UNIT I :AMPLITUDE MODULATION

Communication is the transfer of information from one place (known as the source of information) to another place (known as the destination of information).It is used in Teleconferencing, teleshopping, telebanking, internet, computer networks and mobile etc.

Basic Communication System:

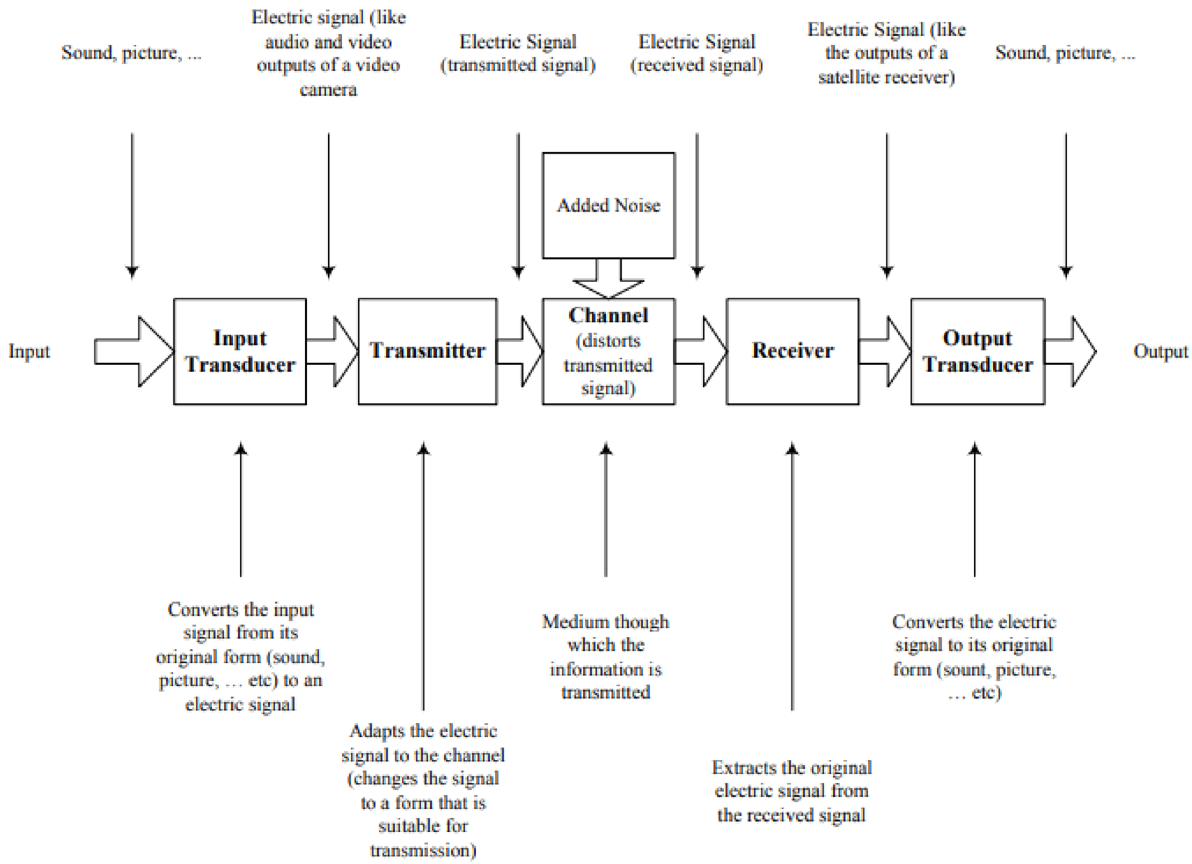


Fig.1 Communication system

Advantages of Digital Communication over Analog Communication:

1. Immunity to Noise
2. Efficient use of communication bandwidth Digital communication provides higher security
3. The ability to detect errors and correct them if necessary.
4. Design and manufacturing of electronics for digital communication systems is much easier and much cheaper than the design and manufacturing of electronics for analog communication systems.

Modulation: Modulation is changing one or more of the characteristics of a carrier signal in accordance with the modulating signal to produce a modulated signal.

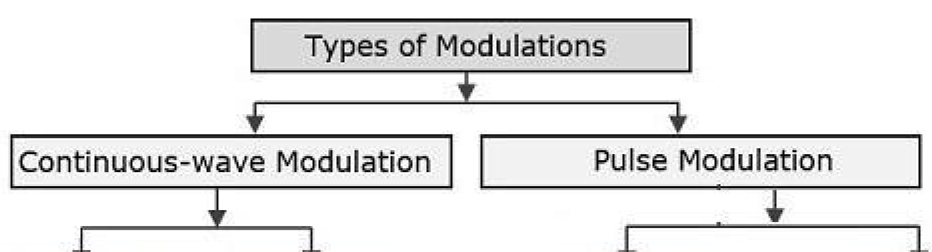


Fig.2 Types of Modulations

Continuous Wave modulation Types:

1. **Amplitude Modulation (AM):**Process of changing the amplitude of the carrier signal in accordance with instantaneous amplitude of the modulating signal.
2. **Frequency Modulation (FM):** Process of changing the frequency of the carrier signal in accordance with instantaneous amplitude of the modulating signal.
3. **Phase Modulation (PM):**Process of changing the phase of the carrier signal in accordance with instantaneous amplitude of the modulating signal.

Need for Modulation:

1. Reduction in the height of antenna
2. Avoids mixing of signals
3. Increases the range of communication
4. Multiplexing is possible
5. Improves quality of reception

MATHEMATICAL REPRESENTATION OF AM:

The method of varying amplitude of a high frequency carrier wave in accordance with theinformation to be transmitted, keeping the frequency and phase of the carrier wave unchanged is called Amplitude Modulation. The information is considered as the modulating signal and it is superimposed on the carrier wave by applying both of them to the modulator. The detailed diagram showing the amplitude modulation process is given.

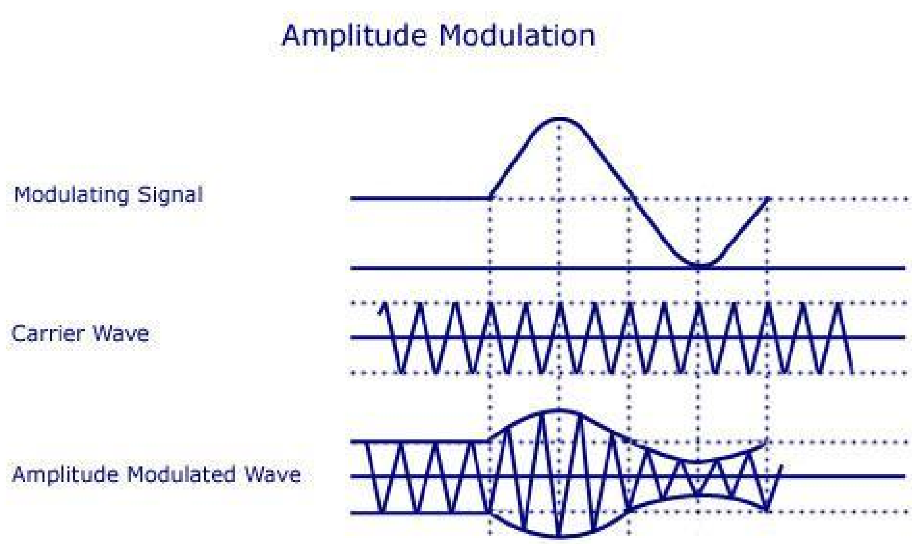


Fig 3. AM modulated wave

From the figure, the amplitude variation of the high frequency carrier is at the sig al frequency and the frequency of the carrierwave is the same as the frequency of the resulting wave.

MATHEMATICAL REPRESENTATION OF AM WAVE: TWO TITLES

Analysis of Amplitude Modulation Carrier Wave:

Let carrier signal be represented as $v_{c(t)} = V_c \text{ Sin } w_c t$ -------------- (1)
Let the message signal be represented as $v_m(t) = Vm \text{ Sin } w_m t$ -----(2)
v_c – Instantaneous value of the carrier V_c – Peak value of the carrier
w_c – Angular velocity of the carrier
v_m – Instantaneous value of the modulating signal
V_m – Maximum value of the modulating signal
w_m – Angular velocity of the modulating signal
f_m – Modulating signal frequency

The amplitude of modulated carrier waveis given by the equation
$A = V_c + v_m = V_c + V_m \text{ Sin } w_m t = V_c \, [1+ (\frac{Vm}{V_c \text{ Sin } W_m t})] = V_c \, (1 + m\text{Sin}w_m t)$

m – Modulation Index. The ratio of $\frac{Vm}{Vc}$.
Instantaneous value of amplitude modulated carrier wave is given by the equation
$Cm(t) = A \text{ Sin } w_c t = Vc \, (1 + m \text{ Sin } w_m t) \text{ Sin } wct$

$= V_c \text{ Sin } w_c t + mVc \, (\text{Sin } w_m t \text{ Sin } w_c t)$

$Cm(t) = V_c \text{ Sin } wct + [\frac{mVc}{2 \, Cos} \, (wc\text{-}wm)t – mVc/2 \text{ Cos } (wc + wm)t]$

The above equation represents the sum of three sine waves. One with amplitude of Vc and a frequency of $w_c/2$, the second one with an amplitude of $mV_c/2$ and frequency of $(w_c – w_m)/2$ and the third one with an amplitude of $mV_c/2$ and a frequency of $(w_c + w_m)/2$.

In practice the angular velocity of the carrier is known to be greater than the angular velocity of the modulating signal ($w_c >> w_m$). Thus, the second and third cosine equations are more close to the carrier frequency. The equation is represented graphically as shown below.

Frequency Spectrum of AM Wave:

Lower side frequency = $\frac{(wc – wm)}{2}$

Upper side frequency = $\frac{(wc + wm)}{2}$

The frequency components present in the AM wave are represented by vertical lines. Thus there will not be any change in the original frequency, but the side band frequencies $(w_c – w_m)/2$ and $(w_c + w_m)/2$ will be changed. The former is called the upper side band (USB)frequency and the later is known as lower side band (LSB) frequency.Since the signal frequency $w_m/2$ is present in the side bands, it is clear that the carrier voltage component does not transmit any information.Two side banded frequencies will be produced when a carrier is amplitude modulated by a singlefrequency. That is, an AM wave has a band width from $(w_c – w_m)/2$ to $(w_c + w_m)/2$, that is, $2w_m/2$ or twice the signal frequency is produced. When a modulating signal has more than one frequency, two side band frequencies are produced by every frequency. Similarly for twofrequencies of the modulating signal 2 LSB‘s and 2 USB‘s frequencies will be produced.

The side band frequencies present above the carrier frequency is known to be the upper side band and all those below the carrier frequency belong to the lower side band. The USB frequencies represent the some of the individual modulating frequencies and the LSB frequenciesrepresent the difference between the modulating frequency and the carrier frequency. The total bandwidth is represented in terms of the higher modulating frequency and is equal to twice this frequency.

Modulation Index (m):

The ratio between the amplitude change of carrier wave to the amplitude of the normal carrier wave is called modulation index. It is represented by the letter _m‘.

It can also be defined as the range in which the amplitude of the carrier wave is varied by the modulating signal.
m = $\frac{Vm}{Vc}$.

Percentage modulation, %m = m*100 = $\frac{Vm}{Vc}$* 100

Amplitude Modulated Carrier Wave:

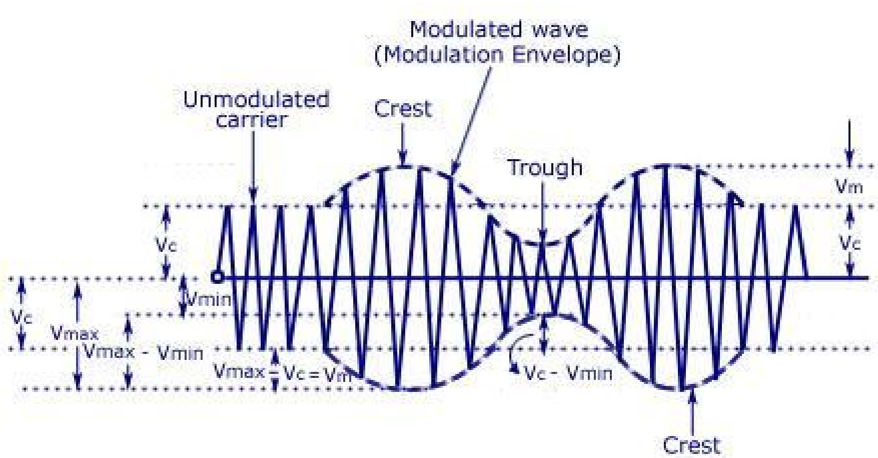


Fig.4 AM modulated wave

$$2\,Vin = Vmax - Vmin$$

$$V_{in} = \frac{(Vmax - Vmin)}{2}$$

$$Vc = Vmax - Vin$$

$$= V_{max} - \frac{(Vmax - Vmin)}{2}$$

$$= \frac{(Vmax + Vmin)}{2}$$

Substituting the values of Vm and Vc in the equation m = Vm/Vc , we get

$$M = \frac{Vmax - Vmin}{Vmax + Vmin}$$

As told earlier, the value of ‗m' lies between 0 and 0.8. The value of m determines the strength and the quality of the transmitted signal. In an AM wave, the signal is contained in the variations of the carrier amplitude. The audio signal transmitted will be weak if the carrier wave is only modulated to a very small degree. But if the value of m exceeds unity, the transmitter output produces erroneous distortion.

Power Relations in an AM wave:

A modulated wave has more power than had by the carrier wave before modulating. The total power components in amplitude modulation can be written as:

$$Ptotal = Pcarrier + PLSB + PUSB$$

Considering additional resistance like antenna resistance R.

$$P_{carrier} = [(V_c/\sqrt{2})/R]^2 = V^2{}_C/2R$$

Each side band has a value of $m/2\ V_c$ and r.m.s value of $mV_c/2\sqrt{2}$. Hence power in LSB and USBcan be written as

$$P_{LSB} = P_{USB} = (mV_c/2\sqrt{2})^2/R = m^2/4*V^2C/2R = m_2/4\ P_{carrier}$$

$$P_{total} = V^2{}_C/2R + [m^2/4*V^2C/2R] + [m^2/4*V^2C/2R] = V^2{}_C/2R\ (1 + m^2/2) = P_{carrier}\ (1 + m^2/2)$$

In some applications, the carrier is simultaneously modulated by several sinusoidal modulatingsignals. In such a case, the total modulation index is given as

$$Mt = \sqrt{}(m1^2 + m2^2 + m3^2 + m4^2 + \ldots..$$

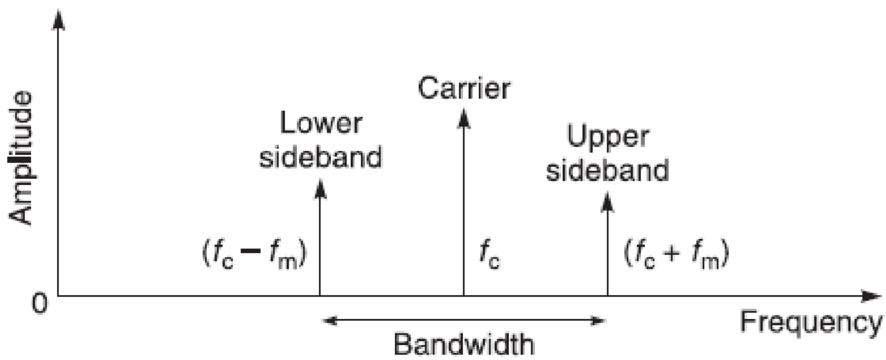
If Ic and It are the r.m.s values of unmodulated current and total modulated current and R is theresistance through which these current flow, then

$$\frac{Ptotal}{Pcarrier} = (\frac{It.R}{Ic.R})^2 = (\frac{It}{Ic})^2$$

$$\frac{Ptotal}{Pcarrier} = (1 + \frac{m2}{2})$$

$$\frac{It}{Ic} = 1 + \frac{m2}{2}$$

Frequency spectrum of AM:



B.W= 2fm

Fig.5 Frequency Spectrum of AM

Limitations of Amplitude Modulation:

2 Low Efficiency- Since the useful power that lies in the small bands is quite small, so the efficiency of AM system is low.

3 Limited Operating Range – The range of operation is small due to low efficiency. Thus, transmission of signals is difficult.

4 Noise in Reception – As the radio receiver finds it difficult to distinguish between the amplitude variations that represent noise and those with the signals, heavy noise is prone to occur in its reception.

4. Poor Audio Quality – To obtain high fidelity reception, all audio frequencies till 15 KiloHertz must be reproduced and this necessitates the bandwidth of 10 KiloHertz to minimise the interference from the adjacent broadcasting stations. Therefore in AM broadcasting stations audio quality is known to be poor.

AM TRANSMITTERS:

Transmitters that transmit AM signals are known as AM transmitters. These transmitters are used in medium wave (MW) and short wave (SW) frequency bands for AM broadcast. The MW band has frequencies between 550 KHz and 1650 KHz, and the SW band has frequencies ranging from 3 MHz to 30 MHz. The two types of AM transmitters that are used based on their transmitting powers are:

1. High Level

2. Low Level

High level transmitters use high level modulation, and low level transmitters use low levelmodulation. The choice between the two modulation schemes depends on the transmitting powerof the AM transmitter. In broadcast transmitters, where the transmitting power may be of the orderof kilowatts, high level modulation is employed. In low power transmitters, where only a fewwatts of transmitting power are required , low level modulation is used.

High-Level and Low-Level Transmitters Below figure's show the block diagram of high-level andlow-level transmitters. The basic difference between the two transmitters is the poweramplification of the carrier and modulating signals

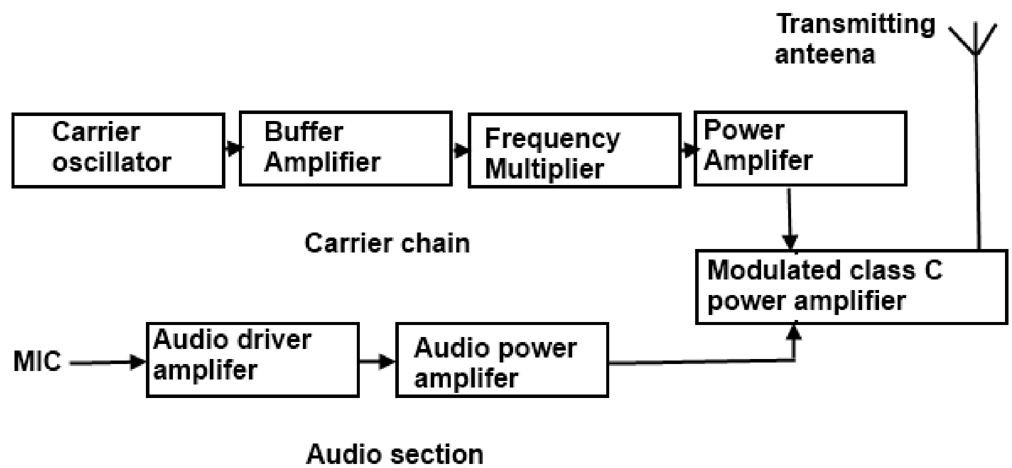


Fig.6 Block diagram of High level AM transmitter

In high-level transmission, the powers of the carrier and modulating signals are amplified before applying them to the modulator stage, as shown in figure (a). In low-level modulation, the powers of the two input signals of the modulator stage are not amplified. The required transmitting power is obtained from the last stage of the transmitter, the class C power amplifier.

The various sections of the figure (a) are:

- Carrier oscillator
- Buffer amplifier
- Frequency multiplier
- Power amplifier
- Audio chain
- Modulated class C power amplifier

1.**Carrier oscillator**

The carrier oscillator generates the carrier signal, which lies in the RF range. The frequency of thecarrier is always very high. Because it is very difficult to generate high frequencies with goodfrequency stability, the carrier oscillator generates a sub multiple with the required carrierfrequency. This sub multiple frequency is multiplied by the frequency multiplier stage to get therequired carrier frequency. Further, a crystal oscillator can be used in this stage to generate a lowfrequency carrier with the best frequency stability. The frequecy multiplier stage then increasesthe frequency of the carrier to its requirements.

2.**Buffer Amplifier**

The purpose of the buffer amplifier is twofold. It first matches the output impedance of the carrier oscillator with the input impedance of the frequency multiplier, the next stage of the carrier oscillator. It then isolates the carrier oscillator and frequency multiplier.

This is required so that the multiplier does not draw a large current from the carrier oscillator. If this occurs, the frequency of the carrier oscillator will not remain stable.

3.**Frequency Multiplier**

The sub-multiple frequency of the carrier signal, generated by the carrier oscillator , is now applied to the frequency multiplier through the buffer amplifier. This stage is also known as harmonic generator. The frequency multiplier generates higher harmonics of carrier oscillator frequency. The frequency multiplier is a tuned circuit that can be tuned to the requisite carrier frequency that is to be transmitted.

4.**Power Amplifier**

The power of the carrier signal is then amplified in the power amplifier stage. This is the basic requirement of a high-level transmitter. A class C power amplifier gives high power current pulses of the carrier signal at its output.

5.**Audio Chain**

The audio signal to be transmitted is obtained from the microphone, as shown in figure (a). The audio driver amplifier amplifies the voltage of this signal. This amplification is necessary to drive the audio power amplifier. Next, a class A or a class B power amplifier amplifies the power of the audio signal.

6.**Modulated Class C Amplifier**

This is the output stage of the transmitter. The modulating audio signal and the carrier signal, afterpower amplification, are applied to this modulating stage. The modulation takes place at thisstage. The class C amplifier also amplifies the power of the AM signal to the reacquiredtransmitting power. This signal is finally passed to the antenna., which radiates the signal intospace of transmission.

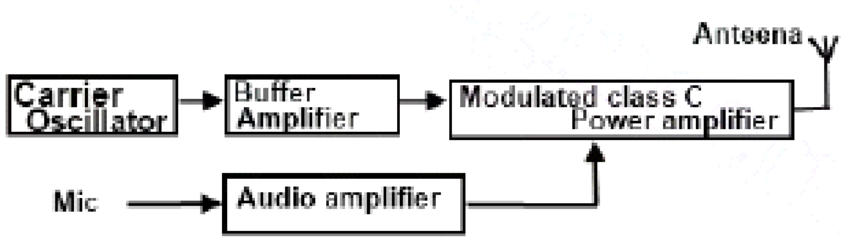


Fig.7 Low level AM transmitter

The low-level AM transmitter shown in above figure is similar to a high-level transmitter, except that the powers of the carrier and audio signals are not amplified. These two signals are directly applied to the modulated class C power amplifier.

Modulation takes place at the stage, and the power of the modulated signal is amplified to the required transmitting power level. The transmitting antenna then transmits the signal.

7.**Coupling of Output Stage and Antenna**

The output stage of the modulated class C power amplifier feeds the signal to the transmitting antenna. To transfer maximum power from the output stage to the antenna it is necessary that the impedance of the two sections match. For this , a matching network is required. The matching between the two should be perfect at all transmitting frequencies. As the matching is required at different frequencies, inductors and capacitors offering different impedance at different frequencies are used in the matching networks.

1. DSBSC MODULATION:

Double-sideband suppressed-carrier transmission (DSB-SC) is transmission in which frequencies produced by amplitude modulation (AM) are symmetrically spaced above and below the carrier frequency and the carrier level is reduced to the lowest practical level, ideally being completely suppressed.

DSB-SC is basically an amplitude modulation wave without the carrier, therefore reducing power waste, giving it a 50% efficiency. This is an increase compared to normal AM transmission (DSB), which has a maximum efficiency of 33.333%, since 2/3 of the power is in the carrier.

Generation of DSB – SC – AM

In DSB – SC, the transmitted wave consists of only upper and lowersidebands.Transmitted power is saved here through the suppression of the carrier wave becauseit does not contain any useful information, but the channel bandwidth required is thesame as before.

Expression for DSB –SC:

Let the modulating signal,

$V_m(t) = V_m \sin sin\, \omega_m t$

The carrier signal be,

$V_c(t) = V_c \sin sin\, \omega_c t$

$V_m(t)_{DSC-SC} = V_m(\mathrm{t}) V_c(\mathrm{t})$

$V_m(t)_{DSC-SC} = V_m \sin sin\, \omega_m t.\ V_c \sin sin\, \omega_c t$

$= V_m V_c \sin sin\, \omega_m t\ \ \sin sin\, \omega_c t$

$V_m(t)_{DSC-SC} = \frac{V_m V_c}{2} \lfloor \cos cos\, (\omega_c - \omega_m) t - \ \cos cos\, (\omega_c + \omega_m) t \rfloor$

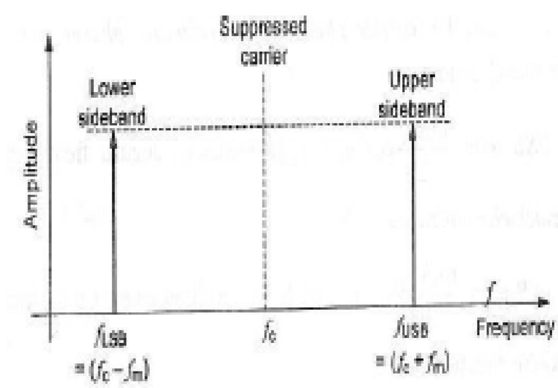


Fig 8. Frequency spectrum of DSBSC

There are two ways of generating **DSB–SC–AM**suchas ,

i) Balanced modulator,
ii) Ring modulator.

Balanced Modulator

This is the circuit that is very commonly used for **DSB–SC generation**. In balanced modulator, two non-linear devices are connected in the balanced mode, so as to suppress the carrier wave. It is assumed that the two transistors are identical and the circuit is symmetrical. Since the operation is confined in non-linear region of its transfer characteristics.

Modulating voltage across the two windings of a centre-tap transformer is equal,opposite in phase, i.e., Vm = V'm

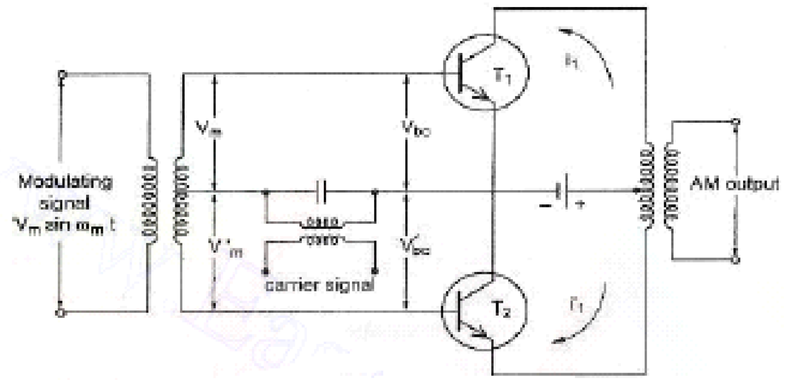


Fig.9 Balanced Modulator

Ring Modulator:

The one of the most popular method of generating a **DSB – SC** wave is ring modulator.
The circuit employs diodes as non-linear devices and the carrier signal is connected between centre taps of the input and output transformers. There is no need for a band pass filter at the output. The four diodes are controlled by a carrierVc(t) of frequency fc. The carrier signal acts as a switching signal to alternate the polarity of the modulating signals at the carrier frequency. For better understanding of the operation, assume that the modulating input is zero. Only carrier signal is present.

