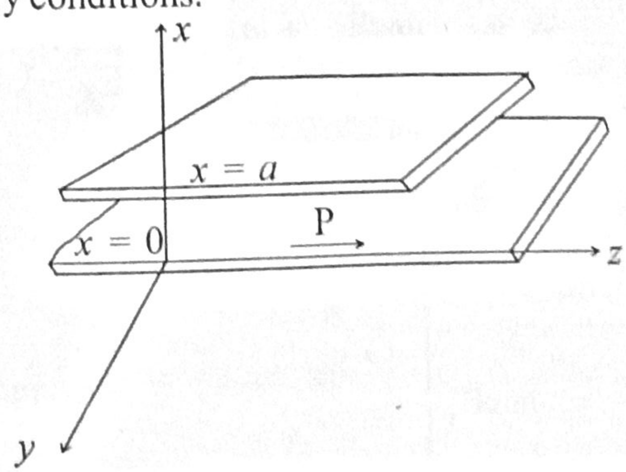


Figure 4.1 Parallel conducting planes.

The boundary conditions are $E_{tangential} = 0$

$H_{normal} = 0$

The elecric field and magnetic field can be expressed as

$$E = [\vec{a}_x E_x(x,y) + \vec{a}_y E_y(x,y) + \vec{a}_z E_z(x,y)]e^{-\gamma z} \quad ...(1)$$

$$H = [\vec{a}_x H_x(x,y) + \vec{a}_y H_y(x,y) + \vec{a}_z H_z(x,y)]e^{-\gamma z} \quad ...(2)$$

Wave propagation is assumed to be in + z direction

We know that

$$\nabla \times H = \sigma E + j\omega\varepsilon E = E(\sigma + j\omega\varepsilon)$$

Assuming $\sigma = 0$

$$\nabla \times H = j\omega\varepsilon E \quad ...(3)$$

$$\nabla \times E = \frac{-\partial B}{\partial t} = -j\omega\mu H \quad ...(4)$$

Equation (3) can be written as

$$\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial_x} & \frac{\partial}{\partial_y} & \frac{\partial}{\partial_z} \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon \quad [\vec{a}_x E_x(x,y) + \vec{a}_y E_y(x,y) + \vec{a}_z E_z(x,y)]$$

$$\vec{a}_x\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) - \vec{a}_y\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) + \vec{a}_z\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

$$= \vec{a}_x \; j\omega\varepsilon E_x + \vec{a}_y \, j\omega\varepsilon E_y + \vec{a}_z \, j\omega\varepsilon E_z$$

Comparing $\vec{a}_x, \vec{a}_y, \vec{a}_z$ on both sides

$$\left.\begin{aligned} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} &= j\omega\varepsilon E_x \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} &= j\omega\varepsilon E_y \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= j\omega\varepsilon E_z \end{aligned}\right\} \quad ...(5)$$

Equation (4) can be written as

$$\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu[\vec{a}_x H_x(x,y) + \vec{a}_y H_y(x,y) + \vec{a}_z H_z(x,y)]$$

$$\vec{a}_x\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) - \vec{a}_y\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) + \vec{a}_z\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

$$= -\vec{a}_x \; j\omega\mu H_x - \vec{a}_y \; j\omega\mu H_y - \vec{a}_z \; j\omega\mu H_z$$

Comparing $\vec{a}_x, \vec{a}_y, \vec{a}_z$ on both sides

$$\left.\begin{aligned} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} &= -j\omega\mu H_x \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} &= -j\omega\mu H_y \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= -j\omega\mu H_z \end{aligned}\right\} \quad ...(6)$$

The wave equation is given by

$$\left.\begin{aligned} \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} &= -\omega^2\mu\varepsilon E \\ \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} &= -\omega^2\mu\varepsilon H \end{aligned}\right\} \quad ...(7)$$

It is assumed that the propagation is in z direction & the variation of field components in this z direction may be expressed in the form $e^{-\gamma z}$

Propagation constant $\gamma = \alpha + j\beta$

If $\alpha = 0$; $\gamma = j\beta$ the wave propagate without attenuation.

If $\beta = 0; \gamma = \alpha$ there is no wave motion but only an exponential decrease in amplitude.

Let $H_y = H_y^{\circ} e^{-\gamma z}$

$$\frac{\partial H_y}{\partial z} = -\gamma e^{-\gamma z} H_y^{\circ} = -\gamma H_y$$

Similarty $\frac{\partial H_x}{\partial z} = -\gamma H_x$

Let $E_y = E_y^{\circ} e^{-\gamma z}$

$$\frac{\partial E_y}{\partial z} = -\gamma E_y$$

Similarly

$$\frac{\partial E_x}{\partial z} = -\gamma E_x$$

There is no variation in y direction, derivative of y is zero. Substitute the values of z derivatives and y derivatives in equation (5), (6) and (7)

$$\gamma H_y = j\omega\varepsilon E_x \quad \text{...(8a)}$$

$$-\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \quad \text{...(8b)}$$

$$\frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z \quad \text{...(8c)}$$

$$\gamma E_y = -j\omega\mu H_x \quad \text{...(9a)}$$

$$-\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \quad \text{...(9b)}$$

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z \quad \text{...(9c)}$$

$$\left.\begin{aligned} \frac{\partial^2 E}{\partial x^2} + \gamma^2 E = -\omega^2\mu\varepsilon E \\ \frac{\partial^2 H}{\partial x^2} + \gamma^2 H = -\omega^2\mu\varepsilon H \end{aligned}\right\} \quad \text{...(10)}$$

To Solve H_x:

From equation (8b)

$$E_y = -\frac{1}{j\omega\varepsilon}\left(\gamma H_x + \frac{\partial H_z}{\partial x}\right) \quad \text{...(11)}$$

From equation (9a)

$$H_x = -\frac{1}{j\omega\mu}\gamma E_y \quad \text{...(12)}$$

Substituting equation (11) in equation (12)

$$H_x = -\frac{\gamma}{j\omega\mu}\left[-\frac{1}{j\omega\varepsilon}\left(\gamma H_x + \frac{\partial H_z}{\partial x}\right)\right]$$

$$= \frac{-\gamma}{\omega^2\mu\varepsilon}\left[\gamma H_x + \frac{\partial H_z}{\partial x}\right]$$

$$H_x = -\frac{\gamma^2 H_x}{\omega^2\mu\varepsilon} - \frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial x}$$

$$H_x\left[1 + \frac{\gamma^2}{\omega^2\mu\varepsilon}\right] = -\frac{\gamma}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial x}$$

$$H_x\left[\omega^2\mu\varepsilon + \gamma^2\right] = -\gamma\frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-\gamma}{\gamma^2 + \omega^2\mu\varepsilon}\frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} \quad \text{...(13)}$$

Where $h^2 = \gamma^2 + \omega^2\mu\varepsilon$

To solve H_y:

From equation (9b)

$$H_y = \frac{1}{j\omega\mu}\left[\gamma E_x + \frac{\partial E_z}{\partial x}\right] \quad \text{...(14)}$$

From equation (8a)

$$E_x = \frac{1}{j\omega\varepsilon}[\gamma H_y] \quad \text{...(15)}$$

Substituting equation (15) in equation (14)

$$H_y = \frac{1}{j\omega\mu}\left[\gamma\left[\frac{\gamma H_y}{j\omega\varepsilon}\right] + \frac{\partial E_z}{\partial x}\right]$$

$$= \frac{1}{j\omega\mu}\left[\frac{\gamma^2 H_y}{j\omega\varepsilon} + \frac{\partial E_z}{\partial x}\right]$$

$$H_y = \frac{-\gamma^2 H_y}{\omega^2\mu\varepsilon} + \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x}$$

$$H_y\left[1 + \frac{\gamma^2}{\omega^2\mu\varepsilon}\right] = \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x}$$

$$H_y\left[\frac{\gamma^2 + \omega^2\mu\varepsilon}{\omega^2\mu\varepsilon}\right] = \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x}$$

$$H_y = -\frac{j\mu\varepsilon}{h^2}\frac{\partial E_z}{\partial x} \qquad ...(16)$$

To solve E_x:

From equation (8a)

$$H_y = \frac{j\omega\varepsilon E_x}{\gamma} \qquad ...(17)$$

From equation (9b)

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y \qquad ...(18)$$

Substituting equation (17)in (18)

$$\gamma E_x + \frac{\partial E_z}{\partial x} = j\omega\mu\left[\frac{j\omega\varepsilon E_x}{\gamma}\right] = -\frac{\omega^2\mu\varepsilon E_x}{\gamma}$$

$$E_x\left[\frac{\gamma + \omega^2\mu\varepsilon}{\gamma}\right] = \frac{-\partial E_z}{\partial x}$$

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} \qquad ...(19)$$

To solve H_y:

$$\gamma H_x + \frac{\partial H_z}{\partial x} = -j\omega\varepsilon E_y \qquad ...(20)$$

From equation (9a)

$$H_x = \frac{-\gamma E_y}{j\omega\mu} \qquad ...(21)$$

Substituting (21) in (20)

$$\gamma\left[\frac{-\gamma E_y}{j\omega\mu}\right] + \frac{\partial H_z}{\partial x} = -j\omega\varepsilon E_y$$

$$\frac{-\gamma^2 E_y}{j\omega\mu} + \frac{\partial H_z}{\partial x} = -j\omega\varepsilon E_y$$

$$E_y\left(j\omega\varepsilon - \frac{\gamma^2}{j\omega\mu}\right) = \frac{-\partial H_z}{\partial x}$$

$$E_y = \frac{-\partial H_z}{\partial x}\;\frac{j\omega\mu}{-\omega^2\mu\varepsilon - \gamma^2}$$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} \qquad ...(22)$$

The components of electricfield and magnetic field strength (E_x, E_y, H_x, H_y) are expressed in terms of E_z and H_z. It is observed that there must be a z component of either E or H otherwise all the components would be zero.

4.2.1 Transverse Electromagnetic Waves (TEM)

These are waves in which both electric field and magnetic field strength are entirely transverse. It has no component of E_z and H_z. It is also called as principal waves.

Characteristics of TEM waves

i) $\beta = \sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2}$

When m = 0

$$\beta = \omega\sqrt{\mu\varepsilon}$$

β varies lineraly with frequency and therefore the wave propagation takes place without dispertion.

ii) Phase velocity $V_{ph} = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}}$

iii) $\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}} = \frac{2\pi}{2\pi f\sqrt{\mu\varepsilon}} = \frac{c}{f\sqrt{\mu_r\varepsilon_r}} = \frac{\lambda_o}{\sqrt{\varepsilon_r}}$

($\because \mu_r = 1$ for most of medium)

iv) $f_c = \frac{m}{2a\sqrt{\mu\varepsilon}}$ When m=0 ; f_c=0

v) $Z_{TEM} = \frac{E_x}{H_y} = \sqrt{\frac{\mu}{\varepsilon}} = 120\pi \text{ (or) } 377\Omega$

4.2.2 Transverse Magnetic Waves (TM)

These are waves in which the magnetic field strength is entirely transverse. It has electric field strength in the direction of propagation and no component of magnetic field strength in same direction. ($H_z = 0$; $E_z \neq 0$)

4.2.3 Transverse Electric Waves (TE)

These are waves in which the electric field strength is entirely trasverse. It has magneticfield strength in the direction of propagation no component of electricfield strength in same direction. ($E_z = 0$; $H_z \neq 0$)

Characteristics of TE and TM waves

We know that

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon$$

$$\gamma^2 = h^2 - \omega^2\mu\varepsilon$$

$$\gamma = \sqrt{h^2 - \omega^2\mu\varepsilon}$$

Substitute $h = \frac{m\pi}{a}$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon}$$

$$\gamma = \alpha + j\beta$$

For very high frequency $\omega^2\mu\varepsilon > \left(\frac{m\pi}{a}\right)^2$

γ becomes imaginary

$$\gamma = j\beta = j\sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2}$$

$$\beta = \sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2}$$

As the frequency is decreased, a critical frequency is reached at which

$$\omega_c^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2$$

$$\omega_c = \frac{m\pi}{a\sqrt{\mu\varepsilon}}$$

$$f_c = \frac{m}{2a\sqrt{\mu\varepsilon}}$$

The frequency at which wave motion ceases is called cut off frequency.

$$f_c = \frac{mv}{2a}$$

Where $v = \frac{1}{\sqrt{\mu\varepsilon}}$

Cut off wave length $\lambda_c = \frac{v}{f_c} = \frac{2a}{m}$

$$v_{ph} = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2}}$$

$$= \frac{\omega}{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}\sqrt{1 - \frac{\omega_c^2}{\omega^2}}}$$

$$v_{ph} = \frac{v}{\sqrt{1 - \frac{f_c^2}{f^2}}}$$

Guide wave length $\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2}}$

$$= \frac{2\pi}{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}}$$

$$= \frac{2\pi}{\omega\sqrt{\mu\varepsilon}\sqrt{1 - \frac{\omega_c^2}{\omega^2}}} = \frac{2\pi}{2\pi f\sqrt{\mu\varepsilon}\sqrt{1 - \frac{f_c^2}{f^2}}}$$

$$= \frac{v}{f\sqrt{1 - \frac{f_c^2}{f^2}}} = \frac{\lambda_o}{\sqrt{1 - \frac{f_c^2}{f^2}}}$$

$$\lambda_g = \frac{\lambda_o}{\sqrt{1 - \left(\frac{\lambda_o}{\lambda_c}\right)^2}}$$

$$= \frac{\lambda_o \lambda_c}{\sqrt{\lambda_c^2 - \lambda_o^2}}$$

4.3 TM WAVES BETWEEN PARALLEL PLATES

TM wave are the waves in which magnetic field strength is entirely transverse. It has electric field strength in the direction of propagation and no component of magnetic field strength in the same direction. ($H_z = 0$; $E_z \neq 0$).

We know that

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} \quad \text{and} \quad E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

Substituting $H_z = 0$

$$H_x = 0; E_y = 0$$

The wave equation for component H_y

$$\frac{\partial^2 H_y}{\partial x^2} + \gamma^2 H_y = -\omega^2\mu\varepsilon H_y \quad \text{...(1)}$$

$$\frac{\partial^2 H_y}{\partial x^2} = -(\omega^2\mu\varepsilon + \gamma^2) H_y = -h^2 H_y$$

$$\frac{\partial^2 H_y}{\partial x^2} + h^2 H_y = 0$$

Let $H_y = H_y^\circ e^{-\gamma z}$ ...(2)

$$\frac{\partial H_y^\circ}{\partial x^2} + h^2 H_y^\circ = 0$$

This is the second order differential equation of simple harmonic motion. Solution of the above equation is,

$$H_y^\circ = C_3 \sinh x + C_4 \cosh x$$

$$H_y = (C_3 \sinh x + C_4 \cosh x)e^{-\gamma z} \quad \text{...(3)}$$

Where C_3 and C_4 are arbitary constants.

Boundary conditions can not be applied directly to H_y because $H_{tangential} \neq 0$ at the surface of conductor.

Differentiating equation (3) with respect to x

$$\frac{\partial H_y}{\partial x} = h(C_3 \cos hx - C_4 \sin hx)e^{-\gamma z}$$

From equation (8c)

$$\frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z$$

$$E_z = \frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial x}$$

$$= \frac{h}{j\omega\varepsilon}(C_3 \cosh x - C_4 \sinh x)e^{-\gamma z} \quad \text{...(4)}$$

Applying first boundary condition.

$$E_z = 0 \text{ at } x=0$$

$$C_3 e^{-\gamma z}\frac{h}{j\omega\varepsilon} = 0$$

$$C_3 = 0$$

Equation (4) becomes

$$E_z = -\frac{hC_4 \sinh xe^{-\gamma z}}{j\omega\varepsilon}$$

Applying second boundary condition.

$$E_z = 0 \text{ at } x = a$$

$$\sin ha = 0$$

$$ha = \sin^{-1}(0) = m\pi$$

$$h = \frac{m\pi}{a} \quad \text{Where m= 1,2...}$$

$$E_z = \frac{-C_4\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)xe^{-\gamma z}}{j\omega\varepsilon}$$

$$E_z = \frac{jC_4 \sin\left(\frac{m\pi x}{a}\right)m\pi}{\omega\varepsilon a}e^{-\gamma z}$$

$$H_y = \int j\omega\varepsilon E_z\, d_x$$

$$= \int j\omega\varepsilon \frac{jw\varepsilon\, jC_4 \sin\left(\frac{m\pi x}{a}\right)m\pi\; e^{-\gamma z}}{\omega\varepsilon a}d_x$$

$$H_y = C_4 \cos\left(\frac{m\pi}{a}\right)xe^{-\gamma z}$$

But $\gamma H_y = j\omega\varepsilon E_x$

$$E_x = \frac{\gamma}{j\omega\varepsilon}H_y$$

$$= \frac{\gamma}{j\omega\varepsilon}\left[C_4 \cos\left(\frac{m\pi}{a}\right)x\, e^{-\gamma z}\right]$$

The field strength for TM waves between parallel planes are

$$H_y = C_4 \cos\left(\frac{m\pi}{a}\right)xe^{-\gamma z}$$

$$E_x = \frac{\gamma}{j\omega\varepsilon}C_4 \cos\left(\frac{m\pi}{a}\right)x\, e^{-\gamma z}$$

$$E_z = \frac{jm\pi}{\omega\varepsilon a}C_4 \sin\left(\frac{m\pi}{a}\right)x\, e^{-\gamma z}$$

Transverse magnetic wave associated with intger m is designated as TM_{mo} wave (or) TM_{mo} mode. If m=0 all the fields will not be equal to zero. E_x and H_y exist and only E_z=0 .In the case of TM waves there is a possibility of m =0.

If the wave propagate without attenuation $\alpha = 0$, $\gamma = j\beta$

$$H_y = C_4 \cos\left(\frac{m\pi}{a}\right)xe^{-j\beta z}$$

$$E_x = \frac{\beta}{\omega\varepsilon}C_4 \cos\left(\frac{m\pi}{a}\right)x\, e^{-j\beta z}$$

$$E_z = \frac{jm\pi}{\omega\varepsilon a}C_4 \sin\left(\frac{m\pi}{a}\right)x\, e^{-j\beta z}$$

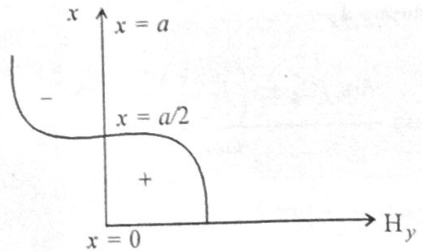


Figure 4.2 H_y *as a function of x for* TM_{10} *mode.*

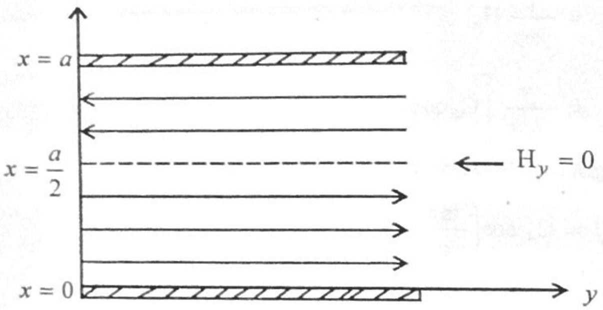


Figure 4.3 H_y *as a function of y for* TM_{10} *mode.*

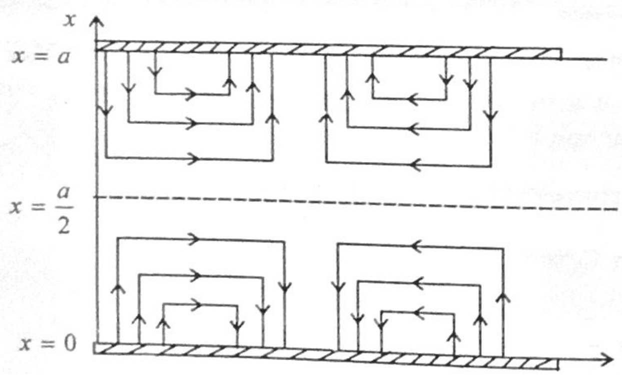


Figure 4.4 Combined field distribution of E_z *and* E_x *for* TM_{10} *mode.*

4.4 TE WAVES BETWEEN PARALLEL PLATES

Transverse electric waves are the waves in which electric field strength is entirely transverse. It has magneticfield strength in the direction of propagation and there is no electricfield component in the direction of propagation. ($E_z = 0; H_z \neq 0$)

We know that

$$E_x = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial x} \; ; H_y = -\frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

Substituting $E_z = 0$

$$E_x = 0 \text{ and } H_y = 0$$

The wave equation for E_y is given by

$$\frac{\partial^2 E_y}{\partial x^2} + \gamma^2 E_y = -\omega^2\mu\varepsilon E_y \quad ...(1)$$

$$\frac{\partial^2 E_y}{\partial x^2} = -(\omega^2\mu\varepsilon + \gamma^2)E_y = -h^2 E_y$$

$$\frac{\partial^2 E_y}{\partial x^2} + h^2 E_y = 0$$

Let $E_y = E_y^{\circ} e^{-\gamma z}$ ...(2)

$$\frac{\partial^2 E_y^{\circ}}{\partial x^2} + h^2 E_y^{\circ} = 0 \quad ...(3)$$

This is the differential equation of simple harmonic motion. The solution of this equation is given by

$$E_y^{\circ} = C_1 \sin hx + C_2 \cos hx \quad ...(4)$$

Where C_1 and C_2 are arbitary constants.

Substituting equation (4) in (2)

$$E_y = (C_1 \sinh x + C_2 \cosh x)e^{-\gamma z} \quad ...(5)$$

C_1 and C_2 can be determined from boundary conditions.

$$E_y = 0 \ at \ x = 0$$

$$E_y = 0 \ at \ x = a$$

Applying first boundary condition

$$0 = 0 + C_2 e^{-\gamma z}$$

$$C_2 = 0$$

Substituting $C_2 = 0$ in equation (5)

$$E_y = C_1 \sin hx \ e^{-\gamma z}$$

Applying second boundary condition

$$C_1 \ \sin h_a \ \ e^{-\gamma z} = 0$$

Assuming $C_1 \neq 0$

$$\sin ha = 0 \Rightarrow ha = \sin^{-1}(0) \Rightarrow ha = m\pi \Rightarrow h = \frac{m\pi}{a}; \quad m = 1, 2, ..$$

$$E_y = C_1 \sin\left(\frac{m\pi}{a}\right) x e^{-\gamma z}$$

From (9a)

$$\gamma E_y = -j\omega\mu H_x$$

$$H_x = \frac{-\gamma E_y}{j\omega\mu} = \frac{-\gamma}{j\omega\mu}\left[C_1 \sin\left(\frac{m\pi}{a}\right) x e^{-\gamma z}\right]$$

From (9c)

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_z$$

$$H_z = \frac{-1}{j\omega\mu}\left[\frac{\partial E_y}{\partial x}\right]$$

$$= \frac{-1}{j\omega\mu}\left[\frac{\partial}{\partial x}\left(C_1 \sin\left(\frac{m\pi}{a}\right) x \cdot e^{-\gamma z}\right)\right]$$

$$= \frac{jm\pi}{\omega\mu a} C_1 \cos\left(\frac{m\pi}{a}\right) x \cdot e^{-\gamma z}$$

Field strength for TE waves between parallel planes are

$$E_y = C_1 \sin\left(\frac{m\pi}{a}\right) x e^{-\gamma z}$$

$$H_x = \frac{-\gamma}{j\omega\mu} C_1 \sin\left(\frac{m\pi}{a}\right) x e^{-\gamma z}$$

$$H_z = \frac{jm\pi}{\omega\mu a} C_1 \cos\left(\frac{m\pi}{a}\right) x e^{-\gamma z}$$

Each value of m specifies a particular field of configuration (or) mode and wave associated with integer m is designated as TE_{mo} mode. The second subscript refers to another integer which varies with y. If m=0 all the fields become zero $E_y = 0$, $H_x = 0$, $H_z = 0$ therefore lowest value of m =1. The lowest order mode is TE_{10}. This is called dominant mode in TE waves.

$\gamma = \alpha + j\beta$ if the wave propagate without attenuation $\alpha = 0$, $\gamma = j\beta$.

$$E_y = C_1 \sin\left(\frac{m\pi}{a}\right) x e^{-\gamma\beta z}$$

$$H_x = \frac{-\beta}{\omega\mu} C_1 \sin\left(\frac{m\pi}{a}\right) x e^{-j\beta z}$$

$$H_z = \frac{jm\pi}{\omega\mu a} C_1 \cos\left(\frac{m\pi}{a} x\right) x e^{-j\beta z}$$

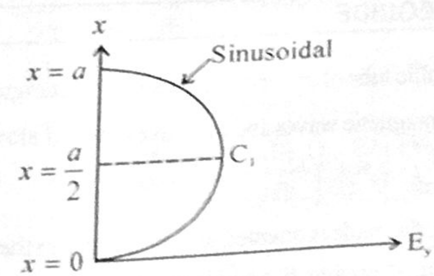


Figure 5.5 E_y as a function of x for TE_{10} mode.

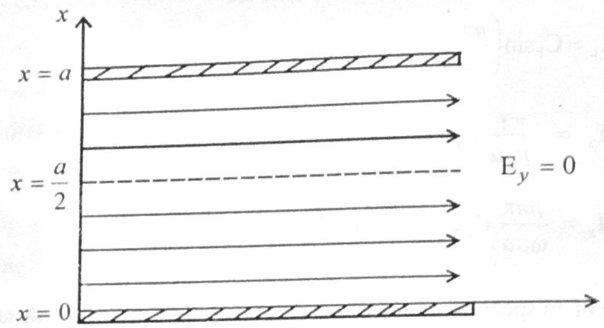


Figure 5.6 E_y as a function of y for TE_{10} mode.