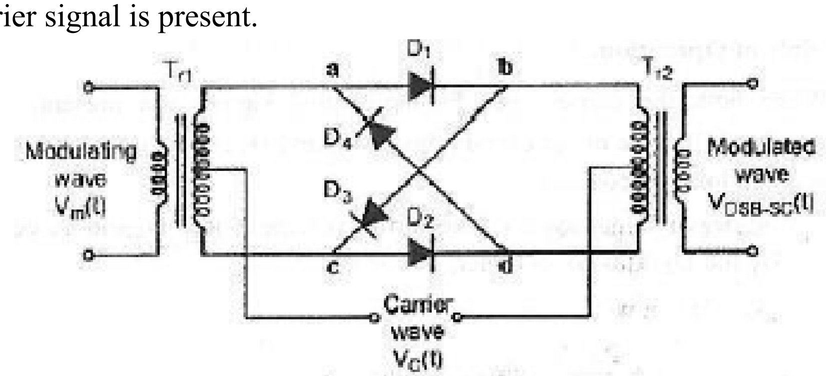


Fig.10 Ring modulator

Positive Half Cycle of Carrier:

Diodes D1 and D2 are form ward biased. AT this time D3 and D4 are reverse biased and act like open circuits. The current divides equally in the upper and lower portions of the primary winding of T12 .

The current in the upper part of the winding produces a magnetic field that is equal and opposite to the magnetic field produced by the current in the lower half of the secondary. Therefore, these magnetic fields cancel each other out and no output is induced in the secondary. Thus the carrier is effectively suppressed.

Negative Half Cycle of Carrier:

When the polarity of the carrier reverses. Diodes D1 and D2 are reverse biased and diodes D3 and D4 conduct. Again the current flows in the secondary winding of Tr1 and the primary winding of Tr2.

The equal and opposite magnetic fields produced in Tr2 cancel each other out and thus result in zero carrier output. The carrier is effectively balanced out.

Principle of Operation:

When both the carrier and the modulating signals are present, during positivehalf cycle of the carrier, diodes D1 and D2 conduct, while diodes D3 and D4 does notconduct.

During negative half cycle of the carrier voltage diodes D3 and D4 conduct andD1 and D2 does not conduct.

Phase Reversal

When polarity of the modulating signal changes, the result is a 180 ° phase reversal.

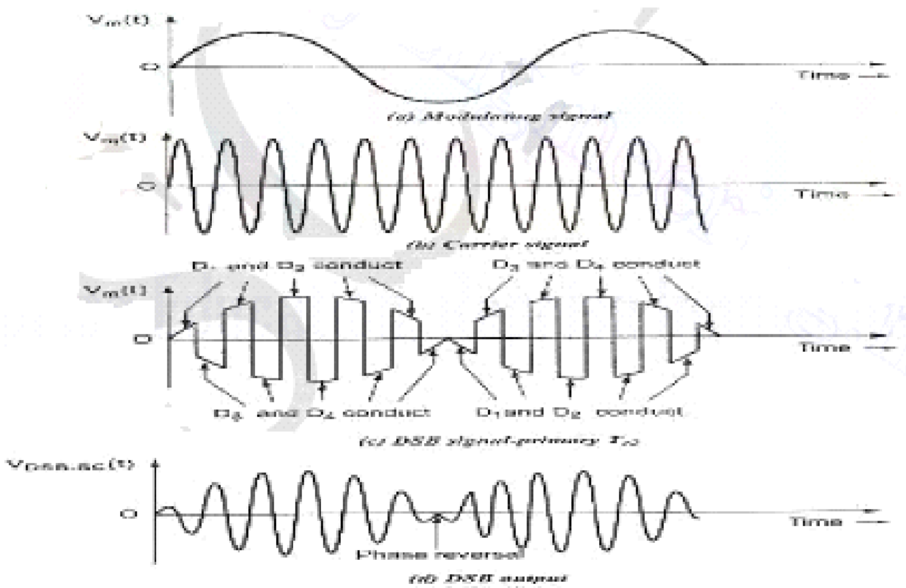


Fig.11 DSB output

At the time, during the positive half cycle of the carrier, diodes D3 and D4 are in forward bias and the negative half cycle of the carrier, diodes D1 and D2 are in reverse bias.

The ring modulator circuit is also known as double balanced modulator because comparing to balanced modulator here two more diodes are used.

Advantages:

1. DSB –SC is more efficient in transmitted power as compared to DSBFC .
2. DSB –SC has better signal to noise ratio as compared to single side band (SSB)transmission.

Disadvantage:

Even though the carrier is suppressed the bandwidth of DSBSC remains sameas DSBFC.

5.GENERATION OF AM USING A NON LINEAR MODULATOR CIRCUIT:

Non linear modulator circuits:

A simple diode can be used as a non linear modulator by restricting itsOperation to non linear operation of its characteristics.

The undesired frequency terms are filtered out by a band pass filter. The methods for generation of AM waves using non linear property are broadly divided into two types

(a) Square law modulator

(b)Balance modulator

(a)Square law modulators:

Any device operated in non linear region of its output characteristics is capable of producing amplitude modulated waves when the carrier and modulating signals are fed at the input. Thus a transistor, a triode tube,a diode etc. may be used as the square law modulator. A square law modulator circuit consists of the following:

i) A non linear device
ii) A bandpass filter
iii) A carrier source and modulating signal

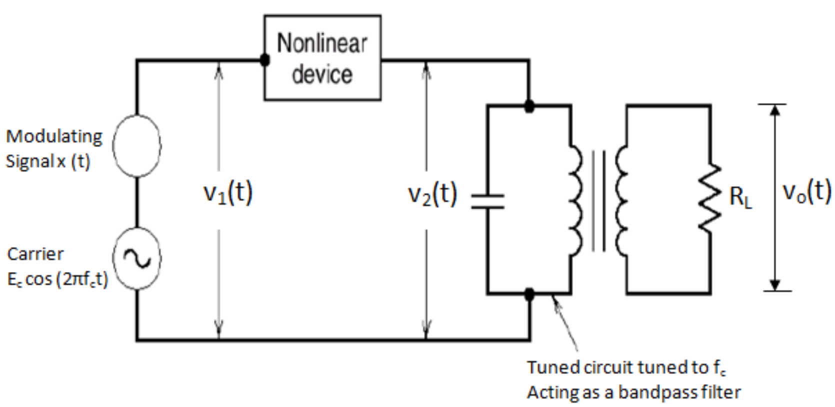


Fig12.Square law modulator.

Consider a non linear device to which a carrier ***c(t)*= *A*cos(2Π*fct*)** and an information signal *m(t)* are fed simultaneously as shown in figure12. The total input to the device at any instant is

$$V_{in} = c(t) + \ m(t)$$

$$V_{in} = A_c \cos cos\ 2\pi f_c t \ + \ m(t)$$

As the level of the input is smll very small, the output can be considered up to square of theinput

$V_{\circ} = a_{\circ} + a_1 V_{in} + a_2 V^2_{in}$

Taking Fourier transform on both sides we get

$$V_o(f) = \left(a_o + \frac{a_2 A_c}{2}\right)\delta(f) + \frac{a_1 A_c}{2}\left[\delta\left(f - f_c\right) + \delta\left(f + f_c\right)\right] + a_1 M(f) + \frac{a_2 A^2_c}{2}\left[\delta\left(f - 2f_c\right) + \delta\left(f + 2f_c\right)\right] + a_2 M$$

Therefore the square law device output Vo consists of the dc component at f=0 The information signal ranging from 0 to W Hz and its second harmonicsSignal at fc and 2fc Frequency band centered at fc with a deviation of ± W Hz.

The required AM signal with a carrier frequency fc can be separated using abandpass filter at the output of the square law device. The filter should have a lower cutoff frequency ranging between 2W and (fc - W) and upper cut-off frequency between (fc+W) and 2fc.
The output AM signal is free from distortion and attenuation only when(fc-W)2W or
fc<3W

6. DSBSC DEMODULATION – ENVELOPE DETECTOR

The most commonly used AM detector is simple diode detector as the shown in Fig. The AM signal at fixed IF is applied to the transformer primary. The signal at secondary is half wave rectified by diode D. This diode is the detector diode. The resistance R is load resistance to rectifier and C is the filter capacitor. In the positivehalf cycle of AM signal diode conducts and currents flows through R, whereas innegative half cycle, the diode is reverse biased and no current flows. Therefore onlypositive half of the AM wave appears across resistance R as shown in figure. Thecapacitor across capital R provides low impedance at the carrier frequency and muchhigher impedance at the modulating frequency. Therefore capacitor reconstructs theoriginal modulating signal shown in figure and high frequency carrier is removed.

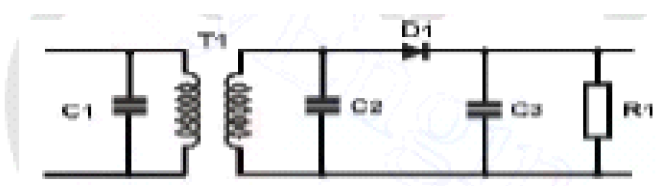


Fig13. Envelope detector

Negative peak clipping in diode detector:

This is the distortion that occurs in the output of diode detector because ofunequal *ac* and *dc* load impedances of the diode. The modulation index is defined as E***m*** /E***c.***

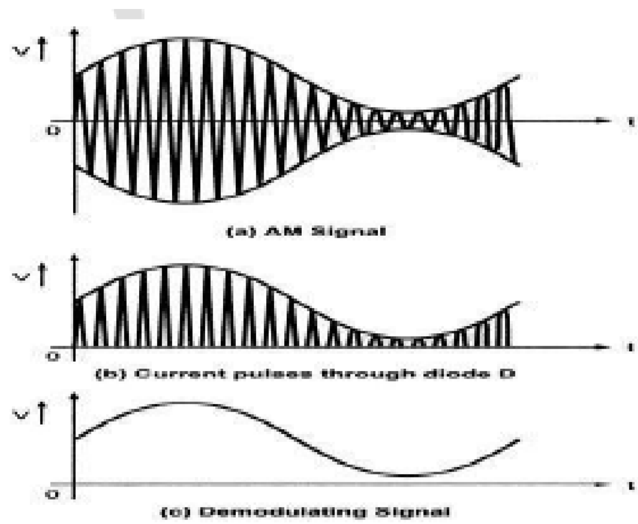
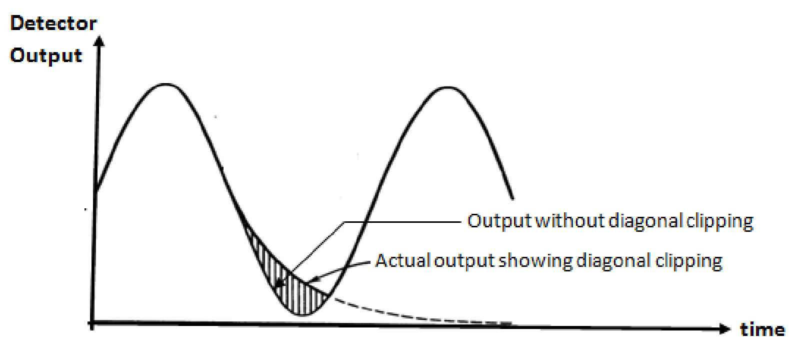


Fig.14 Waveform of negative peak clipping

The audio load resistance of the diode is smaller than the dc resistance. Hence the AF current I_m is larger, in proportion to dc current. This makes the modulation index in the demodulated wave relatively higher than that of modulated wave applied at the detector input. This introduces the distortion due to over modulation in the detector signal for modulation index near 100%. This is illustrated in Fig 14. In the figure observe that the negative peak of the detected signal takes place because of over modulation effect taking place in detector.

Diagonal Clipping in Diode Detector

As the modulating frequency is increased, the diode ac load impedance, Zm does not remain purely resistive. It does have reactive component also. At high modulation depths, the current changes so fast that the time constant of the



load does not follow the changes. Hence the

current decays slowly as shown in fig. The output voltage follo s the discharge law of RC circuit. This introduces distortion in the detected signaland it is called diagonal peak clipping.

Fig.15 Waveform of Diagonal clipping

7. SSB MODULATION

SSB – SC – AM waves can by generated in three ways.

1. Frequency discrimination (or) Filter method
2. Phase discrimination method.
3. Third method or Weaver's method

Filter Method:

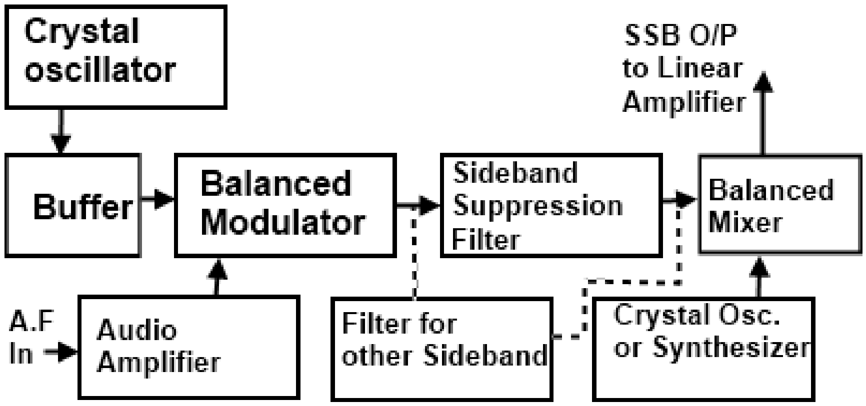


Fig.16 Filter method of SSB generation

Fig.16shows the block diagram of filter method to suppress one sideband. As shown in the block diagram, the balanced modulator produces DSB output. This DSB signal contains both the sideband. The filter must have a flat pass band and extremely high attenuation outside the pass band. In order to have this type of response the Q of the tuned circuits must be very high.

The required value of Q factor increases as the difference between modulating frequency and carrier frequency increases. Carrier frequency is usually same as the transmitter frequency increases. Carrier frequency is usually same as the transmitter frequency.

For higher transmitting frequencies required value of Q is so high that there is no practical way of achieving it. In such situation, initial modulation is carried out at a low frequency carrier say 100 kHz by the balanced modulator. Then the filter suppresses one of the sidebands. The frequency of the SSB signal generated at output of filter is very low as compared to the transmitter frequency.

The frequency is boosted up to the transmitter frequency by the balanced mixer and crystal oscillator. This process of frequency booting is also called as up conversion. The SSB signal having frequency equal to the transmitter frequency is then amplified by the linear amplifiers.

Phase Shift Method to Generate SSB

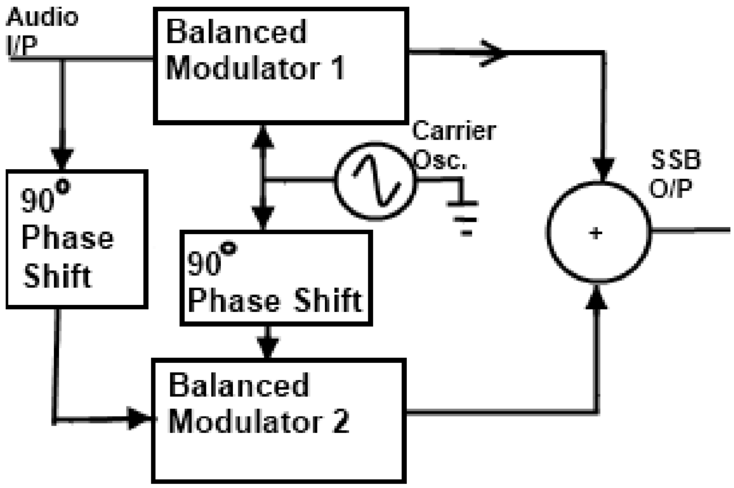


Fig.17 Phase Shift generator

Fig 17 shows the block diagram of phase shift method to generate SSB. The carriersignal is shifted by 90° and applied to the balanced modulator M1. The modulating signal isalso directly applied to the balanced modulator M2. The modulating signal is phase shiftedby 90° and applied to balanced modulator M2.

Both the modulators produce an outputconsisting of only sidebands. The upper balanced modulator (M1) generates uppersideband and lower sideband, but upper sideband is shifted by +90°whereas lowersideband is shifted by -90.The output of balanced modulators are added by the summingamplifier. Since upper sidebands of both the modulators are phase shifted by +900, they are in phase and add to produce double amplitude signal. But lower sideband of the balanced modulator are (+90° ,-90°) 180° out of phase and hence cancel each other.

Thus the output of summing amplifier contains only upper sideband SSB signal. The carrier is already suppressed by balanced modulators.

Weaver's Method:

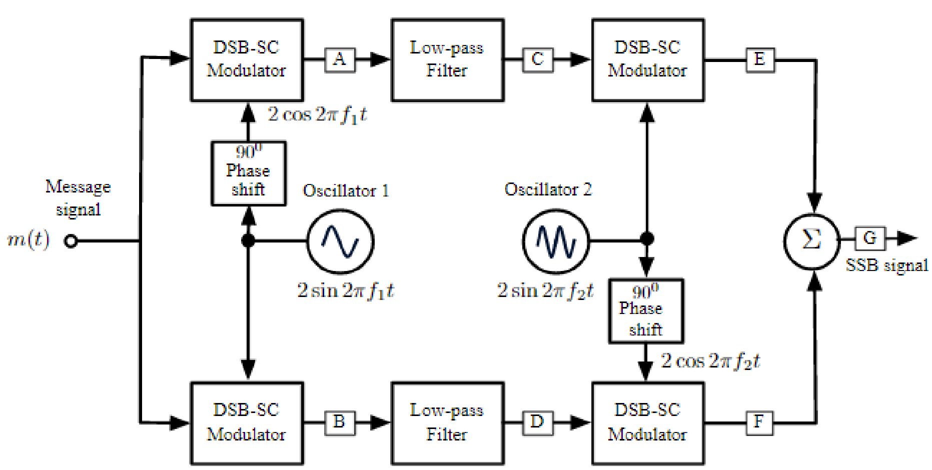


Fig 18. Weaver method

Principle: It uses two carriers: One is audio subcarrier at frequency fo and other is RFcarrier at frequency fc.The input to the modulator is s(t) = cos(2fmt). It is the modulating signal of frequency fm

Frequency spectrum of SSB:

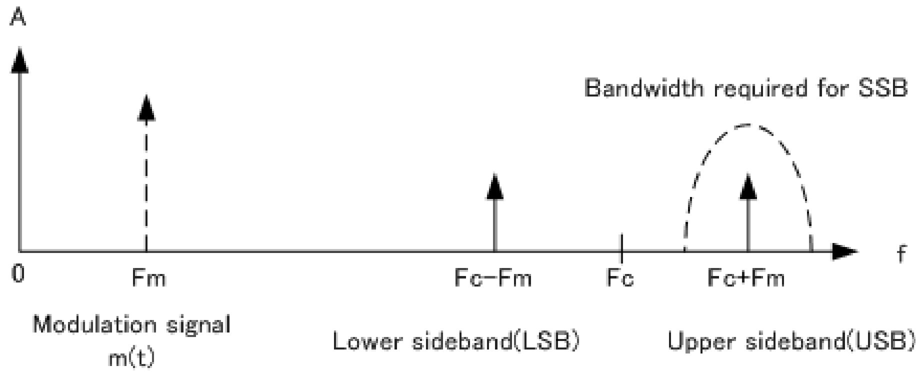


Fig.19 Frequency spectrum of SSB

Above shown is the frequency spectrum of SSB where only one sideband (USB) is retained. Here Bandwidth is fm.

8. SSB DEMODULATION

For demodulation, the same block diagram of a simple DSBSC demodulator can be used. The sideband at the positive and negative frequencies merge (recombine) at zero frequency when the SSB signal is multiplied by the carrier. (Try the exercise of finding the output of the DSBSC demodulator in time– and frequency–domain when its input is a either an USB or a LSB signal).

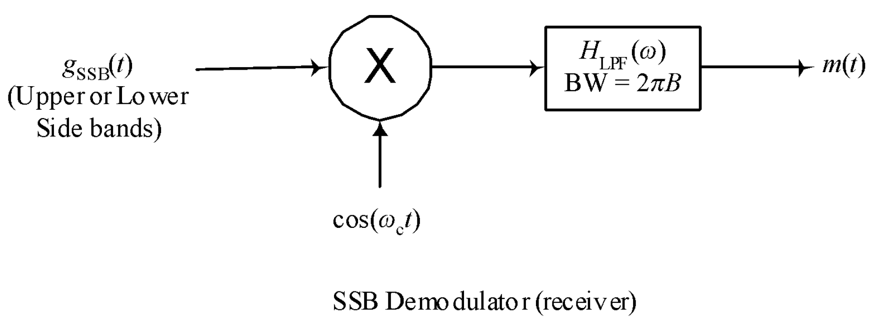


Fig.20 SSB demodulation

If the SSB signal includes a LARGE carrier, it can be demodulated using an envelope detector similar to that used for full AM signals

9. VSB MODULATION AND DEMODULATION

Generation of VSB signals:

A vestigial-sideband system is a comprom se between DSB and SSB It inheritsthe advantages of DSB and SSB but avoids the r SSB. It inherits the advantages ofDSB and SSB but avoids their disadvantages.

VSB signals are relatively easy to generate and their bandwdth is VSB signalsare relatively easy to generate and their bandwidth is only slightly (typically 25 percent)greater than that of SSB signals.

In VSB, instead of rejecting one sideband completely as in SSB, a gradual cut-off of one sideband is accepted. All of the one sideband is transmitted and a small amount (vestige) of the other sideband is transmitted The filter is allowed to have a nonzero transition band.

The roll-off characteristic of the filter is such that the partial suppression of the transmitted sideband in the neighborhood of the carrier is exactly compensated for by the partial transmission of the carrier is exactly compensated for by the partial transmission of the corresponding part of the suppressed sideband.

Our goal is to determine the particular H(f) required to produce a modulated signal s(t) with desired spectral characteristics such with desired spectral characteristics, such that the original baseband signal m(t) may be recovered from s(t) by coherent detection.

$$S(f) = U(f)H(f)$$

$$= \frac{A_c}{2}\lfloor M\left(f - f_c\right) + M\left(f + f_c\right)\rfloor H(f)$$

$$m(t) \xrightarrow{F} M(f)\ ,\ u(t) \xrightarrow{F} U(f)$$

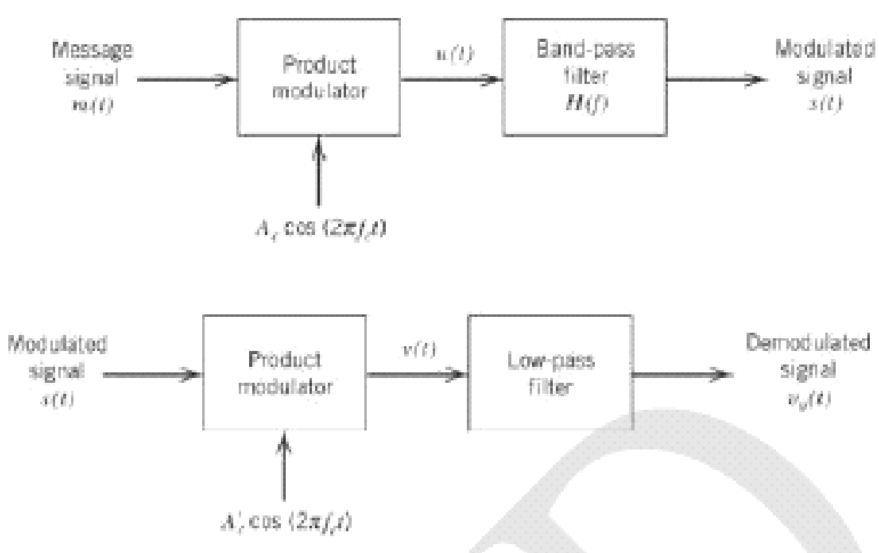


Fig.21Filtering scheme of generation of VSB modulated signal

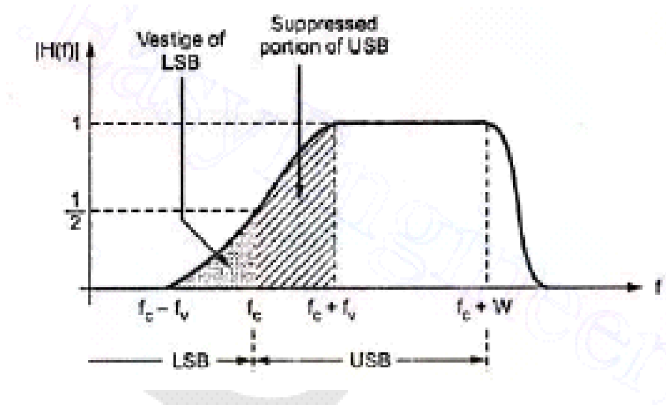


Fig22. Spectrum of VSB SC

Amplitude response of VSB filter

$$V(f) = \frac{A_c}{2}\left[S(f - f_c) + S(f + f_c)\right]$$

$$V(f) = \frac{A_c A_c'}{4} M(f)\left[H(f - f_c) + H(f + f_c)\right]$$

$$+ \frac{A_c A_c'}{4}\left[M(f - 2f_c)H(f - f_c) + M(f + 2f_c)H(f + f_c)\right]$$

Low-pass filter

$$V_0(f) = \frac{A_c A_c'}{4} M(f)\left[H(f - f_c) + H(f + f_c)\right]$$

To obtain baseband signal m(t) at coherent detector output, we require Vo(f) to be a scaled version of M(f),Therefore we can write

$$H\left(f - f_c\right) + H\left(f + f_c\right) = 1, \quad -W \le f \le W$$

Now equation(a) can be written as

$$V_o = \frac{A_c A_c'}{4} m(t)$$

1. COMPARISON OF AM MODULATION SYSTEMS

Description	AM with carrier	DSB – SC – AM	SSB – SC -AM	VSB - AM
Band width	2fm	2fm	fm	fm<BW<2fm
PowerSavingfor Sinusoid al	33.33%	66.66%	83.3%	75%

Power Saving for non - Sinusoidal	33.33%	50%	75%	75%
Generation methods	Easier to generate	Not difficult	More difficult to generate	Difficult.But easier to generate than SSB-SC
Detection methods	Simple& Inexpensive	Difficult	More difficult	Difficult

9.) SUPER HETERODYNE RECEIVER

In a broadcasting system whether it is based on amplitude modulation or frequency modulation, the receivers not only have the task of demodulating the modulated signal, but it is also required to perform some other system functions.

Carrier frequency tuning, the purpose of which is to select the desired signal (i.e.) desired radio or TV station) Filtering, which is required to separate the desire signal from other modulation signals that may be picked up along the way. Amplification, which is intended to compensate for the loss of signal power incurred in the course of transmission.

R.F Section

The incoming amplitude modulated wave is picked up by the receiving antenna and is fed to the RF section. The RF section consists of a pre selector and an RF Amplifier. The pre selector is a band pass filter with an adjustable centre frequency that is tuned to the desired carrier frequency of the incoming signal. The main use of the preselected is to provide sufficient band limiting to prevent undesired ratio in frequency signal or image signal. The effectiveness of suppressing unwanted image signals increases as the number of selective stages in the RF section increases and as the ratio of intermediate to signal frequency increases. R.F amplifiers are used for better selectivity.

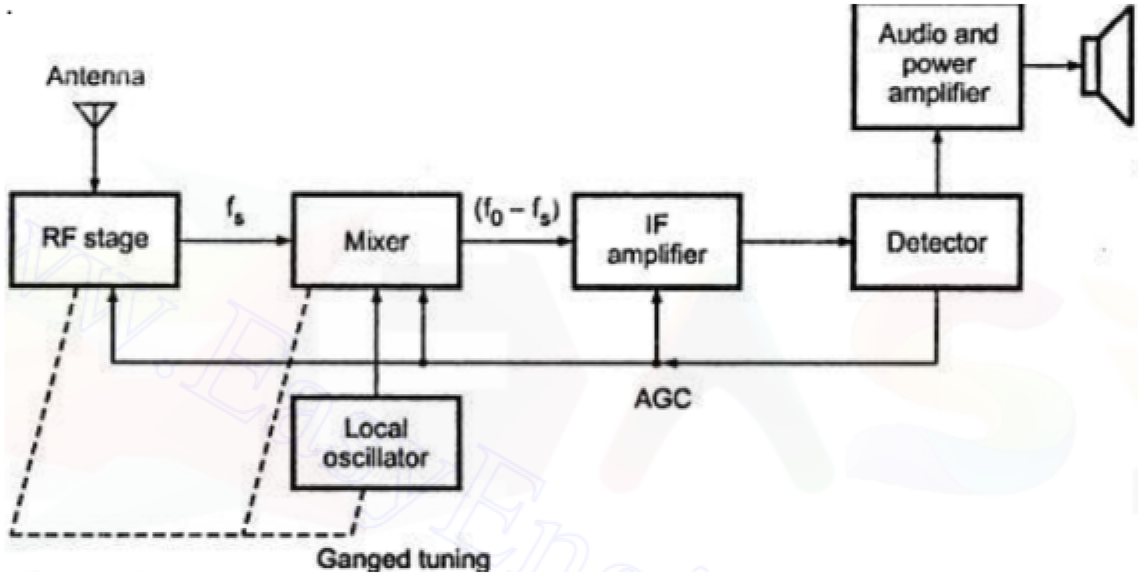


Fig23. Super Heterodyne Receiver

Frequency Changer

The combination of mixer and local oscillator provides a heterodyning functionwhereby the incoming signal is converted to a predetermined fixed intermediatefrequency, usually lower than the incoming carrier frequency. This frequency traslationis achieved without disturbing the relation of the sidebands to the carrier. The result of heterodyning is to produced an intermediate frequency carrier defined by fIF = flo –fRF

Where f_{LO} is the frequency of the local oscillator and fRF is the carrier frequency of the incoming RF signal. Since the output of the frequency his neither the original input frequency not the final baseband frequency, it is called as intermediate frequency. Sometimes the frequency changer circuits are referred to as the first detector, in which case the demodulator i s called as second detector.

IF Section

The IF section consists of one or more stages of turned amplification witha bandwidth corresponding to that required for the particular type of modulation that the receiver intended to handle. The IF section provides most of the amplification purpose of which is to recover the baseband or message signal.

If coherent detection is used, then a coherent signal source must be provided in the receiver.

Audio Amplifiers

The final stage of the super heterodyne receiver consists of one or more audio amplifiers which is used for the power amplification of the recovered message signal.

Image frequency and its rejection ratio

In a super heterodyne receiver, the mixer will develop and intermediate frequency output when the input signal frequency is greater or less than the local oscillator frequency by an amount equal to intermediate frequency

Amplitude Limiter

The basic difference between AM and FM super heterodyne receiver lies in the use of an FM Demodulator such as limiter frequency discriminator. In FM system the message signal is transmitted by the instantaneous value of carrier signal & its amplitude remain constant. Therefore any variation of the carriers amplitude at the receiver input must result from noise or interference.

An amplitude limiter following the IF section is used to remove amplitude variations by clipping the modulated wave is rounded by a band pass filter that suppresses harmonics of the carrier frequency. Thus the filter output is again sinusoidal, with an amplitude that in practically independent ofthe carrier amplitude of the receiver input.

Performance Parameters of Receivers

The performance of Radio receiver is measured on the basic of its selectivity,sensitivity, fidelity and image frequency rejection selectivity.

Selectivity

The selectivity is the ability of the receiver to select a signal of a desiredfrequency while rejecting all others. The selectivity of the receiver is obtained partiallyby RF amplifier and mainly by IF amplifiers. The selectivity shows the attenuation thatthe receiver offers to signals at frequencies near to the one to which it is tuned. Fig.shows the typical selectivity curve of the receiver. The s l ctivity depends upon tunedLC circuits used in RF and IF stages, fr is the resonating (tuned) frequency and Q isquality factor of these LC Circuits, As shown in Fig. bandwidth should be narrow forbetter selectivity. Hence Q of the coil should be high.

Sensitivity

The ability of the receiver to pick up weak signals and simplify them is call d sensitivity. It is often defined in terms of the voltage that must be applied to the receiv r input terminals to give the standard output power, measured at the output terminals.

As the gain of the receiver is increased, sensitivity is also increased. The sensitivity is expressed in micro volts or decibels. Fig.shows the typical sensitivity curve of a receiver. As shown in the Fig., the sensitivity is decreased (i.e., voltage is increased) at high frequencies.

Fidelity

Fidelity is a measure of the ability of a communication system to produce at the output of the receiver, an exact replica of the original source information. This may also be defined as the degree to which the system accurately reproduces at the output,the**essential c**haracteristics of signals that are impressed upon the input

Signal to noise Ratio

Signal to noise Ratio may be defined as the ratio of signal power to noise power at the receiver output. A good receiver should have high signal to noise ratio (SNR) which indicates negligible noise present at the output.

Image Frequency Rejection

We know that local oscillator frequency is made higher than the signal frequency such that f0 – fs = fi . Here fi is IF. That is f0 = fs + fi. The IF stage passes only fi . If the frequency fi = fs +2fi appears at the input of the mixer, then the mixer will produce different frequency equal to fi . This is equal to IF. The frequency fsi is called image frequency and is defined as the signal frequency plus twice the IF. The image frequency is converted in the IF stage and it is also amplified by IF amplifiers. This is the effect of two stations being received simultaneously. The image frequency rejection is done by tuned circuit in the RF stage. It depends upon the selectivity of the RF stage. The image rejection should be done before the RF stage.

10.) **Hilbert Transform**

If every frequency components of a signal f(t) is shifted by (-π/2) the resultant signal fh (t) is the Hilbert transform of f(t).

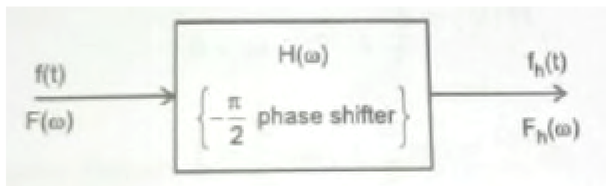


Fig.24 Phase shifting system

A signal f(t) is passed through a phase shift system H(ω) and the output fh (t) shown inabovefig.

The characteristics of the of the system specified as follows:

i) The magnitude frequency components present in f(t) remains unchanged when it ispassed through the system that is H(ω) = 1 and

ii) The phase of the positive frequency components if shifted by -π/2 . Since the phase spectrum θ (ω) has an odd symmetry, the phase of the negative frequency componentsis shifted by π/2. H(ω) and θ (ω) are plotted in Fig by continuous and dotted linesrespectively.

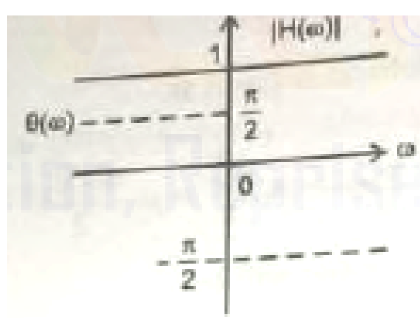


Fig 25.Transfer function of phase shifter

Properties of Hilbert transform

Hilbert transform has the following properties

1. A signal f(t) and its Hilbert transform have the same magnitude spectrum.

2 A signal f(t) and its Hilbert transform sh(t) have the same energy density spectrum.

3 If the Hilbert transform of fh(t) is –f(t), then fh(t) is Hilbert transform of s(t) that is if

$$H[f(t)] = f_H(\text{t})$$

$$H\left[f_H(t)\right] = -f(\text{t'})$$

Where H denotes the Hilbert transform.

2. A signal f(t) and its Hilbert transform fh(t) are mutually orthogonal over the time interval (-∞,∞) that is

$$\int_{-\infty}^{\infty} f(t)f_h(t)dt = 0$$

3. A signal f(t) and its Hilbert transform fh(t) have the same auto correlation function. Some useful Hilbert transform.

Application of Hilbert transform pair

1. Generation of SSB signal

2. Design of minimum phase type filters Representation of band pass signals

1. $\cos cos\, \omega_c t \; H\rightarrow \; \sin sin\, \omega_c t$

2. $\sin sin\, \omega_c t \quad H \quad \rightarrow \quad \cos cos\, \omega_c t$

3. $\sin sin\, (\omega_c t + \theta) \quad H \quad \rightarrow \quad \cos cos \left(\omega_c t\right) + \theta - \frac{\pi}{2}$

4. Let *m(t)*be a low pass signal with cutoff frequency W_1abd*c(t)* a high pass signal with lower cut off frequency $\omega_2 > W_1$. Then

$$m(t)c(t) \quad H \quad \rightarrow m(t)cn(t)$$

Pre envelope:

The pre envelope of a real signal x(t) is the complex function

$$x_+(t) = x(t) + j\,\hat{x}(t).$$

The pre envelope is useful in treating band pass signals and systems. This is due to the result

$$X_+(v) = \begin{cases} 2\,X(v) & , v > 0 \\ X(0) & , v = 0 \\ 0, & v < 0 \end{cases}$$

Complex envelope:

The complex envelope of a band pass signal x(t) is

$X'(t) = x_+(t)$

UNIT 2 ANGLE MODULATION

Angle modulation is a class of analog modulation. These techniques are based on altering the angle (or phase) of a sinusoidal carrier wave to transmit data, as opposed to varying the amplitude, such as in AMtransmission.

Angle Modulation is modulation in which the angle of a sine-wave carrier is varied by a modulating wave. Frequency Modulation (FM) and Phase Modulation (PM) are two types of angle modulation. In frequency modulation the modulating signal causes the carrier frequency to vary. These variations are controlled by both the frequency and the amplitude of the modulating wave. In phase modulation the phase of the carrier is controlled by the modulatingwaveform.

The two main types of angle modulation are:

- Frequency modulation (FM), with its digital correspondence frequency-shift keying(FSK).
- Phase modulation (PM), with its digital correspondence phase-shift keying(PSK).

FREQUENCY MODULATION:

Frequency modulation (FM): the encoding of information in a carrier wave by varying the instantaneous frequency of thewave.

Besides using the amplitude of carrier to carrier information, one can also use the angle of a carrier to carrier information. This approach is called angle modulation, and includes frequency modulation (FM) and phase modulation (PM). The amplitude of the carrier is maintained constant. The major advantage of this approach is that it allows the trade-off between bandwidth and noise performance.

An angle modulated signal can be written as

$$s(t) = A\cos\theta(t)$$

where $\theta(t)$ is usually of the form $\theta(t) = 2\pi f_c t + \varnothing(t)$ and fc is the carrier frequency. Thesignal $\varnothing(t)$ isderivedfromthemessagesignalm(t).If $\varnothing(t) = k_p m(t)$ for some constant k_p,the

resulting modulation is called phase modulation. The parameter$k_p$is called the phase SCE sensitivity.In telecommunications and signal processing, frequency modulation (FM) is the encoding of information in a carrier wave by varying the instantaneous frequency of the wave. (Compare with amplitude modulation, in which the amplitude of the carrier wave varies, while the frequency remains constant.) Frequency modulation is known as phase modulation when the carrier phase modulation is the time integral of the FM signal.

If the information to be transmitted (i.e., the baseband signal) is $x_m(t)$ and the sinusoidal carrier is $x_c(t) = A_c \cos(2\pi f_c t)$, where f_c is the carrier's base frequency, and A_c is the carrier's amplitude, the modulator combines the carrier with the baseband data signal to get the transmitted signal:

$$
\begin{aligned}
y(t) &= A_c \cos\left(2\pi \int_0^t f(\tau) d\tau\right) \\
&= A_c \cos\left(2\pi \int_0^t \left[f_c + f_\Delta x_m(\tau)\right] d\tau\right) \\
&= A_c \cos\left(2\pi f_c t + 2\pi f_\Delta \int_0^t x_m(\tau) d\tau\right)
\end{aligned}
$$

In this equation, $f(\tau)$ is the instantaneous frequency of the oscillator and f_Δ is the frequency deviation, which represents the maximum shift away from f_c in one direction, assuming $x_m(t)$ is limited to the range± 1.

While most of the energy of the signal is contained within $f_c \pm f_\Delta$, it can be shown by Fourier analysis that a wider range of frequencies is required to precisely represent an FM signal. The frequency spectrum of an actual FM signal has components extending infinitely, although their amplitude decreases and higher-order components are often neglected in practical design problems.

Sinusoidal baseband signal:

Mathematically, a baseband modulated signal may be approximated by a sinusoidal continuous wave signal with a frequency f_m.

The integral of such a signal is:

$$\int_0^t x_m(\tau) d\tau = \frac{A_m \cos(2\pi f_m t)}{2\pi f_m}$$

In this case, the expression for y(t) above simplifies to:

$$y(t) = A_c \cos\left(2\pi f_c t + \frac{f_\Delta}{f_m} \cos\left(2\pi f_m t\right)\right)$$

where the amplitude A_m of the modulating sinusoid is represented by the peakdeviation f_Δ

The harmonic distribution of a sine wave carrier modulated by such a sinusoidal signal can be represented with Bessel functions; this provides the basis for a mathematical understanding of frequency modulation in the frequency domain.

- ✔ **Modulationindex:**

As in other modulation systems, the value of the modulation index indicates by how much the modulated variable varies around its unmodulated level. It relates to variations in the carrier frequency:

$$h = \frac{\Delta f}{f_m} = \frac{f_\Delta |x_m(t)|}{f_m}$$

where f_m is the highest frequency component present in the modulating signal $x_m(t)$, and Δf is the peak frequency-deviation—i.e. the maximum deviation of the instantaneous frequency from the carrier frequency. For a sine wave modulation, the modulation index is seen to be the ratio of the amplitude of the modulating sine wave to the amplitude of the carrier wave (hereunity).

If $h \ll 1$, the modulation is called narrowband FM, and its bandwidth is approximately $2f_m$.

For digital modulation systems, for example Binary Frequency Shift Keying (BFSK), where a binary signal modulates the carrier, the modulation index is givenby:

$$h = \frac{\Delta f}{f_m} = \frac{\Delta f}{\frac{1}{2T_s}} = 2\Delta f T_s$$

where T_s is the symbol period, and $f_m = \frac{1}{2T_s}$ is used as the highest frequency of the modulating binary waveform by convention, even though it would be more accurate to say it isthe highest fundamental of the modulating binary waveform. In the case of digital modulation, the carrier f_c is never transmitted. Rather, one of two frequencies is transmitted,either $f_c + \Delta f$ or $f_c - \Delta f$, depending on the binary state 0 or 1 of the modulationsignal.

If $h \gg 1$, the modulation is called wideband FM and its bandwidth is approximately $2f_\Delta$. While wideband FM uses more bandwidth, it can improve the signal-to-noise ratiosignificantly; for example, doubling the value of Δf, while keeping f_m constant, results in an eight-fold improvement in the signal-to-noise ratio. (Compare this with Chirp spread spectrum, which uses extremely wide frequency deviations to achieve processing gains comparable to traditional, better- known spread-spectrummodes).

With a tone-modulated FM wave, if the modulation frequency is held constant and the modulation index is increased, the (non-negligible) bandwidth of the FM signal increases but the spacing between spectra remains the same; some spectral components decrease in strength as others increase. If the frequency deviation is held constant and the modulation frequency increased, the spacing between spectra increases.

Frequency modulation can be classified as narrowband if the change in the carrier frequency is about the same as the signal frequency, or as wideband if the change in the carrier frequency is much higher (modulation index >1) than the signal frequency. [6] For example, narrowband FM is used for two way radio systems such as Family Radio Service, in which the carrier is allowed to deviate only 2.5 kHz above and below the center frequency with speech signals of no more than 3.5 kHz bandwidth. Wideband FM is used for FM broadcasting, in which music and speech are transmitted with up to 75 kHz deviation from the center frequency and carry audio with up to a 20-kHz bandwidth.

Carson's rule:

$$BT = 2\Delta f + fm.$$

2.2 NARROW BAND FMMODULATION:

NarrowbandFM: If the modulation index of *FM* is kept under 1, then the *FM* produced is regarded as narrow band*FM*.

Thecasewhere$|\theta m(t)| \ll 1$foralltiscalled narrow band FM. Using the approximations cosx≃1andsinx ≃x for$|x| \ll 1$,theFMsignalcanbeapproximatedas:

$$s(t) = Ac\cos[\omega ct + \theta m(t)]$$

$$= Ac\cos\omega ct\cos\theta m(t) - Ac\sin\omega ct\sin\theta m(t)$$

$$\simeq Ac\cos\omega ct - Ac\theta m(t)\sin\omega ct$$

or in complex notation

$$st = A_C RE\{e^{jwct}(1 + j\theta mt\}$$

ThisissimilartotheAMsignalexceptthatthediscretecarriercomponentAcco$swc(t)$is90°out of phase with the sinusoid Ac sin$wc(t)$ multiplying the phase angle θm(t). The spectrum of narrow band FM is similar to that ofAM.

✔ **The Bandwidth of an FMSignal:**

The following formula, known as Carson‘s rule is often used as an estimate of the FM signal bandwidth: BT = 2(Δf + fm) Hz

whereΔf is the peak frequency deviation and fm is the maximum baseband message frequencycomponent.

✔ **FM Demodulation by a Frequency Discriminator:**

A frequency discriminator is a device that converts a received FM signal into a voltage that is proportional to the instantaneous frequency of its input without using a local oscillator and, consequently, in a non coherentmanner.

- When the instantaneous frequency changes slowly relative to the time-constantsofthe filter, a quasi-static analysis can beused.
- In quasi-static operation the filter output has the same instantaneous frequency as the input but with an envelope that varies according to the amplitude response of thefilter at the instantaneousfrequency.
- Theamplitudevariationsarethendetectedwithanenvelopedetectorliketheonesusedfor AMdemodulation.

✔ **An FM Discriminator Using the Pre-Envelope:**

When θm(t) is small and band-limited so that cosθm(t) and sinθm(t) are essentially band-limited signals with cut off frequencies less than fc, the pre-envelope of the FM signal is

$$s+(t) = s(t) + j\hat{s}(t) = Ac e^{j(\omega ct+\theta m(t))}$$

The angle of the pre-envelope is φ'(t) = arctan[ˆs(t)/s(t)] = ωct + θm(t)

The derivative of the phase is =ωct+ kθm(t)

$$\frac{d\varphi t}{dt} = \frac{s t \frac{d}{dt} \hat{s} t - \hat{s} \frac{d}{dt} s t}{s^2 t + \hat{s}^2(t)} = \omega ct + k\omega m(t)$$

which is exactly the instantaneous frequency. This can be approximated in discrete-time by using FIR filters to form the derivatives and Hilbert transform. Notice that the denominator is the squared envelope of the FM signal.

This formula can also be derived by observing,

$$\frac{d}{dt} s t = \frac{d}{dt} AC\cos\omega ct + \theta mt = -AC\omega ct + k\omega mt \sin[\omega ct + \theta mt]$$

So, $s\hat{}t = \frac{d}{dt} ACsin\omega ct + \theta mt = \frac{d}{dt} AC\omega ct + k\omega mtcos[\omega ct + \theta mt]$

$$stddts\hat{}(t) - \frac{s\hat{}(t)d}{tdtst} = AC^2\omega ct + k\omega mt * cos 2[wct + \theta mt + sin2[wct + \theta m$$

The bandwidth of an FM discriminator must be at least as great as that of the received FM signal which is usually much greater than that of the baseband message. This limits the degree of noise reduction that can be achieved by preceding the discriminator by a bandpass receive filter.

✔ **Using a Phase-Locked Loop for FMDemodulation:**

A device called a phase-locked loop (PLL) can be used to demodulate an FM signal with better performance in a noisy environment than a frequency discriminator. The block diagram of a discrete-time version of a PLL as shown in figure,

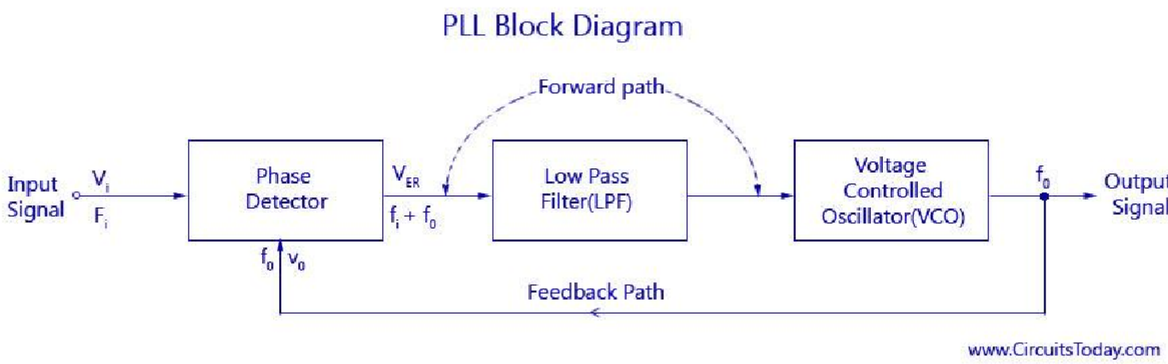


FIG 2.2 PLL Block diagram

The block diagram of a basic PLL is shown in the figure below. It is basically a flip flop consisting of a phase detector, a low pass filter (LPF),and a Voltage Controlled Oscillator (VCO) The input signal Vi with an input frequency fi is passed through a phase detector. A phase detector

basically a comparator which compares the input frequency fiwith the feedback frequency fo .The phase detector provides an output error voltage Ver (=fi+fo),which is a DC

voltage. This DC voltage is then passed on to an LPF. The LPF removes the high frequency noise and produces a steady DC level, Vf (=Fi-Fo). Vf also represents the dynamic characteristics of the PLL.

The DC level is then passed on to a VCO. The output frequency of the VCO (fo) is directly proportional to the input signal. Both the input frequency and output frequency are compared and adjusted through feedback loops until the output frequency equals the input frequency. Thus the PLL works in these stages – free-running, capture and phase lock.

As the name suggests, the free running stage refer to the stage when there is no input voltage applied. As soon as the input frequency is applied the VCO starts to change and begin producing an output frequency for comparison this stage is called the capture stage. The frequency comparison stops as soon as the output frequency is adjusted to become equal to the input frequency. This stage is called the phase locked state.

✔ **PLLPerformance:**

- The frequency response of the linearized loop characteristics of a band-limiteddifferentiator.
- The loop parameters must be chosen to provide a loop bandwidth that passes the desired baseband message signal but is as small as possible to suppress out-of-bandnoise.
- The PLL performs better than a frequency discriminator when the FM signal is corrupted by additive noise. The reason is that the bandwidth of the frequency discriminator must be large enough to pass the modulated FM signal while the PLL bandwidth only has to be large enough to pass the baseband message. With wideband FM, the bandwidth of the modulated signal can be significantly larger than that of the basebandmessage.

✔ **Bandwidth of FM PLL vs. CostasLoop:**

The PLL described in this experiment is very similar to the Costas loop presented in coherent demodulation of DSBSC-AM. However, the bandwidth of the PLL used for FM demodulation must be large enough to pass the baseband message signal, while the Costas loop is used to generate a stable carrier reference signal so its bandwidth should be very small and just wide enough to track carrier drift and allow a reasonable acquisition time.

2.3 WIDE-BAND FM:

$$st= ACcos(2\pi fct + \varphi(t)$$

Finding its FT is not easy: $\phi(t)$ is inside the cosine.

To analyze the spectrum, we use complex envelope.

s(t) can be written as: Consider single tone FM: $s(t) = ACcos(2\pi fct + \beta sin2\pi fm(t))$

Wideband FM is defined as the situation where the modulation index is above 0.5. Under these circumstances the sidebands beyond the first two terms are not insignificant. Broadcast FM stations use wideband FM, and using this mode they are able to take advantage of the wide bandwidth available to transmit high quality audio as well as other services like a stereo channel, and possibly other services as well on a single carrier.

The bandwidth of the FM transmission is a means of categorising the basic attributes for the signal, and as a result these terms are often seen in the technical literature associatedwith frequency modulation, and products using FM. This is one area where the figure for modulation index is used.

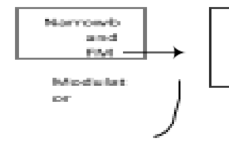
- ✔ **GENERATION OF WIDEBAND**

FMSIGNALS: Indirect Method for Wideband FM

Generation: Consider the following blockdiagram

m(*t*)

g *FM (WB)* (*t*)



$g_{FM\ (NB)}(t)$

Assume a BPF is included in this block to pass the signal with the highest carrier freuqnecy and reject all others

FIG 2.3 Block diagram of FM generation

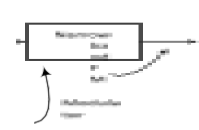
A narrowband FM signal can be generated easily using the block diagram of the narrowband FM modulator that was described in a previous lecture. The narrowband FM modulator generates a narrowband FM signal using simple components such as an integrator (an OpAmp), oscillators, multipliers, and adders. The generated narrowband FM signal can be converted to a wideband FM signal by simply passing it through a non–linear device with power P. Both the carrier frequency and the frequency deviation Δf of the narrowband signal are increased by a factor P. Sometimes, the desired increase in the carrier frequency and the desired increase in Δf are different. In this case, we increase Δf to the desired value and use a frequency shifter (multiplication by a sinusoid followed by a BPF) to change the carrier frequency to the desired value.

- ✔ **System1:**

Frequency Shifter

m(*t*) BW_m = 5 kHz

FM (NB)



$\Delta f_1 = 35$ Hz

$f_{c1} = 300$ kHz

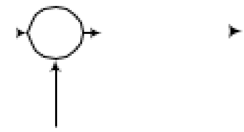
$g_{FM2\,(WB)}(t)$

$\Delta f_2 = 77$ kHz

$f_{c2} = 135$ MHz

$BW_2 = 2(\Delta f_2 + BW_m)$

$= 164$ kHz



BW = 2*5 = 10kHz B

FIG 2.4 Block diagram of FM generation

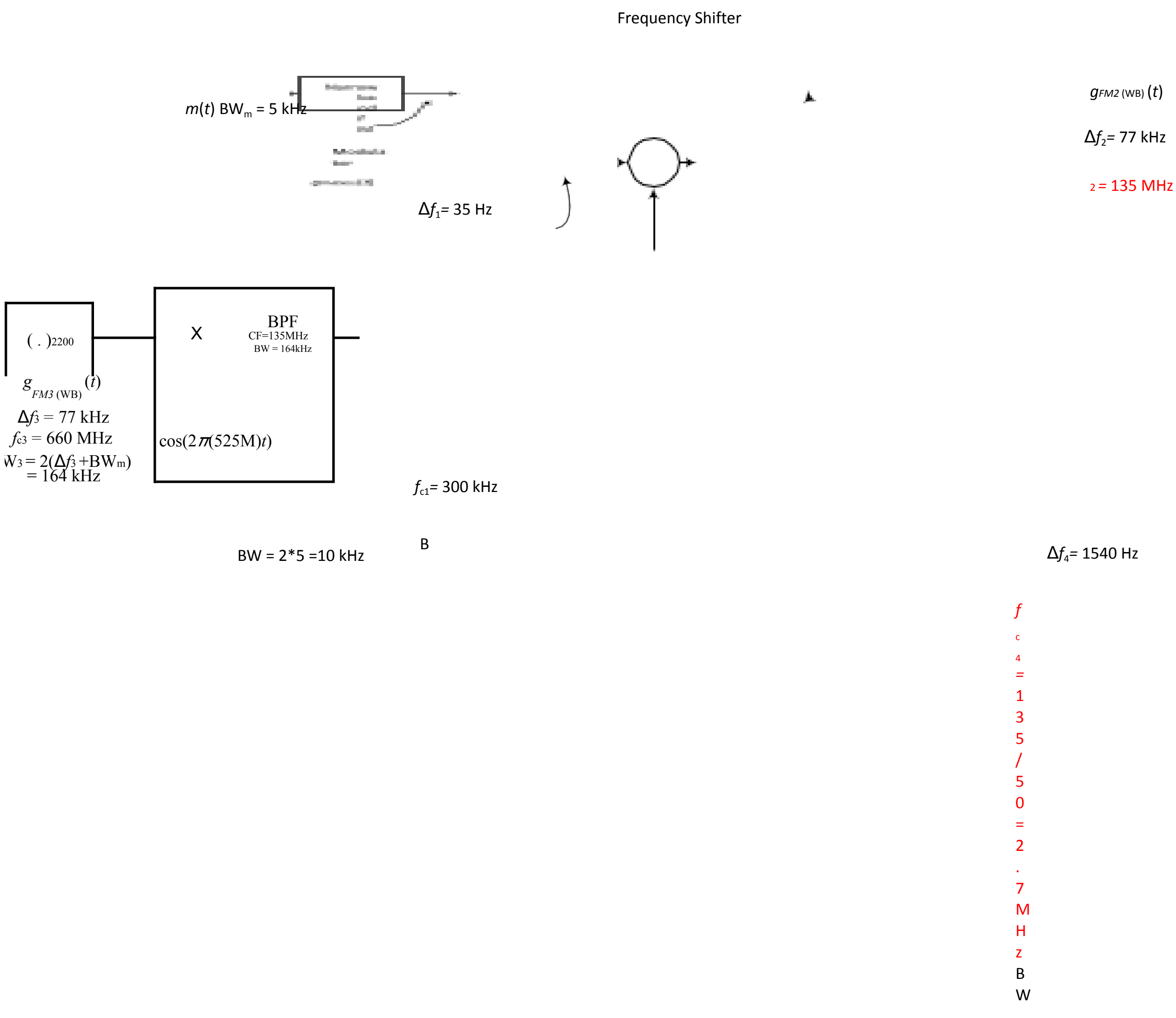
In this system, we are using a single non–linear device with an order of 2200 or multiple devices withacombinedorderof2200.Itisclearthattheoutputofthenon–lineardevicehasthecorrect

Δf but an incorrect carrier frequency which is corrected using a the frequency shifter with an

oscillator that has a frequency equal to the difference between the frequency of its input signaland

the desired carrier frequency. We could also have used an oscillator with a frequency that is the sum of the frequencies of the input signal and the desired carrier frequency. This system is characterized by having a frequency shifter with an oscillator frequency that is relatively large.