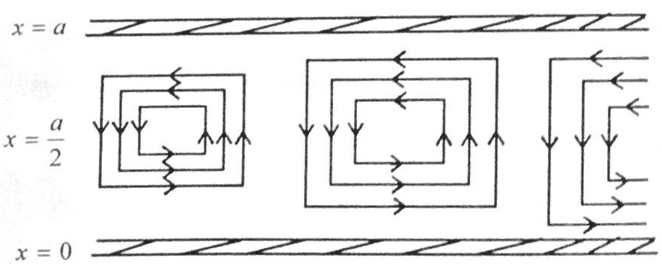


Figure 5.7 Combined field distribution of H_x & H_z for TE_{10} mode.

5.5 RECTANGULAR WAVEGUIDE.

A hollow conducting metallic tube of uniform cross section (rectangular or circular) is used for propagating electromagnetic waves and the waves guided along the walls of the tubee is called wave guide

In rectangular waveguide, the guide is oriented with two faces in the planes of axes and that dimension a is height in x direction, b is width in y direction. The guide is made up of perfectly metal walls and is filled with dielectric with constants μ_1, ε_1.

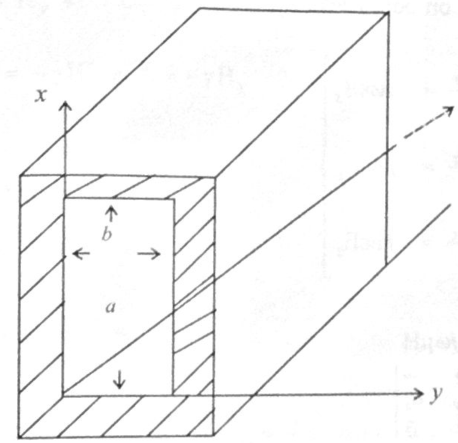


Figure 5.8 Cross section of rectangular waveguide.

The propagation of energy takes place in z- direction with length of guide being infinite in z direction. The field components of electric field and magnetic field are obtained by solving Maxwell's equations by applying appropriate boundary conditions.

We know that

$$\nabla \times H = j\omega\varepsilon E$$

$$\nabla \times H = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = j\omega\varepsilon\left[\vec{a}_x E_x + \vec{a}_y E_y + \vec{a}_z E_z\right]$$

$$\vec{a}_x\left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right] - \vec{a}_y\left[\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right] + \vec{a}_z\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]$$

$$= j\omega\varepsilon\left[\vec{a}_x E_x + \vec{a}_y E_y + \vec{a}_z E_z\right]$$

Equating $\vec{a}_x, \vec{a}_y, \vec{a}_z$ on both sides

$$\left.\begin{aligned} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} &= j\omega\varepsilon E_x \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} &= j\omega\varepsilon E_y \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= j\omega\varepsilon E_z \end{aligned}\right\} \quad ...(1)$$

Similarly

$$\nabla \times E = -j\omega\mu H$$

$$\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu\left[\vec{a}_x H_x + \vec{a}_y H_y + \vec{a}_z H_z\right]$$

$$\vec{a}_x\left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right] - \vec{a}_y\left[\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right] + \vec{a}_z\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right]$$

$$= -j\omega\mu\left[\vec{a}_x H_x + \vec{a}_y H_y + \vec{a}_z H_z\right]$$

Equating $\vec{a}_x, \vec{a}_y, \vec{a}_z$ on both sides

$$\left.\begin{aligned} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} &= -j\omega\mu H_x \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} &= -j\omega\mu H_y \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= -j\omega\mu H_z \end{aligned}\right\} \quad ...(2)$$

Similarly for wave equation

$$\left.\begin{aligned} \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} &= -\omega^2\mu\varepsilon E \\ \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} &= -\omega^2\mu\varepsilon E \end{aligned}\right\} \quad ...(3)$$

Let $H_y = H_y^\circ \, e^{-\gamma z}$

$$\frac{\partial H_y}{\partial z} = -\gamma H_y^\circ \, e^{-\gamma z} = -\gamma H_y$$

Similarly

$$\frac{\partial H_x}{\partial z} = -\gamma H_x$$

Let $E_y = E_y^\circ \, e^{-\gamma z}$

$$\frac{\partial E_y}{\partial z} = -\gamma E_y^\circ \, e^{-\gamma z} = -\gamma E_y$$

Similarly

$$\frac{\partial E_x}{\partial z} = -\gamma E_x$$

Sbustituting these derivatives in equation (1) and (2)

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x \quad ...(4)$$

$$\gamma H_x + \frac{\partial H_z}{\partial x} = -j\omega\varepsilon E_y \quad ...(5)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \quad ...(6)$$

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x \quad ...(7)$$

$$\frac{\partial E_z}{\partial x} + \gamma E_x = j\omega\mu H_y \quad ...(8)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad ...(9)$$

Wave equation becomes

$$\left.\begin{aligned} \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z &= -\omega^2\mu\varepsilon E_z \\ \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z &= -\omega^2\mu\varepsilon H_z \end{aligned}\right\} \quad ...(10)$$

Solving equation (5) and (7)

From equation (7)

$$H_x = \frac{-1}{j\omega\mu}\left[\frac{\partial E_z}{\partial y} + \gamma E_y\right]$$

Substituting in equation (5)

$$\frac{\partial H_z}{\partial x} + \gamma\left(\frac{-1}{j\omega\mu}\left(\frac{\partial E_z}{\partial y} + \gamma E_y\right)\right) = -j\omega\varepsilon E_y$$

$$\frac{\partial H_z}{\partial x} - \frac{\gamma\partial E_z}{j\omega\mu\partial y} - \frac{\gamma^2 E_y}{j\omega\mu} = -j\omega\varepsilon E_y$$

$$\frac{\partial H_z}{\partial x} - \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial y} = \left(\frac{\gamma^2}{j\omega\mu} - j\omega\varepsilon\right)E_y$$

$$j\omega\mu\frac{\partial H_z}{\partial x} - \gamma\frac{\partial E_z}{\partial y} = (\gamma^2 + \omega^2\mu\varepsilon)E_y = h^2 E_y$$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x} - \frac{\gamma}{h^2}\frac{\partial E_z}{\partial y} \quad \ldots(11)$$

From equation (5)

$$E_y = \frac{-1}{j\omega\varepsilon}\left[\frac{\partial H_z}{\partial x} + \gamma H_x\right]$$

Substituting in (7)

$$\frac{\partial E_z}{\partial y} + \gamma\left[\frac{-1}{j\omega\varepsilon}\left(\frac{\partial H_z}{\partial x}\right) + \gamma H_x\right] = -j\omega\mu H_x$$

$$\frac{\partial E_z}{\partial y} - \frac{\gamma}{j\omega\varepsilon}\frac{\partial H_z}{\partial x} - \frac{\gamma^2 H_z}{j\omega\varepsilon} = -j\omega\mu H_x$$

$$\frac{\partial E_z}{\partial y} - \frac{\gamma}{j\omega\varepsilon}\frac{\partial H_z}{\partial x} = \left(\frac{\gamma^2}{j\omega\varepsilon} - j\omega\mu\right)H_x$$

$$\frac{j\omega\varepsilon\partial E_z}{\partial y} - \gamma\frac{\partial H_z}{\partial x} = (\gamma^2 + \omega^2\mu\varepsilon)H_x$$

$$H_x = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y} - \frac{\gamma}{h^2}\frac{\partial H_z}{\partial x} \quad \ldots(12)$$

Solving equations(4) and (8)

From equation (4)

$$E_x = \frac{\gamma H_y}{j\omega\varepsilon} + \frac{1}{j\omega\varepsilon}\frac{\partial H_z}{\partial y}$$

Substituting in equation (8)

$$\frac{\partial E_z}{\partial x} + \gamma\left[\frac{\gamma H_y}{j\omega\varepsilon} + \frac{1}{j\omega\varepsilon}\frac{\partial H_z}{\partial y}\right] = j\omega\mu H_y$$

$$\frac{\partial E_z}{\partial x} + \frac{\gamma^2 H_y}{j\omega\varepsilon} + \frac{\gamma}{j\omega\varepsilon}\frac{\partial H_z}{\partial y} = j\omega\mu H_y$$

$$\frac{j\omega\varepsilon\partial E_z}{\partial x} + \gamma^2 H_y + \gamma\frac{\partial H_z}{\partial y} = -\omega^2\mu\varepsilon H_y$$

$$\frac{j\omega\varepsilon\partial E_z}{\partial x} + \gamma\frac{\partial H_z}{\partial y} = -(\omega^2\mu\varepsilon + \gamma^2)H_y = -h^2 H_y$$

$$H_y = -\frac{\gamma}{h^2}\frac{\partial H_z}{\partial y} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x} \ldots(13)$$

From equation (8)

$$H_y = \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\mu}E_x$$

Substituting in equation (4)

$$\frac{\partial H_z}{\partial y} + \gamma\left[\frac{1}{j\omega\mu}\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\mu}E_x\right] = j\omega\varepsilon E_x$$

$$\frac{\partial H_z}{\partial y} + \frac{\gamma^2}{j\omega\mu}E_x + \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial x} = j\omega\varepsilon E_x$$

$$\frac{j\omega\mu\partial H_z}{\partial y} + \gamma^2 E_x + \gamma\frac{\partial E_z}{\partial x} = -\omega^2\mu\varepsilon E_x$$

$$\frac{j\omega\mu\partial H_z}{\partial y} + \gamma\frac{\partial E_z}{\partial x} = -(\omega^2\mu\varepsilon + \gamma^2)E_x = -h^2 E_x$$

$$E_x = -\frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} \quad \ldots(14)$$

The above equations are in terms of E_z and H_z. For wave propagation either E_z or H_z should exist. If both E_z and H_z are zero, all the fields with in the guide will vanish. wave propagation within the guide is divided into two sets.

✓ Transverse electric waves ($E_z = 0$; $H_z \neq 0$)

✓ Transverse magnetic waves ($H_z = 0$; $E_z \neq 0$)

5.5.1 TM waves in Rectangular Waveguide

Wave equation for E_z is given by,

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu\varepsilon E_z \quad ...(1)$$

Let

$$\frac{\partial E_z}{\partial z} = -\gamma E_z$$

$$\frac{\partial^2 E_z}{\partial z^2} = \gamma^2 E_z$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \mu\varepsilon E_z \quad ...(2)$$

The wave equation is second order partial differential equation that can be solved by product solution.

E_z can be written as

$$E_z(x, y, z) = E_z^\circ(x, y) e^{-\gamma z} \quad ...(3)$$

Let $E_z^\circ = XY$ ...(4)

Where X is a function of x alone

Y is a function of y alone

Substituting the value of E_z° in equation (2)

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + \gamma^2 XY = -\omega^2\mu\varepsilon XY$$

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + (\gamma^2 + \omega^2\mu\varepsilon)XY = 0$$

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + h^2 XY = 0$$

Divide by XY

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + h^2 = 0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + h^2 = -\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} \quad ...(5)$$

The above equation equates a function of x alone to a function of y alone and the only way for above equation to be true is to have each of these function equal to some constant A^2.

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + h^2 = A^2$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + h^2 - A^2 = 0$$

Let $B^2 = h^2 - A^2$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + B^2 = 0 \quad ...(6)$$

The solution of the equation (6) is given by

$$X = C_1 \cos Bx + C_2 \sin Bx \quad ...(7)$$

$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -A^2 \quad ...(8)$$

The solution of the equation (8) is given by

$$Y = C_3 \cos Ay + C_4 \sin Ay \quad ...(9)$$

Substituting equation (7) and (9) in (4)

$$E_z^\circ = XY = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay)$$

$$= C_1 C_3 \cos Bx \cos Ay + C_1 C_4 \cos Bx \sin Ay + C_2 C_3 \sin Bx \cos Ay + C_2 C_4 \sin Bx \sin Ay \quad ...(10)$$

The constants C_1, C_2, C_3, C_4, A and B are to be determined by using appropriater boundary conditions.

$E_z^\circ = 0$ When $x = 0,\ x = a,\ y = 0,\ y = b$

When $x = 0$, equation (10)become

$E_z^\circ = C_1 C_3 \cos Ay + C_1 C_4 \sin Ay = 0$

For the above equation to be true for all values of y , $C_1 = 0$

Equation (10) becomes

$$E_z^\circ = C_2 C_3 \sin Bx \cos Ay + C_2 C_4 \sin Bx \sin Ay \quad ...(11)$$

When $y = 0$ equation (11) becomes

$E_z^\circ = C_2 C_3 \sin Bx = 0$ (Assuming $B \neq 0$)

This is possible only if either $C_2 = 0$, or $C_3 = 0$. If $C_2 = 0$, E_z° is identially zero.

so substituting $C_3 = 0$ in equation (11)

$E_z^\circ = C_2 C_4 \sin Bx \sin Ay$

Let $C = C_2 C_4$

$$E_z^\circ = C \sin Bx \sin Ay \quad ...(12)$$

Applying boundary conditions in order to evaluate the constants A and B

It $x = a$; $E_z^\circ = 0$ equation (12) become

$C \sin Ba \sin Ay = 0$

The above equation is ture for all values of y when $A \neq 0$

$\sin Ba = 0$

$Ba = \sin^{-1}(0)$

$Ba = m\pi$

$B = \dfrac{m\pi}{a}$ When m = 1,2....

Substituting $B = \dfrac{m\pi}{a}$ in equation (12)

$$E_z^\circ = C \sin\left(\frac{m\pi}{a}\right) x \sin Ay \quad ...(13)$$

If $y = b$; $E_z^\circ = 0$

$E_z^\circ = C \sin\left(\dfrac{m\pi x}{a}\right) \sin Ab = 0$

The above equation is true if

$\sin Ab = 0$

$Ab = \sin^{-1}(0)$

$Ab = n\pi$

$A = \dfrac{n\pi}{b}$; n = 1,2 ...

Substituting $A = \dfrac{n\pi}{b}$ in equation (13)

$$E_z^\circ = C \sin\left(\frac{m\pi}{a}\right) x \sin\left(\frac{n\pi}{b}\right) y$$

The general field components with $H_z = 0$ and $\gamma = j\beta$ is given by

$$E_x^\circ = \frac{-j\beta}{h^2} \frac{\partial E_z^\circ}{\partial x}$$

$$= \frac{-j\beta}{h^2} CB \cos Bx \sin Ay$$

$$E_y^\circ = \frac{-j\beta}{h^2} \frac{\partial E_z^\circ}{\partial y}$$

$$= \frac{-j\beta}{h^2} AC \sin Bx \cos Ay$$

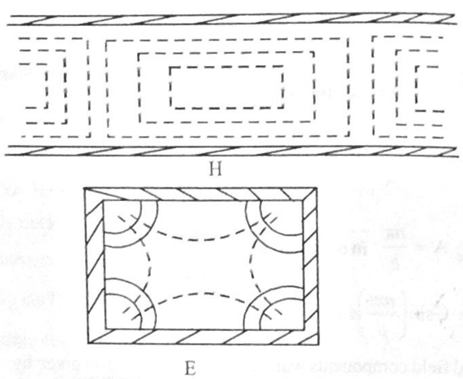
$$H_x^\circ = \frac{j\omega\varepsilon}{h^2} \frac{\partial E_z^\circ}{\partial y}$$

$$= \frac{j\omega\varepsilon AC}{h^2} \sin Bx \cos Ay$$

$$H_y^\circ = \frac{-j\omega\varepsilon}{h^2} \frac{\partial E_z^\circ}{\partial x} = \frac{-j\omega\varepsilon}{h^2} CB \cos Bx \sin Ay$$

Where $A = \dfrac{n\pi}{b}$; $B = \dfrac{m\pi}{a}$

If $m = 0$ or $n = 0$ the fields for TM wave will be identically zero. So the lowest possible value for m or n is unity. ($m = 1$; $n = 1$) for TM waves. This particular wave is called TM_{11} mode.



TM$_{11}$ Wave

Figure 5.9 Electric field and magnetic field configuration for TM$_{11}$ in rectangular waveguide

5.5.2 TE Waves in Rectangular Waveguide.

Wave equation for H_z is given by

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} = -\omega^2 \mu\varepsilon H_z \quad ...(1)$$

Let $$\frac{\partial H_z}{\partial z} = -\gamma H_z$$

$$\frac{\partial^2 H_z}{\partial z^2} = \gamma^2 H_z$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z = -\omega^2 \mu\varepsilon H_z \quad ...(2)$$

The wave equation is second order partial differential equation that can be solved by product solution.

H_z can be written as

$$H_z(x,y,z) = H_z^\circ(x,y)e^{-\gamma z} \quad ...(3)$$

Let $H_z^\circ = XY$ ...(4)

Where X is a function of x alone

Y is a function of y alone

Substituting the value of H_z° in equation (2)

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + \gamma^2 XY = -\omega^2 \mu\varepsilon XY$$

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + (\gamma^2 + \omega^2\mu\varepsilon)XY = 0$$

$$Y\frac{\partial^2 X}{\partial x^2} + x\frac{\partial^2 Y}{\partial y^2} + h^2 XY = 0$$

Divide by XY

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + h^2 = 0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + h^2 = -\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} \quad ...(5)$$

The above equation equates a function of x alone to a function of y alone and the only way for the above equation to be true is to have each of these function equal to some constant A^2

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + h^2 = A^2$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + h^2 - A^2 = 0$$

Let $B^2 = h^2 - A^2$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + B^2 = 0 \quad ...(6)$$

The solution of the equation (6) is given by

$$X = C_1 \cos Bx + C_2 \sin Bx \quad ...(7)$$

$$\frac{1}{Y}\frac{\partial^2 y}{\partial y^2} = -A^2 \quad ...(8)$$

The solution of the equation (8) is given by

$$Y = C_3 \cos Ay + C_4 \sin Ay \quad ...(9)$$

Substituting equation (7) and (9) in (4)

$$H_z^\circ = XY = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay)$$

$$= C_1 C_3 \cos Bx \cos Ay + C_1 C_4 \cos Bx \sin Ay + C_2 C_3 \sin Bx \cos Ay + C_2 C_4 \sin Bx \sin Ay \quad ...(10)$$

C_1, C_2, C_3, C_4, A and B are to be determined using boundary conditions. The tangential component of electric field is continuous and $E_{tan} = 0$ at the boundary for a perfect conductor.

$$E_x^\circ = \frac{-j\omega\mu}{h^2}\frac{\partial H_z^\circ}{\partial y}$$

$$= \frac{-j\omega\mu}{h^2}\begin{bmatrix} -AC_1 C_3 B \sin Ay \cos By + AC_1 C_4 \sin Bx \sin Ay \\ -C_2 C_3 A \cos Bx \cos Ay + A C_2 C_4 \cos Bx \sin Ay \end{bmatrix} \quad ...(11)$$

$$E_y^\circ = \frac{j\omega\mu}{h^2}\frac{\partial H_z^\circ}{\partial x}$$

$$= \frac{j\omega\mu}{h^2}\begin{bmatrix} -C_1 C_3 B \sin Bx \cos Ay - C_1 C_4 B \sin Bx \sin Ay \\ +C_2 C_3 B \cos Bx \cos Ay + B C_2 C_4 \cos Bx \sin Ay \end{bmatrix} \quad ...(12)$$

$E_y^\circ = 0$ at $x = 0$

Equation (12) becomes

$$E_y^\circ = \frac{j\omega\mu}{h^2}(C_2 C_3 B \cos Ay + B C_2 C_4 \sin Ay) = 0$$

The above equation is true only if $C_2 = 0$

Substituting $C_2 = 0$ in equation (12)

$$E_y^\circ = \frac{j\omega\mu}{h^2}[-C_1 C_3 B \sin Bx \cos Ay - C_1 C_4 B \sin Bx \sin Ay] \quad ...(13)$$

$E_y^\circ = 0$ at $x = a$

Equation (13) becomes

$$E_y^\circ = \frac{j\omega\mu}{h^2}[-C_1 C_3 B \sin Ba \cos Ay - C_1 C_4 B \sin Ba \sin Ay] = 0$$

This is true only if

$$\sin Ba = 0$$

$$Ba = \sin^{-1}(0) = m\pi$$

$$B = \frac{m\pi}{a} \quad \text{Where } m = 1,2,...$$

Substituting $B = \frac{m\pi}{a}$ in equation (13)

$$E_y^\circ = \frac{j\omega\mu}{h^2}\left[-C_1 C_3\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x \cos Ay - C_1 C_4\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}\right)x \sin Ay\right] \quad ...(14)$$

$E_x^\circ = 0 \; ; \; y = 0$

Equation (11) becomes

$$E_x^\circ = \frac{-j\omega\mu}{h^2}[A C_1 C_4 \cos Bx \cos Ay + A C_2 C_4 \sin Bx \cos Ay] = 0$$

The above equation is true only for $C_4 = 0$

Substituting $C_4 = 0$ in equation (11)

$$E_x^\circ = \frac{-j\omega\mu}{h^2}A[-C_1 C_3 \cos Bx \sin Ay - C_2 C_3 \sin Bx \sin Ay] \quad ...(15)$$

$E_x^\circ = 0$ at $y = b$

Equation (15) becomes

$$E_x^\circ = \frac{-j\omega\mu}{h^2}A[-C_1 C_3 \cos Bx \sin Ab - C_2 C_3 \sin Bx \sin Ab] = 0$$

This is true only if, $\sin Ab = 0$

$$Ab = \sin^{-1}(0) = n\pi$$

$$A = \frac{n\pi}{b} \quad \text{Where } n = 1,2,...$$

Equation (15) becomes

$$E_x^\circ = \frac{j\omega\mu}{h^2} A \left[C_1C_3 \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y + C_2C_3 \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y \right] \quad ...(16)$$

Using $C_2 = 0$, $C_4 = 0$, $A = \frac{n\pi}{b}$, $B = \frac{m\pi}{a}$, $C_1C_3 = C$, $\gamma = j\beta$ in equation (10)

$$H_z^\circ = C \cos Bx \cos Ay$$

The general field components become

$$E_x^\circ = \frac{-j\omega\mu}{h^2}\frac{\partial H_z^\circ}{\partial y}$$

$$= \frac{-j\omega\mu}{h^2}[C \cos Bx(-\sin Ay)A] = \frac{j\omega\mu AC}{h^2}\cos Bx \sin Ay$$

$$H_x^\circ = \frac{-j\beta}{h^2}\frac{\partial H_z^\circ}{\partial x}$$

$$= \frac{-j\beta}{h^2} CB \sin Bx \cos Ay$$

$$H_y^\circ = \frac{-j\beta}{h^2}\frac{\partial H_z^\circ}{\partial y}$$

$$= \frac{j\beta}{h^2} CA \cos Bx \sin Ay$$

$$E_y^\circ = \frac{-j\omega\mu}{h^2} BC \sin Bx \cos Ay$$

Where $A = \frac{n\pi}{b}$, $B = \frac{m\pi}{a}$

TE_{10} mode is dominant mode in rectangular wave.

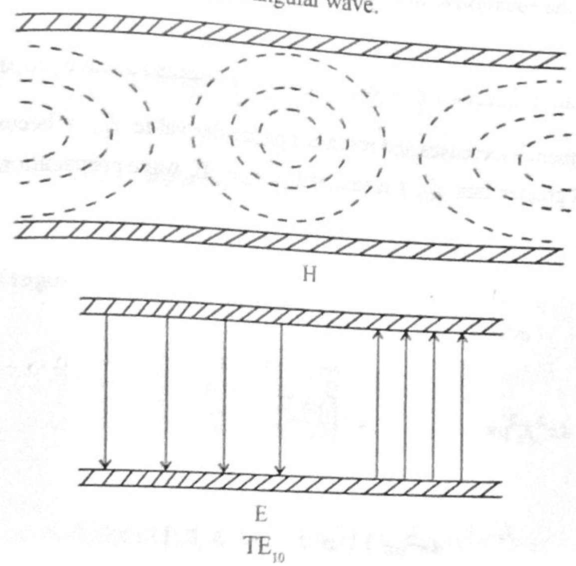


Figure 5.10 Electric field and magnetic field configuration for TE_{10} in rectangular wave guide.

5.5.3 Characteristics of TE and TM Waves in Rectangular Waveguide.

We know that

$$A^2 + B^2 = h^2$$

Where $A = \frac{n\pi}{b}$, $B = \frac{m\pi}{a}$, m and n are integers

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon$$

$$\gamma^2 = h^2 - \omega^2\mu\varepsilon$$

$$\gamma = \sqrt{h^2 - \omega^2\mu\varepsilon}$$

$$= \sqrt{A^2 + B^2 - \omega^2\mu\varepsilon}$$

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\varepsilon}$$

This is the equation of propagation constant in rectangular waveguide for TE or TM waves.

For small frequencies $\gamma = \alpha$, γ is real and there is no wave propagation.

As frequency increases and reaches a particular value f_c, γ becomes zero. For all values of f greater than f_c, γ is imaginary, $\gamma = j\beta$, wave propagation takes place.

At $f = f_c$, $\gamma = 0$; $h^2 = \omega_c^2\mu\varepsilon$

$$\omega_c^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$4\pi^2 f_c^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$f_c^2 = \frac{1}{4\pi^2\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]$$

Cut off frequency $f_c = \dfrac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\dfrac{m\pi}{a}\right)^2 + \left(\dfrac{n\pi}{b}\right)^2}$

$$= \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$f < f_c, \gamma = \alpha = \sqrt{h^2 - \omega^2\mu\varepsilon}$$

$$f > f_c, \gamma = j\beta$$

$$= \sqrt{-(\omega^2\mu\varepsilon - h^2)} = j\sqrt{\omega^2\mu\varepsilon - h^2}$$

$$j\beta = j\sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

Phase constant $\beta = \sqrt{\omega^2\mu\varepsilon - \left(\dfrac{m\pi}{a}\right)^2 - \left(\dfrac{n\pi}{b}\right)^2} = \sqrt{\omega^2\mu\varepsilon - \left[\left(\dfrac{m\pi}{a}\right)^2 + \left(\dfrac{n\pi}{b}\right)^2\right]}$

$$= \sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}$$

$$= \omega\sqrt{\mu\varepsilon}\sqrt{1 - \frac{\omega_c^2}{\omega^2}}$$

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \frac{f_c^2}{f^2}}$$

Cut-off wavelength $\lambda_c = \dfrac{c}{f_c} = \dfrac{c \times 2\sqrt{\mu\varepsilon}}{\sqrt{\left(\dfrac{m}{a}\right)^2 + \left(\dfrac{n}{b}\right)^2}}$

$$= \frac{\frac{1}{\sqrt{\mu\varepsilon}} \times 2\sqrt{\mu\varepsilon}}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

Guide wavelength $\lambda_g = \dfrac{2\pi}{\beta}$

$$= \frac{2\pi}{\omega\sqrt{\mu\varepsilon}\sqrt{1 - \frac{f_c^2}{f^2}}}$$

$$= \frac{2\pi}{2\pi f\sqrt{\mu\varepsilon}\sqrt{1 - \frac{f_c^2}{f^2}}}$$

$$= \frac{c}{f\sqrt{1 - \frac{f_c^2}{f^2}}}$$

$$\lambda_g = \frac{\lambda_o}{\sqrt{1 - \frac{f_c^2}{f^2}}}$$

Where $\lambda_o = \dfrac{c}{f}$ is free space wavelength

Phase velocity $v_{ph} = \frac{\omega}{\beta}$

$$= \frac{\omega}{\omega\sqrt{\mu\varepsilon}\sqrt{1-\frac{f_c^2}{f^2}}}$$

$$v_{ph} = \frac{c}{\sqrt{1-\frac{f_c^2}{f^2}}}$$

Group velocity $v_g = \frac{c^2}{v_{ph}} = \frac{c^2}{c}\sqrt{1-\frac{f_c^2}{f^2}}$

$$v_g = c\sqrt{1-\frac{f_c^2}{f^2}}$$

Wave impedance or characteristic impedance

$$Z_{TE} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\omega\mu}{\beta}$$

$$= \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}\sqrt{1-\frac{f_c^2}{f^2}}}$$

$$= \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt{1-\frac{f_c^2}{f^2}}}$$

$$Z_{TE} = \frac{\eta}{\sqrt{1-\frac{f_c^2}{f^2}}}$$

$$Z_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\beta}{\omega\varepsilon}$$

$$= \frac{\omega\sqrt{\mu\varepsilon}\sqrt{1-\frac{f_c^2}{f^2}}}{\omega\varepsilon}$$

$$= \sqrt{\frac{\mu}{\varepsilon}}\sqrt{1-\frac{f_c^2}{f^2}}$$

$$Z_{TM} = \eta\sqrt{1-\frac{f_c^2}{f^2}}$$

5.6 BESSEL'S DIFFERENTIAL EQUATION AND BESSEL FUNCTION

In solving for the electromagnetic fields within the circular wave guides, a differential equation known as Bessel's equation is encountered. The solution of the Bessel's equation leads to Bessel function.

Bessel's differential equation involved in circular waveguide has the form

$$\frac{d^2P}{d\rho^2} + \frac{1}{\rho}\frac{dP}{d\rho} + \left(1-\frac{n^2}{\rho^2}\right)P = 0 \quad ...(1)$$

Where n is an integer.

The solution of this Bessel's equation can be obtained by assuming a power series solution

$$P = a_o' + a_1\rho + a_2\rho^2 + ... \quad ...(2)$$

For special case $n = 0$, the Bessel's equation becomes

$$\frac{d^2P}{d\rho^2} + \frac{1}{\rho}\frac{dP}{d\rho} + P = 0 \quad ...(3)$$

Substituting equation (2) in eqation (3) and equating the sum of the coefficients of each power of ρ to zero.

$$P = C_1\left[1-\left(\frac{\rho}{2}\right)^2 + \frac{\left(\frac{1}{2}\rho\right)^4}{(2!)^2} - \frac{\left(\frac{1}{2}\rho\right)^6}{(3!)^2} + ...\right] \quad ...(4)$$

$$= C_1 \sum_{r=0}^{\infty} (-1)^r \frac{\left(\frac{1}{2}\rho\right)^{2r}}{(r!)^2} = J_o(\rho) \qquad \ldots(5)$$

The series is convergent for all values of ρ either real or complex. Equation (5) is called Bessel's function of first kind of order zero and is denoted by $J_0(\rho) = P_0$ for $n = 0$. The corresponding solution for $n = 1,2,3\ldots$ are designated as $J_1(\rho)$, $J_2(\rho)$, $J_3(\rho)\ldots$ where 'n' denotes the order of Bessel's function. Figure 5.11 shows the Bessel function of first kind of different orders.

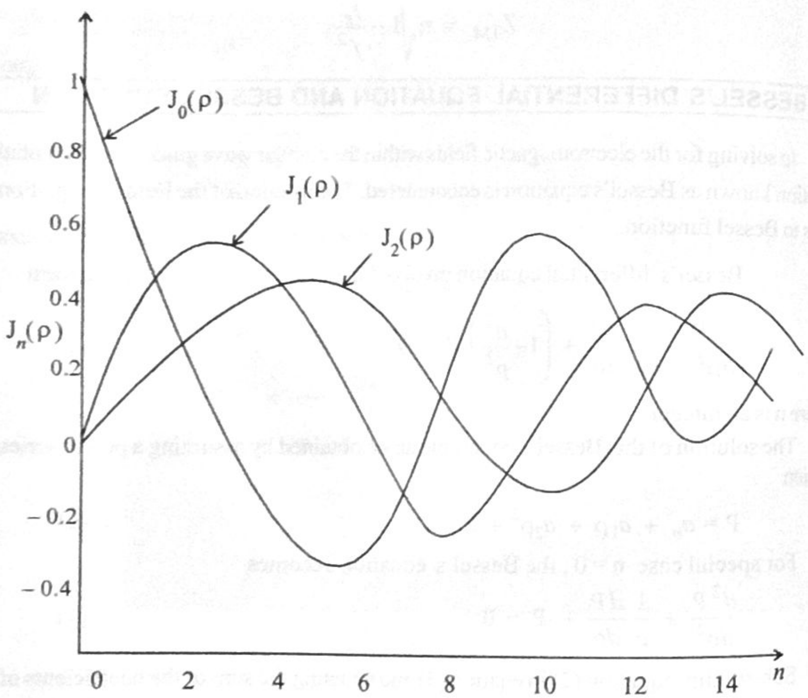


Figure 5.11 Bessel's function of first kind of different orders

Since equation (3) is a second order differential equation, there must be two linearly independent solutions for each value of n. The second solution may be obtained in a similar manner that is used for first but starting with a slightly different series. This second solution is known as Bessel's function of second kind or Neumann's function denoted as $N_n(\rho)$, where 'n' indicates the order of function.

For the zero order of the second kind, the following series is obtained,

$$N_0(\rho) = \frac{2}{\pi}\left[ln\left(\frac{\rho}{2}\right) + \gamma\right] J_0(\rho) - \frac{2}{\pi}\sum_{r=1}^{\infty}(-1)^r \frac{\left(\frac{1\rho}{2}\right)^{2r}}{(r!)^2}\left[1 + \frac{1}{2} + \frac{1}{3} + \ldots \frac{1}{r}\right]$$

The complete solution of equation (3) is given by

$$P = A\,J_0(\rho) + B\,N_0(\rho)$$

Figure 5.12 shows the zero order Bessel function of first and second kind.

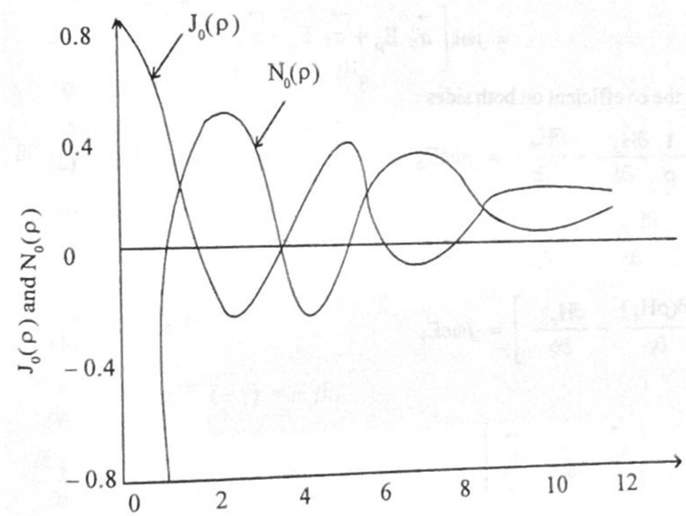


Figure 5.12 Zero order Bessel function of first and second kind

5.7 CIRCULAR (OR) CYLINDRICAL WAVEGUIDE

In order to simplify the application of boundary conditions cylindrical co- ordinates are used.

Maxwell's equations is

$$\nabla \times H = j\omega\varepsilon E$$

$$\nabla \times H = \begin{vmatrix} \frac{\vec{a}_\rho}{\rho} & \vec{a}_\phi & \frac{\vec{a}_z}{\rho} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ H_\rho & \rho H_\phi & H_z \end{vmatrix} = j\omega\varepsilon E \qquad ...(1)$$

$$\frac{\vec{a}_\rho}{\rho}\left[\frac{\partial H_z}{\partial\phi} - \frac{\partial(\rho H_\phi)}{\partial z}\right] - \vec{a}_\phi\left[\frac{\partial H_z}{\partial\rho} - \frac{\partial H_\rho}{\partial z}\right] + \frac{\vec{a}_z}{\rho}\left[\frac{\partial(\rho H_\phi)}{\partial\rho} - \frac{\partial H_\rho}{\partial\phi}\right]$$

$$= j\omega\varepsilon\left[\vec{a}_\rho E_\rho + \vec{a}_\phi E_\phi + \vec{a}_z E_z\right]$$

Comparing the co efficient on both sides

$$\frac{1}{\rho}\frac{\partial H_z}{\partial\phi} - \frac{\partial H_\phi}{\partial z} = j\omega\varepsilon E_\rho \qquad ...(2)$$

$$\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial\rho} = j\omega\varepsilon E_\phi \qquad ...(3)$$

$$\frac{1}{\rho}\left[\frac{\partial(\rho H_\phi)}{\partial\rho} - \frac{\partial H_z}{\partial\phi}\right] = j\omega\varepsilon E_z \qquad ...(4)$$

Similarly

$$\nabla \times E = \begin{vmatrix} \frac{\vec{a}_\rho}{\rho} & \vec{a}_\phi & \frac{\vec{a}_z}{\rho} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ E_\rho & \rho E_\phi & E_z \end{vmatrix} = -j\omega\mu\left[\vec{a}_\rho H_\rho + \vec{a}_\phi H_\phi + \vec{a}_z H_z\right] \qquad ...(15)$$

$$\frac{\vec{a}_\rho}{\rho}\left[\frac{\partial E_z}{\partial\phi} - \frac{\partial(\rho E_\phi)}{\partial z}\right] - \vec{a}_\phi\left[\frac{\partial E_z}{\partial\rho} - \frac{\partial E_\rho}{\partial z}\right] + \frac{\vec{a}_z}{\rho}\left[\frac{\partial(\rho E_\phi)}{\partial\rho} - \frac{\partial E_\rho}{\partial\phi}\right]$$

$$= -j\omega\mu\left[\vec{a}_\rho H_\rho + \vec{a}_\phi H_\phi + \vec{a}_z H_z\right]$$

Comparing the co efficient on both sides

$$\frac{1}{\rho}\frac{\partial E_z}{\partial\varphi} - \frac{\partial E_\varphi}{\partial z} = -j\omega\mu H_\rho \qquad ...(6)$$

$$\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial\rho} = -j\omega\mu H_\phi \qquad ...(7)$$

$$\frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho E_\phi) - \frac{\partial E_\rho}{\partial\phi}\right] = -j\omega\mu H_z \qquad ...(8)$$

Let

$$H_\rho = H_\rho^\circ e^{-\gamma z}$$

$$\frac{\partial H_\rho}{\partial z} = H_\rho^\circ e^{-\gamma z}(-\gamma) = -\gamma H_\rho \qquad ...(9)$$

$$lll^{ly} \quad \frac{\partial H_\phi}{\partial z} = -\gamma H_\phi \qquad ...(10)$$

$$\frac{\partial H_z}{\partial z} = -\gamma H_z \qquad ...(11)$$

Let

$$E_\rho = E_\rho^\circ e^{-\gamma z}$$

$$\frac{\partial E_\rho}{\partial z} = E_\rho^\circ e^{-\gamma z}(-\gamma) = -\gamma E_\rho \qquad ...(12)$$

$$\frac{\partial E_\phi}{\partial z} = -\gamma E_\phi \qquad ...(13)$$

$$\frac{\partial E_z}{\partial z} = -\gamma E_z \qquad ...(14)$$

Substituting equations(12), (13) & (14) into equation (6), (7) & (8)

$$\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} + \gamma E_\phi = -j\omega\mu H_\rho \quad ...(15)$$

$$\gamma E_\rho + \frac{\partial E_z}{\partial \rho} = j\omega\mu H_\phi \quad ...(16)$$

$$\frac{1}{\rho}\left[\frac{\partial(\rho E_\phi)}{\partial \rho} - \frac{\partial E_\rho}{\partial \phi}\right] = -j\omega\mu H_z \quad ...(17)$$

Substituting equations (9), (10) & (11) into equations (2), (3) & (4)

$$\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} + \gamma H_\phi = j\omega\varepsilon E_\rho \quad ...(18)$$

$$\gamma H_\rho + \frac{\partial H_z}{\partial \rho} = -j\omega\varepsilon E_\phi \quad ...(19)$$

$$\frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho H_\phi) - \frac{\partial H_\rho}{\partial \phi}\right] = j\omega\varepsilon E_z \quad ...(20)$$

From equation (16)

$$H_\phi = \frac{\gamma E_\rho}{j\omega\mu} + \frac{\partial E_z}{\partial \rho}\left(\frac{1}{j\omega\mu}\right) \quad ...(21)$$

Substituting equation (21) in (18)

$$j\omega\varepsilon E_\rho = \frac{1}{\rho}\frac{\partial H_z}{\partial \phi} + \gamma\left[\frac{\gamma E_\rho}{j\omega\mu} + \frac{\partial E_z}{\partial_\rho} \times \frac{1}{j\omega\mu}\right]$$

$$= \frac{1}{\rho}\frac{\partial H_z}{\partial \phi} + \frac{\gamma^2 E_\rho}{j\omega\mu} + \frac{\gamma \partial E_z}{j\omega\mu\partial\rho}$$

$$\left(j\omega\varepsilon - \frac{\gamma^2}{j\omega\mu}\right)E_\rho = \frac{1}{\rho}\frac{\partial H_z}{\partial \phi} + \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial \rho}$$

$$-\left(\frac{\omega^2\mu\varepsilon + \gamma^2}{j\omega\mu}\right)E_\rho = \frac{1}{\rho}\frac{\partial H_z}{\partial \phi} + \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial \rho}$$

$$\frac{-h^2 E_\rho}{j\omega\mu} = \frac{1}{\rho}\frac{\partial H_z}{\partial \phi} + \frac{\gamma}{j\omega\mu}\frac{\partial E_z}{\partial \rho}$$

$$E_\rho = -\frac{j\omega\mu}{\rho h^2}\frac{\partial H_z}{\partial \phi} - \frac{\gamma}{h^2}\frac{\partial E_z}{\partial \rho} \quad ...(22)$$

From equation (15)

$$H_\rho = \frac{-1}{\rho j\omega\mu}\frac{\partial E_z}{\partial \phi} - \frac{\gamma}{j\omega\mu}E_\phi$$

Substitute the above equation in equation (19)

$$-j\omega\varepsilon E_\phi = \gamma H_\rho + \frac{\partial H_z}{\partial \rho}$$

$$= \frac{-\gamma}{\rho j\omega\mu}\frac{\partial E_z}{\partial \phi} - \frac{\gamma^2 E_\phi}{j\omega\mu} + \frac{\partial H_z}{\partial \rho}$$

$$j\omega\varepsilon E_\phi = \frac{\gamma}{\rho j\omega\mu}\frac{\partial E_z}{\partial \phi} + \frac{\gamma^2 E_\phi}{j\omega\mu} - \frac{\partial H_z}{\partial \rho}$$

$$\frac{(\omega^2\mu\varepsilon - \gamma^2)}{j\omega\mu}E_\phi = \frac{\gamma}{\rho j\omega\mu}\frac{\partial E_z}{\partial \phi} - \frac{\partial H_z}{\partial \rho}$$

$$E_\phi = \frac{-\gamma}{\rho h^2}\frac{\partial E_z}{\partial \phi} + \frac{j\omega\mu}{h^2}\cdot\frac{\partial H_z}{\partial \rho} \quad ...(23)$$

Substitute equation (23) in equation (15)

$$-j\omega\mu H_\rho = \frac{1}{\rho}\frac{\partial E_z}{\partial \phi} + \gamma E_\phi$$

$$= \frac{1}{\rho}\frac{\partial E_z}{\partial \phi} + \gamma\left[\frac{-\gamma}{\rho h^2}\frac{\partial E_z}{\partial \phi} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial \rho}\right]$$

$$= \frac{1}{\rho}\frac{\partial E_z}{\partial \phi} - \frac{\gamma^2}{\rho h^2}\frac{\partial E_z}{\partial \phi} + \frac{\gamma j\omega\mu}{h^2}\frac{\partial H_z}{\partial \rho}$$

$$H_\rho = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial \rho} - \frac{\partial E_z}{\rho d\phi\, j\omega\mu} + \frac{\gamma^2 \partial E_z}{\rho h^2 d\phi\, j\omega\mu}$$

$$= \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial \rho} + \frac{\partial E_z}{\partial \phi}\left[\frac{\gamma^2}{\rho h^2 j\omega\mu} - \frac{1}{\rho j\omega\mu}\right]$$

$$= \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial \rho} + \frac{\partial E_z}{\partial \phi}\left[\frac{\gamma^2 - h^2}{\rho h^2 j\omega\mu}\right]$$

$$= \frac{\partial E_z}{\partial \phi}\left(\frac{-\omega^2\mu\varepsilon}{\rho h^2 j\omega\mu}\right) - \frac{\gamma}{h^2}\frac{\partial H_z}{\partial \rho}$$

$$= \frac{\partial E_z}{\partial \rho}\left(\frac{j\omega\varepsilon}{\rho h^2}\right) - \frac{\gamma}{h^2}\frac{\partial H_z}{\partial \rho} \qquad \text{...(24)}$$

Substitute the equation (22) in equation (16)

$$j\omega\mu H_\varphi = \gamma E_\rho + \frac{\partial E_z}{\partial \rho}$$

$$= \gamma\left[\frac{-j\omega\mu}{\rho h^2}\frac{\partial H_z}{\partial \phi} - \frac{\gamma}{h^2}\frac{\partial E_z}{\partial \rho}\right] + \frac{\partial E_z}{\partial \rho}$$

$$= \frac{-j\omega\mu\gamma}{\rho h^2}\frac{\partial H_z}{\partial \phi} - \frac{\gamma^2}{h^2}\frac{\partial E_z}{\partial \rho} + \frac{\partial E_z}{\partial \rho}$$

$$= \frac{-j\omega\mu\gamma}{\rho h^2}\frac{\partial H_z}{\partial \phi} + \frac{\partial E_z}{\partial \rho}\left[1 - \frac{\gamma^2}{h^2}\right]$$

$$= \frac{-j\omega\mu\gamma}{\rho h^2}\frac{\partial H_z}{\partial \phi} + \frac{\partial E_z}{\partial \rho}\left[\frac{h^2 - \gamma^2}{h^2}\right]$$

$$= \frac{-j\omega\mu\gamma}{\rho h^2}\frac{\partial H_z}{\partial \phi} + \frac{\partial E_z}{\partial \rho}\left[\frac{\omega^2\mu\varepsilon}{h^2}\right]$$

$$H_\phi = \frac{-\gamma}{\rho h^2}\frac{\partial H_z}{\partial \varphi} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial \rho} \qquad \text{...(25)}$$

The wave equation becomes

$$\nabla^2 E = -\omega^2\mu\varepsilon E$$

$$\nabla^2 E = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[\rho\frac{\partial E}{\partial \rho}\right] + \frac{1}{\rho^2}\frac{\partial^2 E}{\partial \phi^2} + \frac{\partial^2 E}{\partial z^2}$$

Assume the wave propagation in z direction

$$\frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} + \frac{1}{\rho}\frac{\partial E_z}{\partial \rho} = -\omega^2\mu\varepsilon E_z$$

Let the general solution of this equation be

$$E_z = P(\rho)\,Q(\phi)\,e^{-\gamma z} = PQe^{-\gamma z} = E_z^\circ e^{-\gamma z}$$

Where $P(\rho)$ is a function of ρ alone

$Q(\phi)$ is a function of ϕ alone

Substituting E_z in the wave equation

$$Q\frac{\partial^2 P}{\partial \rho^2} + \frac{Q}{\rho}\frac{\partial P}{\partial \rho} + \frac{P}{\rho^2}\frac{\partial^2 Q}{\partial \phi^2} + PQ\gamma^2 + \omega^2\mu\varepsilon PQ = 0$$

Divide by PQ

$$\frac{1}{P}\frac{\partial^2 P}{\partial \rho^2} + \frac{1}{P\rho}\frac{\partial P}{\partial \rho} + \frac{1}{Q\rho^2}\frac{\partial^2 Q}{\partial \phi^2} + h^2 = 0$$

Let $\frac{\partial^2 Q}{\partial \phi^2} = -n^2 Q$

$$\frac{\partial^2 Q}{\partial \phi^2} + n^2 Q = 0$$

The solution of this equation is given by

$$Q = A_n \cos n\phi + B_n \sin n\phi$$

$$\frac{1}{P}\frac{\partial^2 P}{\partial \rho^2} + \frac{1}{P\rho}\frac{\partial P}{\partial \rho} + \frac{1}{Q\rho^2} \times -n^2 Q + h^2 = 0$$

$$\frac{1}{P}\frac{\partial^2 P}{\partial \rho^2} + \frac{1}{P\rho}\frac{\partial P}{\partial \rho} + h^2 - \frac{n^2}{\rho^2} = 0$$

Multiply by P

$$\frac{\partial^2 P}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial P}{\partial \rho} + \left(h^2 - \frac{n^2}{\rho^2}\right)P = 0$$

Divide by h^2

$$\frac{\partial^2 P}{\partial(\rho h)^2} + \frac{1}{(\rho h)}\frac{\partial P}{\partial(\rho h)} + \left(1 - \frac{n^2}{(\rho h)^2}\right)P = 0$$

This is the standard from of Bessel's equation in terms of (ρh)

$$P(\rho h) = J_n(\rho h)$$

Where $J_n(\rho h)$ is the Bessel's function of first kind of order 'n'

$$E_z = J_n(\rho h)(A_n \cos n\phi + B_n \sin n\phi)e^{-\gamma z}$$

$$H_z = J_n(\rho h)(C_n \cos n\phi + D_n \sin n\phi)e^{-\gamma z}$$

We have two constants A_n and B_n. Eventhough both may exist in general by proper choice of excitation of waveguide we can make one of them to be zero. Hence making $B_n = 0$

$$E_z = J_n(\rho h) A_n \cos n\phi \; e^{-\gamma z}$$

$$H_z = J_n(\rho h) C_n \cos n\phi \; e^{-\gamma z}$$

5.7.1 TM (Transverse Magnetic) Waves in Circular Wave Guide

For TM waves $H_z = 0$

$$E_\rho = \frac{-j\omega\mu}{\rho h^2}\frac{\partial H_z}{\partial\phi} - \frac{\gamma}{h^2}\frac{\partial E_z}{\partial\rho}$$

Substituting $H_z = 0$

$$E_\rho = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial\rho}$$

$$= \frac{-\gamma}{h^2}\frac{\partial}{\partial\rho}[J_n(\rho h) A_n \cos n\phi]$$

$$= \frac{-\gamma}{h^2} J_n'(\rho h) A_n \cos n\phi \, h$$

$$= \frac{-\gamma}{h} J_n'(\rho h) A_n \cos n\phi$$

$$= \frac{-j\beta}{h} J_n'(\rho h) A_n \cos n\phi$$

From equation (23)

$$E_\varphi = \frac{-\gamma}{\rho h^2}\frac{\partial E_z}{\partial\phi}$$

$$= \frac{-\gamma}{\rho h^2}\frac{\partial}{\partial\phi}[J_n(\rho h) A_n \cos n\phi]$$

$$= \frac{-\gamma}{\rho h^2} J_n(\rho h) A_n(-\sin n\phi)n$$

$$= \frac{\gamma n}{\rho h^2} A_n \sin n\phi J_n(\rho h)$$

$$= \frac{j\beta n}{\rho h^2} A_n \sin n\phi J_n(\rho h)$$

From equation (24)

$$H_\rho = \frac{\partial E_z}{\partial\phi}\left(\frac{j\omega\varepsilon}{\rho h^2}\right)$$

$$= \frac{j\omega\varepsilon}{\rho h^2}\frac{\partial}{\partial\phi}[J_n(\rho h) A_n \cos n\phi]$$

$$= \frac{j\omega\varepsilon}{\rho h^2} J_n(\rho h) A_n(-\sin n\phi)n$$

$$= \frac{-j\omega\varepsilon n A_n \sin n\phi J_n(\rho h)}{\rho h^2}$$

From equation (25)

$$H_\phi = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial\rho}$$

$$= \frac{-j\omega\varepsilon}{h^2}\frac{\partial}{\partial\rho}[J_n(\rho h) A_n \cos n\phi]$$

$$= \frac{-j\omega\varepsilon}{h^2} J_n'(\rho h) h A_n \cos n\phi$$

$$= \frac{-j\omega\varepsilon J_n'(\rho h) A_n \cos n\phi}{h}$$

Let us now apply the boundary condition $E_{tan} = 0$ for E_z component replacing ρ by a where 'a' is radius of cylindrical waveguide.

$$J_n(ha) A_n \cos n\phi = 0$$

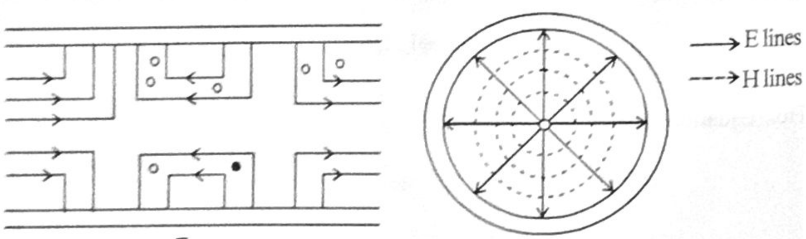
For E_z to vanish at ρ=a, we must have $J_n(ha) = 0$ for arbitary values of $\cos n\phi$. For any value of n, $J_n(ha)$ becomes zero for a number of value of ha.