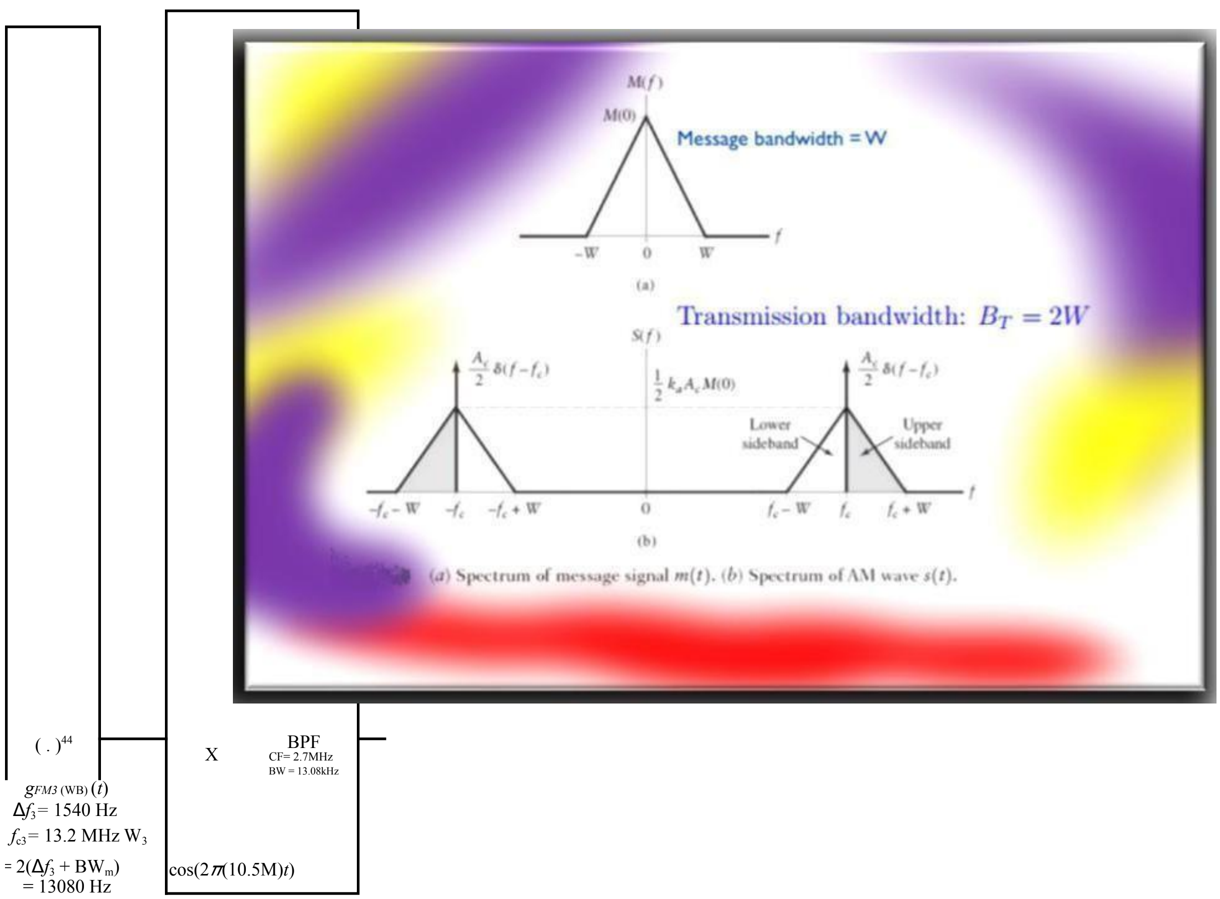
- ✔ **System2:**



$$= 2(\Delta f_4 + BW_m) = 130800 \text{ Hz}$$

FIG 2.5 Block diagram of FM generation

In this system, we are using two non–linear devices (or two sets of non–linear devices) with orders 44 and 50 (44*50 = 2200). There are other possibilities for the factorizing 2200 such as 2*1100,4*550,8*275,10*220.. Depending on the available components, one of these factorizations may be better than the others. In fact, in this case, we could have used the same factorization but put 50 first followed by 44. We want the output signal of the overall system to be as shown in the block diagram above, so we have to insure that the input to the non–linear device with order 50 has the correct carrier frequency such that its output has a carrier frequency of 135 MHz. This is done by dividing the desired output carrier frequency by the non–linearity order of 50, which gives 2.7 Mhz. This allows us to figure out the frequency of the require oscillator which will be in this case either 13.2–2.7 = 10.5 MHz or 13.2+2.7 = 15.9 MHz. We are generally free to choose which ever we like unless the available components dictate the use of one of them and not the other. Comparing this system with System 1 shows that the frequency of the oscillator that is required here is significantly lower (10.5 MHz compared to 525 MHz), which is generally an advantage.



2.4 TRANSMISSIONBANDWIDTH:

FIG 2.6 Spectrum of FM Bandwidth

2.5 FMTRANSMITTER

- ✔ **Indirect method (phase shift) of modulation**

The part of the Armstrong FM transmitter (Armstrong phase modulator) which is expressed in dotted lines describes the principle of operation of an Armstrong phase modulator. It should be noted, first that the output signal from the carrier oscillator is supplied to circuits that perform the task of modulating the carrier signal. The oscillator does not change frequency, as is the case of direct FM. These points out the major advantage of phase modulation (PM), or indirect FM, over direct FM. That is the phase modulator is crystal controlled for frequency.

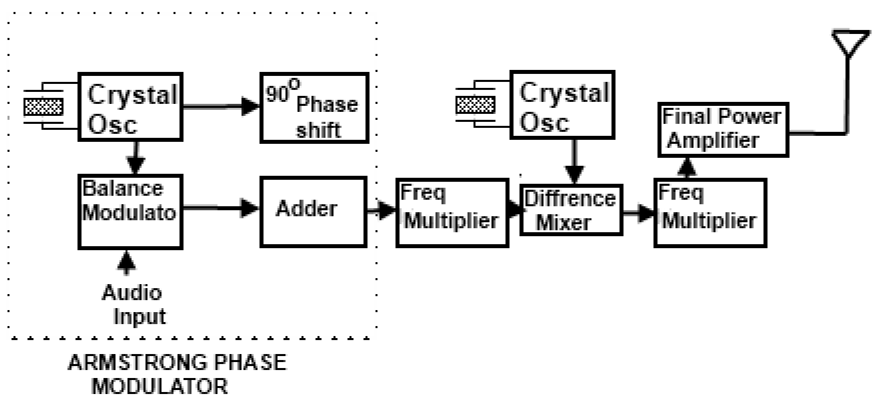


FIG 2.7 Armstrong Modulator

The crystal-controlled carrier oscillator signal is directed to two circuits in parallel. This signal (usually a sine wave) is established as the reference past carrier signal and is assigned a value 0°.The balanced modulator is an amplitude modulator used to form an envelope of double side-bands and to suppress the carrier signal (DSSC). This requires two input signals, the carrier signal and the modulating message signal. The output of the modulator is connected to the adder circuit; here the 90° phase-delayed carriers signal will be added back to replace the suppressed carrier. The act of delaying the carrier phase by 90° does not change the carrier frequency or its wave- shape. This signal identified as the 90° carriersignal.

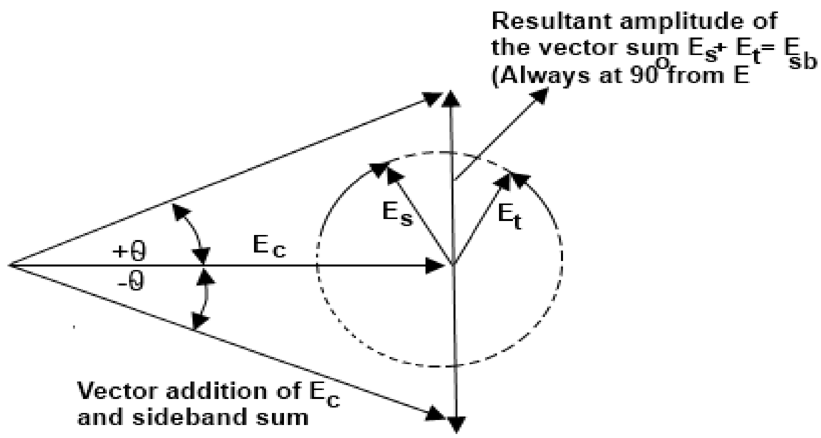


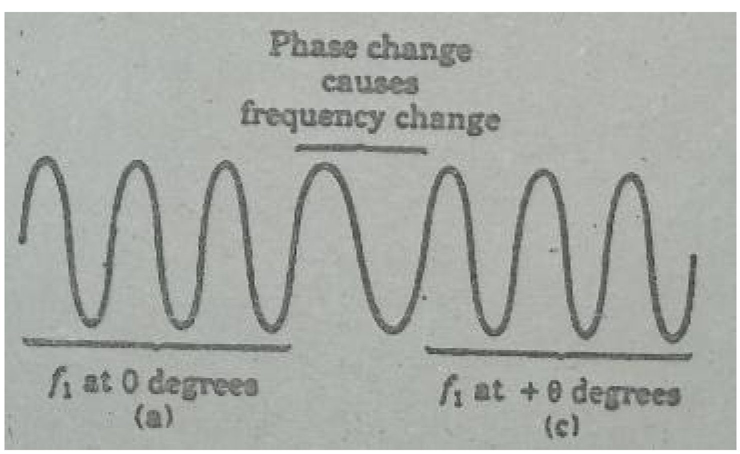
FIG 2.8 Phasor diagram of Armstrong Modulator

$$\% \text{ of modulation} = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} \times 100$$

The carrier frequency change at the adder output is a function of the output phase shift and is found by. $fc = \Delta\theta f_s$ (in hertz)

When θ is the phase change in radians and f_s is the lowest audio modulating frequency. In most FM radio bands, the lowest audio frequency is 50Hz. Therefore, the carrier frequency change at the adder output is 0.6125 x 50Hz = ± 30Hz since 10% AM represents the upper limit of carrier voltage change, then ± 30Hz is the maximum deviation from the modulator for PM.

The 90° phase shift network does not change the signal frequency because the components and resulting phase change are constant with time. However, the phase of the adder output voltage is in a continual state of change brought about by the cyclical variations of the message signal, and during the time of a phase change, there will also be a frequencychange.



In figure. (c). during time (a), the signal has a frequency f_1, and is at the zero reference phase.

During time (c), the signal has a frequency f_1 but has changed phase to θ. During time (b) when the phase is in the process of changing, from 0 to θ. the frequency is less than f_1.

✔ **Using Reactance modulator directmethod**

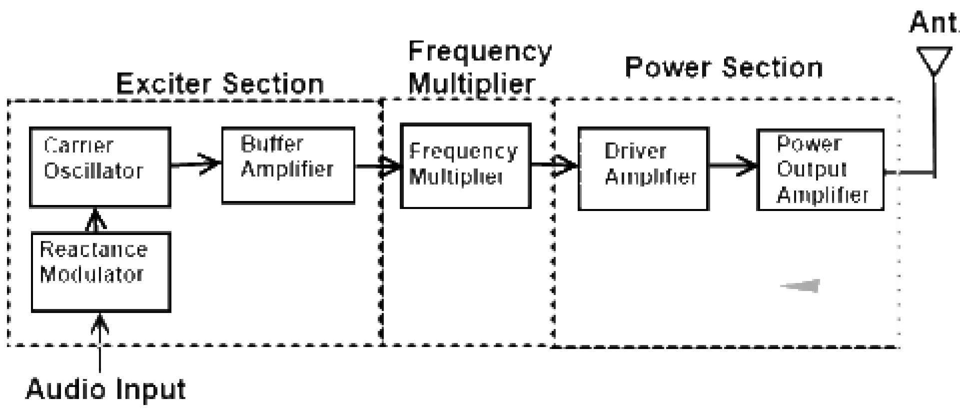


FIG 2.9 Reactance Modulator

The FM transmitter has three basic sections.

1. The exciter section contains the carrier oscillator, reactance modulator and thebuffer amplifier.
2. The frequency multiplier section, which features several frequencymultipliers.
3. The poweroutputection, which includes alow-level power amplifier, the final power amplifier, and the impedance matching network to properly load the power section with the antenna impedance.

The essential function of each circuit in the FM transmitter may be described as follows.

✔ **TheExciter**

1. The function of the carrier oscillator is to generate a stable sine wave signal at the rest frequency, when no modulation is applied. It must be able to linearly change frequency when fully modulated, with no measurable change inamplitude.
2. The buffer amplifier acts as a constant high-impedance load on the oscillator to help stabilize the oscillator frequency. The buffer amplifier may have a smallgain.
3. The modulator acts to change the carrier oscillator frequency by application of the message signal. The positive peak of the message signal generally lowers the oscillator's frequency to a point below the rest frequency, and the negativemessage peak raises the oscillator frequency to a value above the rest frequency. Thegreater the peak-to-peak message signal, the larger the oscillatordeviation.

✔ Frequency multipliers are tuned-input, tuned-output RF amplifiers in which the output resonant circuit is tuned to a multiple of the input frequency. Common frequency multipliers are 2x, 3x and 4x multiplication. A 5x Frequency multiplier is sometimes seen, but its extreme low efficiency forbids widespread usage. Note that multiplicationis by whole numbers only. There can not a 1.5x multiplier, forinstance.

- ✔ The final power section develops the carrier power, to be transmitted and often has a low-power amplifier driven the final power amplifier. The impedance matchingnetwork is the same as for the AM transmitter and matches the antenna impedance to the correct load on the final overamplifier.
- ✔ **FrequencyMultiplier**

A special form of class C amplifier is the frequency. multiplier. Any class C amplifier is capable of performing frequency multiplidàtion if the tuned circuit in the collector resonates at some integer multiple of the inputfrequency.

For example a frequency doubler can be constructed by simply connecting a parallel tuned circuit in the collector of a class C amplifier that resonates at twice the input frequency. When the collector current pulse occurs, it excites or rings the tuned circuit at twice the input frequency. A current pulse flows for every other cycle of the input.

A Tripler circuit is constructed in the same way except that the tuned circuit resonates at 3 times the input - frequency. In this way, the tuned circuit receives one input pulse for every three cycles of oscillation it produces Multipliers can be constructed to increase the input

frequency by any integer factor up to approximately 10. As' the multiplication factor gets higher, the power output of the multiplier decreases. For most practical applications, the best result is obtained with multipliers of 2 and 3.

Another way to look the operation of class C multipliers is .to .remember that the non-sinusoidal current pulse is rich in harmonics. Each time the pulse occurs, the second, third, fourth, fifth, and higher harmonics are generated. The purpose of the tuned circuit in the collector is to act as a filter to select the desired harmonics.

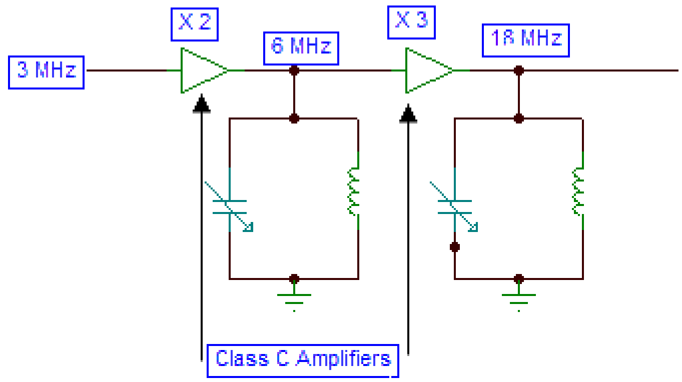


FIG 2.10 Block Diagram of Frequency Multiplier -1

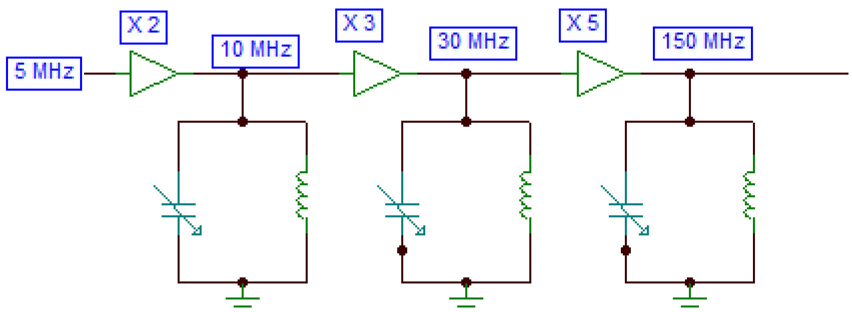


FIG 2.10 Block Diagram of Frequency Multiplier -2

In many applications a multiplication factor greater than that achievable with a single multiplier stage is required. In such cases two or more multipliers are cascaded to produce an overall multiplication of 6. In the second example, three multipliers provide an overall multiplication of 30. The total multiplication factor is simply the product of individual stage multiplicationfactors.

- ✔ **ReactanceModulator**

The reactance modulator takes its name from the fact that the impedance of the circuit acts as a reactance (capacitive or inductive) that is connected in parallel with the resonant circuit of the Oscillator. The varicap can only appear as a capacitance that becomes part of the frequency determining branch of the oscillator circuit. However, other discrete devices can appear as a capacitor or as an inductor to the oscillator, depending on how the circuit is arranged. A colpitts oscillator uses a capacitive voltage divider as the phase-reversing feedback path and would most likely tapped coil as the phase-reversing element in the feedback loop and most commonly uses a modulator that appears inductive.

2.6 COMPARISION OF VARIOUSMODULATIONS:

- ✔ **Comparisons of Various Modulations:**

Amplitude modulation	Frequency modulation	Phase modulation

1. Amplitude of the carrier wave is varied in accordance with the message signal.	1. Frequency of thecarrier wave is varied in accordance with the message signal.	1. Phase of the carrier wave is varied in accordance with the message signal.
2.Much affected by noise.	2.More immune to the noise.	2. Noise voltage is constant.
3.System fidelity is poor.	3.Improved system fidelity.	Improved system fidelity.
4.Linear modulation	4.Non Linear modulation	4.Non Linear modulation

✔ **Comparisons of Narrowband and WidebandFM:**

Narrowband FM	**Wideband FM**
Modulation index > 1.	Modulation index < 1.
Occupies more bandwidth.	Occupies less bandwidth.
Used in entertainment broadcastings	Used in FM Mobile communication services.

2.7 APPLICATION & ITS USES:

- Magnetic TapeStorage.
- Sound
- Noise FmReduction
- Frequency Modulation (FM) stereo decoders, FM Demodulation networks for FM operation.
- Frequency synthesis that provides multiple of a reference signalfrequency.
- Used in motor speed controls, trackingfilters.

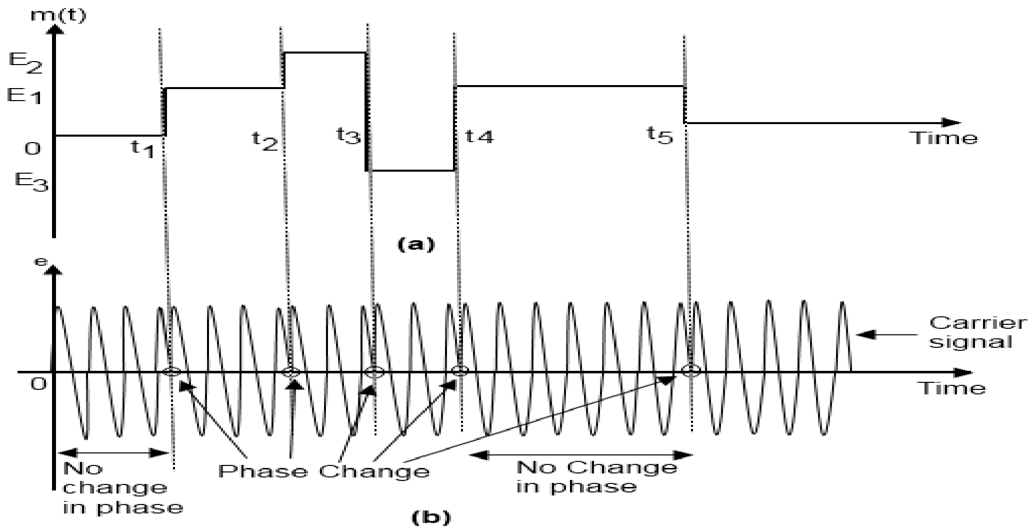
1. Frequency modulation(FM), with its digital correspondence frequency-shift keying(FSK).
2. Phase modulation (PM), with its digital correspondence phase-shift keying(PSK).
3. In PM, the total phase of the modulated carrier changes due to the changes in the instantaneous phase of the carrier keeping the frequency of the carrier signalconstant.
4. A device called a phase-locked loop (PLL) can be used to demodulate an FM signal with better performance in a noisy environment than a frequencydiscriminator.

5. As in other modulation systems, the value of the modulation index indicates by how much the modulated variable varies around its unmodulatedlevel.
6. Amplitude Limiters, are used to keep the output constant despite changes in the input signal to removedistortion.

2.8 PHASEMODULATION:

Phase Modulation (PM) is another form of angle modulation. PM and FM are closely related to each other. In both the cases, the total phase angle θ of the modulated signal varies. In an FM wave, the total phase changes due to the change in the frequency of the carrier corresponding to the changes in the modulating amplitude.

In PM, the total phase of the modulated carrier changes due to the changes in the instantaneous phase of the carrier keeping the frequency of the carrier signal constant. These two types of modulation schemes come under the category of angle modulation. However, PM is not as extensively used as FM.



Phase Modulation Waveforms

(a) Non-Sinusoidal Modulating Signal,m(t). (b) Phase-Modulated Carrier Signal e

At time t1, the amplitude of m(t) increases from zero to E1. Therefore, at t1, the phase modulated carrier also changes corresponding to E1, as shown in Figure (a). This phase remains to this attained value until time t2, as between t1 and t2, the amplitude of m(t) remains constant at El. At t2, the amplitude of m(t) shoots up to E2, and therefore the phase of the carrier again increases corresponding to the increase in m(t). This new value of the phase attained at time t2remains constant up to time t3. At time t3, m(t) goes negative and its amplitude becomes E3. Consequently, the phase of the carrier also changes and it decreases from the previous value attained at t2.

The decrease in phase corresponds to the decrease in amplitude of m(t). The phase of the carrier remains constant during the time interval between t3 and t4. At t4, m(t) goes positive to reach the amplitude El resulting in a corresponding increase in the phase of modulated carrier at time t4. Between t4 and t5, the phase remains constant. At t5 it decreases to the phase of the unmodulated carrier, as the amplitude of m(t) is zero beyondt5.

- ✔ **Equation of a PM Wave:**

To derive the equation of a PM wave, it is convenient to consider the modulating signal as a pure sinusoidal wave. The carrier signal is always a high frequency sinusoidal wave. Consider the modulating signal, em and the carrier signal ec, as given by, equation 1 and 2, respectively.

$$e_m = E_m \cos\omega_m t \text{------------}(1)$$

$$e_c = E_c \sin \omega_c t \text{--------------}(2)$$

The initial phases of the modulating signal and the carrier signal are ignored in Equations (1) and

(2) because they do not contribute to the modulation process due to their constant values. After PM, the phase of the carrier will not remain constant. It will vary according to the modulating signal em maintaining the amplitude and frequency as constants. Suppose, after PM, the equation of the carrier is represented as:

$$e = E_c \, \text{Sin}\, \theta \text{ ------------------(3)}$$

Where θ, is the instantaneous phase of the modulated carrier, and sinusoid ally varies in proportion to the modulating signal. Therefore, after PM, the instantaneous phase of the modulated carrier can be written as:

$$\theta = \omega_c \, t + K_p e_m \text{-------------------(4)}$$

Where, kp is the constant of proportionality for phase modulation. Substituting Equation (1) in Equation (4), yonget:

$$\theta = \omega_c \, t + K_p E_m \, \text{Cos}\, \omega_m \, t \text{ ---------------------(5)}$$

In Equation (5), the factor, kpEm is defined as the modulation index, and is given as:

$$m_p = K_p E_m \text{ ------------------------(6)}$$

where, the subscript p signifies; that mp is the modulation index of the PM wave. Therefore, equation (5) becomes

$$\theta = \omega_c \, t + m_p \, \text{Cos}\, \omega_m \, t \text{ ---------------------(7)}$$

Substituting Equation (7) and (3), you get:

$$e = E_c \sin (\omega_c t + m_p \cos \omega_m t) \text{ --------------------(8)}$$

Unit – II

Angle Modulation

Instantaneous Frequency

The frequency of a cosine function $x(t)$ that is given by

$$x(t) = \cos\left(\omega_c t + \theta_0\right)$$

is equal to ω_c since it is a constant with respect to t, and the phase of the cosine is the constant θ_0. The angle of the cosine $\theta(t) = \omega_c t + \theta_0$ is a linear relationship with respect to t (a straight line with slope of ω_c and y–intercept of θ_0). However, for other sinusoidal functions, the frequency may itself be a function of time, and therefore, we should not think in terms of the constant frequency of the sinusoid but in terms of the INSTANTANEOUS frequency of the sinusoid since it is not constant for all t. Consider for example the following sinusoid

$$y(t) = \cos\left[\theta(t)\right],$$

where $\theta(t)$ is a function of time. The frequency of $y(t)$ in this case depends on the function of $\theta(t)$ and may itself be a function of time. The instantaneous frequency of $y(t)$ given above is defined as

$$\omega_i(t) = \frac{d\theta(t)}{dt}.$$

As a checkup for this definition, we know that the instantaneous frequency of $x(t)$ is equal to its frequency at all times (since the instantaneous frequency for that function is constant) and is equal to ω_c. Clearly this satisfies the definition of the instantaneous frequency since $\theta(t) = \omega_c t + \theta_0$ and therefore $\omega_i(t) = \omega_c$.

If we know the instantaneous frequency of some sinusoid from $-\infty$ to sometime t, we can find the angle of that sinusoid at time t using

$$\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha)\,d\alpha.$$

Changing the angle $\theta(t)$ of some sinusoid is the bases for the two types of angle modulation: Phase and Frequency modulation techniques.