These values of ha for which $J_n(ha)$ is zero give the roots of equation $J_n(ha)$ for different values of n and designated as q_{nm} (or) h_{nm}

Where n is order of Bessel function, m is roots of $J_n(ha)$

Table 5.1 Roots of $J_n(ha)$

n	m = 1	m = 2	m = 3
0	2.405 TM_{01}	5.52 TM_{02}	8.654 TM_{02}
1	3.832 TM_{11}	7.016 TM_{12}	10.174 TM_{13}



$\beta_z = 0$ $\beta_z = \frac{\pi}{2}$ $\beta_z = \pi$

Figure 5.13 Field distribution for TM_{01} mode

5.7.2 TE (Transverse Electric Waves) in Circular Waveguide

For TE waves $E_z = 0$

$$E_\rho = \frac{-j\omega\mu}{\rho h^2}\frac{\partial H_z}{\partial \phi} - \frac{\gamma}{h^2}\frac{\partial E_z}{\partial \rho}$$

Substituting $E_z = 0$

$$E_\rho = \frac{-j\omega\mu}{\rho h^2}\frac{\partial H_z}{\partial \phi}$$

$$= \frac{-j\omega\mu}{\rho h^2}\frac{\partial}{\partial \phi}[J_n(\rho h) C_n \cos n\phi]$$

$$= \frac{-j\omega\mu}{\rho h^2}[J_n(\rho h) C_n(-\sin n\phi) n]$$

$$= \frac{J_n(\rho h) n C_n \sin n\phi \; j\omega\mu}{\rho h^2}$$

From equation (23)

$$E_\phi = \frac{-\gamma}{\rho h_2}\frac{\partial E_z}{\partial \phi} + \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial \rho}$$

Substituting $E_z = 0$

$$E_\phi = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial \rho}$$

$$= \frac{j\omega\mu}{h^2}\frac{\partial}{\partial \rho}[J_n(\rho h) C_n \cos n\phi]$$

$$= \frac{j\omega\mu}{h^2} J'_n(\rho h)\, h\, C_n \cos n\phi$$

$$= \frac{j\omega\mu J'_n(\rho h) C_n \cos n\phi}{h}$$

From equation (24)

$$H_\rho = \frac{\partial E_z}{\partial \phi}\left(\frac{j\omega\varepsilon}{\rho h^2}\right) - \frac{\gamma}{h^2}\frac{\partial H_z}{\partial \rho}$$

Substituting $E_z = 0$

$$H_\rho = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial \rho}$$

$$= \frac{-\gamma}{h^2}\frac{\partial}{\partial \rho}(J_n(\rho h)\, c_n \cos n\phi)$$

$$= \frac{-\gamma}{h^2} J_n'(\rho h)\, h\, c_n \cos n\phi = \frac{-\gamma Jn'(\rho h) c_n \cos n\phi}{h}$$

From equation (25)

$$H_\phi = \frac{-\gamma}{\rho h^2}\frac{\partial H_z}{\partial \phi} - \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial \rho}$$

Substituting $E_z = 0$

$$H_\phi = \frac{-\gamma}{\rho h^2}\frac{\partial H_z}{\partial \phi}$$

$$= \frac{-\gamma}{\rho h^2}\frac{\partial}{\partial \phi}(J_n(\rho h)\, c_n \cos n\phi)$$

$$= \frac{-\gamma}{\rho h^2} J_n(\rho h)\, c_n(-\sin n\phi) n$$

$$= \frac{\gamma}{\rho h^2} J_n(\rho h) n c_n \sin n\phi$$

Let us apply boundary condition since H_z is not equal zero the boundary condition can not be applied on H_z since it does not vanish at the walls. Hence we will choose E_ϕ

$$E_\phi = 0 \; at \; \rho = a$$

$$J_n'(ha) = 0$$

Table 5.2 Roots of $J_n'(ha)$

n	m = 1	m = 2	m = 3
0	3·82 (TE_{01})	7·016 (TE_{02})	10·174 (TE_{03})
1	1·841 (TE_{11})	5·331 (TE_{12})	8·536 (TE_{13})

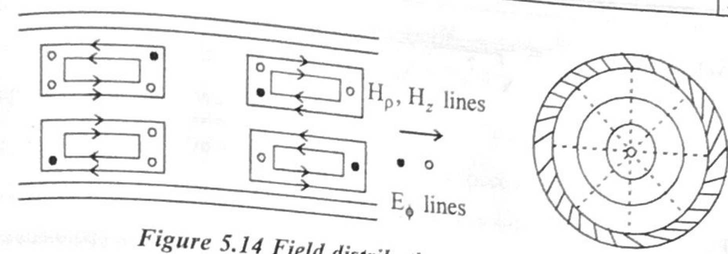


Figure 5.14 Field distribution for TE_{01} mode.

5.7.3 Characteristics of TE and TM Waves

We know that

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon \Rightarrow \gamma = \sqrt{h^2 - \omega^2\mu\varepsilon}$$

For propagation $\gamma = j\beta$

$$j\beta = j\sqrt{\omega^2\mu\varepsilon - h^2}$$

$$\beta = \sqrt{\omega^2\mu\varepsilon - h_{nm}^2}$$

$$\omega_c^2\mu\varepsilon = h_{nm}^2$$

$$4\pi^2 f_c^2 \mu\varepsilon = h_{nm}^2$$

$$f_c^2 = \frac{h_{nm}^2}{4\pi^2\mu\varepsilon}$$

$$f_c = \frac{h_{nm}}{2\pi\sqrt{\mu\varepsilon}}$$

$$h_{nm}\, a = q_{nm}$$

$$h_{nm} = \frac{q_{nm}}{a} = \frac{(ha)_{nm}}{a}$$

$$f_c = \frac{(ha)_{nm}}{2\pi a\sqrt{\mu\varepsilon}}$$

$$f_c = \frac{c\,(ha)_{nm}}{2\pi a} \qquad \therefore c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\lambda_c = \frac{c}{f_c} = \frac{2\pi a}{(ha)_{nm}}$$

Phase velocity $v_{ph} = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\omega^2\mu\varepsilon - h_{nm}^2}} = \frac{\omega}{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}}$

$$= \frac{\omega}{\omega\sqrt{\mu\varepsilon}\sqrt{1-\frac{\omega_c^2}{\omega^2}}}$$

$$v_{ph} = \frac{c}{\sqrt{1-\frac{f_c^2}{f^2}}}$$

$$\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{\omega^2\mu\varepsilon - \omega_c^2\mu\varepsilon}} = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}\sqrt{1-\frac{\omega_c^2}{\omega^2}}}$$

$$= \frac{2\pi}{2\pi f\sqrt{\mu\varepsilon}\sqrt{1-\frac{f_c^2}{f^2}}}$$

$$\lambda_g = \frac{c}{f\sqrt{1-\frac{f_c^2}{f^2}}}$$

$$\lambda_g = \frac{\lambda_o}{\sqrt{1-\frac{f_c^2}{f^2}}}$$

where $\lambda_o = \frac{c}{f}$

Group velocity $v_g = \frac{c^2}{v_{ph}} = \frac{c^2}{c}\sqrt{1-\frac{f_c^2}{f^2}}$

$$v_g = c\sqrt{1-\frac{f_c^2}{f^2}}$$

5.8 RESONANT CAVITIES

When one end of the waveguide is terminated in a shorting plate, there will be complete reflection of waves. When one more shorting plate is kept at a distance of multiple of $\frac{\lambda_g}{2}$ from first shorting plate, then hollow space so formed can now support a signal which bounches back and forth between the two shorting plates. The waves appear to be stationary and hence they are called standing waves.

Applications of cavity resonators.

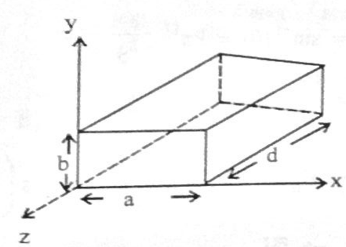
✓ It is used in microwave amplifiers, filters

✓ It is used in cavity magnetron for generation of microwave signals.

✓ It is used in cavity wave meter for measurement of frequency of microwave signal.

5.8.1 Rectangular Cavity Resonators

Rectangular cavity resonators are formed by shorting two ends of section of a waveguide as shown in figure 5.15



Figurer 5.15 Rectangular cavity resonator

The field components inside the cavity can be computed from the wave equations which satisfy the boundary condition of zero tangential electric field at all conducting walls. Because of metallic surface at boundary the field distribution posses standing wave pattern in all 3 directions.

5.8.1.1 TE_{mnp} Mode

$$H_z = C\cos\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y\ e^{-j\beta z}$$

Considering incident and reflected wave for this component

$$H_z = (C_i\,e^{-j\beta z} + C_r\,e^{j\beta z})\cos\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y$$

$$C_i + C_r = 0 \Rightarrow C_i = -C_r$$

$$H_z = C_i\cos\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y\,(e^{-j\beta z} - e^{j\beta z})$$

$$= -2jc_i\sin\beta z\cos\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y$$

$$= H_o\sin\beta z\cos\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y$$

Applying boundary condition $H_z = 0$ at $z = d$

$$\sin\beta d = 0$$

$$\beta d = \sin^{-1}(0) = P\pi$$

$$\beta = \frac{P\pi}{d} \quad \text{where } P = 1, 2...$$

$$H_z = H_o\sin\left(\frac{P\pi}{d}\right)z\cos\left(\frac{m\pi}{a}\right)x\cos\left(\frac{n\pi}{b}\right)y$$

$$H_y = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial y}$$

In the above equation γ occurs because of differentiation of $e^{-\gamma z}$ with respect to z. For a resonator this is to be replaced by trigonometric function.

$$H_y = \frac{1}{h^2}\frac{\partial^2 Hz}{\partial y\,\partial z}$$

$$= \frac{1}{h^2}\frac{\partial}{\partial y}\left[\frac{\partial Hz}{\partial z}\right]$$

$$= \frac{1}{h^2}\frac{\partial}{\partial y}\left[H_z\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{d}\right)\left(\frac{p\pi}{d}\right)\right]$$

$$= \frac{-H_o}{h^2}\left(\frac{p\pi}{d}\right)\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{p\pi z}{d}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$H_x = \frac{1}{h^2}\frac{\partial^2 H_z}{\partial x\,\partial z}$$

$$= \frac{1}{h^2}\frac{\partial}{\partial x}\left[\frac{\partial H_z}{\partial z}\right]$$

$$= \frac{1}{h^2}\frac{\partial}{\partial x}\left[H_o\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{d}\right)\left(\frac{p\pi}{d}\right)\right]$$

$$= -\frac{H_o}{h^2}\left(\frac{p\pi}{d}\right)\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{d}\right)$$

$$E_z = 0$$

$$E_y = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial x}$$

$$= -\frac{j\omega\mu}{h^2}H_o\sin\left(\frac{m\pi x}{a}\right)\left(\frac{m\pi}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{d}\right)$$

$$E_x = -\frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial y}$$

$$= \frac{j\omega\mu H_o}{h^2}\left(\frac{n\pi}{b}\right)\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{d}\right)$$

5.8.1.2 TM_{mnp} Mode

$$E_z = C\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{-j\beta z}$$

Considering incident and reflected wave for this component

$$E_z = (C_i\,e^{-j\beta z} + C_r\,e^{j\beta z})\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

Assuming $C_t = C_r$

$$= 2C_t \cos\beta z \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$E_z = E_o \cos\beta z \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$E_y = \frac{1}{h^2}\frac{\partial^2 E_z}{\partial y \partial z}$$

$$= \frac{1}{h^2}\frac{\partial}{\partial y}\left[\frac{\partial E_z}{\partial z}\right]$$

$$= \frac{1}{h^2}\frac{\partial}{\partial x}\left[E_o(-\sin\beta z)\beta\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\right]$$

$$= \frac{1}{h^2}\left[-E_o \sin\beta z\cdot\beta\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cdot\frac{n\pi}{b}\right]$$

Applying boundary condition

$E_y = 0$ at $z = d$

$\sin\beta d = 0$

$$\beta d = \sin^{-1}(o) = p\pi$$

$$\beta = \frac{p\pi}{d} \qquad \text{Where } p = 1, 2 \ldots.$$

$$E_z = E_o \cos\left(\frac{p\pi z}{d}\right)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)$$

$$E_x = \frac{1}{h^2}\frac{\partial^2 E_z}{\partial x \partial z}$$

$$= \frac{1}{h^2}\frac{\partial}{\partial x}\left[\frac{\partial E_z}{\partial z}\right]$$

$$= \frac{1}{h^2}\frac{\partial}{\partial x}\left[-E_o \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{d}\right)\left(\frac{p\pi}{d}\right)\right]$$

$$= \frac{1}{h^2}\left[-E_o \cos\left(\frac{m\pi x}{a}\right)\left(\frac{m\pi}{a}\right)\left(\frac{p\pi}{d}\right)\sin\left(\frac{n\pi y}{b}\right)\sin\left(\frac{p\pi z}{d}\right)\right]$$

$H_z = 0$

$$H_y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

$$= \frac{-j\omega\varepsilon E_o}{h^2}\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{d}\right)$$

$$H_x = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$$

$$= \frac{j\omega\varepsilon}{h^2}E_o\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{p\pi z}{d}\right)$$

For either TE_{mnp} (or) TM_{mnp} mode :

At resonance

$$\omega_r^2\,\mu\varepsilon = h^2$$

$$= \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$

$$\omega_r = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

$$f_r = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

$$f_r = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

The dominant mode of rectangular cavity depends on dimension of cavity. For $b < a < d$ the dominant mode is TE_{101}

5.8.2 Circular Cavity Resonators

It is a circular waveguide with two ends closed by a metal wall as shown in figure 5.16 Electromagnetic field analysis shows that due to ϕ symmetric structure of circular cavity field solution posses harmonic solution in ϕ and standing waves in radial and z directions.

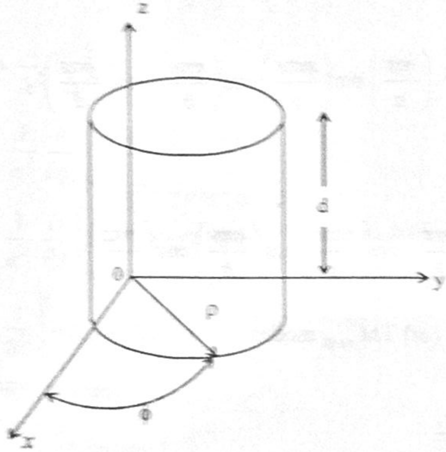


Figure 5.16 Circular cavity resonator

5.8.2.1 TE$_{mnp}$ Mode

$$H_z = C_n J_n(\rho h)\cos n\phi$$

$$= (C_{in}\, e^{-j\beta z} + C_{nr}\, e^{j\beta z})\, J_n(\rho h)\cos n\phi$$

$$C_{in} = -C_{nr}$$

$$= C_{ni}\,[e^{-j\beta z} - e^{j\beta z}]J_n(\rho h)\cos n\phi$$

$$= -2jC_{ni}\,\sin\beta z\, J_n(\rho h)\cos n\phi$$

$$= H_o\,\sin\beta z\cos n\phi\, J_n(\rho h)$$

$$= H_o\,\sin\beta z\cos n\varphi\, J_n\left(\frac{\rho q'_{nm}}{a}\right)$$

Applying boundary condition

$$H_z = 0\,;\, Z = d$$

$$\sin\beta d = 0$$

$$\beta d = \sin^{-1}(0) = p\pi$$

$$\beta = \frac{p\pi}{d}$$

$$H_z = H_o\,\sin\left(\frac{p\pi z}{d}\right)\cos n\phi\; J_n\left(\frac{\rho q'_{nm}}{a}\right)$$

$$H_\rho = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial\rho} = \frac{1}{h^2}\frac{\partial^2 H_z}{\partial\rho\,\partial z}$$

$$= \frac{1}{h^2}\frac{\partial}{\partial\rho}\left(\frac{\partial H_z}{\partial z}\right)$$

$$= \frac{1}{h^2}\frac{\partial}{\partial\rho}\left[H_o\cos\left(\frac{p\pi z}{d}\right)\left(\frac{p\pi}{d}\right)\cos n\phi\cdot J_n\left(\frac{\rho q'_{nm}}{a}\right)\right]$$

$$= \frac{1}{h^2}\left[H_o\cos\left(\frac{p\pi z}{d}\right)\left(\frac{p\pi}{d}\right)\cos n\phi\; J'_n\left(\frac{\rho q'_{nm}}{a}\right)\left(\frac{q'_{nm}}{a}\right)\right]$$

$$= H_o\left(\frac{p\pi}{d}\right)\left(\frac{a}{q'_{nm}}\right)J'_n\left(\frac{q'_{nm}\rho}{a}\right)\cos n\varphi\cos\left(\frac{p\pi z}{d}\right)$$

$$H_\phi = \frac{-\gamma}{\rho h^2}\frac{\partial H_z}{\partial\phi} = \frac{1}{\rho h^2}\frac{\partial^2 H_z}{\partial\phi\,\partial z}$$

$$= \frac{1}{\rho h^2}\frac{\partial}{\partial\phi}\left[\frac{\partial H_z}{\partial z}\right]$$

$$= \frac{1}{\rho h^2}\frac{\partial}{\partial\varphi}\left[H_o\cos\left(\frac{p\pi z}{d}\right)\left(\frac{p\pi}{d}\right)\cos n\phi\, J_n\left(\frac{\rho q'_{nm}}{a}\right)\right]$$

$$= \frac{-1}{\rho h^2}\left[H_o\cos\left(\frac{p\pi z}{d}\right)\left(\frac{p\pi}{d}\right)\sin n\varphi\cdot n\,J_n\left(\frac{\rho q'_{nm}}{a}\right)\right]$$

$$= -H_o\left(\frac{p\pi}{d}\right)\frac{n}{\rho}\left(\frac{a}{q'_{nm}}\right)^2 J_n\left(\frac{\rho q'_{nm}}{a}\right)\sin n\phi\cos\left(\frac{p\pi z}{d}\right)$$

$$E_\rho = \frac{-j\omega\mu}{\rho h^2}\frac{\partial H_z}{\partial\phi}$$

$$= \frac{j\omega\mu}{\rho h^2} H_o \sin\left(\frac{p\pi z}{d}\right)\sin n\phi\cdot n J_n\left(\frac{\rho q'_{nm}}{a}\right)$$

$$= \frac{n\, j\omega\mu}{\rho h^2}\left(\frac{a}{q'_{nm}}\right)^2 H_o \sin\left(\frac{p\pi z}{d}\right)\sin n\phi\cdot J_n\left(\frac{\rho q'_{nm}}{a}\right)$$

Where

n = 0, 1... is the number of periodicity in ϕ

m = 1, 2... is the number of zeros of fields in ρ direction

p = 1, 2.... is the number of halfwaves in z direction.

$$E_\phi = \frac{j\omega\mu}{h^2}\frac{\partial H_z}{\partial\rho} = \frac{j\omega\mu}{h^2} H_o \sin\left(\frac{p\pi z}{d}\right)\cos n\phi\, J'_n\left(\frac{\rho q'_{nm}}{a}\right)\left(\frac{q'_{nm}}{a}\right)$$

$$= j\omega\mu\left(\frac{a}{q'_{nm}}\right) H_o \sin\left(\frac{p\pi z}{d}\right)\cos n\phi\, J'_n\left(\frac{\rho q'_{nm}}{a}\right)$$

$$E_z = 0$$

5.8.2.2 TM_{mnp} Mode

$$E_z = J_n(\rho h) A_n \cos n\phi\, e^{-j\beta z}$$

$$= (A_{ni} e^{-j\beta z} + A_{nr} e^{j\beta z}) J_n(\rho h)\cos n\phi$$

$$A_{ni} = A_{nr}$$

$$= A_{ni}(e^{-j\beta z} + e^{j\beta z}) J_n(\rho h)\cos n\phi$$

$$= 2A_{ni}\cos\beta z\, J_n(\rho h)\cos n\phi$$

$$E_z = E_o \cos\beta z\, J_n\left(\frac{\rho q_{nm}}{a}\right)\cos n\phi$$

$$E_\rho = 0 \text{ at } z = d$$

$$E_\rho = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial\rho} = \frac{1}{h^2}\frac{\partial^2 E_z}{\partial\rho\,\partial z} = \frac{1}{h^2}\frac{\partial}{\partial\rho}\left[\frac{\partial E}{\partial z}\right]$$

$$= \frac{1}{h^2}\frac{\partial}{\partial\rho}\left[E_o(-\sin\beta z)\beta\, J_n\left(\frac{\rho q_{nm}}{a}\right)\cos n\phi\right]$$

$$= \frac{-E_o}{h^2}\sin\beta z\cdot\beta J'_n\left(\frac{\rho q_{nm}}{a}\right)\left(\frac{q_{nm}}{a}\right)\cos n\phi$$

Applying boundary condition $E_\rho = 0$ at $z = d$

$$\sin\beta d = 0$$

$$\beta d = \sin^{-1}(0)$$

$$\beta d = p\pi$$

$$\beta = \frac{p\pi}{d}$$ Where P = 1, 2....

$$E_z = E_o \cos\left(\frac{p\pi z}{d}\right) J_n\left(\frac{\rho q_{nm}}{a}\right)\cos n\phi$$

$$E_\phi = \frac{-\gamma}{\rho h^2}\frac{\partial E_z}{\partial\phi} = \frac{1}{\rho h^2}\frac{\partial^2 E_z}{\partial\phi\,\partial z}$$

$$= \frac{1}{\rho h^2}\frac{\partial}{\partial\phi}\left[\frac{\partial E_z}{\partial z}\right]$$

$$= \frac{1}{\rho h^2}\frac{\partial}{\partial\phi}\left[-E_o\sin\left(\frac{p\pi z}{d}\right)\left(\frac{p\pi}{d}\right)J_n\left(\frac{\rho q_{nm}}{a}\right)\cdot\cos n\phi\right]$$

$$= \frac{1}{\rho h^2} E_o \sin\left(\frac{p\pi z}{d}\right)\left(\frac{p\pi}{d}\right)J_n\left(\frac{\rho q_{nm}}{a}\right)\sin n\phi\, n$$

$$= \left(\frac{a}{q_{nm}}\right)^2\left(\frac{n}{\rho}\right)E_o\left(\frac{p\pi}{d}\right)J_n\left(\frac{\rho q_{nm}}{a}\right)\sin\left(\frac{p\pi z}{d}\right)$$

$$E_\rho = \frac{-\gamma}{h^2}\frac{\partial E_z}{\partial \rho}$$

$$= \frac{1}{h^2}\frac{\partial^2 E_z}{\partial \rho\, \partial z}$$

$$= \frac{1}{h^2}\frac{\partial}{\partial \rho}\left[\frac{\partial E_z}{\partial z}\right]$$

$$= \frac{1}{h^2}\frac{\partial}{\partial \rho}\left[-E_o \sin\left(\frac{p\pi z}{d}\right)\left(\frac{p\pi}{d}\right) J_n\left(\frac{\rho q_{nm}}{a}\right)\cos n\phi\right]$$

$$= -\frac{1}{h^2} E_o \sin\left(\frac{p\pi z}{d}\right)\left(\frac{p\pi}{d}\right) J'_n\left(\frac{\rho q_{nm}}{a}\right)\left(\frac{q_{nm}}{a}\right)\cdot\cos n\phi$$

$$= -E_o \sin\left(\frac{p\pi z}{d}\right)\left(\frac{p\pi}{d}\right) J'_n\left(\frac{\rho q_{nm}}{a}\right)\left(\frac{a}{q_{nm}}\right)\cos n\phi$$

$$H_z = 0$$

$$H_\phi = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial \rho}$$

$$= \frac{-j\omega\varepsilon}{h^2} E_o \cos\left(\frac{p\pi z}{d}\right) J'_n\left(\frac{\rho q_{nm}}{a}\right)\cos n\phi\left(\frac{q_{nm}}{a}\right)$$

$$= -j\omega\varepsilon\, E_o \cos\left(\frac{p\pi z}{d}\right) J'_n\left(\frac{\rho q_{nm}}{a}\right)\cos n\phi\left(\frac{a}{q_{mm}}\right)$$

$$H_\rho = \frac{j\omega\varepsilon}{\rho h^2}\frac{\partial E_z}{\partial \phi} = \frac{j\omega\varepsilon}{\rho h^2} E_o J_n\left(\frac{\rho q_{nm}}{a}\right)\cos\left(\frac{p\pi z}{d}\right)(-\sin n\phi).n$$

$$= \frac{-j\omega\varepsilon}{\rho}\left(\frac{a}{q_{nm}}\right)^2 E_o J_n\left(\frac{\rho q_{nm}}{a}\right)\cos\left(\frac{p\pi z}{d}\right)\sin n\phi\, n$$

For TE_{nmp}

At resonance $\omega_o^2 \mu\varepsilon = h^2$

$$h^2 = \left(\frac{q'_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$

$$\omega_o^2 \mu\varepsilon = \left(\frac{q'_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2$$

$$\omega_o = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{q'_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

$$f_o = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{q'_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

For TM_{mnp}

$$f_o = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{q_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2}$$

The dominant mode in circular cavity will depend on dimension of cavity

For $d < 2a$, TM_{010} mode is dominant mode

$d \geq 2a$, TE_{111} mode is dominant mode.

Example 1

A pair of perfectly conducting planes are separated by 8cm in air. For a frequency of 5000 MHz with the TM_1 mode excited find the following i) cut off frequency ii) Characteristic impedance iii) Attenuation constant for $f = 0.95fc$ iv) phase constant v) Phasevelocity and group velocity vi) wavelength measured along the guided walls vii) Cut off wavelength viii) Angle of incidence.