Phase Modulation (PM)

In this type of modulation, the phase of the carrier signal is directly changed by the message signal. The phase modulated signal will have the form

$$g_{PM}(t) = A \cdot \cos\left[\omega_c t + k_p m(t)\right],$$

where A is a constant, ω_c is the carrier frequency, $m(t)$ is the message signal, and k_p is a parameter that specifies how much change in the angle occurs for every unit of change of $m(t)$. The phase and instantaneous frequency of this signal are

$$\theta_{PM}(t) = \omega_c t + k_p m(t),$$
$$\omega_i(t) = \omega_c + k_p \frac{dm(t)}{dt} = \omega_c + k_p \dot{m}(t).$$

So, the frequency of a PM signal is proportional to the derivative of the message signal.

Frequency Modulation (FM)

This type of modulation changes the frequency of the carrier (not the phase as in PM) directly with the message signal. The FM modulated signal is

$$g_{FM}(t) = A \cdot \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha\right],$$

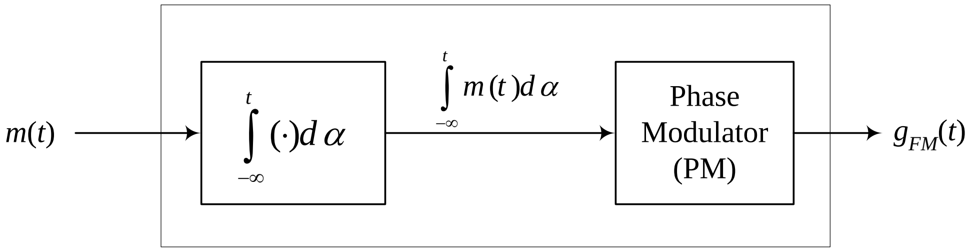
where k_f is a parameter that specifies how much change in the frequency occurs for every unit change of $m(t)$. The phase and instantaneous frequency of this FM are

$$\theta_{FM}(t) = \omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha,$$
$$\omega_i(t) = \omega_c + k_f \frac{d}{dt}\left[\int_{-\infty}^{t} m(\alpha) d\alpha\right] = \omega_c + k_f m(t).$$

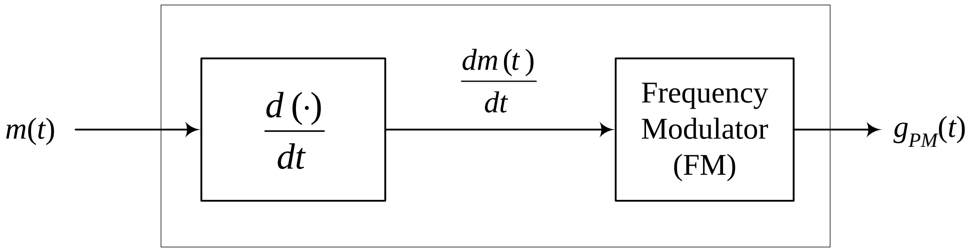
Relation between PM and FM

PM and FM are tightly related to each other. We see from the phase and frequency relations for PM and FM given above that replacing $m(t)$ in the PM signal with $\int_{-\infty}^{t} m(\alpha) d\alpha$ gives an FM signal and replacing $m(t)$ in the FM signal with $\frac{dm(t)}{dt}$ gives a PM signal. This is illustrated in the following block diagrams.

Frequency Modulator (FM)

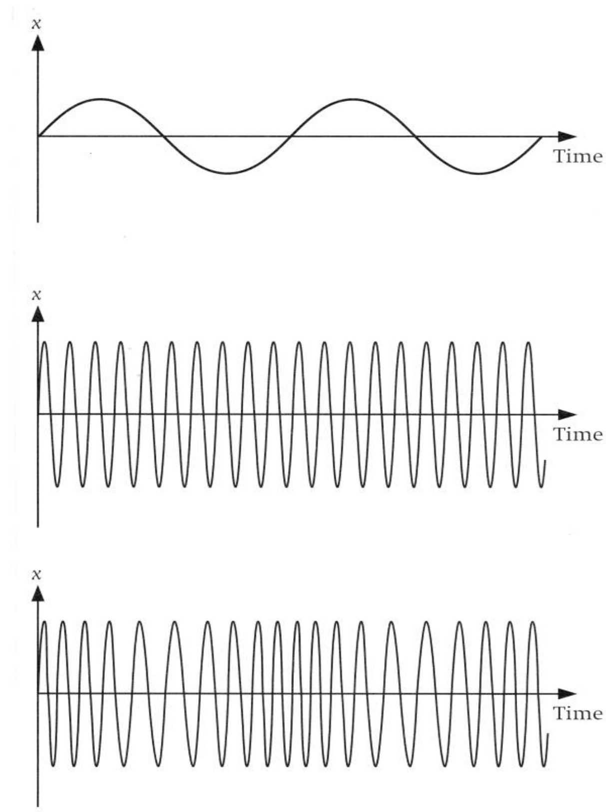


Phase Modulator (PM)



Frequency Modulation

In **F**requency **M**odulation (**FM**) the instantaneous value of the information signal controls the frequency of the carrier wave. This is illustrated in the following diagrams.



Notice that as the information signal increases, the frequency of the carrier increases, and as the information signal decreases, the frequency of the carrier decreases.

The frequency f_i of the information signal controls the rate at which the carrier frequency increases and decreases. As with AM, f_i must be less than f_c. The amplitude of the carrier remains constant throughout this process.

When the information voltage reaches its maximum value then the change in frequency of the carrier will have also reached its maximum deviation above the nominal value. Similarly when the information reaches a minimum the carrier will be at its lowest frequency below the nominal carrier frequency value. When the information signal is zero, then no deviation of the carrier will occur.

The maximum change that can occur to the carrier from its base value f_c is called the frequency deviation, and is given the symbol Δf_c. This sets the dynamic range (i.e. voltage range) of the transmission. The dynamic range is the ratio of the largest and smallest analogue information signals that can be transmitted.

Bandwidth of FM and PM Signals

The bandwidth of the different AM modulation techniques ranges from the bandwidth of the message signal (for SSB) to twice the bandwidth of the message signal (for DSBSC and Full AM). When FM signals were first proposed, it was thought that their bandwidth can be reduced to an arbitrarily small value. Compared to the bandwidth of different AM modulation techniques, this would in theory be a big advantage. It was assumed that a signal with an instantaneous frequency that changes over of range of Δf Hz would have a bandwidth of Δf Hz. When experiments were done, it was discovered that this was not the case. It was discovered that the bandwidth of FM signals for a specific message signal was at least equal to the bandwidth of the corresponding AM signal. In fact, FM signals can be classified into two types: Narrowband and Wideband FM signals depending on the bandwidth of each of these signals

Narrowband FM and PM

The general form of an FM signal that results when modulating a signals $m(t)$ is

$$g_{FM}(t) = A \cdot \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha) d\alpha\right].$$

A narrow band FM or PM signal satisfies the condition

$$\left|k_f a(t)\right| \ll 1$$

For FM and

$$\left| k_p \cdot m(t) \right| \ll 1$$

For PM, where

$$a(t) = \int_{-\infty}^{t} m(\alpha) d\alpha \,,$$

such that a change in the message signal does not results in a lot of change in the instantaneous frequency of the FM signal.

Now, we can write the above as

$$g_{FM}(t) = A \cdot \cos\left[\omega_c t + k_f a(t)\right].$$

Starting with FM, to evaluate the bandwidth of this signal, we need to expand it using a power series expansion. So, we will define a slightly different signal

$$\hat{g}_{FM}(t) = A \cdot e^{j\{\omega_c t + k_f a(t)\}} = A \cdot e^{j\omega_c t} \cdot e^{jk_f a(t)}.$$

Remember that

$$\hat{g}_{FM}(t) = A \cdot e^{j\{\omega_c t + k_f a(t)\})} = A \cdot \cos\left[\omega_c t + k_f a(t)\right] + jA \cdot \sin\left[\omega_c t + k_f a(t)\right],$$

so

$$g_{FM}(t) = \text{Re}\left\{\hat{g}_{FM}(t)\right\}.$$

Now we can expand the term $e^{jk_f a(t)}$ in $\hat{g}_{FM}(t)$, which gives

$$\begin{aligned}\hat{g}_{FM}(t) &= A \cdot e^{j\omega_c t} \cdot \left[1 + jk_f a(t) + \frac{j^2 k_f^2 a^2(t)}{2!} + \frac{j^3 k_f^3 a^3(t)}{3!} + \frac{j^4 k_f^4 a^4(t)}{4!} + \ldots\right] \\ &= A \cdot \left[e^{j\omega_c t} + jk_f a(t) e^{j\omega_c t} - \frac{k_f^2 a^2(t)}{2!} e^{j\omega_c t} - \frac{jk_f^3 a^3(t)}{3!} e^{j\omega_c t} + \frac{k_f^4 a^4(t)}{4!} e^{j\omega_c t} + \ldots\right]\end{aligned}$$

Since k_f and $a(t)$ are real ($a(t)$ is real because it is the integral of a real function $m(t)$), and since $\text{Re}\{e^{j\omega ct}\} = \cos(\omega_c t)$ and $\text{Re}\{ je^{j\omega ct}\} = -\sin(\omega_c t)$, then

$$g_{FM}(t) = \mathrm{Re}\{\hat{g}_{FM}(t)\}$$
$$= A \cdot \left[\cos(\omega_c t) - k_f a(t)\sin(\omega_c t) - \frac{k_f^{\,2} a^2(t)}{2!}\cos(\omega_c t) + \frac{k_f^{\,3} a^3(t)}{3!}\sin(\omega_c t) + \frac{k_f^{\,4} a^4(t)}{4!}\cos(\omega_c t) + \ldots \right]$$

The assumption we made for narrowband FM is ($|k_f a(t)| \ll 1$). This assumption will result in making all the terms with powers of $k_f a(t)$ greater than 1 to be small compared to the first two terms. So, the following is a reasonable approximation for $g_{FM}(t)$

$$\boxed{g_{FM\,(Narrowband)}(t) \approx A \cdot \left[\cos(\omega_c t) - k_f a(t)\sin(\omega_c t)\right]}, \qquad \text{when } |k_f a(t)| \ll 1.$$

It must be stressed that the above approximation is only valid for narrowband FM signals that satisfy the condition ($|k_f a(t)| \ll 1$). The above signal is simply the addition (or actually the subtraction) of a cosine (the carrier) with a DSBSC signal (but using a sine as the carrier). The message signal that modulates the DSBSC signal is not $m(t)$ but its integration $a(t)$. One of the properties of the Fourier transform informs us that the bandwidth of a signal $m(t)$ and its integration $a(t)$ (and its derivative too) are the same (verify this). Therefore, the bandwidth of the narrowband FM signal is

$$\boxed{BW_{FM\,(Narrowband)} = BW_{DSBSC} = 2 \cdot BW_{m(t)} =}.$$

We will see later that when the condition ($k_f \ll 1$) is not satisfied, the bandwidth of the FM signal becomes higher that twice the bandwidth of the message signal. Similar relationships hold for PM signals. That is

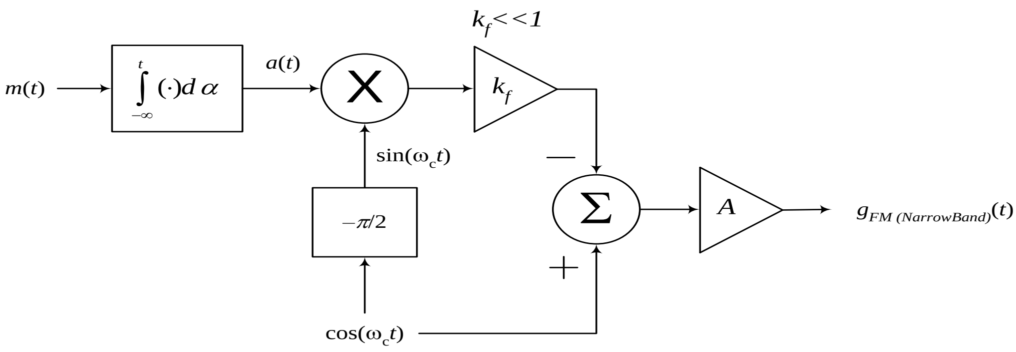
$$\boxed{g_{PM\,(Narrowband)}(t) \approx A \cdot \left[\cos(\omega_c t) - k_p m(t)\sin(\omega_c t)\right]}, \qquad \text{when } |k_p \cdot m(t)| \ll 1,$$

and

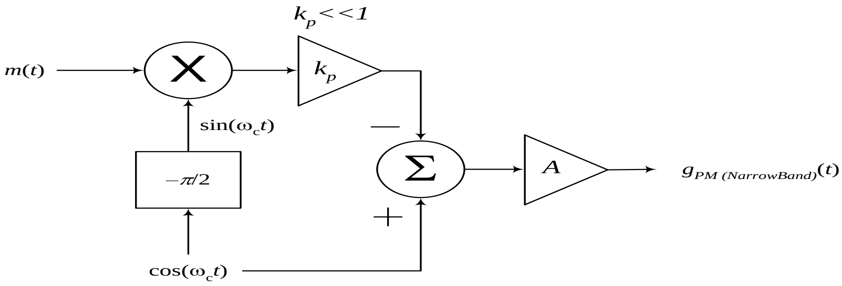
$$\boxed{BW_{PM\,(Narrowband)} = BW_{DSBSC} = 2 \cdot BW_{m(t)} =}.$$

Construction of Narrowband Frequency and Phase Modulators

The above approximations for narrowband FM and PM can be easily used to construct modulators for both types of signals



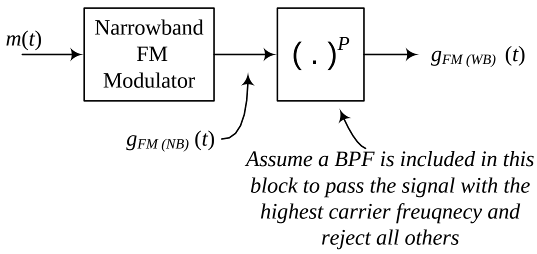
Narrowband FM Modulator



Narrowband PM Modulator

Generation of Wideband FM Signals

Consider the following block diagram



A narrowband FM signal can be generated easily using the block diagram of the narrowband FM modulator that was described in a previous lecture. The narrowband FM modulator generates a narrowband FM signal using simple components such as an integrator (an OpAmp), oscillators, multipliers, and adders. The generated narrowband FM signal can be converted to a wideband FM signal by simply passing it through a non–linear device with power *P*. Both the carrier frequency and the frequency deviation Δf of the narrowband signal are increased by a factor P. Sometimes, the desired increase in the carrier frequency and the desired increase in Δf are different. In this case, we increase Δf to the desired value and use a frequency shifter (multiplication by a sinusoid followed by a BPF) to change the carrier frequency to the desired value.

SINGLE-TONE FREQUENCY MODULATION

Time-Domain Expression

Since the FM wave is a nonlinear function of the modulating wave, the frequency modulation is a nonlinear process. The analysis of nonlinear process is the difficult task. In this section, we will study single-tone frequency modulation in detail to simplify the analysis and to get thorough understanding about FM.

Let us consider a single-tone sinusoidal message signal defined by

$$n(t) = A_n \cos(2\pi f_n t) \qquad (5.13)$$

The instantaneous frequency from Eq. (5.8) is then

$$f(t) = f_c + k_f A_n \cos(2\pi f_n t) = f_c + \Delta f \cos(2\pi f_n t) \qquad (5.14)$$

where

$$\Delta f = k_f A_n$$

$$\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t A_m \cos(2\pi f_m t) dt$$

$$= 2\pi f_c t + 2\pi k_f \frac{A_m}{2\pi f_m} \sin(2\pi f_m t)$$

$$= 2\pi f_c t + \frac{k_f A_m}{f_m} \sin(2\pi f_m t)$$

$$= 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)$$

$$\therefore \theta(t) = 2\pi f_c t + \beta_f \sin(2\pi f_m t)$$

Where

$$\beta_f = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}$$

is the modulation index of the FM wave. Therefore, the single-tone FM wave is expressed by

$$s_{FM}(t) = A_c \cos[2\pi f_c t + \beta_f \sin(2\pi f_m t)] \qquad (5.18)$$

This is the desired time-domain expression of the single-tone FM wave

Similarly, **single-tone phase modulated wave** may be determined from Eq.as

$$s_{PM}(t) = A_c \cos[2\pi f_c t + k_p A_m \cos(2\pi f_m t)]$$

$$\text{or, } s_{PM}(t) = A_c \cos[2\pi f_c t + \beta_p \cos(2\pi f_m t)] \qquad (5.19)$$

where

$$\beta_p = k_p A_m \qquad (5.20)$$

is the modulation index of the single-tone phase modulated wave.
The frequency deviation of the single-tone PM wave is

$$s_{FM}(t) = A_c \cos[2\pi f_c t + \beta_f \sin(2\pi f_m t)]$$

Spectral Analysis of Single-Tone FM Wave

The above Eq. can be rewritten as

$$s_{FM}(t) = \mathrm{Re}\{A_c e^{j2\pi f_c t} e^{j\beta \sin(2\pi f_m t)}\}$$

For simplicity, the modulation index of FM has been considered as β instead of β_f afterward. Since $\sin(2\pi f_m t)$ is periodic with fundamental period $T = 1/f_m$, the complex expontial $e^{j\beta \sin(2\pi f_m t)}$ is also periodic with the same fundamental period. Therefore, this complex exponential can be expanded in Fourier series representation as

$$e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_m t}$$

where the Fourier series coefficients c_n are obtained as

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} e^{j\beta\sin(2\pi f_m t)} e^{-j2\pi n f_m t} dt \qquad (5.24)$$

Let $2\pi f_m t = x$, then Eq. (5.24) reduces to

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\beta\sin(x)} e^{-jnx} dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin(x)-nx)} dx \qquad (5.25)$$

The integral on the right-hand side is known as the nth order Bessel function of the first kind and is denoted by $J_n(\beta)$. Therefore, $c_n = J_n(\beta)$ and Eq. (4.23) can be written as

$$e^{j\beta\sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t} \qquad (5.26)$$

By substituting Eq. (5.26) in Eq. (5.22), we get

$$\begin{aligned} s_{FM}(t) &= \mathrm{Re}\left\{A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}\, e^{j2\pi f_c t}\right\} \\ &= A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi(f_c + n f_m)t] \qquad (5.27) \end{aligned}$$

Taking Fourier transform of Eq. (5.27), we get

$$S(f) = \frac{1}{2}A_c \sum_{n=-\infty}^{\infty} J_n(\beta)[\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m)] \qquad (5.28)$$

From the spectral analysis we see that there is a carrier component and a number of side-frequencies around the carrier frequency at $\pm n f_m$.

The Bessel function may be expanded in a power series given by

$$J_n(\beta) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1}{2}\beta\right)^{n+2k}}{k!\,(k+n)!} \qquad (5.29)$$

Plots of Bessel function $J_n(\beta)$ versus modulation index β for $n = 0, 1, 2, 3, 4$ are shown in Figure 5.3.

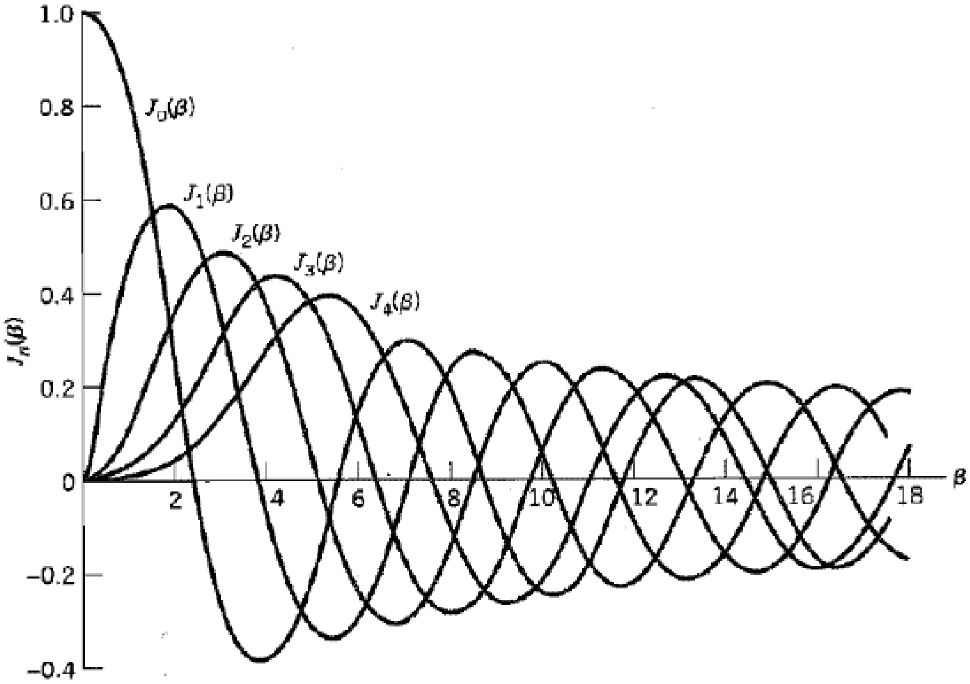


Figure 5.3 Plot of Bessel function as a function of modulation index.

Figure 5.3 shows that for any fixed value of n, the magnitude of $J_n(\beta)$ decreases as β increases. One property of Bessel function is that

$$J_{-n}(\beta) = \begin{cases} J_n(\beta), & n \text{ even} \\ -J_n(\beta), & n \text{ odd} \end{cases} \tag{5.30}$$

One more property of Bessel function is that

$$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1 \tag{5.31}$$

(iii) The average power of the FM wave remains constant. To prove this, let us determine the average power of Eq. (5.27) which is equal to

$$P = \frac{1}{2}A_c^2 \sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

Using Eq. (5.31), the average power P is now

$$P = \frac{1}{2}A_c^2$$

TRANSMISSION BANDWIDTH OF FM WAVE

The transmission bandwidth of an FM wave depends on the modulation index þ. The modulation index, on the other hand, depends on the modulating amplitude and modulating frequency. It is almost impossible to determine the exact bandwidth of the FM wave. Rather, we use a rule-of-thumb expression for determining the FM bandwidth.

For single-tone frequency modulation, the approximated bandwidth is determined by the expression

$$B = 2(\Delta f + f_m) = 2(\beta + 1)f_m = 2\Delta f\left(1 + \frac{1}{\beta}\right)$$

This expression is regarded as the Carson's rule. The FM bandwidth determined by this rule accommodates at least 98 % of the total power.

For an arbitrary message signal n(t) with bandwidth or maximum frequency W, the bandwidth of the corresponding FM wave may be determined by Carson's rule as

$$B = 2(\Delta f + W) = 2(D + 1)W = 2\Delta f\left(1 + \frac{1}{D}\right)$$

GENERATION OF FM WAVES

FM waves are normally generated by two methods: indirect method and direct method.

Indirect Method (Armstrong Method) of FM Generation

In this method, narrow-band FM wave is generated first by using phase modulator and then the wideband FM with desired frequency deviation is obtained by using frequency multipliers.

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right]$$

$$\text{or, } s(t) = A_c \cos[2\pi f_c t + \emptyset(t)]$$

$$\emptyset(t) = 2\pi k_f \int_0^t m(t)dt$$

$$s(t) = A_c \cos(2\pi f_c t)\cos[\emptyset(t)] - A_c \sin(2\pi f_c t)\sin[\emptyset(t)]$$

The above eq is the expression for narrow band FM wave

In this case

$$\cos[\emptyset(t)] \approx 1 \text{ and } \sin[\emptyset(t)] \approx \emptyset(t)$$

$$s(t) = A_c \cos(2\pi f_c t) - A_c \sin(2\pi f_c t)\emptyset(t)$$

$$\text{or}, s(t) = A_c \cos(2\pi f_c t) - 2\pi A_c k_f \sin(2\pi f_c t) \int_0^t m(t)dt$$

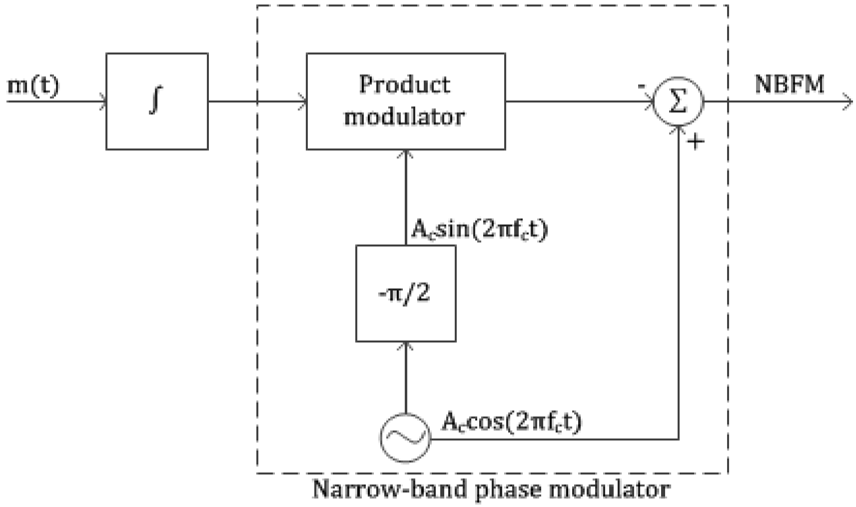


Fig: Narrowband FM Generator

The frequency deviation Δf is very small in narrow-band FM wave. To produce wideband FM, we have to increase the value of Δf to a desired level. This is achieved by means of one or multiple frequency multipliers. A frequency multiplier consists of a nonlinear device and a bandpass filter. The n^{th} order nonlinear device produces a dc component and n number of frequency modulated waves with carrier frequencies f_c, $2f_c$, … nf_c and frequency deviations Δf, $2\Delta f$, … $n\Delta f$, respectively. If we want an FM wave with frequency deviation of $6\Delta f$, then we may use a 6^{th} order nonlinear device or one 2^{nd} order and one 3^{rd} order nonlinear devices in cascade followed by a bandpass filter centered at $6f_c$. Normally, we may require very high value of frequency deviation. This automatically increases the carrier frequency by the same factor which may be higher than the required carrier frequency. We may shift the carrier frequency to the desired level by using mixer which does not change the frequency deviation.

The narrowband FM has some distortion due to the approximation made in deriving the expression of narrowband FM from the general expression. This produces some amplitude modulation in the narrowband FM which is removed by using a limiter in frequency multiplier.

Direct Method of FM Generation

In this method, the instantaneous frequency f(t) of the carrier signal c(t) is varied directly with the instantaneous value of the modulating signal n(t). For this, an oscillator is used in which any one of the reactive components (either C or L) of the resonant network of the oscillator is varied linearly with n(t). We can use a varactor diode or a varicap as a voltage-variable capacitor whose capacitance solely depends on the reverse-bias voltage applied across it. To vary such capacitance linearly with n(t), we have to reverse-bias the diode by the fixed DC voltage and operate within a small linear portion of the capacitance-voltage characteristic curve. The unmodulated fixed capacitance C_0 is linearly varied by n(t) such that the resultant capacitance becomes

$$C(t) = C_0 - kn(t)$$

where the constant k is the sensitivity of the varactor diode (measured in capacitance per volt).

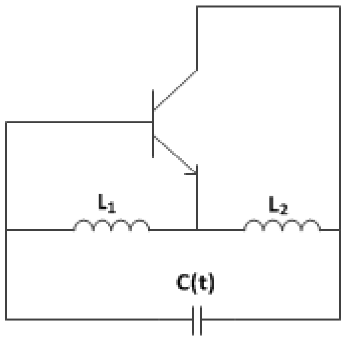


fig: Hartley oscillator for FM generation

The above figure shows the simplified diagram of the Hartley oscillator in which is implemented the above discussed scheme. The frequency of oscillation for such an oscillator is given

$$f(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)C(t)}}$$

$$f(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)(C_0 - km(t))}}$$

$$= \frac{1}{2\pi\sqrt{(L_1 + L_2)C_0}} \frac{1}{\sqrt{1 - \frac{km(t)}{C_0}}}$$

$$\text{or,} \qquad f(t) = f_c\left(1 - \frac{km(t)}{C_0}\right)^{-1/2}$$

where f_c is the unmodulated frequency of oscillation. Assuming,

$$\frac{km(t)}{C_0} \ll 1$$

we have from binomial expansion,

$$\left(1 - \frac{km(t)}{C_0}\right)^{-1/2} \approx 1 + \frac{km(t)}{2C_0}$$

$$f(t) \approx f_c\left(1 + \frac{km(t)}{2C_0}\right)$$

$$= f_c + \frac{kf_c m(t)}{2C_0}$$

$$\text{or, } f(t) = f_c + k_f m(t)$$

$$k_f = \frac{kf_c}{2C_0}$$

is the frequency sensitivity of the modulator. The Eq. (5.42) is the required expression for the instantaneous frequency of an FM wave. In this way, we can generate an FM wave by direct method.