Solution:

Given data,

$a = 8cm = 0.08m$, $f = 5000MHz$ TM_1 hence $m = 1$

i) Cut off frequancy $f_c = \frac{m}{2a}\frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{2\times 0.08}\times\frac{1}{\sqrt{\mu_o\mu_r\varepsilon_o\varepsilon_r}}$

$$= \frac{1}{2\times 0.08}\times\frac{1}{\sqrt{4\pi\times 10^{-7}\times 1\times 8.854\times 10^{-12}\times 1}}$$

$f_c = 1.875$ GHz

ii) Characteristic impedance $Z_{TM} = \eta\sqrt{1-\frac{fc^2}{f^2}} = \sqrt{\frac{\mu_o \mu_r}{\varepsilon_o \varepsilon_r}}\sqrt{1-\frac{fc^2}{f^2}}$

$$= \sqrt{\frac{4\pi \times 10^{-7} \times 1}{8.854 \times 10^{-12} \times 1}}\sqrt{1-\frac{(1.875 \times 10^9)^2}{(5000 \times 10^6)^2}}$$

$$Z_{TM} = 349 \cdot 41\Omega$$

iii) Attenuation constant for $f = 0 \cdot 95 f_c$

Propagation constant becomes attenuation constant if the operating frequency is less than cut off frequency ($\gamma = \alpha$)

$$\gamma = \alpha = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon} = \sqrt{\left(\frac{m\pi}{a}\right)^2 - 4\pi^2 f^2 \mu_o \mu_r \varepsilon_o \varepsilon_r}$$

$$= \sqrt{\left(\frac{1 \times \pi}{0 \cdot 08}\right)^2 - 4\pi^2 \times (5000 \times 10^6)^2 \times 4\pi \times 10^{-7} \times 1 \times 8.854 \times 10^{-12} \times 1}$$

$$\alpha = 12 \cdot 26 \text{ Neper}/m$$

iv) Phase constant $\beta = \sqrt{\omega^2\mu\varepsilon - \left(\frac{m\pi}{a}\right)^2}$

$$= \sqrt{4\pi^2 f^2 \mu_o \mu_r \varepsilon_o \varepsilon_r - \left(\frac{m\pi}{a}\right)^2}$$

$$= \sqrt{4 \times \pi^2 \times (5000 \times 10^6)^2 \times 4\pi \times 10^{-7} \times 1 \times 8.854 \times 10^{-12} \times 1 - \left(\frac{1 \times \pi}{0 \cdot 08}\right)^2}$$

$$\beta = 97 \cdot 08 \text{ Radian}$$

v) Phase velocity $v_{ph} = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2 \times 3.14 \times 5000 \times 10^6}{97 \cdot 08}$

$$v_{ph} = 3 \cdot 236 \times 10^8 \, m/s$$

Group velocity $v_g = \frac{c^2}{v_{ph}} = \frac{(3 \times 10^8)^2}{3 \cdot 236 \times 10^8} = 2 \cdot 78 \times 10^8 \, m/s$

vi) $\lambda_g = \frac{2\pi}{\beta} = \frac{2 \times \pi}{97 \cdot 08} = 0.0647 \, m$

vii) $\lambda_c = \frac{2a}{m} = \frac{2 \times 0.08}{1} = 0.16 \, m$

viii) Angle of incidence $\tan\theta = \sqrt{\left(\frac{f}{f_c}\right)^2 - 1}$

$$\theta = \tan^{-1}\sqrt{\left(\frac{5000 \times 10^6}{1.875 \times 10^9}\right)^2 - 1}$$

$$\theta = 67.97$$

Example 2

For a frequency of 6000 MHz and plane separation 7cm. Find the following for TE_1 mode. i) Cut off frequency ii) Angle of incidence on the planes iii) Phase velocity and group velocity. Is it possible to propagate TE_3 mode?

Solution:

Given Data,

m = 1, a = 7cm = 0.07m, f = 6000 MHz

i) Cut off frequency $f_c = \frac{m}{2a} \times \frac{1}{\sqrt{\mu\varepsilon}}$

$$= \frac{1}{2 \times 0.07} \times \frac{1}{\sqrt{4\pi \times 10^{-7} \times 1 \times 8.854 \times 10^{-12} \times 1}}$$

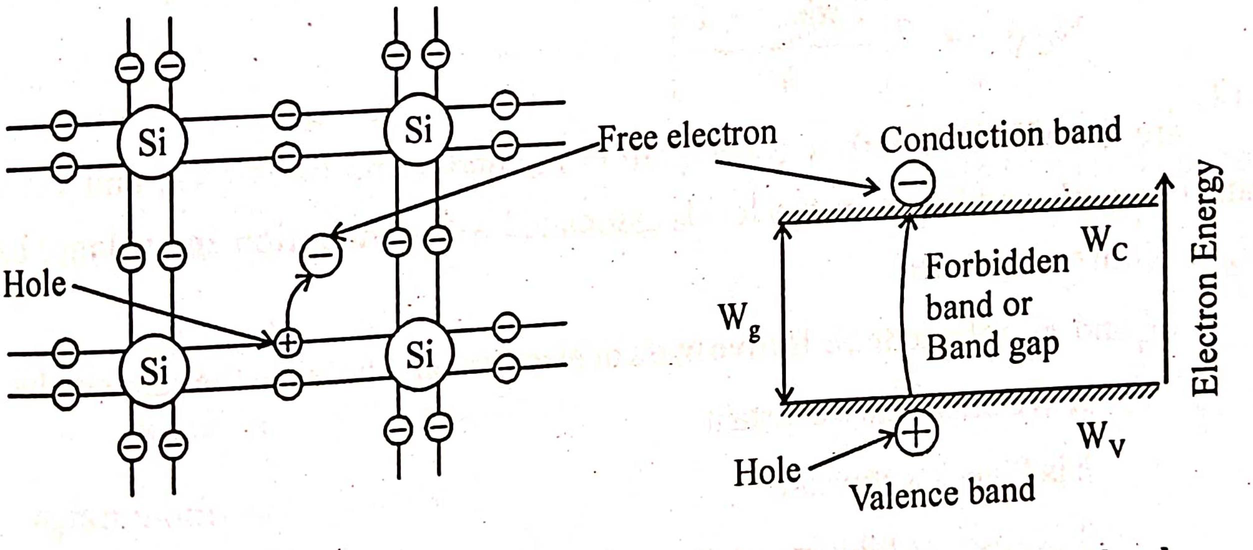
$$f_c = 2 \cdot 142 \, \text{GHz}$$

5 RF SYSTEM DESIGN CONCEPTS

5.1 ACTIVE RF COMPONENTS: SEMICONDUCTOR BASICS IN RF

5.1.1 Physical Properties of Semiconductors

The operation of semiconductor devices is dependent on physical behaviour of semiconductor themselves. Three most commonly used semiconductors are silicon (si), germanium (Ge) and gallium arsenide (GaAs). Figure 5.1(a) shows the bonding structure of pure silicon. Each silicon atom shares its four valance electrons with four neighboring atoms, forming four covalent bonds.



(a) Planar representation of covalent bonds

(b) Energy band levels

Figure 5.1 Lattice structure and energy levels of silicon

In the absence of thermal energy i.e., when the temperature is equal to zero degree Kelvin ($T = 0K = -273.15°C$) all electrons are bonded to the corresponding atoms and semiconductor is not conductive. When the temperature increases, some of electrons obtain sufficient energy to break up the covalent bond and cross the energy gap $W_g = W_C - W_V$ as shown in figure 5.1(b). At room temperature $T = 300$ K, the band gap energy is equal to 1.12 eV for Si, 0.62 eV for Ge, 1.42 eV for GaAs. These free electrons form negative charge carriers that allow electric current conduction. The concentration of conduction electrons semiconductor is denoted as n. when electron breaks the covalent bond, it leaves behind a positively charged vacancy, which can be occupied by another free electron. These type of vacancies are called holes and their concentration is denoted by p. In thermal equilibrium, we have equal number of recombination and generation of holes and electrons.

The concentration obey the Fermi statistics according to

$$n = N_C e^{-\left(\frac{W_C - W_F}{KT}\right)} \quad \text{... (1)}$$

$$p = N_V e^{-\left(\frac{W_F - W_V}{KT}\right)} \quad \text{... (2)}$$

where

$$N_{C,V} = 2\left[\frac{2 m_{n,p}^{*} \pi KT}{h^2}\right]^{3/2} \quad \text{... (3)}$$

are the effective carrier concentration in conduction band (N_C) and valance band (N_V). W_C and W_V is energy levels associated with conduction and valance band W_F is Fermi energy level.

m_n^* and m_p^* refer to the effective mass of electrons and holes in the semiconductor.

K is Boltzmann's constant

h is Planck's constant

T is absolute temperature measured in Kevin

The electron and hole concentrations are described by concentration law

$$np = n_i^2 \quad \text{... (4)}$$

where n_i is the intrinsic concentration

Substitution of equation (1) and (2) in (4) results in the expression for intrinsic carrier concentration

$$n_i = \sqrt{N_C N_V}\, e^{-\left(\frac{W_C - W_V}{2KT}\right)} = \sqrt{N_C N_V}\, e^{-\left(\frac{W_g}{2KT}\right)} \quad \text{... (5)}$$

Classical electromagnetic theory specifies electrical conductivity in a material to be $\sigma = \frac{J}{E}$ where J is current density & E is applied electric field

$$\sigma = \frac{q\,NV_d}{E} \qquad \text{... (6)}$$

where N is carrier concentration

q is elementary charge

V_d is drift velocity

E is applied electric field

We can rewrite equation (6) as

$$\sigma = qn\mu_n + qp\mu_p \qquad \text{... (7)}$$

where μ_n, μ_p are mobilities of electrons & holes

simplify the equation (7) using $n = p = n_i$

$$\sigma = qn_i(\mu_n + \mu_p)$$

$$= q\sqrt{N_C N_V}\, e^{-\left(\frac{W_g}{2KT}\right)}(\mu_n + \mu_p) \qquad \text{... (8)}$$

Consider n-type semiconductor in which the electron concentration is related to the hole concentration as

$$n_n = N_D + p_n \qquad \text{... (9)}$$

where N_D is the donor concentration

p_n is minority hole concentration

To find n_n and p_n solve equation (9) in conjunction with (4). The result is

$$n_n = \frac{N_D + \sqrt{N_D^2 + 4n_i^2}}{2} \qquad \text{... (10)}$$

$$p_n = \frac{-N_D + \sqrt{N_D^2 + 4n_i^2}}{2} \qquad \text{... (11)}$$

If the donor concentration N_D is much greater than the intrinsic electron concentration n_i then

$$n_n = N_D \qquad \text{... (12)}$$

$$p_n \simeq \frac{-N_D + N_D\left(1 + \frac{2n_i^2}{N_D^2}\right)}{2} = \frac{n_i^2}{N_D} \quad \text{... (13)}$$

Consider p type semiconductor with

$$p_p = N_A + n_p \quad \text{... (14)}$$

where N_A, n_p are acceptor and minority electron concentrations. Solving equation (14) together with (4)

$$p_p = \frac{N_A + \sqrt{N_A^2 + 4n_i^2}}{2} \quad \text{... (15)}$$

$$n_p = \frac{-N_A + \sqrt{N_A^2 + 4n_i^2}}{2} \quad \text{... (16)}$$

5.1.2 The pn-Junction

The physical contact of a p-type with n-type semiconductor leads to one of the most important concepts when dealing with active semiconductor devices pn junction. Because of difference in the carrier concentration between two types of semiconductor, a current flow will be initiated across the interface. This current is known as diffusion current and is composed of electrons and holes. Consider one dimensional model of pn Junction as shown in Figure 5.2.

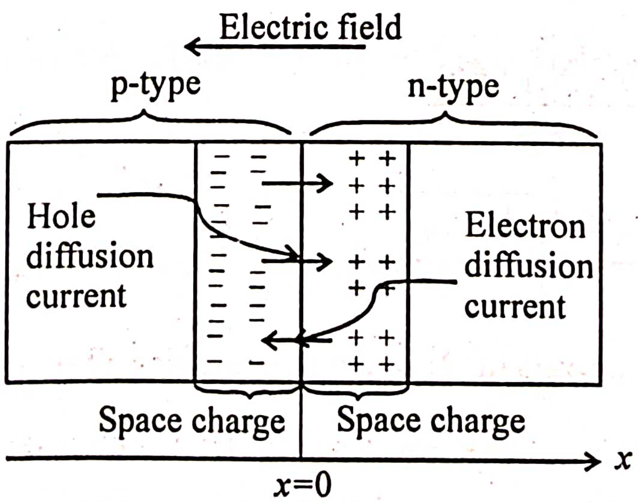


Figure 5.2 Current flows in the pn-Junction

The diffusion current is composed of $I_{n\,diff}$ and $I_{p\,diff}$ components

$$I_{diff} = I_{n\,diff} + I_{p\,diff} = qA\left(D_n\frac{dn}{dx} + D_p\frac{dp}{dx}\right) \qquad \text{... (1)}$$

where A is the semiconductor cross-sectional area orthogonal to the x-axis and D_n, D_p are diffusion constants for electrons and holes in the form

$$D_{n,p} = \mu_{n,p}\frac{KT}{q} = \mu_{n,p}V_T \qquad \text{... (2)}$$

The thermal potential $V_T = \dfrac{KT}{q}$ is approximately 26 mV at room temperature of 300K.

Since p type semiconductor was initially neutral, the diffusion current of holes is going to leave behind a negative space charge. Similarly electron current flow from n semiconductor will leave behind positive space charges. As the diffusion current flow takes place an electric field E is crated between net positive charge in the n semiconductor and net negative charge in the p semiconductor. This field in turn induces a current $I_F = \sigma AE$ which opposes the diffusion current such that $I_F + I_{diff} = 0$.

Substituting equation (7) for the conductivity

$$I_F = qA(n\mu_n + p\mu_p)E = I_{nF} + I_{pF} \qquad \text{... (3)}$$

since total current is equal to zero, the electron portion of the current is also equal to zero; that is

$$I_{ndiff} + I_{nF} = q\,D_n A\frac{dn}{dx} + q_n\,\mu_n\,AE = q\,\mu_n A\left(V_T\frac{dn}{dx} - \frac{ndV}{dx}\right) = 0 \qquad \text{... (4)}$$

where electric field E has been replaced by the derivative of the potential $E = \dfrac{-dV}{dx}$.

Integrating equation (4) we obtain the diffusion barrier voltage or built in potential

$$\int_0^{V_{diff}} dV = V_{diff} = V_T\int_{n_p}^{n_n} n^{-1}\,dn = V_T\ln\left[\frac{n_n}{n_p}\right] \qquad \text{... (5)}$$

where n_n is electron concentration in n-type

n_p is electron concentration in p-type

$$V_{diff} = V_T \ln\left[\frac{p_p}{p_n}\right] \qquad \dots (6)$$

If the concentration of acceptor in the p-semiconductor is $N_A >> n_i$, and the concentration of donors in the n semiconductor is $N_D >> n_i$, then $n_n = N_D$, $n_p = \frac{n_i^2}{N_A}$.

By using $n_p = \dfrac{-N_A + N_A\left(1+\dfrac{2n_i^2}{N_A^2}\right)}{2} \simeq \dfrac{n_i^2}{N_A}$ and equation (5) we obtain

$$V_{diff} \simeq V_T \ln\left(\frac{N_A N_D}{n_i^2}\right) \qquad \dots (7)$$

If we desire to determine the potential distribution along the x-axis, we can employ Poisson's equation which for one-dimensional analysis is written as

$$\frac{d^2 V(x)}{dx^2} = \frac{-\rho(x)}{\varepsilon_r \varepsilon_o} = -\frac{dE}{dx} \qquad \dots (8)$$

where $\rho(x)$ is the charge density and ε_r is the relative dielectric constant of the semiconductor

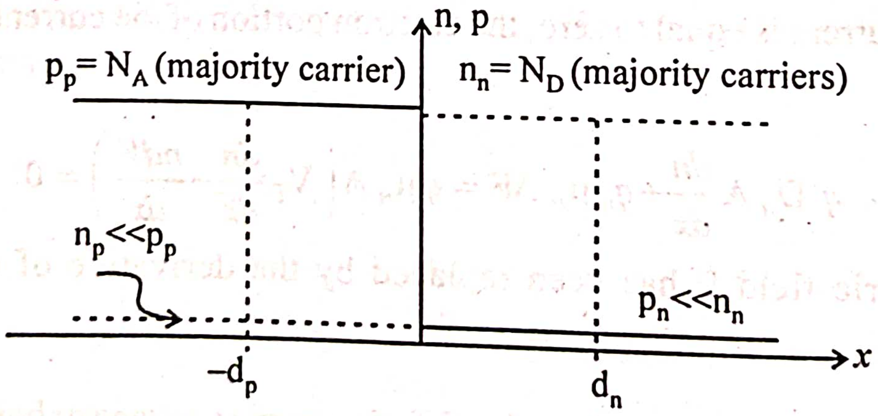


Figure 5.3 Acceptor and donor concentration

Assuming uniform doping and abrupt junction approximation as shown in figure 5.3 the charge density in each material is

$$\rho(x) = -qN_A, \text{ for } -d_p \le x \le 0 \qquad \dots (9)$$

$$\rho(x) = q\,N_D, \text{ for } 0 \le x \le d_n \qquad \ldots (10)$$

where d_p and d_n are extents of space charges in p and n type semiconductor

The electric field in the semiconductor is found by integrating equation (8) with spatial limits $-d_p \le x \le d_n$ such that

$$E(x) = \int_{-d_p}^{x} \frac{\rho(x)dx}{\varepsilon_r\,\varepsilon_o} = \begin{cases} \dfrac{-q\,N_A}{\varepsilon_r\,\varepsilon_o}(x+d_p), & \text{for } -d_p \le x \le 0 \\ \dfrac{-q\,N_D}{\varepsilon_r\,\varepsilon_o}(d_n - x), & \text{for } 0 \le x \le d_n \end{cases} \qquad \ldots (11)$$

To obtain voltage distribution profile, integrating equation (11) as follows

$$V(x) = \int_{-d_p}^{x} E(x)\,dx = \begin{cases} \dfrac{q\,N_A}{2\varepsilon_r\,\varepsilon_o}(x+d_p)^2, \text{ for} - d_p \le x \le 0 \\ \dfrac{q}{2\varepsilon_r\,\varepsilon_o}(N_A\,d_p^2 + N_D\,d_n^2)\ \dfrac{-q\,N_D}{2\varepsilon_r\,\varepsilon_o}(d_n - x)^2, \text{for } 0 \le x \le d_n \end{cases}$$

$$\ldots (12)$$

Since total voltage drop must be equal to diffusion voltage V_{diff} it is found that

$$V(d_n) = V_{diff} = \frac{q\,N_A\,d_p^2}{2\varepsilon_r\,\varepsilon_o} + \frac{q\,N_D\,d_n^2}{2\varepsilon_r\,\varepsilon_o} \qquad \ldots (13)$$

substituting $d_p = \dfrac{d_n\,N_D}{N_A}$ and solving (13) for d_n

$$d_n = \left[\frac{2\varepsilon\,V_{diff}}{q}\,\frac{N_A}{N_D}\left(\frac{1}{N_A + N_D}\right)\right]^{\frac{1}{2}} \qquad \ldots (14)$$

where $\varepsilon = \varepsilon_o\,\varepsilon_r$. An identical derivation involving $d_n = \dfrac{d_p\,N_A}{N_D}$ gives us the space charge extent into p-semiconductor

$$d_p = \left[\frac{2\varepsilon\,V_{diff}}{q}\,\frac{N_D}{N_A}\left(\frac{1}{N_A + N_D}\right)\right]^{1/2} \qquad \ldots (15)$$

Entire length is addition of (14) & (15)

$$d_s = d_n + d_p = \left[\frac{2\varepsilon\, V_{diff}}{q}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)\right]^{1/2} \qquad \dots (16)$$

Junction capacitance $C = \dfrac{\varepsilon A}{d_s}$ ... (17)

substituting equation (16) in (17)

$$C = A\left[\frac{q\varepsilon}{2V_{diff}}\,\frac{N_A N_D}{N_A + N_D}\right]^{1/2} \qquad \dots (18)$$

5.2 BIPOLAR JUNCTION TRANSISTORS (BJT)

Transistor Definition: A non linear three terminal active semiconductor device where the flow of electrical current between two of the terminals is controlled by the third terminal. The name is an acronym for transfer resistor.

Transistors may be used in circuits as amplifiers oscillators, detector, switches and so on.

Bipolar Junction transistor consists of three layers of semiconductors and two junctions. The semiconductor layer may be alternate N-type and P-type or vice versa. Thus BJTs are either emitter, base and collector and two Junctions are emitter base junction (EBJ) and collector base Junction (CBJ) as shown in figure 5.4 and 5.5

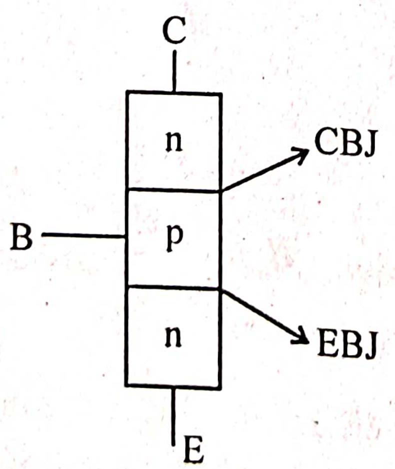
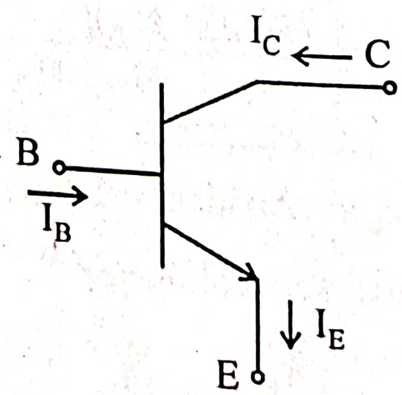
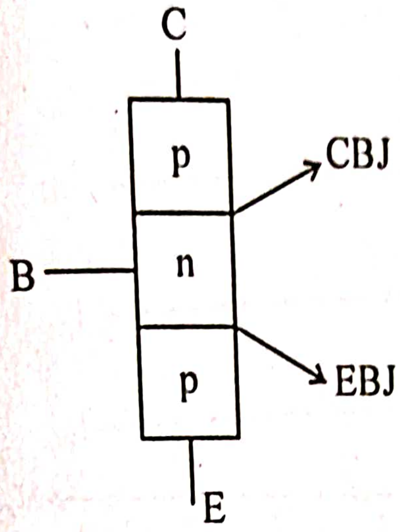


Figure 5.4 (a) NPN Transistor



(b) An NPN circuit symbol



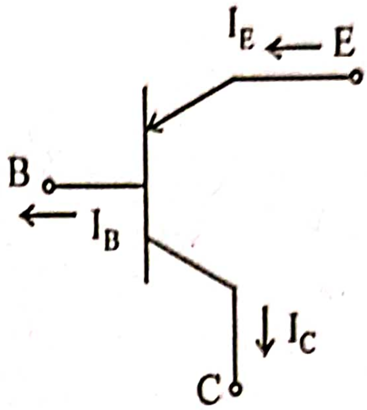


Figure 5.5 (a) PNP Transistor *(b) PNP circuit symbol*

5.2.1 Graphical Representation of BJT Characteristics

It is often useful to describe a BJT graphically in terms of its I-V characteristics. Consider the following conceptual circuit as shown in figure 5.6 where the transistor is connected in common emitter configuration. In this setup the DC voltage sources are independent variables and create three dependent variables (I_C, I_B, V_{CE}).

To obtain a single transistor characteristic curve that is plotted in the $I_C - V_{CE}$ plane with I_B as a parameter, DC voltage source is set to a voltage and then V_{CE} voltage source setting is theoretically varied from 0 to infinity while measuring corresponding collector current (I_C). This is shown in figure 5.7.

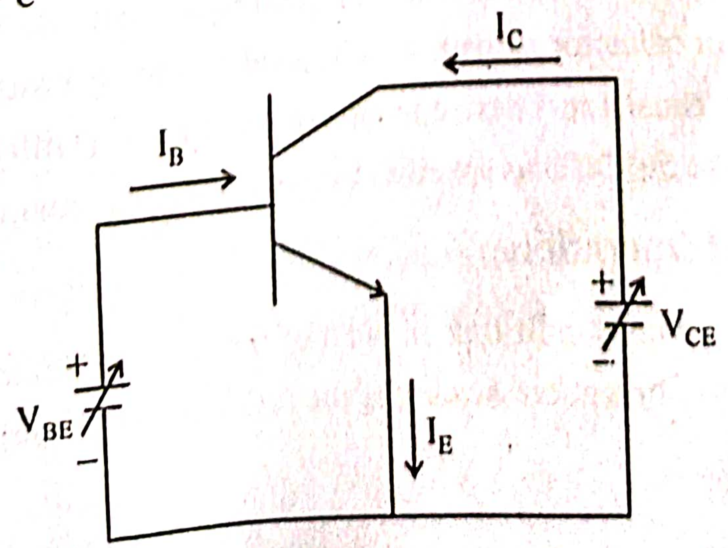


Figure 5.6 A conceptual circuit

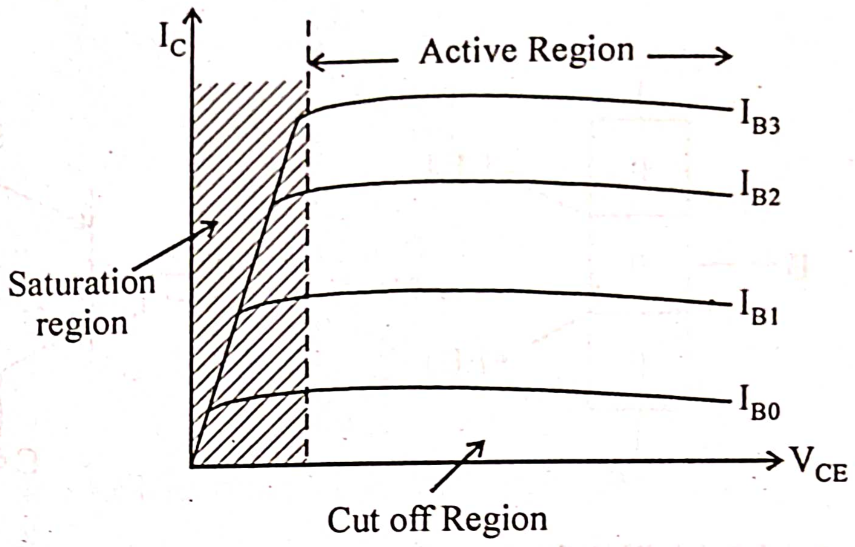


Figure 5.7 $I_C - V_{CE}$ characteristics of a BJT

$I_C - V_{CE}$ family of characteristic curves can be subdivided into several regions active, saturation and cut off.

5.2.2 BJT DC Biasing

The DC biasing of a transistor plays a major role in the operation and proper function of an active circuit.

DC Biasing Definition: It is the setting of the DC voltages at each of the two transistor junctions (EBJ or CBJ) such that the transistor will perform in a stable fashion in the intended mode (e.g., active mode for amplifiers, etc). Setting the proper values for each of the two transistor junction voltages can be translated equivalently into terminal current values such as emitter or collector current, which can alternatively be used to specify the DC bias values of the transistor. These currents in conjunction with the bias voltage values of junctions make the DC bias specification of a transistor complete.

5.2.3 BJT Modes of Operation

Depending on the bias conditions on each of the two Junctions (EBJ or CBJ), there can be four modes of operation. Assuming the following notations,

FWD = forward bias

REV = reverse bias

these four modes of operation are shown in figure 5.8.

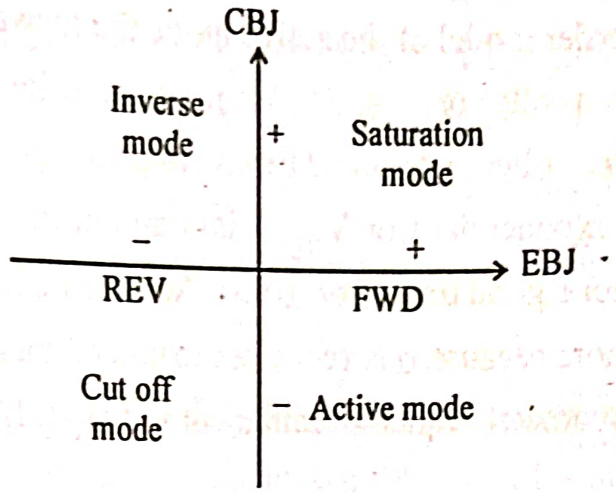


Figure 5.8 Four modes of operation of a BJT

Each mode can be defined as follows.

Saturation mode is the mode in which both EBJ and CBJ are forward biased. In this mode an increase in Base current (I_B) produces no further increase in collector current (I_C).

Cut off mode is the mode in which both EBJ and CBJ are reverse biased. Thus there is no current of any kind through the circuit. $I_E = 0$, $I_C = 0$ and $I_B = 0$. Active mode is the mode where EBJ is forward biased and CBJ is reverse biased. In this mode the collector current (I_C) is proportional to base current.

$$I_C = \beta I_B$$

where β is common emitter current gain

KCL gives $I_E = I_B + I_C = \left(1+\frac{1}{\beta}\right)I_C$

$$I_C = \frac{\beta}{1+\beta} I_E = \alpha I_E$$

where $\alpha = \frac{\beta}{1+\beta}$ is called common base current gain

A first order model for the operation of transistor in the active mode can be represented by hybrid−π equivalent circuit as shown in figure 5.9

$$I_C = I_S\, e^{\left(\frac{V_{BE}}{V_T}\right)}$$

where I_S is the reverse saturation current, V_T is thermal voltage defined to be $V_T = \frac{KT}{q}$, where K is Boltzmann's constant (1.38×10^{-23} J/K), T is absolute temperature in Kelvin and q is magnitude of electronic charge (1.602×10^{-19} C).

In the first order model of the active mode the forward voltage (V_{BE}) causes an exponentially related collector current (I_C) to flow, and as long as the CBJ remains reverse biased ($V_{CB} > 0$), the collector terminal behave as a non linear voltage controlled current source depending exponentially on V_{BE}. β is common emitter current gain and is much larger than unity for a good transistor, $\beta >> 1$. As a result I_B is seen to be much smaller than I_C. Furthermore because α is very close to unity for a good transistor, the collector current I_C is approximately equal to emitter current (I_E): $I_C = I_E$.

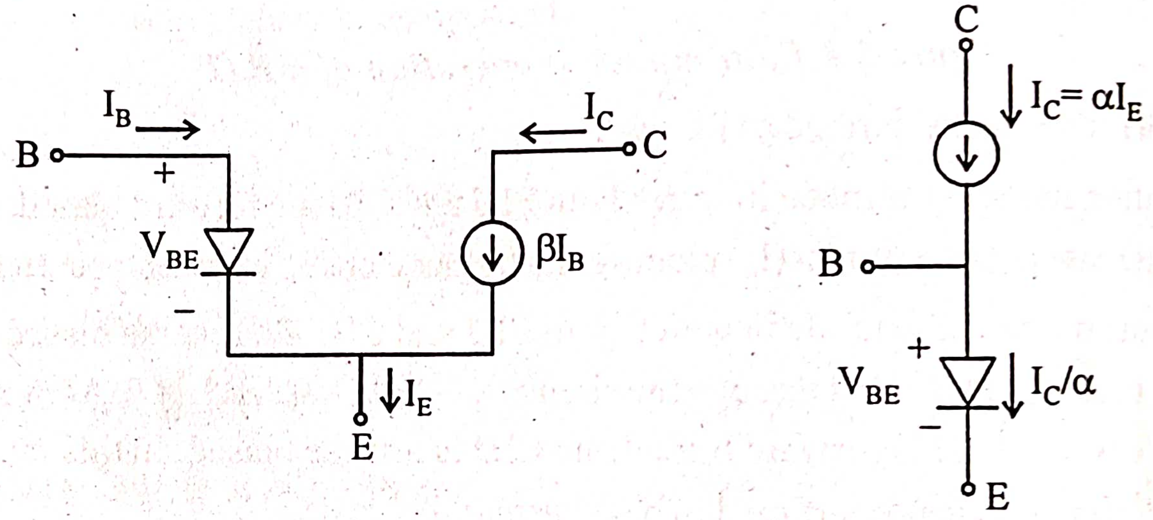


Figure 5.9 Two possible large-signal equivalent circuit models of NPN BJT in active mode.

Inverse mode is a mode in which the EBJ is reverse biased and CBJ is forward biased; that is emitter's and collector's roles are reversed. This mode may theoretically be used in the same manner as active mode.

5.3 RF FIELD EFFECT TRANSISTORS

Field effect transistors are monopolar devices meaning that only one carrier type, either holes or electrons contributes to the current flow through the channel. If hole contributions are involved it is of p-channel otherwise of n-channel FETs. FET is voltage controlled device. A variable electric field controls the current flow from source to drain by changing the applied voltage on the gate electrode.

5.3.1 Construction

FETs are classified according to how the gate is connected to the conducting channel. Specifically the following four types are used.

1. **Metal Insulator semiconductor FET (MISFET):** Here gate is separated from the channel through an insulation layer one of the most widely used types, the metal oxide semiconductor FET (MOSFET) belongs to this class.

2. **Junction FET(JFET):** This type relies on a reverse biased pn-junction that isolates gate from the channel.

3. **Metal semiconductor FET(MESFET):** If the reverse biased pn-junction is replaced by a schottky contact, the channel can be controlled just as in the JFET case.

4. **Hetero FET:** As the name implies heterostructure utilize abrupt transition between layers of different semiconductor materials. Examples are GaAlAs to GaAs. The High electron mobility Transistor (HEMT) belongs to this class.

Due to presence of a large capacitance formed by the gate electrode and the insulator or reverse biased pn junction, MISFET and JFET have a relatively low cut off frequency and are usually operated in low and medium frequency ranges of typically up to 1GHz. GaAs MESFET find application up to 60 – 70 GHz, and HEMT can operate beyond 100 GHz.

5.3.2 Functionality

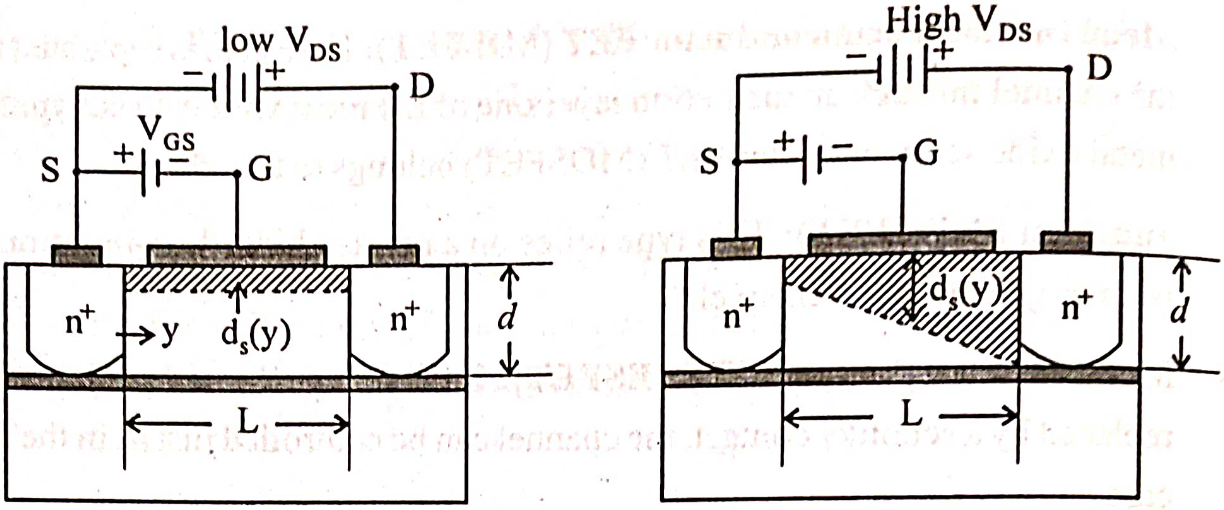
Because of its importance in RF and microwave amplifier, mixer and oscillator circuits, we focus our analysis on the MESFET, whose physical behaviour is in many ways similar to the JFET. The analysis is based on the geometry shown in figure 5.10 where the transistor is operated in depletion mode.

The schottky contact builds up a channel space charge domain that affect the current flow from source to drain. The space charge extent d_s can be controlled via the gate voltage.

$$d_s = \left(\frac{2\varepsilon}{q}\frac{V_d - V_{GS}}{N_D}\right)^{1/2} \quad \text{... (1)}$$

For instance the barrier voltage V_d is 0.9 V for GaAs–Au interface. The resistance R between source and drain is predicted by

$$R = \frac{L}{\sigma(d - d_s)W} \quad \text{... (2)}$$



(a) Operation in linear region (b) Operation in saturation region

Figure 5.10 Functionality of MESFET for different drain-source voltages

$\sigma = q\mu_n N_D$, W being gate width.

substituting (1) in (2) yields the drain current equation

$$I_D = \frac{V_{DS}}{R} = G_0\left[1-\left(\frac{2\varepsilon}{qd^2}\frac{V_d - V_{GS}}{N_D}\right)^{1/2}\right]V_{DS} \quad \text{... (3)}$$

where $G_0 = \frac{\sigma Wd}{L}$. This equation shows that the drain current depends linearly on drain source voltage, a fact that is only true for small V_{DS}.

As the drain source voltage increases, the space change domain near the drain contact increases as well, resulting in a non uniform distribution of the deflection region along the channel.

If we assume that the voltage along the channel changes from 0 at the source location to V_{DS} at the drain end then we can compute the drain current for non uniform space charge region. This approach is also known as gradual-channel approximation. The approximation rests primarily on the assumption that the cross sectional area at a particular location y along the channel is given by $A(y) = (d - d_s(y))\,W$ and electric field E is only y-directed.

The channel current is thus

$$I_D = -\sigma EA(y) = \sigma\frac{dV(y)}{dy}(d - d_s(y))\,W \quad \text{... (4)}$$

where the difference between V_d and V_{GS} in the expression for $d_s(y)$ has to be augmented by the additional drop in voltage V(y) along the channel. So equation (1) becomes

$$d_s(y) = \left[\frac{2\varepsilon}{q N_D}(V_d - V_{GS} + V(y)\right]^{1/2} \quad \text{... (5)}$$

substituting (5) in (4) and carrying out the integration on both sides of equation yields

$$\int_0^L I_D\, d_y = I_D L = \sigma W \int_0^{V_{DS}} \left[d - \left[\frac{2\varepsilon}{q N_D}(V + V_d - V_{GS})\right]^{1/2}\right] dV \quad \text{... (6)}$$

The result is the output characteristic of the MESFET in terms of drain current as a function of V_{DS} and V_{GS} or

$$I_D = G_0\left[V_{DS} - \frac{2}{3}\sqrt{\frac{2\varepsilon}{q N_D d^2}}\left[V_{DS} + V_d - V_{GS}\right]^{3/2} - (V_d - V_{GS})^{3/2}\right] \quad \text{... (7)}$$

We note that this equation reduces to (3) for small V_{DS}. When the space charge extends over the entire channel depth d, the drain source voltage for this situation is called drain saturation voltage $V_{DS\,sat}$ and is given by

$$d_s(L) = d = \sqrt{\frac{2\varepsilon}{q N_D}(V_d - V_{GS} + V_{DSsat})} \quad \text{... (8)}$$

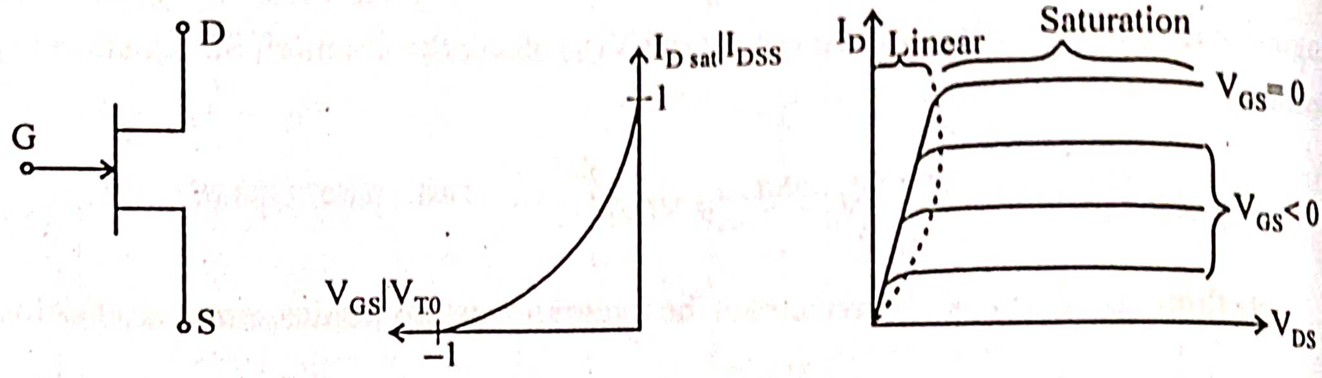
or explicitly

$$V_{DS\,sat} = \frac{q N_D d^2}{2\varepsilon} - (V_d - V_{GS}) = V_p - V_d + V_{GS} = V_{GS} - V_{T0} \quad \text{... (9)}$$

where V_p pinch off voltage $= \dfrac{q N_D d^2}{2\varepsilon}$ and threshold voltage $V_{T0} = V_d - V_p$. The associated drain saturation current is found by inserting (9) into (7) with result

$$I_{D\,sat} = G_0\left[\frac{V_p}{3} - (V_d - V_{GS}) + \frac{2}{3\sqrt{V_p}}(V_d - V_{GS})^{3/2}\right] \quad \text{... (10)}$$

The maximum saturation current in equation (10) is obtained when $V_{GS} = 0$, which we define as $I_{Dsat}(V_{GS} = 0) = I_{DSS}$. Input and output transfer as well as output characteristic behaviour is shown in figure 5.11.



(a) Circuit symbol (b) Transfer characteristics (c) Output characteristics

Figure 5.11 Transfer and output characteristics of an n-channel MESFET

The saturation drain current is often approximated by simple relation.

$$I_{Dsat} = I_{DSS}\left(1-\frac{V_{GS}}{V_{T0}}\right)^2 \qquad \text{...(11)}$$

Example 1

For a particular Si pn-junction, the doping concentrations are given as $N_A = 10^{18}\ cm^{-3}$ and $N_D = 5\times10^{15} cm^{-3}$, with an intrinsic concentration of $n_i = 1.5\times10^{10}\ cm^{-3}$. Find the barrier voltage for $T = 300$ K

☞ ***Solution:***

Given data,

$N_A = 10^{18}\ \text{cm}^{-3}$, $N_D = 5\times10^{15}\text{cm}^{-3}$

$n_i = 1.5\times10^{10}\ \text{cm}^{-3}$, $T = 300$ K

$$V_{diff} = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$= \frac{KT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$V_{diff} = 0.796\ \text{V}.$$

Example 2

A GaAs MESFET has following parameters $N_D = 10^{16}\ cm^{-3}$, $d = 0.75\ \mu m$, $W = 10\ \mu m$, $L = 2\ \mu m$, $\varepsilon_r = 12.0$, $V_d = 0.8\ V$ and $\mu_n = 8500\ cm^2/VS$. Determine

(i) The pinch off voltage

(ii) Threshold voltage

(iii) Maximum saturation current.

Solution:

Given data:

$N_D = 10^{16}\ cm^{-3}$, $d = 0.75\ \mu m$, $W = 10\ \mu m$, $L = 2\ \mu m$, $\varepsilon_r = 12.0$ $V_d = 0.8\ V$ &

$\mu_n = 8500\ cm^2/VS$

(i) Pinch off voltage

$$V_p = \frac{q N_D d^2}{2\varepsilon}$$

$$V_p = 4.24\ V$$

(ii) Threshold voltage

$$V_{T0} = V_d - V_p$$

$$= 0.8 - 4.24$$

$$V_{T0} = -3.44\ V$$

(iii) Maximum saturation current

$$I_{DSS} = G_0 \left[\frac{V_p}{3} - V_d + \frac{2}{3\sqrt{V_p}} V_d^{3/2}\right]$$

$$G_0 = \frac{\sigma q N_D W d}{L} = \frac{q^2 \mu_n N_D^2 W d}{L} = 8.16$$

$$I_{DSS} = 6.89\ A$$

5.4 HIGH ELECTRON MOBILITY TRANSISTORS

It is also known as modulation doped field effect transistor exploits the differences in bandgap energy between dissimilar semiconductor materials such as GaAlAs and GaAs in an effort to substantially surpass the upper frequency limit of MESFET while maintaining low noise performance and high power rating. Transition frequencies of 100 GHz and above have been achieved. The high frequency behaviour is due to a separation of carrier electrons from their donor sites at the interface between doped GaAlAs and undoped GaAs layer, where they are confined to a very narrow layer in which motion is possible only parallel to the interface. Here a two dimensional electron gas (2DEG) or plasma of very high mobility upto 9000 cm^2/V-S.

5.4.1 Construction

The basic heterostructure is shown in figure 5.12 where a GaAlAs n-doped semiconductor is followed by an undoped GaAlAs spacer layer, an undoped GaAs layer and a highly resistive semi-insulating GaAs substrate.

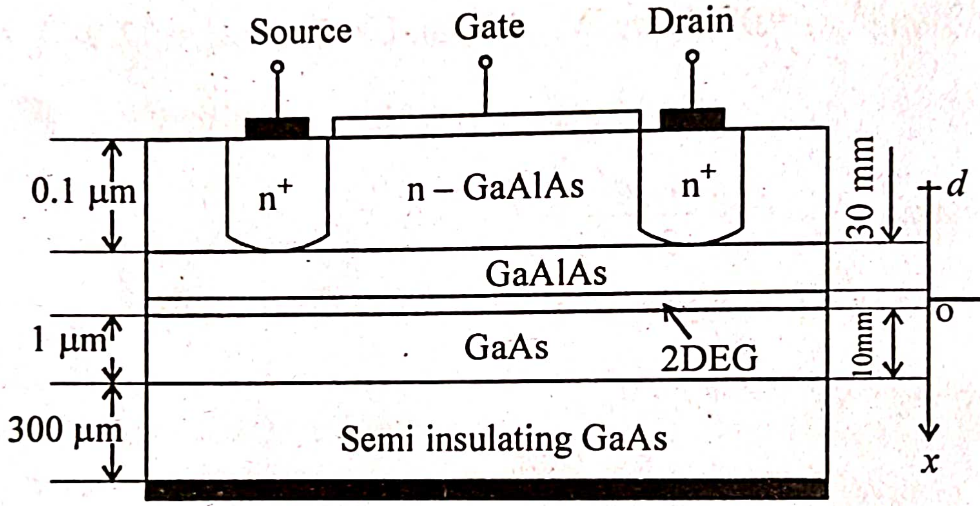


Figure 5.12 Generic heterostructure of a depletion-mode HEMT

The 2DEG forms in the undoped GaAs layer at zero gate bias condition because the fermi level is above the conduction band, so that electrons accumulate in this narrow potential well. The electron concentration can be depleted by applying an increasingly negative gate voltage.

HEMTS are primarily constructed of heterostructures with matching lattice constants to avoid mechanical tensions between layers. Examples are GaAlAs-GaAs and InGaAs-Inp interfaces. A larger In GaAs lattice is compressed onto a smaller GaAs lattice. Such a device configuration are known as pseudomorphic HEMTs or PHEMTs.

5.4.2 Functionality

The key issue that determines the drain current flow in a HEMT is the narroe interface between GaAlAs & GaAs layers.

A mathematical model can be developed by writing the one dimensional poisson equation in the form

$$\frac{\partial^2 V}{\partial x^2} = \frac{-qN_D}{\varepsilon_H} \quad \text{... (1)}$$

where N_D & ε_H are donor concentration and dielectric constant in the GaAlAs heterostructure. The boundary condition for the potential are imposed such that $V(x=0) = 0$ and at the metal semiconductor side $V(x=-d) = -V_b + V_G + \Delta W_C / q$. Here V_b is barrier voltage, ΔW_C is energy difference in the conduction levels between the n-doped GaAlAs and GaAs and V_G is composed of the gate source voltage as well as the channel voltage drop $V_G = -V_{GS} + V(y)$. To find potential equation (1) is integrated twice. At the metal-semiconductor interface, we set

$$V(-d) = \frac{qN_D}{2\varepsilon_H} x^2 - E_y(0)d \quad \text{... (2)}$$

which yields

$$E(0) = \frac{1}{d}\left(V_{GS} - V(y) - V_{T0}\right) \quad \text{... (3)}$$

where defined the HEMT threshold voltage V_{T0} as $V_{T0} = \frac{V_b - \Delta W_C}{q - V_p}$

$$\text{where } V_p = \frac{q\,N_D\,d^2}{2\varepsilon_H}$$

$$I_D = \sigma E_y A = -q\mu_n N_D EWd = q\mu_n N_D \left(\frac{dV}{dy}\right) Wd \quad \text{... (4)}$$

the current flow is restricted to a very thin layer so that it is appropriate to carry out the integration over a surface charge density Q_s at $x = 0$.

The result is $\sigma = \frac{-\mu_n Q}{WL\,d} = \frac{-\mu_n Q_s}{d}$. For the surface charge density, find with Gauss's law $Q_s = \varepsilon_H E(0)$

substitute in (4)

$$\int_0^L I_D\,dy = \mu_n W \int_0^{V_{DS}} Q_s\,dV \quad \text{... (5)}$$

Using (3) It is seen that drain current can be found

$$I_D L = \mu_n W \int_0^{V_{DS}} \frac{\varepsilon_H}{d}(V_{GS} - V - V_{T0})\, dV \quad \text{... (6)}$$

or

$$I_D = \mu_n \frac{W \varepsilon_H}{Ld}\left[V_{DS}(V_{GS} - V_{T0}) - \frac{V_{DS}^2}{2}\right] \quad \text{... (7)}$$

pinch off occur when drain-source voltage is equal to or greater than difference of gate–source and threshold voltages $\left(V_{DS} \geq V_{GS} - V_{T0}\right)$. If the equality of this condition is substituted in (7) it is seen that

$$I_D = \mu_n \frac{W \varepsilon_H}{2Ld}(V_{GS} - V_{T0})^2 \quad \text{... (8)}$$

The, threshold voltage allows us to determine if the HEMT is operated as an enhancement or depletion type. For the depletion type, we require $V_{T0} < 0$ or $V_b - \left(\frac{\Delta W_C}{a}\right) - V_p < 0$.

substituting the pinch off voltage $V_p = \frac{q N_D d}{2\varepsilon_H}$ and solving for d, this implies

$$d > \left[\frac{2\varepsilon_H}{q N_D}\left(V_b - \frac{\Delta W_C}{q}\right)\right]^{1/2} \quad \text{... (19)}$$

and if d is less than the $V_{T0} > 0$, we deal with an enhancement HEMT.

5.5 BASIC CONCEPTS OF RF DESIGN

5.5.1 RF Circuit Design Consideration

Low RF circuits have to go through a three – step design process. In this design process, the effect of wave propagation on the circuit operation is negligible and the following facts in connection with design process can be stated:

1. The length of the circuit (l) is generally much smaller than wavelength $(l \ll \lambda)$
2. Propagation delay time (t_d) is approximately zero ($t_d = 0$)
3. Maxwell's equations simplify into all of the low frequency laws such as KVL, KCL. Therefore at RF frequencies, the delay time of propagation is approximately zero when $l \ll \lambda$ and all elements in the circuit can be considered to be lumped.

The design process has following three steps

Step 1: The design process starts with selecting a suitable device and performing a DC design to obtain a proper Q-point.

Step 2: The device will be characterized to obtain its AC small signal parameters based on the specific DC operating point selected earlier.

Step 3: It consist of designing two matching circuits that transition this device to the outside world, the signal source at one end and load at the other.

The design process for RF circuit is summarized and shown in figure 5.13.