Direct FM may be generated also by a device in which the inductance of the resonant circuit is linearly varied by a modulating signal $n(t)$; in this case the modulating signal being the current.

The main advantage of the direct method is that it produces sufficiently high frequency deviation, thus requiring little frequency multiplication. But, it has poor frequency stability. A feedback scheme is used to stabilize the frequency in which the output frequency is compared with the constant frequency generated by highly stable crystal oscillator and the error signal is feedback to stabilize the frequency.

DEMODULATION OF FM WAVES

The process to extract the message signal from a frequency modulated wave is known as frequency demodulation. As the information in an FM wave is contained in its instantaneous frequency, the frequency demodulator has the task of changing frequency variations to amplitude variations. Frequency demodulation method is generally categorized into two types: direct method and indirect method. Under direct method category, we will discuss about limiter discriminator method and under indirect method, phase-locked loop (PLL) will be discussed.

Limiter Discriminator Method

Recalling the expression of FM signal,

$$s(t) = A_c \cos\,[2\pi f_c t + 2\pi k_f \int_0^t m(t)dt]$$

In this method, extraction of m(t) from the above equation involves the three steps: amplitude limit, discrimination, and envelope detection.

A. Amplitude Limit

During propagation of the FM signal from transmitter to receiver the amplitude of the FM wave (supposed to be constant) may undergo changes due to fading and noise. Therefore, before further processing, the amplitude of the FM signal is limited to reduce the effect of fading and noise by using limiter as discussed in the section 5.9. The amplitude limitation will not affect the message signal as the amplitude of FM does not carry any information of the message signal.

B. Discrimination/ Differentiation

In this step we differentiate the FM signal as given by

$$\frac{ds(t)}{dt} = \frac{d}{dt}\left\{A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right]\right\}$$

$$= \frac{d\left\{A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right]\right\}}{d\left\{2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right\}} \frac{d\left\{2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right\}}{dt}$$

$$= -A_c\left[2\pi f_c + 2\pi k_f m(t)\right] \sin\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right]$$

Here both the amplitude and frequency of this signal are modulated.

In this case, the differentiator is nothing but a circuit that converts change in frequency into corresponding change in voltage or current as shown in Fig. 5.11. The ideal differentiator has transfer function

$$H(jw) = j2\pi f$$

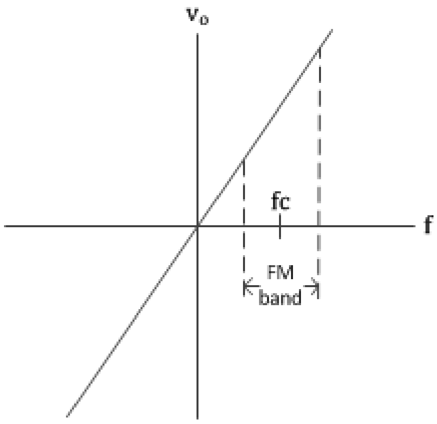


Figure : Transfer function of ideal differentiator.

Instead of ideal differentiator, any circuit can be used whose frequency response is linear for some band in positive slope. This method is known as slope detection. For this, linear segment with positive slope of RC high pass filter or LC tank circuit can be used. Figure 5.13 shows the use of an LC circuit as a differentiator. The drawback is the limited linear portion in the slope of the tank circuit. This is not suitable for wideband FM where the peak frequency deviation is high.

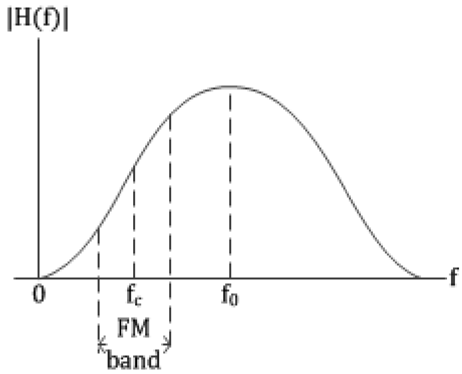


Figure : Use of LC tank circuit as a differentiator.

A better solution is the ratio or balanced slope detector in which two tank circuits tuned at $f_c + \Delta f$ and $f_c - \Delta f$ are used to extend the linear portion as shown in below figure.

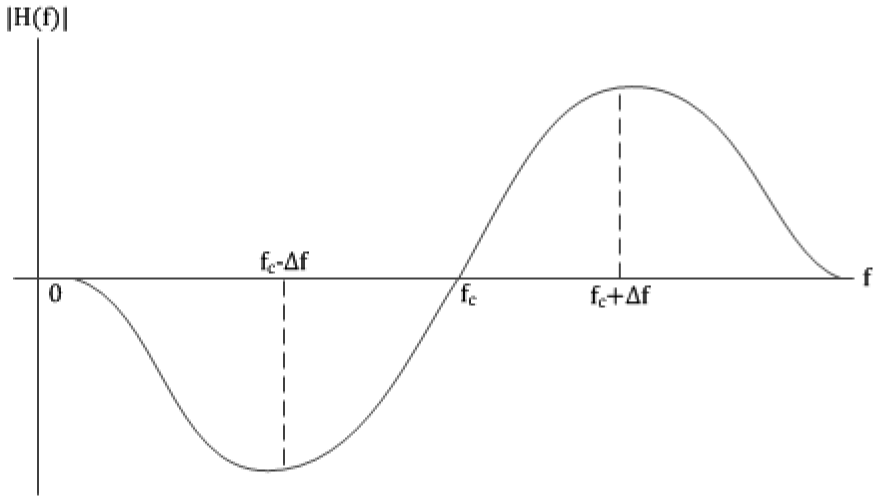


Figure : Frequency response of balanced slope detector.

Another detector called Foster-seely discriminator eliminates two tank circuits but still offer the same linear as the ratio detector.

C. Envelope Detection

The third step is to send the differentiated signal to the envelope detector to recover the message signal.

Phase-Locked Loop (PLL) as FM Demodulator

A PLL consists of a multiplier, a loop filter, and a VCO connected together to form a feedback loop as shown in Fig. 5.15. Let the input signal be an FM wave as defined by

$$s(t) = A_c \cos[2\pi f_c t + \emptyset_1(t)]$$

$$\emptyset_1(t) = 2\pi k_f \int_0^t m(t)dt$$

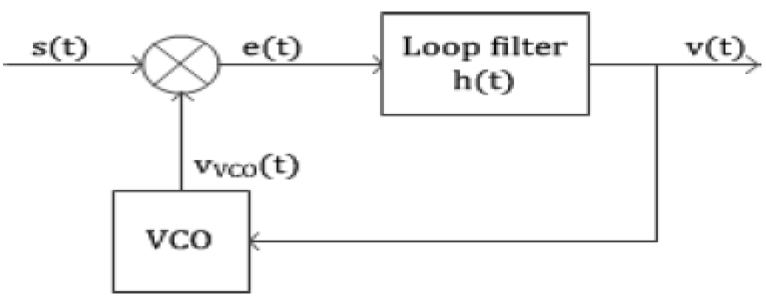


Fig: PLL Demodulator

Let the VCO output be defined by

$$v_{VCO}(t) = A_v \sin[2nf_c t + \emptyset_2(t)]$$

where

$$\emptyset_2(t) = 2nk_v \int_0^t v(t)dt$$

Here k_v is the frequency sensitivity of the VCO measured in hertz per volt. The multiplication of $s(t)$ and $v_{VCO}(t)$ results

$$s(t)v_{VCO}(t) = A_c \cos[2\pi f_c t + \emptyset_1(t)]\, A_v \sin[2\pi f_c t + \emptyset_2(t)]$$

$$= \frac{A_c A_v}{2} \sin[4\pi f_c t + \emptyset_1(t) + \emptyset_2(t)] + \frac{A_c A_v}{2} \sin[\emptyset_2(t) - \emptyset_1(t)]$$

The high-frequency component is removed by the low-pass filtering of the loop filter. Therefore, the input signal to the loop filter can be considered as

$$e(t) = \frac{A_c A_v}{2} \sin[\emptyset_2(t) - \emptyset_1(t)]$$

The difference $\emptyset_2(t) - \emptyset_1(t) = \emptyset_e(t)$ constitutes the phase error. Let us assume that the PLL is in phase lock so that the phase error is very small. Then,

$$\sin[\emptyset_2(t) - \emptyset_1(t)] \approx \emptyset_2(t) - \emptyset_1(t)$$

$$\emptyset_e(t) = 2\pi k_v \int_0^t v(t)dt - \emptyset_1(t)$$

$$e(t) = \frac{A_c A_v}{2} \emptyset_e(t)$$

Differentiating Eq. (5.48) with respect to time, we get

$$\frac{d\emptyset_e(t)}{dt} = 2\pi k_v v(t) - \frac{d\emptyset_1(t)}{dt}$$

Since

$$v(t) = e(t) * h(t) = \frac{A_c A_v}{2} [\emptyset_e(t) * h(t)]$$

Eq. (5.50) becomes

$$\frac{d\emptyset_e(t)}{dt} = 2\pi k_v \frac{A_c A_v}{2} [\emptyset_e(t) * h(t)] - \frac{d\emptyset_1(t)}{dt}$$

$$or, \pi k_v A_c A_v [\emptyset_e(t) * h(t)] - \frac{d\emptyset_e(t)}{dt} = \frac{d\emptyset_1(t)}{dt}$$

Taking Fourier transform of Eq. (5.52), we get

$$\pi k_v A_c A_v \emptyset_e(f) H(f) - j2\pi f \emptyset_e(f) = j2\pi f \emptyset_1(f)$$

$$or, \emptyset_e(f) = \frac{j2\pi f}{\pi k_v A_c A_v H(f) - j2\pi f} \emptyset_1(f)$$

$$or, \emptyset_e(f) = \frac{1}{\frac{\pi k_v A_c A_v}{j2\pi f} H(f) - 1} \emptyset_1(f)$$

Fourier transform of Eq. (5.51) is

$$V(f) = \frac{A_c A_v}{2} \emptyset_e(f) H(f)$$

Substituting Eq. (5.53) into (5.54), we get

$$V(f) = \frac{A_c A_v}{2} \frac{1}{\frac{\pi k_v A_c A_v}{j2\pi f} H(f) - 1} \emptyset_1(f) H(f)$$

We design $H(f)$ such that

$$\left| \frac{\pi k_v A_c A_v}{j2\pi f} H(f) \right| \gg 1$$

in the frequency band $|f| < W$ of the message signal.

$$\therefore V(f) = \frac{A_c A_v}{2} \frac{1}{\frac{\pi k_v A_c A_v}{j2\pi f} H(f)} \emptyset_1(f) H(f)$$

$$or, V(f) = \frac{1}{2\pi k_v} j2\pi f \emptyset_1(f)$$

Taking inverse Fourier transform of Eq. (4.56), we get

$$v(t) = \frac{1}{2\pi k_v} \frac{d\emptyset_1(t)}{dt}$$

$$= \frac{1}{2\pi k_v} \frac{d}{dt} \left\{ 2\pi k_f \int_0^t m(t) dt \right\}$$

$$= \frac{1}{2\pi k_v} 2\pi k_f m(t)$$

$$\therefore v(t) = \frac{k_f}{k_v} m(t)$$

Since the control voltage of the VCO is proportional to the message signal, $v(t)$ is the demodulated signal.

We observe that the output of the loop filter with frequency response $H(f)$ is the desired message signal. Hence the bandwidth of $H(f)$ should be the same as the bandwidth W of the message signal. Consequently, the noise at the output of the loop filter is also limited to the bandwidth W. On the other hand, the output from the VCO is a wideband FM signal with an instantaneous frequency that follows the instantaneous frequency of the received FM signal.

PREEMPHASIS AND DEEMPHASIS NETWORKS

In FM, the noise increases linearly with frequency. By this, the higher frequency components of message signal are badly affected by the noise. To solve this problem, we can use a preemphasis filter of transfer function $H_p(f)$ at the transmitter to boost the higher frequency components before modulation. Similarly, at the receiver, the deemphasis filter of transfer function $H_d(f)$can be used after demodulator to attenuate the higher frequency components thereby restoring the original message signal.

The preemphasis network and its frequency response are shown in Figure 5.19 (a) and (b) respectively. Similarly, the counter part for deemphasis network is shown in Figure 5.20.

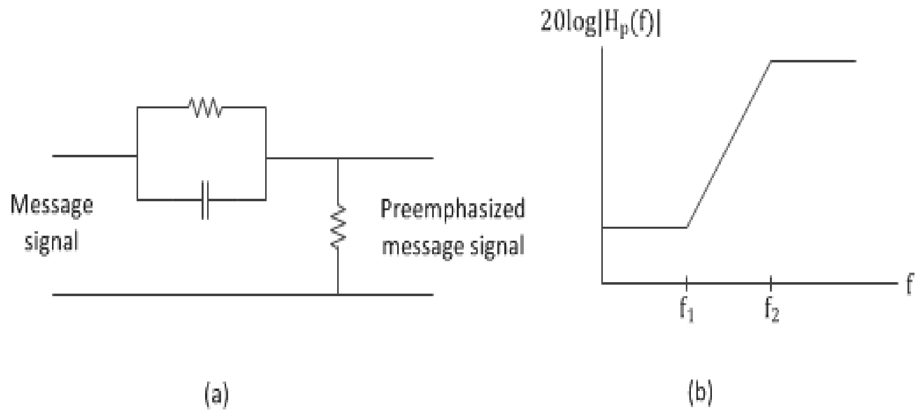


Figure ;(a) Preemphasis network. (b) Frequency response of preemphasis network.

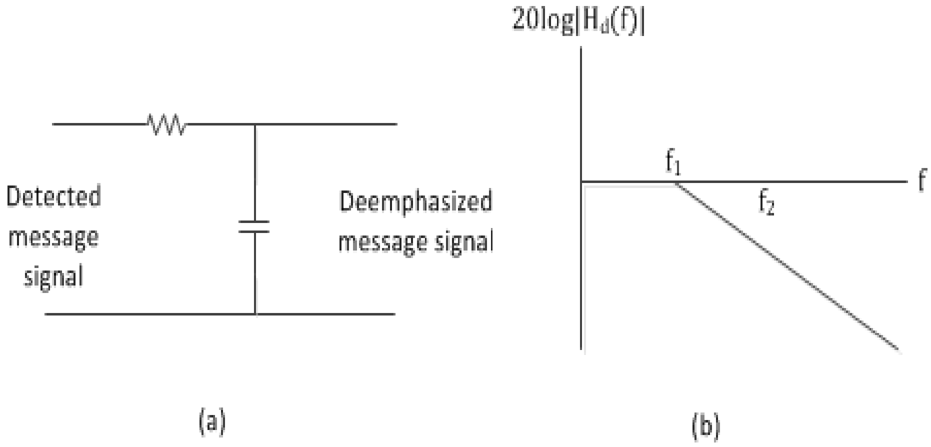


Figure (a) Deemphasis network. (b) Frequency response of Deemphasis network.

In FM broadcasting, f_1 and f_2 are normally chosen to be 2.1 kHz and 30 kHz respectively.

The frequency response of preemphasis network is

$$H_p(f) = \left(\frac{w_2}{w_1}\right)\frac{jw + w_1}{jw + w_2}$$

Here, $w = 2\pi f$ and $w_1 = 2\pi f_1$. For $w \ll w_1$,

$$H_p(f) \approx 1$$

And for $w_1 \ll w \ll w_2$,

$$H_p(f) \approx \frac{j2\pi f}{w_1}$$

So, the amplitude of frequency components less than 2.1 kHz are left unchanged and greater than that are increased proportional to f.

The frequency response of deemphasis network is

$$H_d(f) = \frac{w_1}{j2\pi f + w_1}$$

For $w \ll w_2$,

$$H_p(f) \approx \frac{j2\pi f + w_1}{w_1}$$

such that

$$H_p(f)H_d(f) \approx 1$$

over the baseband of 0 to 15 KHz.

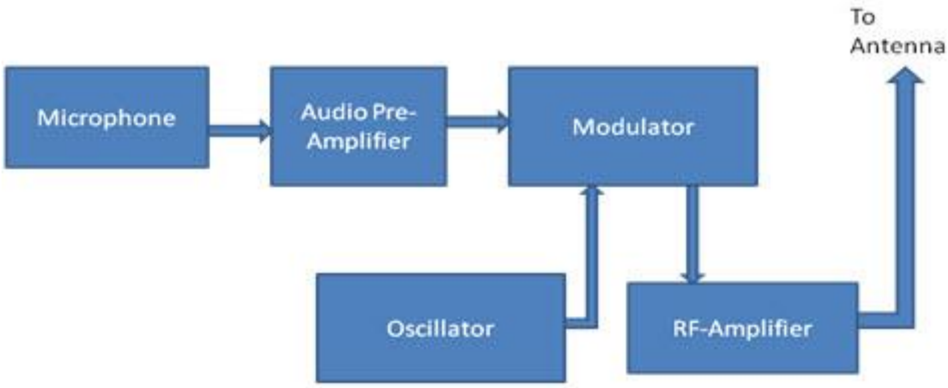
Comparison of AM and FM:

S.NO	AMPLITUDE MODULATION	FREQUENCY MODULATION
1.	Band width is very small which is one of the biggest advantage	It requires much wider channel (7 to 15 times) as compared to AM.
2.	The amplitude of AM signal varies depending on modulation index.	The amplitude of FM signal is constant and independent of depth of the modulation.
3.	Area of reception is large	The are of reception is small since it is limited to line of sight.
4.	Transmitters are relatively simple & cheap.	Transmitters are complex and hence expensive.
5.	The average power in modulated wave is greater than carrier power. This added power is provided by modulating source.	The average power in frequency modulated wave is same as contained in un-modulated wave.
6.	More susceptible to noise interference and has low signal to noise ratio, it is more difficult to eliminate effects of noise.	Noise can be easily minimized amplitude variations can be eliminated by using limiter.
7.	it is not possible to operate without interference.	it is possible to operate several independent transmitters on same frequency.
8.	The maximum value of modulation index = 1, other wise over-modulation would result in distortions.	No restriction is placed on modulation index.

FM Transmitter

The FM transmitter is a single transistor circuit. In the telecommunication, the frequency modulation (FM)transfers the information by varying the frequency of carrier wave according to the message signal. Generally, the FM transmitter uses VHF radio frequencies of 87.5 to 108.0 MHz to transmit & receive the FM signal. This transmitter accomplishes the most excellent range with less power. The performance and working of the wireless audio transmitter circuit is depends on the induction coil & variable capacitor. This article will explain about the working of the FM transmitter circuit with its applications.

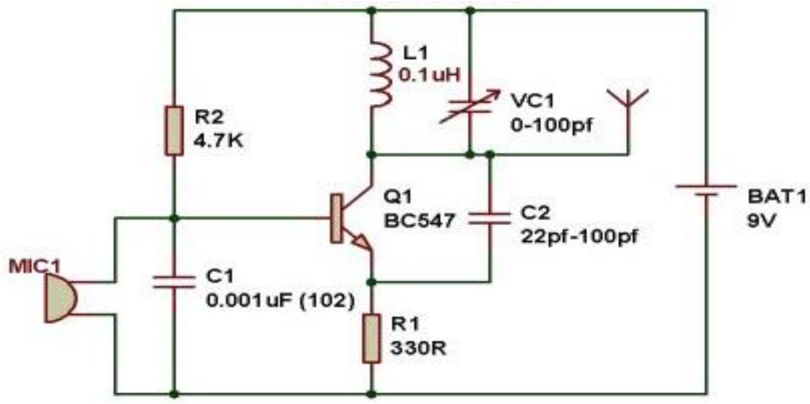
The FM transmitter is a low power transmitter and it uses FM waves for transmitting the sound, this transmitter transmits the audio signals through the carrier wave by the difference of frequency. The carrier wave frequency is equivalent to the audio signal of the amplitude and the FM transmitter produce VHF band of 88 to 108MHZ.Plese follow the below link for: Know all About Power Amplifiers for FM Transmitter



Block Diagram of FM Transmitter

Working of FM Transmitter Circuit

The following circuit diagram shows the FM transmitter circuit and the required electrical and electronic components for this circuit is the power supply of 9V, resistor, capacitor, trimmer capacitor, inductor, mic, transmitter, and antenna. Let us consider the microphone to understand the sound signals and inside the mic there is a presence of capacitive sensor. It produces according to the vibration to the change of air pressure and the AC signal.



FM Transmitter circuit

The formation of the oscillating tank circuit can be done through the transistor of 2N3904 by using the inductor and variable capacitor. The transistor used in this circuit is an NPN transistor used for general purpose amplification. If the current is passed at the inductor L1 and variable capacitor then the tank circuit will oscillate at the resonant carrier frequency of the FM modulation. The negative feedback will be the capacitor C2 to the oscillating tank circuit.

To generate the radio frequency carrier waves the FM transmitter circuit requires an oscillator. The tank circuit is derived from the LC circuit to store the energy for oscillations.

The input audio signal from the mic penetrated to the base of the transistor, which modulates the LC tank circuit carrier frequency in FM format. The variable capacitor is used to change the resonant frequency for fine modification to the FM frequency band. The modulated signal from the antenna is radiated as radio waves at the FM frequency band and the antenna is nothing but copper wire of 20cm long and 24 gauge. In this circuit the length of the antenna should be significant and here you can use the 25-27 inches long copper wire of the antenna.

Application of Fm Transmitter

- The FM transmitters are used in the homes like sound systems in halls to fill the sound with the audio source.
- These are also used in the cars and fitness centers.
- The correctional facilities have used in the FM transmitters to reduce the prison noise in common areas.

Advantages of the FM Transmitters

- The FM transmitters are easy to use and the price is low
- The efficiency of the transmitter is very high
- It has a large operating range
- This transmitter will reject the noise signal from an amplitude variation.

CHAPTER–III

RANDOM PROCESS

3.1 INTRODUCTION

Random signals are encountered in every practical communication system. Random signals, unlike deterministic signals are unpredictable. For example, the received signal in the receiver consists of message signal from the source, noise and interference introduced in the channel. All these signals are random in nature, i.e. these signals are uncertain or unpredictable. These kind of random signals cannot be analyzed using Fourier transform, as they can deal only deterministic signals. The branch of mathematics which deals with the statistical characterization of random signals is probability theory.

3.1.1 Basic Terms in Probability Theory

- **Experiment:** Any activity with an observable result is called as experiment. Examples are rolling a die, tossing a coin or choosing a card.
- **Outcome:** The result of an experiment is called as outcome.
- **Sample space:** The set of all possible outcomes of an experiment is known as the sample space of the experiment and is denoted by S. For example, in tossing a coin experiment, the sample space S = {Head, Tail}
- **Event:** Any subset of the sample space is known as an event. For example, in tossing a coin experiment, E = {Head} is the event that a head appears (or) E = {Tail} is the event that a tail appears.
- **Independent event:** Two events are said to be independent, when the occurrence of one event is not affected by the occurrence of other event. If A and B are independent events, then

$$P(A \cap B) = P(A) * P(B)$$

- **Mutually exclusive events:** Two events are said to be mutually exclusive or disjoint events, when they cannot occur simultaneously (None of the outcomes are common). If A and B are mutually exclusive events, then

$$P(A \cap B) = 0$$

- **Probability:** It is the measure of possibility or chance that an event will occur. It is defined as:

$$\textbf{Probability} = \frac{\textbf{Number of desirable outcome}}{\textbf{Total number of possible outcomes}} = \frac{\textbf{E}}{\textbf{S}}$$

For example, in coin tossing experiment the probability of getting "Head" is 1/2.

- **Axioms of Probability:**

(a) Probability of sample space is 1, i.e. $\mathbf{P(S)=1}$

(b) Probability of an event P(E) is nonnegative real number ranges from 0 to 1.

$$\mathbf{0 \le P(E) \le 1}$$

(c) Probability of the mutually exclusive events is equal to sum of their individual probability.

$$\mathbf{P(A \cup B) = P(A) + P(B)}$$

If A and B are non-mutually exclusive events, then

$$\mathbf{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

- **Conditional Probability:** The conditional probability of an event B is the probability that the event will occur given that an event A has already occurred. It is defined as,

$$\mathbf{P(B / A) = \frac{P(A \cap B)}{P(A)}}$$

Similarly, the conditional probability of event A given that an event B has already occurred is given by,

$$\mathbf{P(A / B) = \frac{P(A \cap B)}{P(B)}}$$

If the event **A** ***and*** **B** ***are independent***, then $\mathbf{P(A \cap B) = P(A) * P(B)}$**,** then the Conditional probability **of B given A** is simply P(B) as,

$$\mathbf{P(B/A) = [\ P(A) * P(B)] / P(A) = P(B)}$$

Similarly, conditional probability of A given B is simply P(A) as,

$$\mathbf{P(A/B) = [\ P(A) * P(B)] / P(B) = P(A)}$$

- **Law of Total Probability:** If the events $A_1, A_2, \ldots. A_k$ be mutually exclusive, then the probability of other event B is given by,

$$\mathbf{P(B) = \sum_{\mathit{i}=1}^{\mathit{k}} P(B/A_{\mathit{i}})\, P(A_{\mathit{i}})}$$

- **Baye's Theorem:** Baye's theorem or Baye's rule describes the probability of an event based on the prior knowledge of the related event. It is defined as:

$$P(A_i / B) = \frac{P(B/A_i)\,P(A_i)}{P(B)} = \frac{P(B/A_i)\,P(A_i)}{\sum_{i=1}^{k} P(B/A_i)\,P(A_i)}$$

where,

$P(A_i)$	–	Prior probability or Marginal probability of A, as it does not takes into account any information about B.
P(B)	–	Prior probability or marginal probability of B,as it does not takes into account any information about A.
$P(A_i/B)$	–	Conditional probability of A given B. It is also called as posterior probability, as it depends on the value of B
$P(B/A_i)$	–	Conditional probability of B given A

3.2 RANDOM VARIABLES

The outcomes of a random experiment are not the convenient representation for mathematical analysis. For example, 'Head' and 'Tail' in tossing the coin experiment.It will be convenient, if we assign a number or a range of values to the outcomes of a random experiment. For example, a 'Head' corresponds to 1 and a 'Tail'corresponds to 0.

A random variable is defined as a process of mapping the sample space Q to the set of real numbers. In other words, a random variable is an assignment of real numbers to the outcomes of a random experiment as shown in Figure 3.1.Random variables are denoted by capital letters, i.e., X, Y, and so on, and individual values of the random variable X are X(*w*).