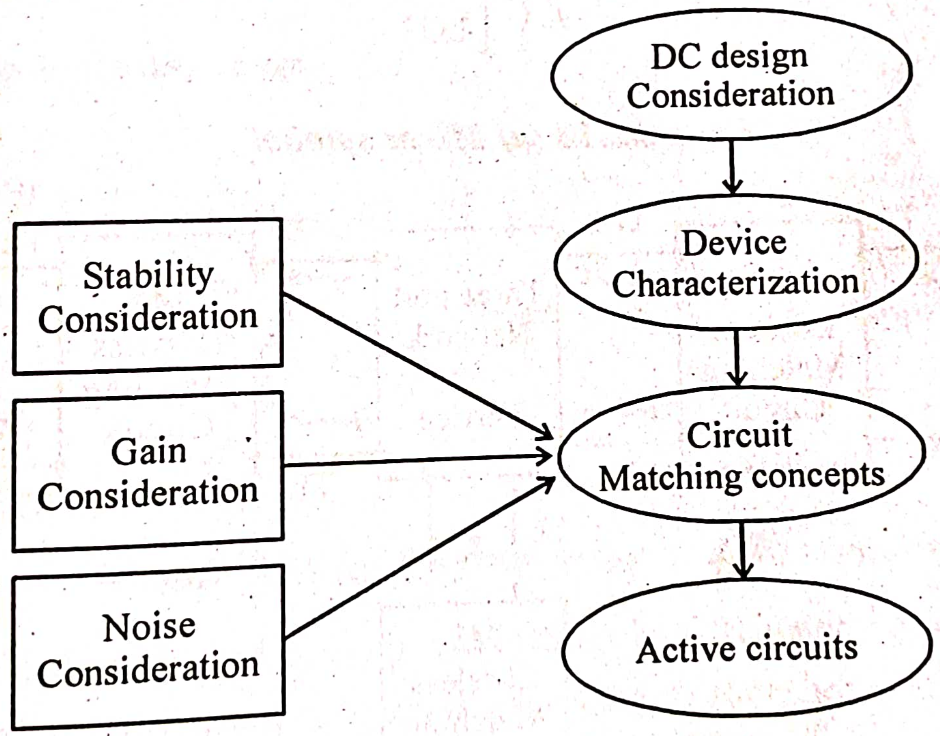


Figure 5.13 RF circuit design steps

5.5.2 High RF and Microwave Circuit Design Consideration

Step 1: The design process starts with the design of DC circuit to establish a stable operating point.

Step 2: Characterize the device at the operating point using electrical waves to measure the percentage of reflection and transmission that device presents at each port.

Step 3: Designing the matching network that transition the device to the outside world such that the required specification such as stability, overall gain, etc., are satisfied.

5.6 MIXER

Definition: A non linear 3 port circuit (Two inputs and one output) that generates a spectrum of output frequencies equal to sum or difference of two input frequencies and their harmonics. The two input ports are referred as RF and LO where as the output is called IF port.

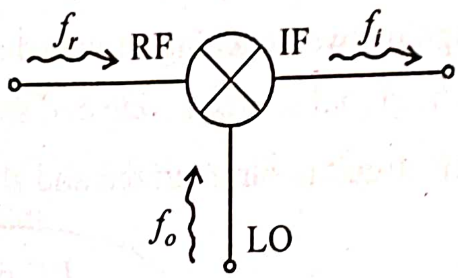


Figure 5.14 (a) Mixer symbol

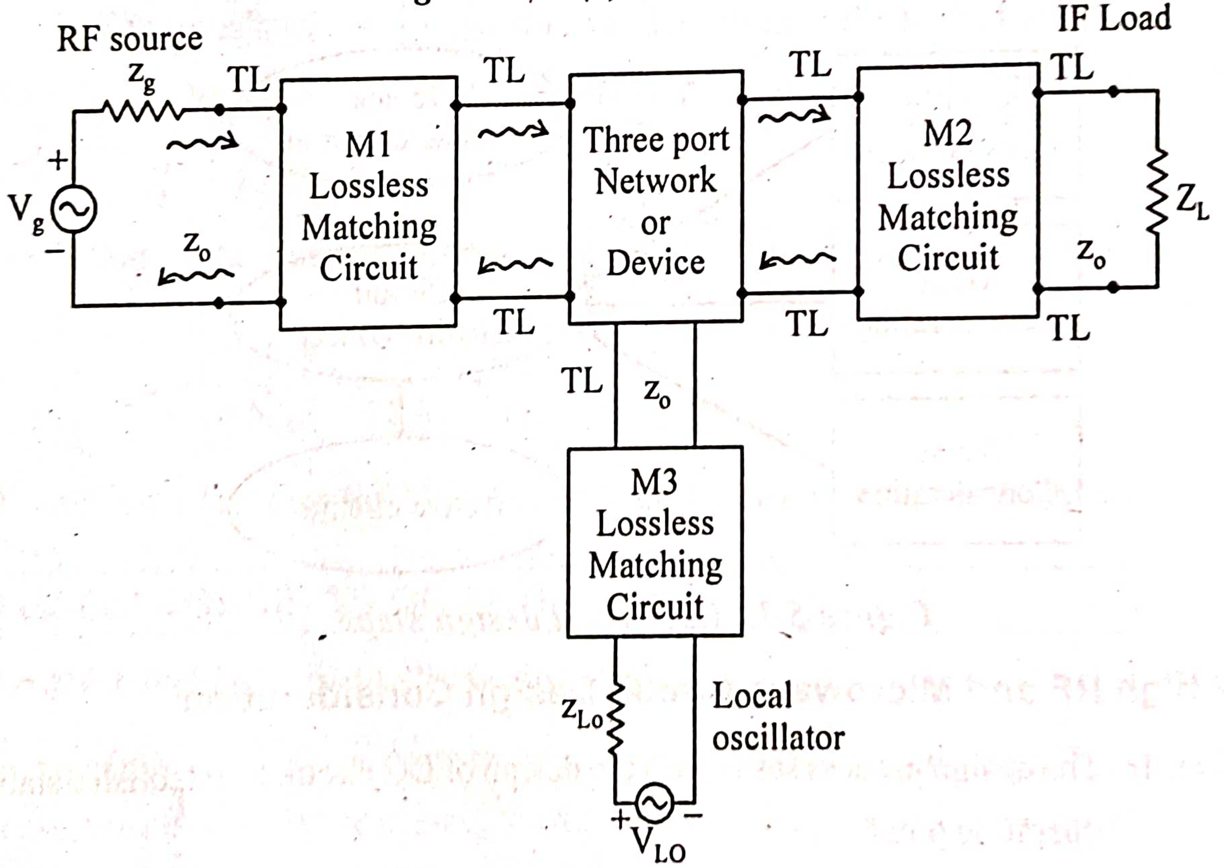


Figure 5.14 (b) Mixer block diagram

Mixer uses the non linearity of a device to generate a spectrum of frequencies (at the IF port) based on sum and difference of harmonics of the RF signal and local oscillator signal frequencies as shown in figure 5.14. The non linear device is flanked on three sides by three matching circuits,, which need to be designed properly for maximum conversion efficiency.

In general, mixers utilize one or more non linear device, properly pumped with a relatively large signal (called LO) to mix with an RF signal in order to generate a spectrum of frequencies based on the sum and difference of harmonics of the RF & LO frequencies.

$$\omega_i = m\omega_r \pm n\omega_o$$

Where m & n are positive integers.

The most important terms for mixer operation are those with frequencies at $\omega_r + \omega_o$ and at $\omega_r - \omega_o$..

5.6.1 Types of Mixers

5.6.1.1 Up - Converters

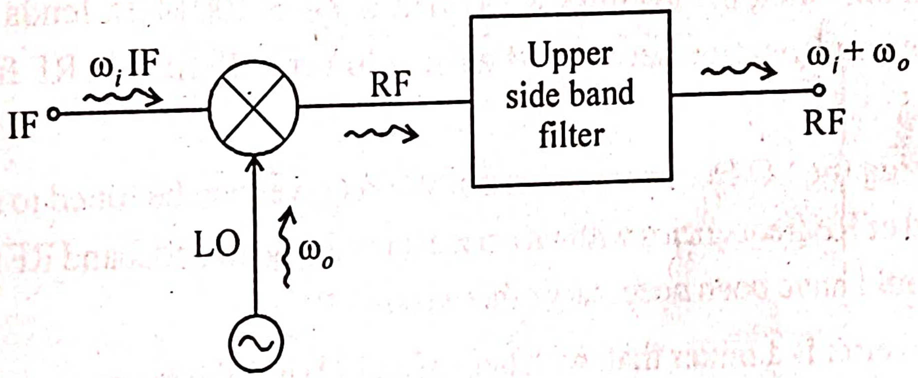


Figure 5.15 (a) Up conversion in a transmitter

When a mixer is used as an up converter the sum frequency $f_i = f_o + f_r$ is utilized and the difference frequency is rejected. In an up converter shown in figure 5.15 (a) the IF oscillator is modulated with desired information signal which when mixed with LO signal will generate the desired frequency conversion. Use of mixer as an up converter particularly in case of a radar or a transceiver is advantageous because it allows the use of a single local oscillator for both receiver and transmitter.

$$\omega_r = \omega_o + \omega_i$$

Use of proper filtering or an image rejection mixer is needed to generate the sum frequency ($\omega_r = \omega_o + \omega_i$). An up converters is used in a transmitter to modulate a carrier wave (LO signal) with an information bearing signal (IF signal) in order to generate an RF signal for transmission.

5.6.1.2 Down Converter

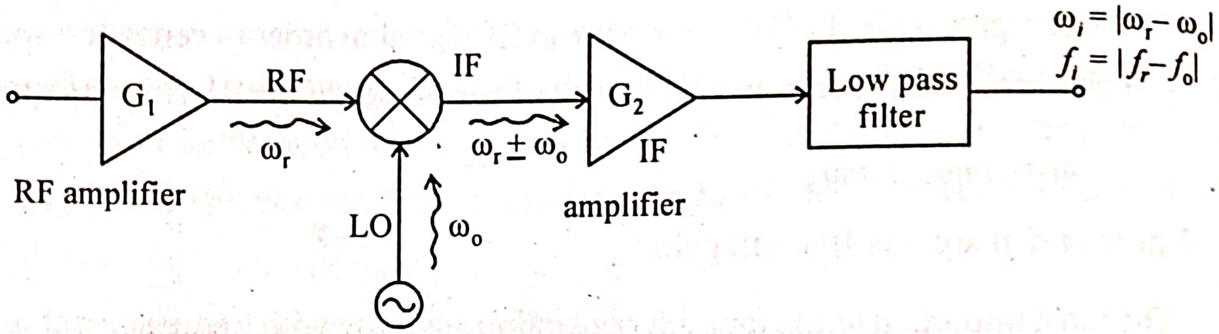


Figure 5.15 (b) Down conversion in a heterodyne receiver

When application requires a down – conversion the difference frequency $f_i = f_o - f_r$ is used and sum frequency is filtered out as shown in figure 5.15 (b). Use of mixer as a down converter in heterodyne receiver has several advantages.

- ✓ The IF signal being in the range of 10 MHz $\leq f_{IF} \leq$ 100 MHz lends itself for low noise amplification because 1/f noise is lower in IF than in RF frequency range.
- ✓ By changing the LO frequency, a heterodyne receiver can be tuned to receive a wide band of RF frequencies without a need for a high gain wideband RF amplifier which would have been necessary otherwise.

A down converter is a mixer that, with help of an LO shift the frequency of the RF signal substantially down to an IF signal ready for further signal processing. The IF signal frequency is given by

$$\omega_i = |\omega_r - \omega_o|$$

Such a mixer is used in receiver as a demodulator to remove the carrier wave from the transmitted signal in order to obtain the information carrying signal.

5.6.1.3 Harmonic Mixers

A simple method of down converting a high frequency RF signal when only a low frequency LO exist is the use of harmonic mixers. Frequency down conversion is achieved by mixing the high frequency signal with an appropriate harmonic of LO frequency.

$$\omega_i = n\omega_o - \omega_r$$

Where $n = 1, \ldots N$ is an integer, when $n = 1$ a fundamental down converter is obtained.

Applications of harmonic mixers are in millimeter wave instrumentation where the use of high frequency LO source, which can generate substantial power to satisfy LO power needs is impractical or very expensive.

5.7 LOW NOISE AMPLIFIERS

A Low noise amplifier is an electronic amplifier that amplifies very low power signal without significantly degrading its signal to noise ratio. LNAs are designed to amplify a signal while minimizing additional noise.

LNA are found in radio communication system, medical instruments and electronic test equipment.

A typical LNA may supply a power gain of 100 while decreasing the signal to noise ratio by less than factor of two. LNA as shown in figure 5.16.

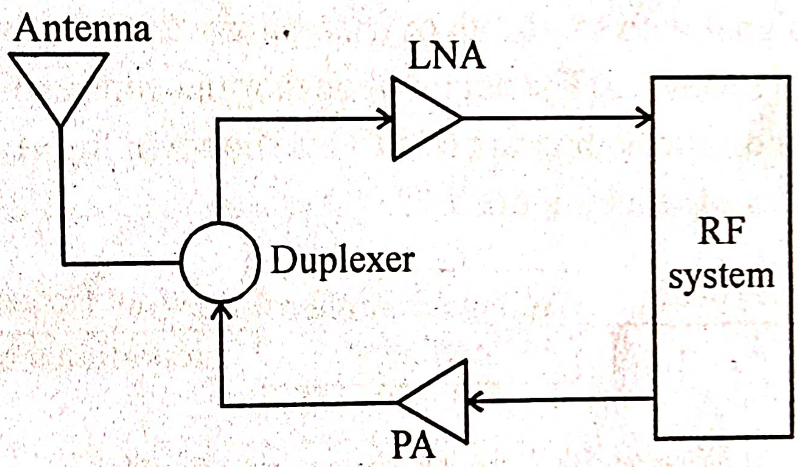


Figure 5.16 Block diagram of LNA

The low noise amplifier of receiver path and power amplifier of transmit path connected to the antenna via duplexer, which separate the two signals and prevents the relatively powerful power amplifier output from overloading the sensitive LNA input.

For LNA, the primary parameters are noise figure (NF), gain, and non linearity. Noise is due to thermal and other sources with typical noise figure 0.5 to 1.5 dB range. The noise figure helps determine the efficiency of particular LNA. Low noise figure results in better signal reception. With low noise figure, an LNA must have high gain. Typical gain is between 10 and 20 dB for a single stage.

LNAs are used in applications such as industrial scientific and medical band (ISM) radios, cellular telephones, GPS receivers, wireless LANs, satellite communication.

5.8 VOLTAGE CONTROL OSCILLATORS

A voltage controlled oscillator is an oscillator with an output signal whose output can be varied over a range, which is controlled by the input DC voltage. It is an oscillator whose output frequency is directly related to the voltage at its input. The oscillation frequency varies from few hert Z to hundred of GHz.

Types of voltage controlled oscillators.

Harmonic oscillators. The output is a signal with sinusoidal waveform. Examples are crystal oscillators and tank oscillators.

Relaxation oscillators:

The output is a signal with saw tooth or triangular waveform and provides a wide range of operational frequencies. The output frequency depends on time of charging and discharging of capacitor. Block diagram of VCO is shown in figure 5.17 and saw tooth wave generator VCO is shown in figure 5.18.

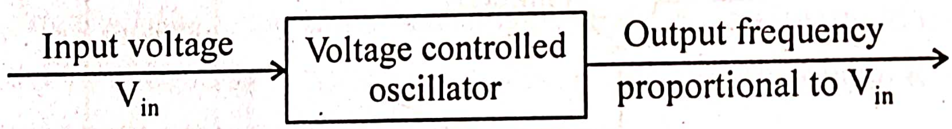


Figure 5.17 Block diagram of VCO

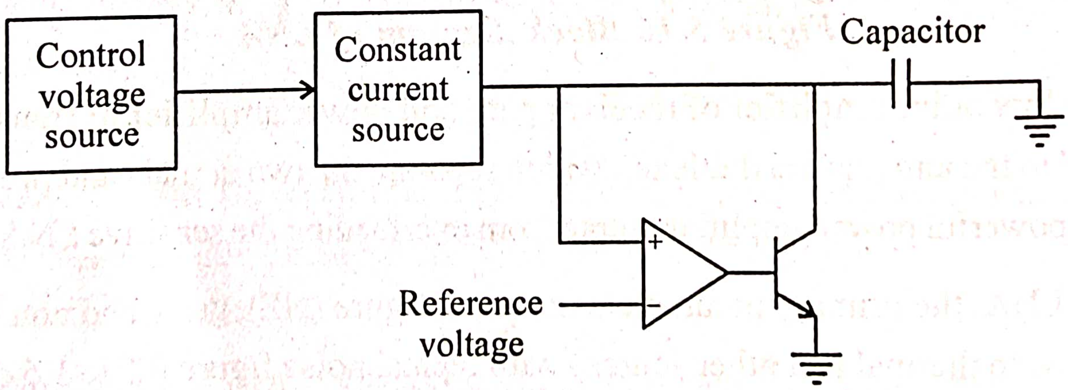


Figure 5.18 Basic working principle of saw tooth wave generator VCO

For a voltage controlled oscillator generating a saw tooth waveform, main component is capacitor who's charging & discharging actually decides formation of output waveform. The input is given in the form of voltage which can be controlled. This voltage is converted into a current signal and is applied to capacitor. As current passes through capacitor it starts charging and voltage starts building across it. As the capacitor charges and voltage across it increases gradually, the voltage is compared with a reference voltage using a comparator.

When capacitor voltage exceeds reference voltage, the comparator generate high logic output which trigger the transistor and capacitor is connected to ground and starts discharging. Thus the output waveform generated is the representation of charging and discharging of capacitor and frequency is controlled by input dc voltage.

Applications of VCO

- Electronic Jamming equipment
- Function generator
- Phase locked loop
- Frequency synthesizer, used in communication circuits.

5.9 POWER AMPLIFIER

The RF power amplifier is the last component of transmitter chain. The purpose of transmitter is to deliver an RF signal with required properties and specified power level to the antenna and need for the PA (Power Amplifier) is in amplification of that signal to level expected at antenna port.

Requirements of power amplifier

✓ It has to have sufficient gain

✓ It has to have sufficient power handling capability.

✓ It has to be stable.

5.10 AMPLIFIER POWER RELATIONS

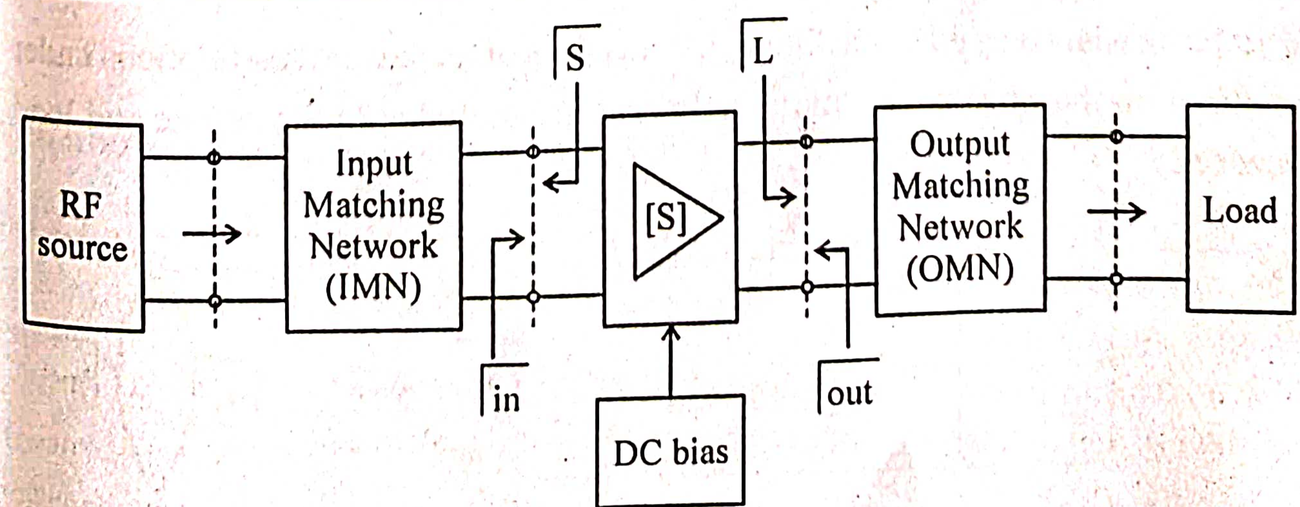


Figure 5.19 Generic amplifier system

A generic single stage amplifier configuration embedded between input and output matching networks is shown in figure 5.19. Input and output matching networks are needed to reduce undesired reflections and thus improve the power flow capabilities. The amplifier is characterized through its S parameter matrix at a particular DC bias point. The following list constitute a set of key amplifier parameters:

- ✓ Gain and gain flatness (in dB)
- ✓ Operating frequency and bandwidth (in Hz)
- ✓ Output power (in dBm)
- ✓ Power supply requirements (in V and A)
- ✓ Noise figure (in dB)

5.10.1 RF Source

There are various power gain definitions that are critical to the understanding of how an RF amplifier functions.

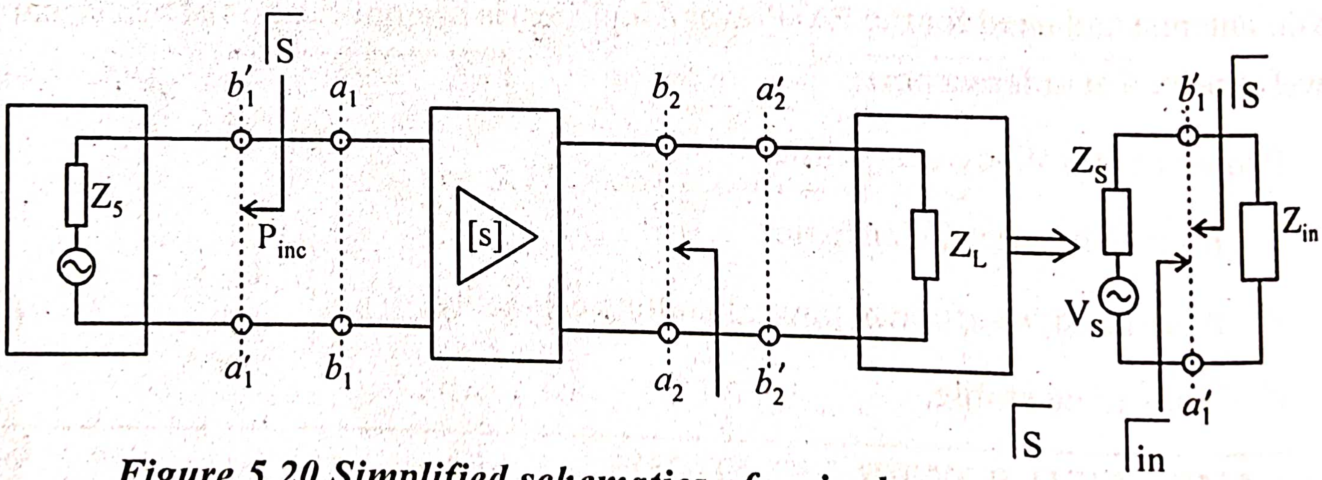


Figure 5.20 Simplified schematics of a single stage amplifier

For this reason let us examine figure 5.19 in terms of its power flow relations under the assumption that the two matching networks are included in the source and load impedances.

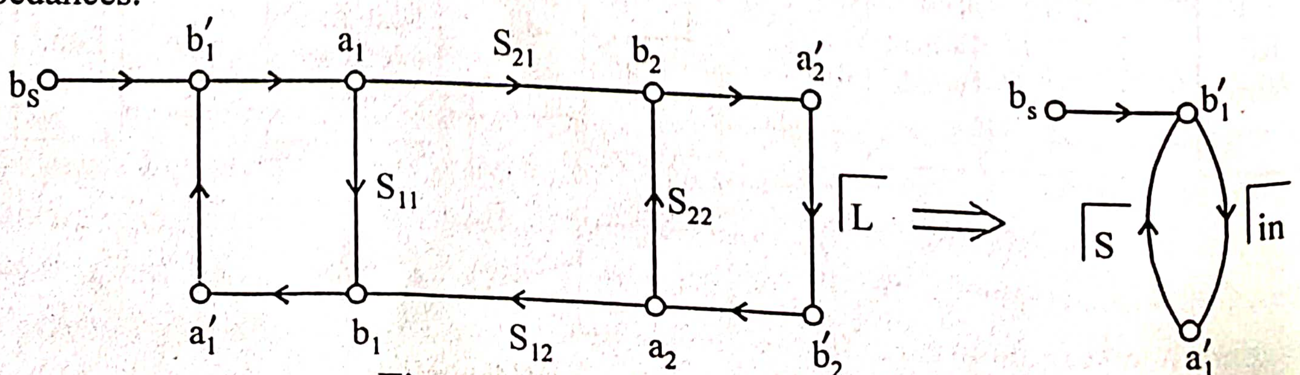


Figure 5.21 Signal flow graph

$$b_S = \frac{\sqrt{Z_o}}{Z_S + Z_o} V_S = b_1' - a_1' \Gamma_S = b_1' (1 - \Gamma_{in} \Gamma_S) \qquad ...(1)$$

The incident power wave associated with b_1' is given as

$$P_{inc} = \frac{|b_1'|^2}{2} = \frac{1}{2} \frac{|b_S'|^2}{|1 - \Gamma_{in} \Gamma_S|^2} \qquad ...(2)$$

Which is the power launched towards the amplifier. The actual input power P_{in} observed at the input terminal of the amplifier is composed of the incident and reflected power waves with the aid of the input reflection coefficient Γ_{in}.

$$P_{in} = P_{inc} \left(1 - |\Gamma_{in}|^2\right) = \frac{1}{2} \frac{|b_S|^2}{|1 - \Gamma_{in} \Gamma_S|^2} \left(1 - |\Gamma_{in}|^2\right) \qquad ...(3)$$

The maximum power transfer from the source to the amplifier is achieved if the input impedance is complex conjugate matched $(Z_{in} = Z_s')$ or in terms of the reflection coefficients, if $\Gamma_{in} = \Gamma_S^*$. Under maximum power transfer condition, the available power P_A as

$$P_A = P_{in}\Big|_{\Gamma_{in} = \Gamma_S^*} = \frac{1}{2} \frac{|b_S|^2}{|1 - \Gamma_{in} \Gamma_S|^2}\Bigg|_{\Gamma_{in} = \Gamma_S^*} \left(1 - |\Gamma_{in}|^2\right)$$

$$= \frac{1}{2} \frac{|b_S|^2}{1 - |\Gamma_S|^2} \qquad ...(4)$$

This expression makes clear the dependence on Γ_S. If $\Gamma_{in} = 0$ and $\Gamma_S \neq 0$, it is seen from (2) that $P_{inc} = \frac{|b_S|^2}{2}$.

5.10.2 Transducer Power Gain

It quantifies the gain of the amplifier placed between source and load.

$$G_T = \frac{\text{Power delivered to the load}}{\text{Available power from the source}} = \frac{P_L}{P_A}$$

or with $P_L = \frac{1}{2} |b_2|^2 \cdot \left(1 - |\Gamma_L|^2\right)$ we obtain

$$G_T = \frac{P_L}{P_A} = \frac{|b_2|^2}{|b_S|^2}\left(1-|\Gamma_L|^2\right)\left(1-|\Gamma_S|^2\right) \qquad \text{...(5)}$$

In this expression, the ratio b_2/b_S has to be determined, with help of signal flow graph and based on figure we establish

$$b_2 = \frac{S_{21}\, a_1}{1 - S_{22}\Gamma_L} \qquad \text{...(6)}$$

$$b_S = \left[1 - \left(S_{11} + \frac{S_{21} S_{12}\Gamma_L}{1 - S_{22}\Gamma_L}\right)\Gamma_S\right] a_1 \qquad \text{...(7)}$$

The required ratio is given by

$$\frac{b_2}{b_S} = \frac{S_{21}}{\left(1 - S_{11}\Gamma_S\right)\left(1 - S_{22}\Gamma_L\right) - S_{21} S_{12}\Gamma_L\Gamma_S} \qquad \text{...(8)}$$

Inserting (8) into (5) results in

$$G_T = \frac{\left(1-|\Gamma_L|^2\right)|S_{21}|^2\left(1-|\Gamma_S|^2\right)}{\left|\left(1-S_{11}\Gamma_S\right)\left(1-S_{22}\Gamma_L\right) - S_{21} S_{12}\Gamma_L\Gamma_S\right|^2} \qquad \text{...(9)}$$

Which can be rearranged by defining the input and output reflection coefficients

$$\Gamma_{in} = S_{11} + \frac{S_{21} S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} \qquad \text{...(10)}$$

$$\Gamma_{out} = S_{22} + \frac{S_{12} S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \qquad \text{...(11)}$$

With these two definitions two more transducer power gain expressions can be derived. First by incorporating (10) into (9) it is seen that

$$G_T = \frac{\left(1-|\Gamma_L|^2\right)|S_{21}|^2\left(1-|\Gamma_S|^2\right)}{\left|1-\Gamma_S\Gamma_{in}\right|^2\left|1 - S_{22}\Gamma_L\right|^2} \qquad \text{...(12)}$$

Second using (11) into (9) results in

$$G_T = \frac{\left(1-|\Gamma_L|^2\right)|S_{21}|^2\left(1-|\Gamma_S|^2\right)}{\left|1-\Gamma_L\Gamma_{out}\right|^2\left|1 - S_{11}\Gamma_S\right|^2} \qquad \text{...(13)}$$

An often employed approximation for the transducer power gain is so called unilateral power gain G_{TU}, which neglect the feedback effect of the amplifier ($S_{12} = 0$). This simplifies (13) into

$$G_{TU} = \frac{\left(1-|\Gamma_L|^2\right)|S_{21}|^2\left(1-|\Gamma_S|^2\right)}{|1-\Gamma_L S_{22}|^2\,|1-S_{11}\Gamma_S|^2} \qquad ...(14)$$

5.10.3 Additional Power Relations

The transducer power gain is a fundamental expression from which additional important power relations can be derived. For instance, the available power gain for load side matching $\left(\Gamma_L = \Gamma_{out}^*\right)$ is defined as

$$G_A = G_T\Big|_{\Gamma_L = \Gamma_{out}^*}$$

$$= \frac{\text{Power available from the network}}{\text{Power available from the source}} = \frac{P_N}{P_A}$$

or

$$G_A = \frac{|S_{21}|^2\left(1-|\Gamma_S|^2\right)}{\left(1-|\Gamma_{out}|^2\right)|1-S_{11}\Gamma_S|^2} \qquad ...(15)$$

Further power gain (operating power gain) is defined as the ratio of the power delivered to the load to the power supplied to the amplifier.

$$G = \frac{\text{Power deliverd to load}}{\text{Power supplied to the amplifier}}$$

$$= \frac{P_L}{P_{in}} = \frac{P_L}{P_A}\cdot\frac{P_A}{P_{in}} = G_T\frac{P_A}{P_{in}}$$

Combining (3), (4) and (12)

$$G = \frac{\left(1-|\Gamma_L|^2\right)|S_{21}|^2}{\left(1-|\Gamma_{in}|^2\right)|1-S_{22}\Gamma_L|^2} \qquad ...(16)$$

Example: 3

An RF amplifier has the following S-parameters:

$S_{11} = 0.3 \angle -70^o$, $S_{21} = 3.5 \angle 85^o$, $S_{12} = 0.2 \angle -10^o$, $S_{22} = 0.4 \angle -45^o$. Furthermore, the amplifier is connected to a voltage source with $V_s = 5V \angle 0^o$ and source impedance $Z_S = 40\ \Omega$. The output is utilized to drive an antenna, which has an impedance of $Z_L = 73\ \Omega$. Assuming that the S parameters of the amplifier are measured with reference to a $Z_0 = 50\Omega$.

Find the following quantities.

i) Transducer gain G_T

ii) Unilateral transducer gain G_{TU}

iii) Available gain G_A

iv) Operating power gain G

v) Incident power to the amplifier P_{inc}

vi) Available power from the source P_A

vii) Power delivered to the load P_L

☞ ***Solution:***

Source reflection coefficient $\Gamma_S = \dfrac{Z_S - Z_0}{Z_S + Z_0}$

$$= \frac{40 - 50}{40 + 50}$$

$$\Gamma_S = -0.111$$

Load reflection coefficient $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$$= \frac{73 - 50}{73 + 50}$$

$$\Gamma_L = 0.187$$

$$\Gamma_{in} = S_{11} + \frac{S_{21} S_{12} \Gamma_L}{1 - S_{22} \Gamma_L}$$

$$= 0.146 - j\,0.151$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}\, S_{21}\, \Gamma_S}{1 - S_{11}\, \Gamma_S}$$

$$= 0.265 - j\,0.358$$

i) $$G_T = \frac{\left(1 - |\Gamma_L|^2\right) |S_{21}|^2 \left(1 - |\Gamma_S|^2\right)}{|1 - \Gamma_L\, \Gamma_{out}|^2\, |1 - S_{11}\Gamma_S|^2} = 12.56 \text{ or } 10.99 \text{ dB}$$

ii) $$G_{TU} = \frac{\left(1 - |\Gamma_L|^2\right) |S_{21}|^2 \left(1 - |\Gamma_S|^2\right)}{|1 - \Gamma_L\, S_{22}|^2\, |1 - S_{11}\Gamma_S|^2} = 12.67 \text{ or } 11.03 \text{ dB}$$

iii) $$G_A = \frac{|S_{21}|^2 \left(1 - |\Gamma_S|^2\right)}{\left|1 - |\Gamma_{out}|^2\right|\, |1 - S_{11}\Gamma_S|^2} = 14.74 \text{ or } 11.68 \text{ dB}$$

iv) $$G = \frac{\left(1 - |\Gamma_L|^2\right) |S_{21}|^2}{\left|1 - |\Gamma_{in}|^2\right|\, |1 - S_{22}\Gamma_L|^2} = 13.74 \text{ or } 11.38 \text{ dB}$$

v) $$P_{inc} = \frac{1}{2} \frac{|b_S|^2}{|1 - \Gamma_{in}\Gamma_S|^2} = \frac{1}{2} \frac{Z_0}{(Z_S + Z_0)} \frac{|V_S|^2}{|1 - \Gamma_{in}\Gamma_S|^2} = 74.7 \text{ mw}$$

$$P_{inc}(dBm) = 10 \log (P_{inc}/(1 \text{ mw})) = 18.73 \text{ dBm}$$

vi) $$P_A = \frac{1}{2} \frac{|b_S|^2}{1 - |\Gamma_S|^2} = 78.1 \text{mw or } 18.93 \text{ dBm}$$

vi) $$P_L = P_A\, G_T = 981.4 \text{ mw or } 29.92 \text{ dBm}$$

5.11 STABILITY CONSIDERATIONS

5.11.1 Stability Circles

An amplifier circuit must be stable over the entire frequency range. The RF circuit tend to oscillate depending on operating frequency and termination. The phenomenon of oscillation can be understood in the context of voltage wave along a transmission line.

If $|\Gamma| > 1$ then return voltage increases in magnitude possibly causing instability.

If $|\Gamma| < 1$ causes a diminished return voltage wave (negative feedback).

Amplifier as a two port network characterized through its S - parameters. Amplifier is stable, when magnitude of reflection coefficient are less than unity.

$$|\Gamma_L| < 1, \; |\Gamma_S| < 1 \qquad \text{...(1)}$$

$$|\Gamma_{in}| = \left| \frac{S_{11} - \Gamma_L \Delta}{1 - S_{22}\Gamma_S} \right| < 1 \qquad \text{...(2)}$$

$$|\Gamma_{out}| = \left| \frac{S_{22} - \Gamma_S \Delta}{1 - S_{11}\Gamma_S} \right| < 1 \qquad \text{...(3)}$$

where $\quad \Delta = S_{11}S_{22} - S_{12}S_{21} \qquad \text{...(4)}$

Since S parameters are fixed for a particular frequency, the only factor that have a parametric effect on stability are Γ_L and Γ_S

In terms of amplifier output port, the condition is established for which (2) is satisfied.

To this end, the complex quantities

$$\left.\begin{array}{ll} S_{22} = S_{11}^R + j\,S_{11}^I, & S_{22} = S_{22}^R + j\,S_{22}^I \\ \Delta = \Delta^R + j\Delta^I, & \Gamma_L = \Gamma_L^R + j\Gamma_L^I \end{array}\right\} \qquad \text{...(5)}$$

substituting equation (5) into (2) output stability circle equation is

$$\left(\Gamma_L^R - C_{out}^R\right)^2 + \left(\Gamma_L^I - C_{out}^I\right)^2 = r_{out}^2 \qquad \text{...(6)}$$

where circle radius is given by

$$r_{out} = \frac{|S_{12}S_{21}|}{\left||S_{22}|^2 - |\Delta|^2\right|} \qquad \text{...(7)}$$

and center of circle is located at

$$C_{out} = C_{out}^R + jC_{out}^I = \frac{(S_{22} - S_{11}^* \Delta)^*}{|S_{22}|^2 - |\Delta|^2} \qquad \text{...(8)}$$

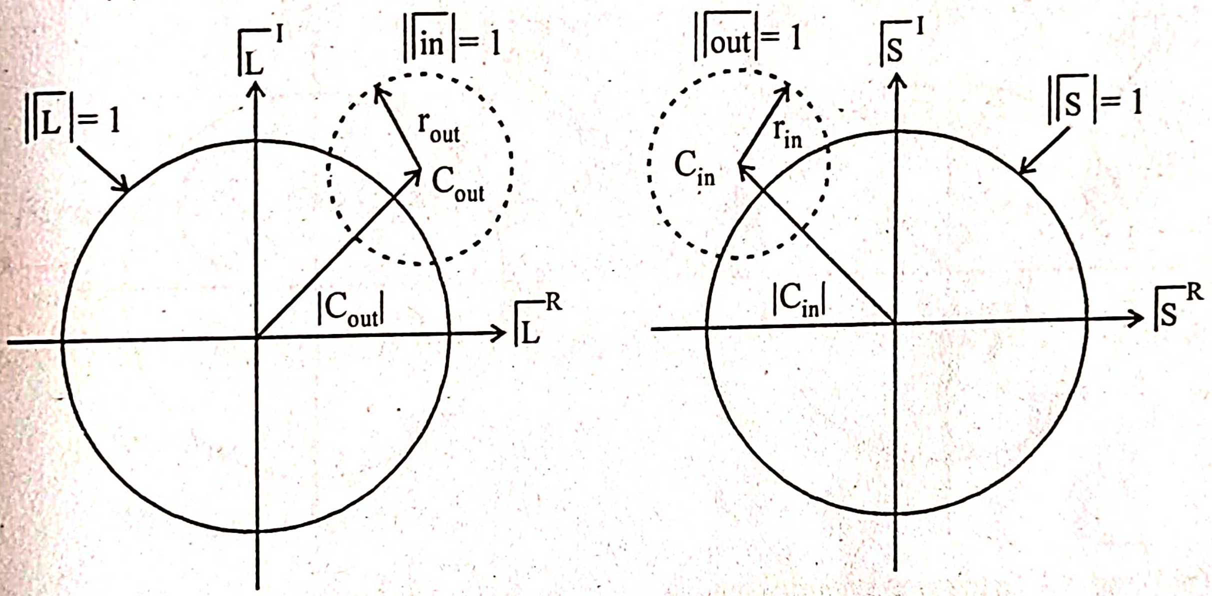
as depicted in figure 5.22(a) Interms of input port, substituting (5) into (3) yields the input stability circle equation

$$\left(\Gamma_S^R - C_{in}^R\right)^2 + \left(\Gamma_S^I - C_{in}^I\right)^2 = r_{in}^2 \qquad \text{...(9)}$$

where $$r_{in} = \frac{S_{12}\,S_{21}}{\left|\,|S_{11}|^2 - |\Delta|^2\right|} \qquad \text{...(10)}$$

and $$C_{in} = \frac{(S_{11} - S_{22}^{*}\,\Delta)^{*}}{|S_{11}|^2 - |\Delta|^2} = C_{in}^{R} + j\,C_{in}^{I} \qquad \text{...(11)}$$

When plotted in Γ_S plane we obtain response as schematically shown in figure 5.22 (b).



(a) output stability circle (b) Input stability circle

Figure 5.22 Stability circle $|\Gamma_{in}| = 1$ in the complex Γ_L-plane and stability circle $|\Gamma_{out}| = 1$ in the complex Γ_S-plane

5.11.2 Unconditional Stability

Unconditional stability refers to the situation where the amplifier remains stable for any passive source and load at the selected frequency and bias conditions.

For $|S_{11}| < 1$ and $|S_{22}| < 1$, it is stated as

$$\left|\,|C_{in}| - r_{in}\right| > 1 \qquad \text{...(12)}$$

$$\left|\,|C_{out}| - r_{out}\right| > 1 \qquad \text{...(13)}$$

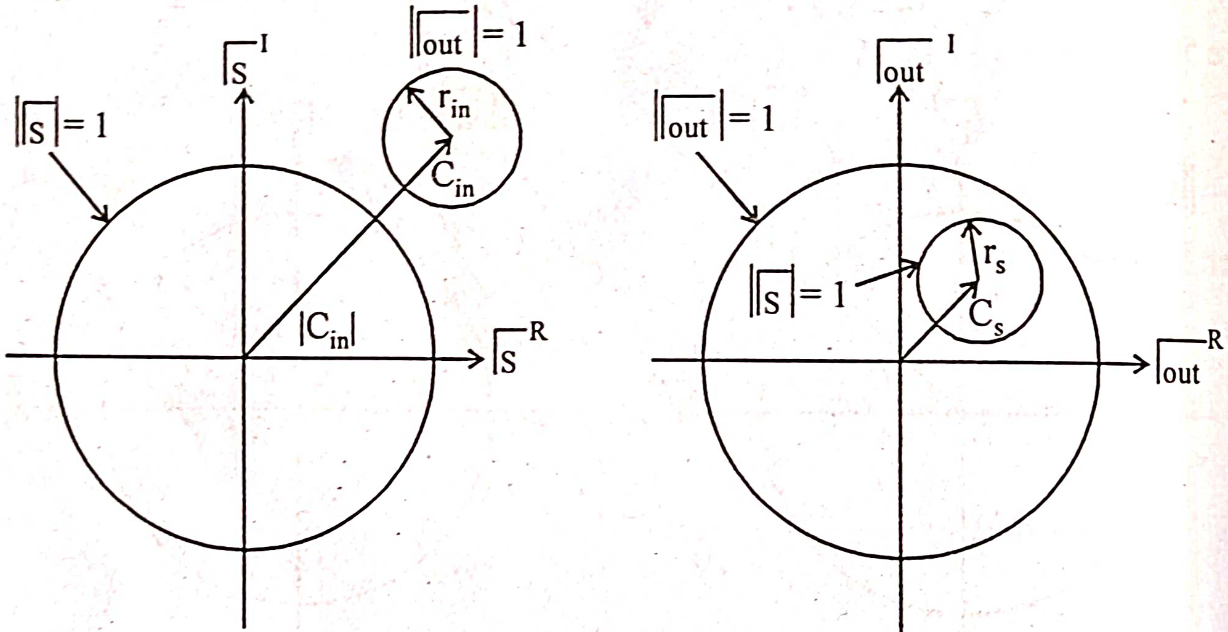
In other words, the stability circles have to reside completely outside the $|\Gamma_S| = 1$ and $|\Gamma_L| = 1$ circles.

The condition for stability is expressed in terms of stability factor K as

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}||S_{21}|} > 1 \qquad ...(14)$$

The stability factor K is also referred as Rollet factor.

It applies for both input and output ports.



(a) $|\Gamma_{out}| = 1$ circle must reside outside (b) $|\Gamma_S| = 1$ circle must reside inside

Figure 5.23 Unconditional stability in the Γ_S and Γ_{out} planes for $|S_{11}| < 1$

5.11.3 Stabilization Methods

If the operation of a FET or BJT is found to be unstable, an attempt can be made to stabilize the transistor. The conditions $|\Gamma_{in}| > 1$ and $|\Gamma_{out}| > 1$ can be written in terms of input and output impedances.

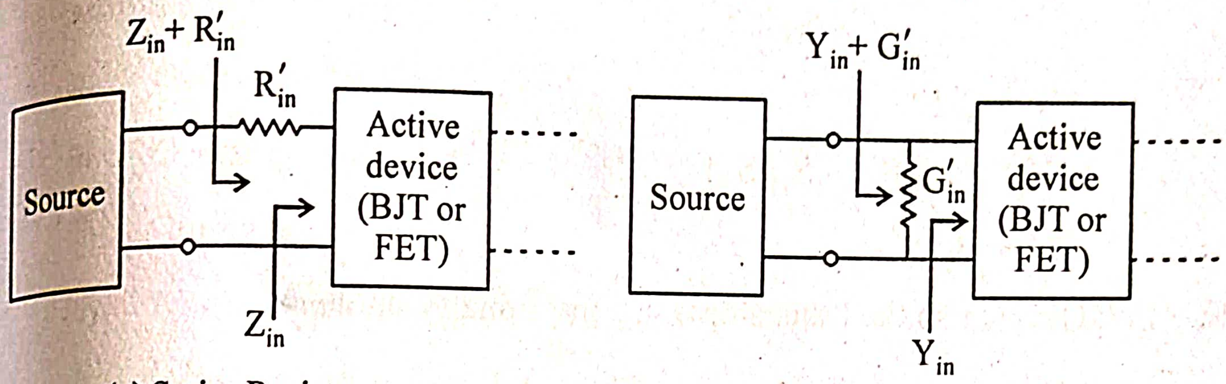
$$|\Gamma_{in}| = \left|\frac{Z_{in} - Z_o}{Z_{in} + Z_o}\right| > 1 \quad |\Gamma_{out}| = \left|\frac{Z_{out} - Z_o}{Z_{out} + Z_o}\right| > 1$$

Which imply $Re(Z_{in}) < 0$ and $Re(Z_{out}) < 0$. One way to stabilize the active device is to add a series resistance or a shunt conductance to port.

Figure 5.24 shows the configuration for the input port. This loading in conjunction with $Re(Z_S)$ must compensate the negative contribution of $Re(Z_{in})$. Thus we require

$$\text{Re}(Z_{in} + R'_{in} + Z_S) > 0 \text{ (or)}$$

$$\text{Re}(Y_{in} + G'_{in} + Y_S) > 0$$



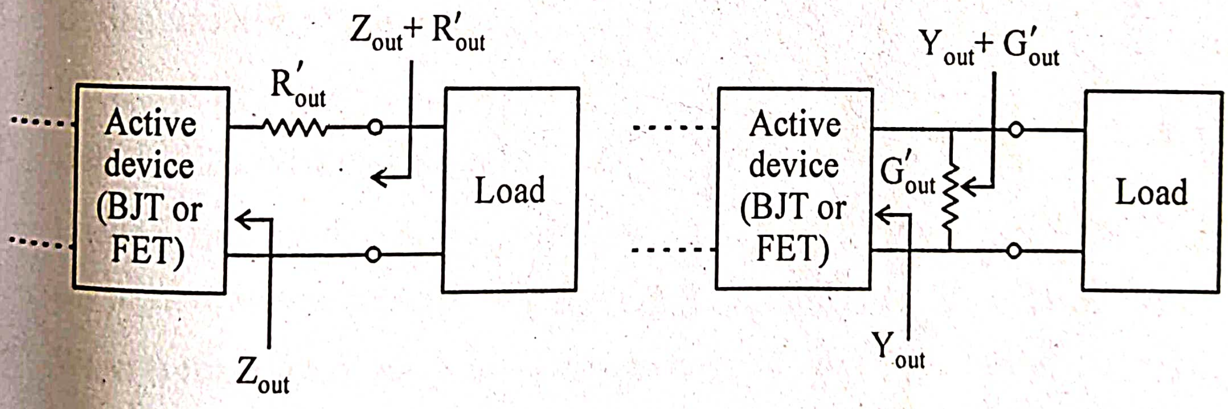
(a) Series Resistance (b) Shunt conductance

Figure 5.24 Stabilization of input port through series resistance or shunt conductance

Figure 5.25 shows the stabilization of the output port. The corresponding condition is

$$\text{Re}(Z_{out} + R'_{out} + Z_L) > 0 \text{ (or)}$$

$$\text{Re}(Y_{out} + G'_{out} + Y_L) > 0$$



(a) Series resistance (b) Shunt conductance

Figure 5.25 Stabilization of output port through series resistance or shunt conductance

Example: 4

A MESFET operated at 5.7 GHz has following S parameters. $S_{11} = 0.5\angle -60^o$, $S_{12} = 0.02\angle 0^o$, $S_{21} = 6.5\angle 115^o$, $S_{22} = 0.6\angle -35^o$. Verify the circuit whether it is unconditionally stable or not?

Solution:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}||S_{21}|}$$

$$K = 2.17$$

$$|\Delta| = |S_{11}\, S_{22} - S_{12}\, S_{21}|$$

$$|\Delta| = 0.42$$

$K > 1$ and $|\Delta| < 1$ so the transistor is unconditionally stable.

TWO MARK QUESTIONS AND ANSWERS

1. **What are the key parameters used to evaluate the performance of an amplifier?**

   i) Gain and gain flatness

   ii) Operating frequency and bandwidth

   iii) Output power

   iv) Power supply requirements

   v) Input and output reflection coefficient

   vi) Noise figure.

2. **Define transducer power gain.**

   It is gain of the amplifier when placed between source and load.

$$G_T = \frac{\text{Power delivered to the load}}{\text{Available power from the source}} = \frac{P_L}{P_A}$$

3. **Define unilateral power gain.**

   It is amplifier power gain when feedback effect of amplifier in neglected $S_{12} = 0$.

4. **Define unconditional stability.**

   Unconditional stability refers to the situation where the amplifier remains stable for any passive source and load at the selected frequency and bias conditions.

5. **What is the need of matching network?**

   It can help stabilize the amplifier by keeping the source and load impedances in the appropriate range.

6. **What are factors used for selecting a matching network?**

   ✓ Complexity

   ✓ Bandwidth requirement

   ✓ Adjustability

   ✓ Implementation

7. Define operating power gain.

It is defined as ratio of power delivered to the load to the power supplied to the amplifier

$$G = \frac{\text{Power deliverd to load}}{\text{Power supplied to the amplifier}} = \frac{P_L}{P_{in}}$$

8. What are the advantages of microwave transistors?

Microwave transistors are miniaturized designs to reduce device and package parasitic capacitances and inductances and to overcome the finite transit time of the charge carriers in the semiconductor materials.

9. What is bipolar transistor?

Bipolar is a three semiconductor (pnp or npn) region structure where charge carriers of both negative (electrons) and positive (holes) polarities are involved in transistor operation.

10. Write the applications of bipolar transistors.

Bipolar transistors are suitable for oscillator and power amplifier applications in addition to small signal amplifiers.

11. What are the different modes of bipolar transistor?

- ✓ Normal (active) mode
- ✓ Saturation mode
- ✓ Cut off mode
- ✓ Inverse (Inverted mode)

12. What is referred as unipolar transistor?

In field effect transistors, the current flow is carried by majority carriers either electrons or holes, this type is referred to as unipolar transistor.

13. Write the advantages of unipolar transistor?

✓ Efficiency is higher

✓ Noise figure is low

✓ It may have voltage gain in addition to current gain.

✓ Its operating frequency is upto X band.

✓ Its input resistance is very high upto several mega ohms.

14. What is MESFET?

Field effect transistors at microwave frequencies are mostly fabricated in GaAs and use a metal semiconductor schottky junction for gate contact. This device is referred as MESFET (Metal Semiconductor FET)

15. Define pinch off voltage.

It is the gate reverse voltage that removes all the free charge from the channel.

16. What is called high electron mobility transistor?

The field effect transistor made using hetero junction is called high electron mobility transistor.

17. Define threshold voltage.

A minimum gate voltage is required to induce the channel and it is called threshold voltage.