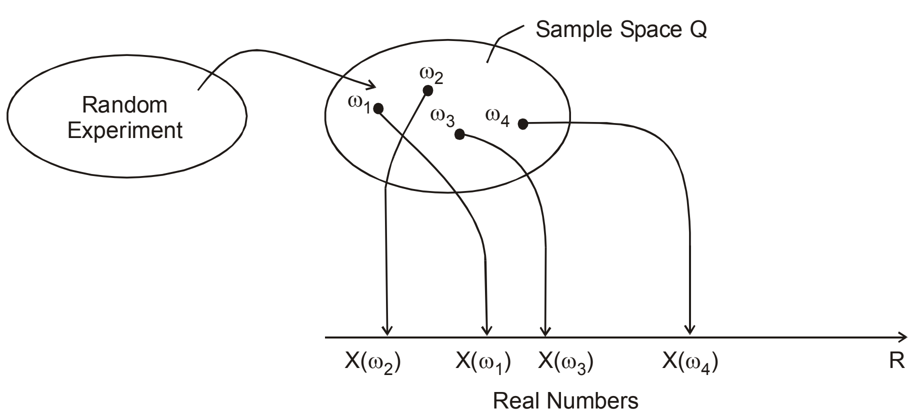


Figure 3.1 Random variable – Mapping from Q to R

For example, consider an experiment of tossing three fair coins. Let, X denote the number of "Heads" that appear, then X is a random variable taking on one of the values 0,1,2,3 with respective probabilities:

$$P\{X = 0\} = P\{(T, T, T)\} = 1/8$$

$$P\{X = 1\} = P\{(T, T, H), (T, H, T), (H, T, T)\} = 3/8$$

$$P\{X = 2\} = P\{(T, H, H), (H, T, H), (H, H, T)\} = 3/8$$

$$P\{X = 3\} = P\{(H, H, H)\} = 1/8$$

3.2.1 Classification of Random Variables

Random variables are classified into continuous and discrete random variables.

- The values of ***continuous random variable*** are continuous in a given continuous sample space. A continuous sample space has infinite range of values. The discrete value of a continuous random variable is a value at one instant of time. For example the Temperature, T at some area is a continuous random variable that always exists in the range say, from T_1 and T_2.

  Examples of continuous random variable – Uniform, Exponential, Gamma and Normal.

- The values of a ***discrete random variable*** are only the discrete values in a given sample space. The sample space for a discrete random variable can be continuous, discrete or even both continuous and discrete points. They may be also finite or infinite. For example the "Wheel of chance" has the continuous sample space. If we define a discrete random variable *n* as integer numbers from 0 to 12, then the discrete random variable is X = {0,1,3,4…..12}.
  Examples of discrete random variable – Bernoulli, Binomial, Geometric and Poisson.

3.2.2 Distribution and Density Function

The **Cumulative Distribution Function (CDF)** of a random variable X is defined as,

$$\mathbf{F_X(x) = P\{\omega \in \Omega : X(w) \le x\}}$$

which can be simply written as, $\mathbf{F_X}(x) = \mathbf{P}(\mathbf{X} \le x)$

Properties of CDF:

- $F_X(x)$ is a non-decreasing function of x.
- Since CDF is a probability, it ranges from 0 to 1,i.e., $0 \le F_X(x) \le 1$
- $\lim_{x \to -\infty} F_X(x) = 0$ and $\lim_{x \to +\infty} F_X(x) = 1$
- $F_X(x)$ is continuous from the right
- $P(a < X \le b) = F_X(b) - F_X(a)$
- $P(X = a) = F_X(a) - F_X(a^-)$

The **Probability Density Function (PDF)**, of a continuous random variable X is defined as the derivative of its CDF. It is denoted by:

i.e., $$\boldsymbol{f_X(x) = \frac{d}{dx} F_X(x).}$$

Properties of PDF:

- $f_X(x) \ge 0$
- $\int_{-\infty}^{+\infty} f_X(x)\,dx = 1$
- $\int_a^b f_X(x)\,dx = P(a < X \le b)$
- $P(X \in A) = \int_A f_X(x)\,dx$
- $F_X(x) = \int_{-\infty}^{x} f_X(u)\,du.$

The **Probability Mass Function (PMF),** of a discrete random variable X is defined as $p_x(x)$, where

$$\boldsymbol{p_x(x) = P(X = x)}$$

Properties of PMF:

- $0 \le p_x(x_i) \le 1,\ i = 1, 2, ...$
- $p_x(x) = 0, \quad \text{if } x \ne x_i \ (i = 1, 2, ...)$
- $\sum_i p_x(x_i) = 1.$

3.2.3 Statistical Averages of Random Variables

Even though the distribution function provides a complete description of the random variable, some other statistical averages such as mean and variance are also used to describe the random variable in detail.

Mean or Expectation of the random variable: The mean of the continuous random variable X with a density function of $f_X(x)$ is defined as:

$$\mathbf{E[X]=\int_{-\infty}^{\infty} x\, f_X(x)\, dx}$$

The mean of the discrete random variable X is defined as the weighted sum of the possible outcomes given as:

$$\mathbf{\mu_x = E[X] = \Sigma_X\, x\, P[X=x]}$$

For example, if X is considered as a random variable representing the observations of the voltage of a random signal, then the ***mean value represents the average voltage or dc offset of the signal***.

Variance of the Random Variable: It provides an estimate of the spread of the distribution about the mean.The variance of the continuous random variables is defined as:

$$\mathbf{\sigma_x^2 = \int_{-\infty}^{\infty} (x-\mu_x)^2\, f_X(x)\, dx}$$

The variance of the discrete random variables is given by the expectation of the squared distance of each outcome from the mean value. It is defined as:

$$\sigma_x^2 = \mathrm{Var(X)} = \mathrm{E}[(\mathrm{X}-\mu_x)^2]$$

We know that, expectation of a variable is

$$\mathrm{E[X]} = \Sigma_{\mathrm{X}}\, x\, \mathrm{P[X}=x]$$

Therefore, the variance is given by,

$$\mathbf{\sigma_x^2 = \Sigma_X (x-\mu_x)^2\, P[X=x]}$$

For example, if X is considered as a random variable representing observations of the voltage of a random signal, then the ***variance represents the AC power of the signal***. The second moment of X, **E[X²]** ***is also called the mean-square value of the random signal and it represents the total power of the signal***.

Covariance of the Random Variable: In communication system, it is very important to compare two signals in order to extract the information. For example, RADAR system compares the transmitted signal to the target with the received (reflected) signal from the target to measure the parameters like range, angle and speed of the object. Correlation and covariance are mainly used for this comparison purpose.

The covariance of two random variables X and Y is defined as the expectation of the product of the two random variables given by:

$$\text{Cov}(X, Y) = E[(X - m_X)(Y - m_Y)]$$

Expanding and simplifying above equation we get:

$$\text{Cov}(X, Y) = E[XY] - \mu_X \mu_Y$$

If Cov(X,Y) = 0, then X and Y are uncorrelated, i.e., $E[XY] = \mu_X \mu_Y = E[X]E[Y]$

If X and Y are independent, then* Cov(X, Y) = 0*, i.e., X and Y are uncorrelated but the converse is not true.

The correlation coefficient of two random variable is defined as,

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

where, the correlation coefficient ranges from [– 1, 1].

Some of the important random variables used in communication systems are,

(a) Binomial Random Variable:

This is a discrete random variable. Suppose there are 'n' independent trials, each of which results in a success and failure with probability p and $1 - p$ respectively. If X represents the number of successes that occur in the n trials, then X is said to be binomial random variable with parameters (n, p).

The probability mass function of a binomial random variable with parameters n and p is given by,

$$P(X = i) = \binom{n}{i} p^i (1 - p)^{n-i}, \qquad 0 \le i \le n$$

The mean and variance of binomial random variable is given by,

$$\mu = E(X) = np$$

$$\sigma^2 = np(1 - p)$$

Application:

Binomial Random Variable can be used to model the total number of bits received in error when sequence of n bits is transmitted over a channel with a bit-error probability of p as shown in Figure 3.2.

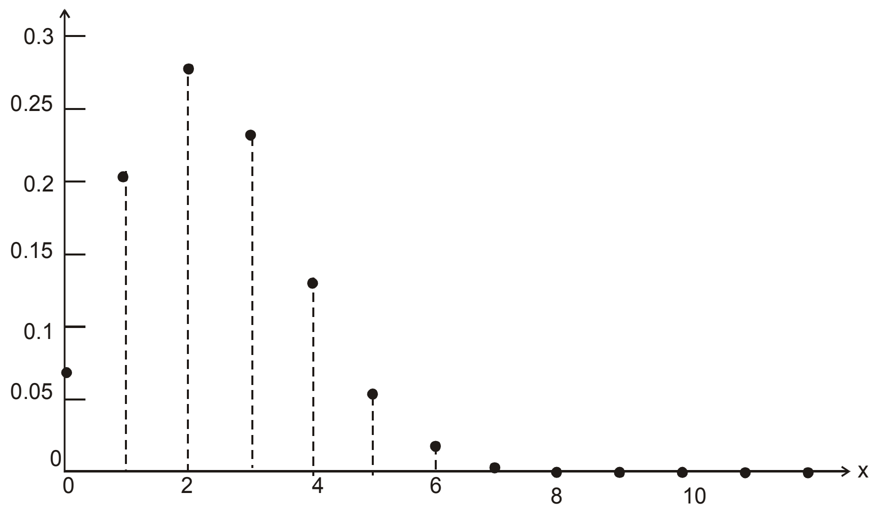


Figure 3.2 PMF for Binomial Random Variable

(b) Uniform Random Variable:

This is a continuous random variable that takes values between a and b with equal probabilities for intervals of equal length. The probability density function is shown in Figure 3.3 defined as:

$$f_X(x) = \begin{cases} \dfrac{1}{b-a}, & a < x < b \\ 0, & \text{Otherwise} \end{cases}$$

The cumulative distribution function is shown in Figure 3.4 defined as:

$$F_X(x) = \begin{cases} 0, & x < a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ 1, & x > b \end{cases}$$

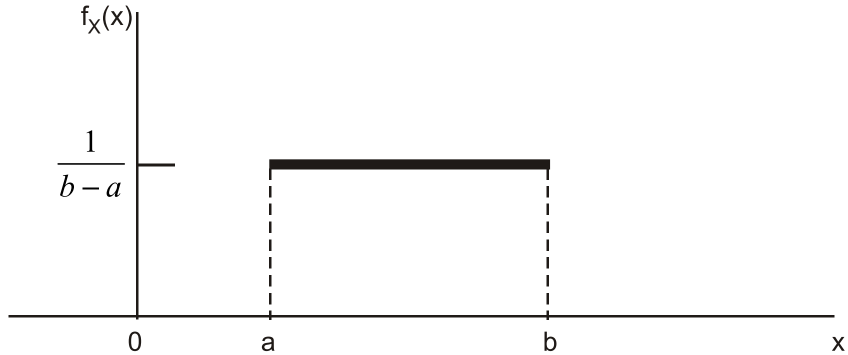


Figure 3.3 PDF for uniform random variable

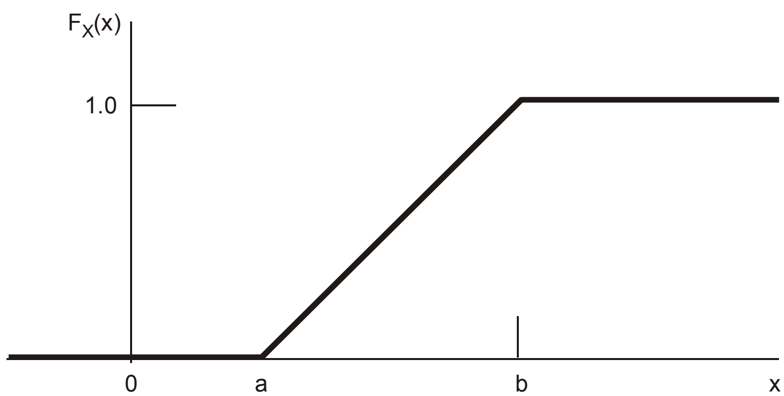


Figure 3.4 CDF for uniform random variable

Application:

Uniform random variable is used to model continuous random variables, whose range is known, but other information like likelihood of the values that the random variable can assume is unknown. For example, the ***phase of a received sinusoid carrier*** is usually modelled as a uniform random variable between 0 and 2π. ***Quantization Errors*** are also modelled as uniform random variable.

(c) Gaussian or Normal Random Variable:

The Gaussian or Normal Random Variable is a continuous random variable described by the probability density function as:

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}$$

The PDF of Gaussian random variable is a bell shaped that is symmetric about the mean m and attains the maximum value of $1/\sqrt{2\pi}\,\sigma$ at $x = m$, as shown in Figure 3.5.

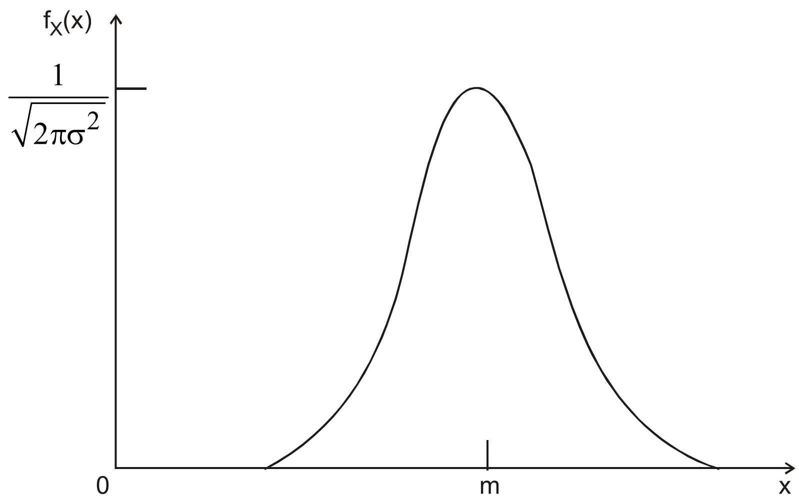


Figure 3.5 PDF for Gaussian Random Variable

Theparameter m is called the ***Mean*** and can assume any finite value. The parameter σ is calledthe ***Standard Deviation*** and can assume any finite and positive value. The square of the standard deviation, i.e., σ^2 is the ***Variance***. A Gaussian random variable with mean m and variance σ^2 is denoted by $N(m, \sigma^2)$. The random variable N(0, 1) is usually called ***standard normal***.

Properties of Gaussian Random Variable:

- It is completely characterized by its mean and variance.
- The sum of two independent Gaussian random variables is also a Gaussian random variable.
- The weighted sum of N independent Gaussian random variables is a Gaussian random variable.
- If two Gaussian random variables have zero covariance (uncorrelated), they are also independent.
- A Gaussian random variable plus a constant is another Gaussian random variable with the mean adjusted by the constant.
- A Gaussian random variable multiplied by a constant is another Gaussian random variable where both the mean and variance are affected by the constant.

Applications:

- The Gaussian random variable is the most important and frequently encountered random variable in communication systems. The reason is that ***thermal noise, which is the major source of noise in communication, has a Gaussian distribution***.
- In Robotics, Gaussian PDF is used to statistically characterize sensor measurements, robot locations and map representations.

3.3 CENTRAL LIMIT THEOREM

An important result in probability theory that is closely related to the Gaussian distribution is the ***Central Limit Theorem (or) Law of large numbers***. Let $X_1, X_2, X_3, \ldots, X_n$ be a set of random variables with thefollowing properties:

- The X_k with $k = 1,2,\ldots n$ are statistically independent.
- The X_k all have the same probability density function.
- Both the mean and the variance exist for each X_k.

We do not assume that the density function of the X_k is Gaussian. Let Y be a new random variable defined as:

$$\mathbf{Y} = \sum_{k=1}^{n} \mathbf{X}_k$$

Then, according to the central limit theorem, the normalized random variable,

$$\mathbf{Z} = \frac{\mathbf{Y} - \mathbf{E[Y]}}{\sigma_{\mathbf{Y}}}$$

approaches a Gaussian random variable with zero mean and unit variance as the number of the random variables $X_1, X_2, X_3, \ldots, X_n$ increases without limit. That is, as n becomes large,the distribution of Z approaches that of a zero-mean Gaussian random variable with unit variance, as shown by:

$$F_Z(z) \to \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{s^2}{2}\right\} ds$$

This is a mathematical statement of the central limit theorem. In words, ***the normalized distribution of the sum of independent, identically distributed random variables***

approaches a Gaussian distribution as the number of random variables increases, regardless of the individual distributions. Thus, Gaussian random variables are common because they characterize the asymptotic properties of many other types of random variables.

When n is finite, the Gaussian approximation is most accurate in the central portion of the density function (hence the name central limit) and less accurate in the "tails" of the density function.

Applications of Central Limit Theorem:

- Channel Modelling
- Finance
- Population Statistics
- Hypothesis Testing
- Engineering Research

3.4 Random Process

A random process or stochastic process is the natural extension of random variables when dealing with signals. In analyzing communication systems, we basically deal with time-varying signals. So far, the assumption is that, all the signals are deterministic. In many situations, it is more appropriate to model signals as ***random rather than deterministic functions***.

One such example is the case of **thermal noise** in electronic circuits. This type of noise is due to the random movement of electrons as a result of thermal agitation, therefore, the resulting current and voltage can only be described statistically. Another situation where modeling by random processes proves useful is in the characterizationof information sources. **An information source**, such as a speech source, generates time-varying signals whose contents are not known in advance. Otherwise there would be no need to transmit them. Therefore, random processes also provide a natural way to model information sources.

A random process is a collection (or ensemble) of random variables $\{\mathbf{X}(t,\omega)\}$ ***that are functions of a real variable, namely time t where*** $\omega \in \mathbf{S}$ ***(Sample space) and*** $t \in \mathbf{T}$ ***(Parameter set or Index set)***.The set of possible values of any individual member of the random process is called state space. Any individual member itself is called a sample function or a realization of the process. A random process can be viewed as a mapping of sample space S to the set of signal waveforms as shown in Figure 3.6.

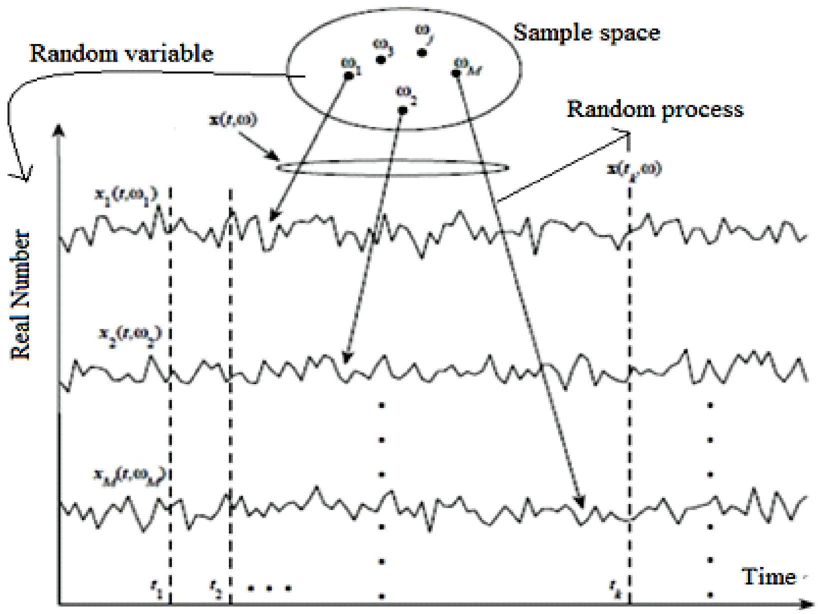


Figure 3.6 Random Process – Mapping of Sample Space to Signal Waveform

The realization of one from the set of possible signals is governed by some probabilistic law. This is similar to the definition of random variables where one from a set of possible values is realized according to some probabilistic law. ***The difference is that in random processes, we have signals (function of time) instead of values (numbers).***

3.4.1 Classification of Random Process

Based on the continuous or discrete nature of the state space S and parameter set T, a random process can be classified into four types:

- ***If both T and S are discrete, the random process is called a Discrete random sequence.***

  For example, if X_n represents the outcome of the n^{th} toss of a fair dice, then $\{X_n,\ n \geq 1\}$ is a discrete random sequence, since $T = \{1, 2, 3, \cdots\}$ and $S = \{1, 2, 3, 4, 5, 6\}$.

- ***If* T *is discrete and S is continuous, the random process is called a continuous random sequence.***

  For example, if X_n represents the temperature at the end of the n^{th} hourof a day, then $\{X_n, 1 \leq n \leq 24\}$ is a continuous random sequence, since temperature can take any value in an interval and hence continuous.

- ***If T is continuous and S is discrete, the random process is called a Discrete random process***.

  For example, if X(t) represents the number of telephone calls received in the interval (0, t) then {X(t)} is a discrete random process, since S = {0, 1, 2, 3, · · ·}.

- ***If both* T *and* S *are continuous, the random process is called a continuous random process.***

  For example, if X(t) represents the maximum temperature at a place in the interval (0, t), {X(t)} is a continuous random process.

  Based on the stationarity, a random process can be classified into stationary and non-stationary random process as shown in Figure 3.7.

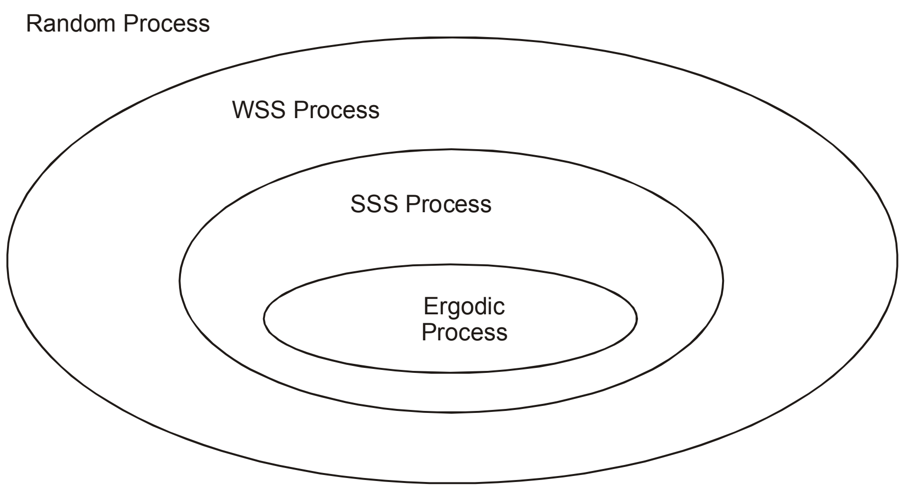


Figure 3.7 Hierarchical Classification of Random Process

- A random process whose ***statistical characteristics do not change with time*** is called as ***Stationary Random Process or Stationary Process***.Example – Noise process as its statistics do not change with time.

- A random process whose ***statistical characteristics changes with time*** is called as ***non-stationary process***. Example – Temperature of a city as temperature statistics depend on the time of the day.

3.5 STATIONARY PROCESS

The random process X(t) is said to be ***Stationary in the Strict Sense* (SSS) *or strictly stationary*** if the following condition holds:

$$F_{X(t_1+\tau),\ldots\ldots,x(t_k+\tau)}(x_1,\ldots\ldots,x_k)=F_{X(t_1),\ldots\ldots,x(t_k)}(x_1,\ldots\ldots,x_k)$$

For all time shifts τ, all k and all possible choices of observation times $t_1, \ldots\ldots t_k$. In other words, if the joint distribution of any set of random variables obtained by observing the random process X(t) **is invariant** with respect to the location at the time origin $t = 0$.

3.5.1 Mean of the Random Process

The mean of process X(t) is defined as the expectation of the random variable obtained by observing the process at some time t given by:

$$\boldsymbol{m_X(t) = E[X(t)] = \int_{-\infty}^{\infty} x\, f_{X(t)}(x)\, dx}$$

where, $f_{X(t)}(x)$ is the first order probability density function of the process.

The ***mean of the strict stationary process is always constant*** given by:

$$\boldsymbol{m_x(t) = m_x} \text{ for all values of } t$$

3.5.2 Correlation of the Random Process

Autocorrelation of the process X(t) is given by the expectation of the product of two random variables X(t_1) and X(t_2) obtained by observing the process X(t) at times t_1 and t_2 respectively. ***It is defined as the measure of similarity of random processses.***

$$R_{XX}(t_1, t_2) = E[X(t_1)\, X(t_2)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X_1\, X_2\, f_{X(t_1), X(t_2)}(x_1, x_2)\, dx_1\, dx_2$$

For a stationary random process $f_{X(t_1), X(t_2)}(x_1, x_2)$ depends only on the difference between the observation time t_1 and t_2. This implies the ***autocorrelation function of a strictly stationary process depends only on the time difference $t_2 - t_1$*** given by:

$$\boldsymbol{R_{XX}(t_1, t_2) = R_{XX}(t_2 - t_1)} \text{ for all values of } t_1 \text{ and } t_2$$

3.5.2.1 Importance of Autocorrelation

Autocorrelation function provides the spectral information of the random process. The frequency content of a process depends on the rapidity of the amplitude change with time. This can be measured by correlating amplitudes at t_1 and $t_1 + \tau$. Autocorrelation function can be used to provide information about the rapidity of amplitude variation with time which in turn gives information about their spectral content.

For example, consider two random process $x(t)$ and $y(t)$, whose autocorrelation function $R_X(\tau)$ and $R_Y(\tau)$as shown in Figure 3.8. We can observe from their autocorrelation function that the random process $x(t)$ is a slowly varying process compared to the process $y(t)$. In fact the power spectral density of random process is obtained from the Fourier Transform of their autocorrelation function.

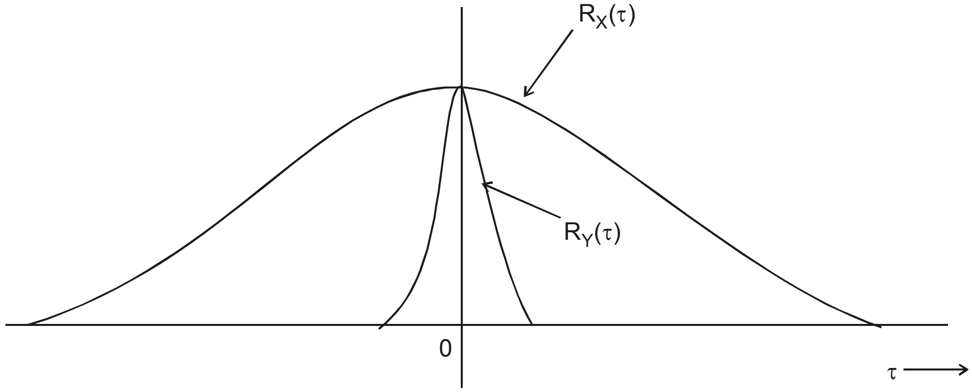


Figure 3.8 Autocorrelation Function of Two Random Process x(t) and y(t)

3.5.2.2 Properties of Autocorrelation Function

- The mean square value of a random process is equal to the value of autocorrelation at $\tau = 0$.

$$R_{XX}(0) = E[X^2(T)]$$

- The autocorrelation function is an even function of τ.

$$R_{XX}(\tau) = R_{XX}(-\tau)$$

- The autocorrelation function is maximum at $\tau = 0$.

$$|R_{XX}(\tau)| \leq R_{XX}(0)$$

- If $E[X(t)] \neq 0$ and $X(t)$ is ergodic with no periodic components, then

$$\lim_{|\tau| \to \infty} R_{XX}(\tau) = \overline{X}^2$$

- If $X(t)$ has a periodic component, then $R_{XX}(\tau)$ will have a periodic component with the same period.

- If $X(t)$ is ergodic, zero mean and has no periodic components, then

$$\lim_{|\tau| \to \infty} R_{XX}(\tau) = 0$$

3.5.3 Covariance of the Random Process

The autocovariance function of X(t) is defined as:

$$C_{XX}(t_1,t_2)=E\{[X(t_1)-m_X(t_1)][X(t_2)-m_X(t_2)]\}$$

$$=R_{XX}(t_1,t_2)-m_X(t_1)\,m_X(t_2)$$

$$\mathbf{C_{XX}(t_1,t_2)=R_{XX}(t_2-t_1)-m_X^2}$$

Like autocorrelation function, the autocovariance function of a strictly stationary process depends on the time difference $t_2 - t_1$.

3.6 WHITE NOISE PROCESS

One of the very important random process is the white noise process. Noises in many practical situations are approximated by the white noise process. Most importantly, the white noise plays an important role in modeling WSS signals.

A random process is said to be white noise process X(*t*), if it is zero mean and power spectral density is defined as:

$$\mathbf{S_X(\omega)=N_0/2}, \qquad \textbf{for all frequencies}$$

where, N_0 is a real constant.

The corresponding autocorrelation function is given by:

$$\boxed{\mathbf{R_X(\tau)=(N_0/2)\,\delta(\tau)}}$$

where, $\delta(\tau)$ is the Dirac delta function.The PSD and autocorrelation function of a white noise is shown in Figure 3.9 (a) and (b) respectively.

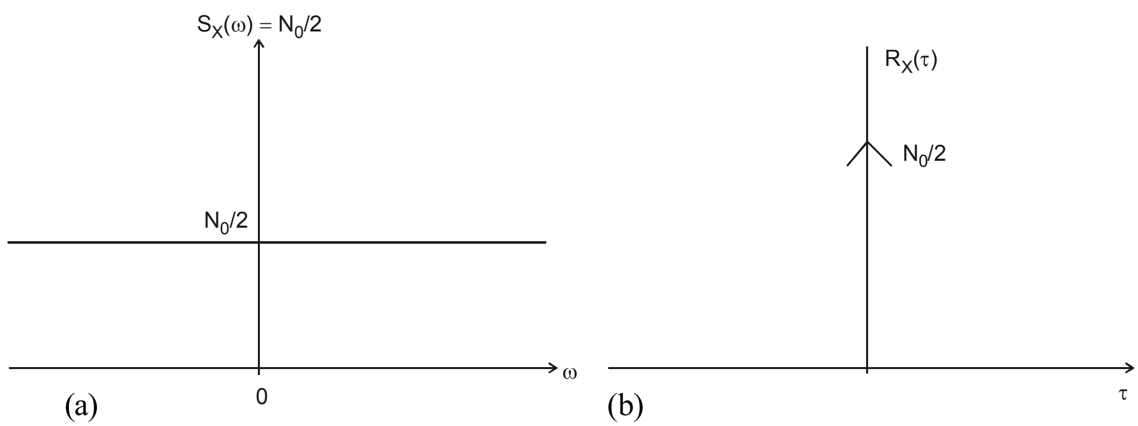


Figure 3.9 (a) PSD of White Noise (b) Autocorrelation Function of White Noise

3.6.1 Properties of White Noise Process

- The term white noise is analogous to white light which contains all visible light frequencies.
- A white noise is a mathematical abstraction; it cannot be physically realized since it has infinite average power.
- A white noise process can have any probability density function.
- The random process X(t) is called as white Gaussian noise process, if X(t) is a stationary Gaussian random process with zero mean and flat power spectral density.
- If the system bandwidth (BW) is sufficiently narrower than the noise BW and noise PSD is flat, we can model it as a white noise process. Thermal noise, which is the noise generated in resistors due to random motion electrons, is generally modelled as white Gaussian noise, since they have very flat PSD over very wide band of frequency.
- A white noise process is called **strict-sense white noise process**, if the noise samples at distinct instants of time are independent.

3.7 WIDE-SENSE STATIONARY PROCESS (WSS)

A process may not be stationary in the strict sense, still it may have mean value $m_x(t)$ and an autocorrelation function which are independent of the shift of time origin.

Ø $m_x(t) = \text{constant}$

Ø $R_{XX}(t_1, t_2) = R_{XX}(t_2 - t_1)$

Such a process is known as ***Wide-Sense Stationary or Weakly Stationary Process (WSS) or Co-Variance Stationary***.

3.8 POWER SPECTRAL DENSITY (PSD)

A random process is a collection of signals and the spectral characteristics of these signals determine the spectral characteristics of the random process.

If the signals of the random process are:

Slowly Varying: Random process will mainly contain low frequencies and its power will be mostly concentrated at **low frequencies**.

Fast Varying: Most of the power in the random process will be at the **high-frequency** components.

A useful function that determines the distribution of the power of the random process at different frequencies is the ***Power Spectral Density or Power Spectrum of the random process***, the power spectral density of a random process X (t) is denoted by $S_X(f)$, and denotes the strength of the power in the random process as a function of frequency. The unit for power spectral density is **watts per Hertz (W/Hz)**.

3.8.1 Expression for power spectral density

The impulse response of a linear time invariant filter is equal to the inverse Fourier Transform of the frequency response of the system. Let H(f) denotes the frequency response of the system, thus

$$h(\tau_1) = \int_{-\infty}^{\infty} H(f)\exp(j2\pi f\tau_1)\,df$$

Substituting $h(\tau_1)$ in $E[Y^2(t)]$, we get

$$E[Y^2(t)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} H(f)\exp(j2\pi f\tau_1)\,df\right] h(\tau_2)R_{XX}(\tau_2-\tau_1)\,d\tau_1\,d\tau_2$$

$$= \int_{-\infty}^{\infty} df\, H(f)\int_{-\infty}^{\infty} d\tau_2\, h(\tau_2)\int_{-\infty}^{\infty} R_{XX}(\tau_2-\tau_1)\exp(j2\pi f\tau_1)\,d\tau_1$$

In the last integral on the right hand side of above equation a new variable is defined :

$$\tau = \tau_2 - \tau_1$$

Then the above equation can be rewritten as:

$$E[Y^2(t)] = \int_{-\infty}^{\infty} df\, H(f)\int_{-\infty}^{\infty} d\tau_2\, h(\tau_2)\exp(j2\pi f\tau_2)\int_{-\infty}^{\infty} R_{XX}(\tau)\exp(-j2\pi f\tau_1)\,d\tau_1$$

The middle integral corresponds to $H^*(f)$, the complex conjugate of the frequency response of the filter and so we may simplify the equation as:

$$E[Y^2(t)] = \int_{-\infty}^{\infty} df\,|H(f)|^2\int_{-\infty}^{\infty} R_{XX}(\tau)\exp(-j2\pi f\tau_1)\,d\tau_1$$

where $|H(f)|$ is the magnitude response of the filter. In the above equation the last integral term is the Fourier transform of the autocorrelation function of the input random process X(t).This provides the definition of a new parameter,

$$\mathbf{S_{XX}(f) = \int_{-\infty}^{\infty} R_{XX}(\tau)\exp(-j2\pi f\tau_1)\,d\tau_1}$$

This function is called $S_{XX}(f)$ is called the power spectral density or power spectrum of the stationary process X(t).

Finally, $\mathbf{E[Y^2(\mathit{t})] = \int_{-\infty}^{\infty} |H(\mathit{f})|^2 S_{XX}(\mathit{f})\, \mathit{df}}$

The mean square of the output of a stable linear time-invariant filter in response to a stationary process is equal to the integral over all frequencies of the power spectral density of the input process multiplied by the squared magnitude response of the filter.

3.8.2 Relationship between power spectral density and autocorrelation

The power spectral density and the autocorrelation function of a stationary process form a Fourier-transform pair with τ and f as the variables of interest,

$$S_{XX}(f) = \int_{-\infty}^{\infty} R_{XX}(\tau) \exp(-j2\pi f\tau)\, d\tau$$

$$R_{XX}(\tau) = \int_{-\infty}^{\infty} S_{XX}(f) \exp(j2\pi f\tau)\, df$$

The above relations provide insight of spectral analysis of random processes and they together called as **Einstein-Wiener-Khintchine Relation.**

3.8.3 Properties of Power Spectral Density

1. The zero-frequency value of the power spectral density of a stationary process equals the total area under the graph of the autocorrelation function (substituting $f = 0$ in $S_{xx}(f)$).

$$S_{XX}(0) = \int_{-\infty}^{\infty} R_{XX}(\tau)\, d\tau$$

2. The mean square value of a stationary process equals the total area under the graph of the power spectral density (substituting $\tau = 0$ in $R_{xx}(\tau)$)

$$E[X^2(t)] = \int_{-\infty}^{\infty} S_{XX}(f)\, df$$

3. The power spectral density of a stationary process is always nonnegative,

$$S_{XX}(f) \leq 0 \quad \text{for all } f$$

4. The power spectral density of a real valued random process is an even function of frequency, that is,

$$S_{XX}(-f) = S_{XX}(f)$$

5. The power spectral density appropriately normalized has the properties usually associated with a probability density function:

$$P_X(f) = \frac{S_{XX}(f)}{\int_{-\infty}^{\infty} S_{XX}(f)\, df}$$

3.9 ERGODIC PROCESS

The expectations or ensemble averages of a random process are averages **across the process** that describes all possible values of the sample functions of the process observed at time t_k. Time averages are defined as long term sample averages that are averages **along the process**.

Time averages are the practical means for the estimation of ensemble averages of the random process. The dc value of $x(t)$ is defined by the time average

$$m_x(T) = \frac{1}{2T} \int_{-T}^{T} x(t)\, dt$$

The time average is a random variable as its value depends on the observation interval and sample function. The mean of the time average is given by,

$$E[m_x(T)] = \frac{1}{2T} \int_{-T}^{T} E[x(t)]\, dt = m_x$$

The process $x(t)$ is **ergodic in the mean** if,

1. The time average $m_x(T)$ approaches the ensemble average m_x in the limit as the observation interval T approaches infinity, that is

$$\lim_{T \to \infty} m_x(T) = m_x$$

2. The variance of $m_x(T)$ approaches zero in the limit as the observation interval T approaches infinity.

$$\lim_{T \to \infty} \operatorname{var}[m_x(T)] = 0$$

The time averaged autocorrelation function of a sample function $x(t)$

$$R_{XX}(\tau, T) = \frac{1}{2T} \int_{-T}^{T} x(t+\tau)\, x(t)\, dt$$

The process $x(t)$ is **ergodic in the autocorrelation** function if,

$$\lim_{T \to \infty} R_{XX}(\tau, T)] = R_{XX}(\tau)$$

$$\lim_{T \to \infty} \operatorname{Var}[R_{XX}(\tau, T)] = 0$$

For a random process to be ergodic, it has to be stationary; however the converse is not true.

3.10 GAUSSIAN PROCESS

Gaussian processes play an important role in communication systems. The fundamental reason for their importance is that ***thermal noise in electronic devices, which is produced by the random movement of electrons due to thermal agitation, can be closely modeled by a Gaussian process.***

In a resistor, free electrons move as a result of thermal agitation. The movement of these electrons is random, but their velocity is a functionof the ambient temperature. The higher the temperature, the higher the velocity of the electrons. The movement of these electrons generates a current with a random value. We can consider each electron in motion as a tiny current source, whose current is a random variable that can be positive or negative, depending on the direction of the movement of the electron. The total current generated by all electrons, which is the generated thermal noise, is the sum of the currents of all these current sources. We can assume that at least a majority of these sources behave independently and, therefore, the total current is the sum of a large number of independent and identically distributed random variables. Now, by applying the central limit theorem, we conclude that this total current has a Gaussian distribution. For this reason, thermal noise can be very well modeled by Gaussian random process.

Gaussian processes provide rather good models for some information sources as well. Some properties of the Gaussian processes, make these process mathematically tractable and easy to use.

Let Y be a random variable obtained by integrating the product of a random process X(t) for a time period of $t = 0$ to $t = \mathrm{T}$ and some function $g(t)$ given by:

$$\mathrm{Y} = \int_0^{\mathrm{T}} \mathrm{X}(t)\, g(t)\, dt$$

The weighting function in above equation is such that the mean-square value of the random variable Y is finite and if the random variable Y is a Gaussian distributed random variable for every g(t), then the process X(t) is said to be Gaussian process.

The random variable Y has a Gaussian distribution if its probability density function has the form,

$$f_{\mathrm{Y}}(y) = \frac{1}{\sqrt{2\pi}\,\sigma_{\mathrm{Y}}} \exp\left[-\frac{(y - m_{\mathrm{Y}})^2}{2\sigma_{\mathrm{Y}}^2}\right]$$

where m_{Y} is the mean and σ_{Y}^2 is the variance of the random variable Y.

A plot of this probability density function is shown in Figure 3.10, for the special case when the Gaussian random variable Y is normalized to have mean m_Y of zero and a variance σ_Y^2 of 1.

$$f_Y(y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right)$$

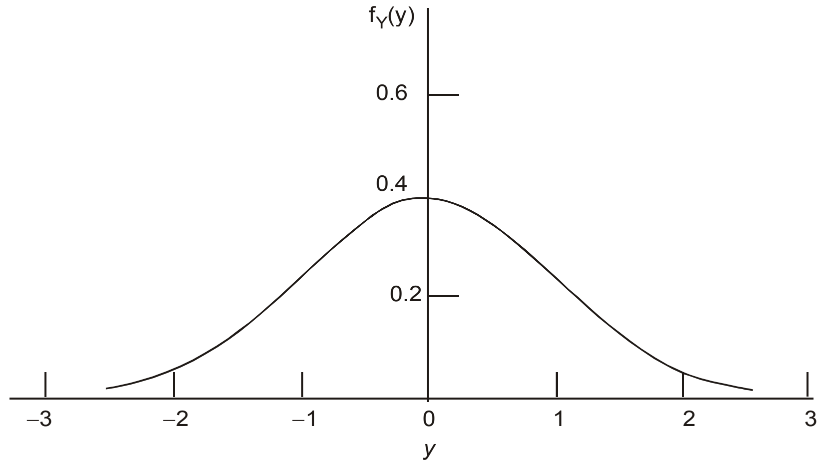


Figure 3.10 Normalized Gaussian Distribution

3.10.1 Advantages of Gaussian Process

1. Gaussian process has many properties that make ***analytic results possible***.
2. Random processes produced by ***physical phenomena are often such that a Gaussian model is appropriate***. Further the use of Gaussian model is confirmed by experiments.

3.10.2 Properties of Gaussian Process

1. If the set of random variables $X(t_1), X(t_2), X(t_3), \dots \dots, X(t_n)$ obtained by observing a random process $X(t)$ at times $t_1, t_2, \dots t_n$. and the process $X(t)$ is Gaussian, then this set of random variables is jointly Gaussian for any n, with their PDF completely specified by the set of means:

$$\boldsymbol{m_{X(t_i)} = E[X(t_i)]}, \text{ where } i = 1, 2, 3 \dots n$$

and the set of covariance functions,

$$\mathbf{C_X(t_k, t_i) = E[(X(t_k)\ m_{X(t_k)})(X(t_i) - m_{X(t_i)})]}\quad k,\ i = 1,2,3 \dots n$$

2. If the set of random variables $X(t_i)$ is uncorrelated, that is,

$$\mathbf{C_{ij} = 0} \qquad \boldsymbol{i \neq j}$$

then $X(t_i)$ are independent.

3. If a Gaussian process is stationary, then the process is also strictly stationary.
4. If a Gaussian process X(t) is passed through LTI filter, then the random process Y(t) at the output of the filter is also Gaussian process.

3.11 TRANSMISSION OF A RANDOM PROCESS THROUGH A LTI FILTER

When a random process X(t) is applied as input to a linear time-invariant filter of impulse response $h(t)$, producing a new random process Y(t) at the filter output as shown in Figure 3.11. It is difficult to describe the probability distribution of the Y(t), even when the probability distribution of the X(t) is completely specified.

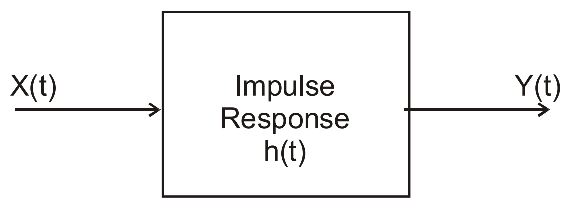


Figure 3.11 Transmission of a random process through a LTI filter

The time domain form of input-output relations of the filter for defining the ***mean and autocorrelation functions*** of the output random process Y(t) in terms of the input X(t), assuming X(t) is a stationary process. The transmission of a process through a LTI filter is governed by the ***convolution integral,*** where the output random process Y(t) is expressed in terms of input random process X(t) as:

$$\mathbf{Y}(t) = \int_{-\infty}^{\infty} h(\tau_1)\,\mathbf{X}(t-\tau_1)\,d\tau_1$$

where, τ_1 is the integration variable.

Mean of Y(t):

$$m_Y(t) = E[Y(t)]$$

$$= E\left[\int_{-\infty}^{\infty} h(\tau_1)\,X(t-\tau_1)\,d\tau_1\right]$$

Provided E[X(t)] is finite for all t and the system is stable.

Interchanging the order of expectation and integration, we get

$$m_Y(t) = \int_{-\infty}^{\infty} h(\tau_1)\,E[X(t-\tau_1)]\,d\tau_1$$

$$= \int_{-\infty}^{\infty} h(\tau_1)\,m_X(t-\tau_1)\,d\tau_1$$

When the input random process X(t) is stationary, the mean $m_X(t)$ is a constant m_X, so

$$m_Y = m_X \int_{-\infty}^{\infty} h(\tau_1)\,d\tau_1$$

$$\boldsymbol{m_Y = m_X\,\mathrm{H}(0)}$$

where, H(0) is the zero-frequency (DC) response of the system.

The mean of the output random process Y(t) produced at the output of a LTI system in response to input process X(t) is equal to the mean of X(t) multiplied by the DC response of the system.

Autocorrelation of Y(t):

$$R_{YY}(t_1,t_2) = E[Y(t_1)\,Y(t_2)]$$

Using convolution integral, we get

$$R_{YY}(t_1,t_2) = E\left[\int_{-\infty}^{\infty} h(\tau_1)\,X(t_1-\tau_1)\,d\tau_1 \int_{-\infty}^{\infty} h(\tau_1)\,X(t_2-\tau_2)\,d\tau_2\right]$$

Provided E[$X^2(t)$] is finite for all t and the system is stable,

$$R_{YY}(t_1,t_2) = \int_{-\infty}^{\infty} h(\tau_1)\,d\tau_1 \int_{-\infty}^{\infty} h(\tau_2)\,d\tau_2\,E[X(t_1-\tau_1)\,X(t_2-\tau_2)]$$

$$= \int_{-\infty}^{\infty} h(\tau_1)\,d\tau_1 \int_{-\infty}^{\infty} h(\tau_2)\,d\tau_2\,R_{XX}(\tau_1-\tau_1, t_2-\tau_2)$$

When the input X(t) is a stationary process, the autocorrelation function of X(t) is only a function of the difference between the observation times $t_1-\tau_1$ and $t_2-\tau_2$. Thus putting $\tau = \tau_1-\tau_2$ in above equation, we get

$$R_{YY}(\tau)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1)h(\tau_2)R_{XX}(\tau-\tau_1+\tau_2)\,d\tau_1\,d\tau_2$$

On combining this result with the mean m_Y ,we see that ***if the input to a stable linear time-invariant filter is a stationary process, then the output of the filter is also a stationary process.***

When $\tau=0$, $R_{YY}(0)=E[Y^2(t)]$ so

$$\mathbf{E[Y^2(t)]=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1)h(\tau_2)R_{XX}(\tau_2-\tau_1)\,d\tau_1\,d\tau_2}$$

which is a constant.

FORMULAE TO REMEMBER

- If A and B are independent events, then

$$P(A \cap B) = P(A) * P(B)$$

- If A and B are mutually exclusive events, then

$$P(A \cap B) = 0$$

- Conditional probability of event A given B,

$$P(A/B)=\frac{P(A\cap B)}{P(B)}$$

- Law of total probability

$$P(B)=\sum_{i=1}^{k} P(B/A_i)\,P(A_i)$$

- Baye's theorem

$$P(A_i/B)=\frac{P(B/A_i)\,P(A_i)}{P(B)}$$

- Cumulative Distribution Function or CDF of a random variable X

$$F_X(x)=P(X\le x)$$

- Probability Density Function or PDF of a continuous random variable X

$$f_X(x)=\frac{d}{dx}F_X(x)$$

- Probability Mass Function or PMF of a discrete random variable X

$$p_X(x) = P(X = x)$$

- Mean of the random variable

$$E[X] = \int_{-\infty}^{\infty} x\, f_x(x)\, dx \text{ (Continuous)}$$

$$E[X] = \Sigma_X\, x\, P[X = x] \text{ (Discrete)}$$

- Variance of the random variable

$$\sigma_X^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f_x(x)\, dx \text{ (Continuous)}$$

$$\sigma_X^2 = E[(X - \mu_x)^2] = \Sigma_x (x - \mu_x)^2\, P[X = x] \text{ (Discrete)}$$

- Covariance of two random variable

$$\text{Cov}(X,Y) = E[XY] - \mu_x \mu_y$$

- If Cov(X,Y) = 0, then X and Y are uncorrelated.
- If X and Y are independent, then Cov(X, Y) = 0.
- PMF of Binomial Random Variable

$$P(X = i) = \binom{n}{i} p^i (1-p)^{n-i}, \quad 0 \le i \le n$$

- PDF of Uniform Random Variable

$$f_x(x) = \begin{cases} \dfrac{1}{b-a}, & a < x < b \\ 0, & \text{Otherwise} \end{cases}$$

- PDF of Gaussian or Normal Random Variable

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}$$

- The random process X(t) is said to be Stationary in the Strict Sense (SSS) or Strictly Stationary if the following condition holds,

$$F_{X(t_1+\tau),\ldots,X(t_k+\tau)}(x_1,\ldots\ldots,x_k) = F_{X(t_1),\ldots,X(t_k)}(x_1,\ldots\ldots,x_k)$$

- Mean of the random process X(t) is defined as the:

$$m_x(t) = \mathrm{E}[\mathrm{X}(t)] = \int_{-\infty}^{\infty} x\, f_{x(t)}(x)\, dx$$

- Autocorrelation of the random process X(t) is:

$$\mathrm{R_{XX}}(t_1, t_2) = \mathrm{E}[\mathrm{X}(t_1)\,\mathrm{X}(t_2)]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2\, f_{x(t_1), x(t_2)}(x_1, x_2)\, dx_1\, dx_2$$

- Autocovariance function of random variable X(t) is:

$$\mathrm{C_{XX}}(t_1, t_2) = \mathrm{R_{XX}}(t_2 - t_1) - m_x^2$$

- Wide-Sense Stationary or Weakly Stationary Process (WSS) or Co-variance Stationary

$$m_{\mathrm{X}}(t) = \text{constant and } \mathrm{R_{XX}}(t_1, t_2) = \mathrm{R_{XX}}(t_2 - t_1)$$

- Power spectral density or power spectrum of the stationary process X(t)

$$\mathrm{S_{XX}}(f) = \int_{-\infty}^{\infty} \mathrm{R_{XX}}(\tau) \exp(-j2\pi f \tau_1)\, d\tau_1$$

- Einstein-Wiener-Khintchine relations

$$\mathrm{S_{XX}}(f) = \int_{-\infty}^{\infty} \mathrm{R_{XX}}(\tau) \exp(-j2\pi f \tau)\, d\tau$$

$$\mathrm{R_{XX}}(\tau) = \int_{-\infty}^{\infty} \mathrm{S_{XX}}(f) \exp(j2\pi f \tau)\, df$$

- $x(t)$ is ergodic in the mean if

$$\lim_{\mathrm{T}\to\infty} m_x(\mathrm{T}) = m_x,$$

$$\lim_{\mathrm{T}\to\infty} \mathrm{var}[m_x(\mathrm{T})] = 0$$

- $x(t)$ is ergodic in the autocorrelation function if:

$$\lim_{\mathrm{T}\to\infty} \mathrm{R_{XX}}(\tau, \mathrm{T}) = \mathrm{R_{XX}}(\tau),$$

$$\lim_{\mathrm{T}\to\infty} \mathrm{Var}[\mathrm{R_{XX}}(\tau, \mathrm{T})] = 0$$

- The mean of the output random process Y(t) produced at the output of a LTI system in response to input process X(t) is equal to the mean of X(t) multiplied by the DC response of the system.

$$m_{\mathrm{Y}} = m_{\mathrm{X}}\, \mathrm{H}(0)$$

SOLVED EXAMPLES

1. A telegraph source generates two symbols: Dot and Dash. The dots were twice as likely to occur as dashes. What is the probability of occurrence of dot and dash?

Solution: Given that,

$$P(\text{Dot}) = 2\,P(\text{Dash})$$

We know that the probability of sample space is 1, so

$$P(\text{Dot}) + P(\text{Dash}) = 1$$

Substituting, P(Dot) = 2 P(Dash) in above equation, we get

$$3\,P(\text{Dash}) = 1$$

Thus, $$P(\text{Dash}) = 1/3 \text{ and } P(\text{Dot}) = 2/3$$

2. Binary data are transmitted over a noisy communication channel in a block of 16 binary digits. The probability that a received digit is in error due to channel noise is 0.01.Assume that the errors occur in various digit positions within a block are independent. Find a) Mean errors per block, *b*) variance of the number of errors per block and c) Probability that the number of errors per block is greater than or equal to 4.

Solution: Let X denote the random variable of number of errors per block. Then, X has a binomial distribution.

(a) Mean $= np$

$$\text{Given, } n = 16 \text{ and } p = 0.01$$

$$\text{Mean error per block} = 16 * 0.01 = 0.16$$

(b) Variance $= np(1 - p)$

Variance of the number of errors per block = (16)(0.01)(0.99) = 0.158.

(c) Probability that the number of errors per block is greater than or equal to 4 is,

$$P(X \geq 4) = 1 - P(X \leq 3)$$

Using binomial distribution, we get,

$$P(X \leq 3) = \sum_{i=0}^{3} \binom{16}{i} (0.01)^i (0.99)^{16-i} = 0.986$$

Hence, $P(X \geq 4) = 1 - 0.986 = 0.014$

3. The PDF of a random variable X is given by $f_X(x) = k$, for $a \leq x \leq b$ and zero elsewhere, where k is a constant. Find the:

(i) Value of k

(ii) If a = – 1 and b = 2. Calculate P(|X| $\leq$ c) for c = 1/2.

Solution:

(i) Given that PDF $f_X(x)$ corresponds to uniform random variable, so the value of k is,

$$k = 1 / (b-a)$$

(ii) Using the value of $k = 1/(2+1) = 1/3$, PDF is given by $f_X(x) = 1/3$, for $-1 \leq x \leq 2$ and zero elsewhere,

$$P(|X| \leq 1/2) = P(-1/2 \leq X \leq 1/2) = \int_{-1/2}^{1/2} \left(\frac{1}{3}\right) dx$$

We get, $P(|X| \leq 1/2) = 1/3.$

4. Find the mean and variance of random variable X that takes the values 0 and 1 with probabilities 0.4 and 0.6 respectively.

Solution:

$$\text{Mean} = E[X] = \sum_X x\,P[X = x] = 0(0.4) + 1(0.6) = 0.6$$

$$\text{Variance} = \sigma_X^2 = \sum_X (x - \mu_X)^2\,P[X = x] = (0 - 0.6)^2(0.4) + (1 - 0.6)^2(0.6) = 0.24$$

5. Find the covariance of X and Y if (a) X and Y are independent (b) Y = aX + b.

Solution:

(a) $\text{Cov}(X,Y) = E[XY] - E[X]E[Y]$

Since, X and Y are independent, $E[XY] = E[X]\,E[Y]$. $\text{Cov}(X, Y) = 0$

(b) $Y = aX + b$

$$E[XY] = E[X(aX + b)] = a\,E[X^2] + b\,E[X] = a\,E[X^2] + b\,E[X]$$

$$E[Y] = E[aX + b] = aE[X] + E[b] = a\,E[X] + b$$

$$\text{Cov}(X,Y) = a\,E[X^2] + b\,E[X] - E[X]\,(aE[X] + b)$$

$$= a\,E[X^2] + b\,E[X] - aE^2[X] - bE[X]$$

$$\text{Cov}(X,Y) = a\,(E[X^2] - E^2[X])$$

6. Show that the random process $X(t) = A\cos(\omega_c t + \theta)$ is a wide-sense stationary process where, θ is a random variable uniformly distributed in the range $(0, 2\pi)$ and A and ω_c are constant.

Solution: For a random process to be WSS, it is necessary to show that,

- Mean is constant
- Autocorrelation function depends only on time difference.

The PDF of the uniform distribution is given by

$$f(\theta) = \frac{1}{2\pi}, \quad 0 \le \theta \le 2\pi$$

(a) Mean

$$E[x(t)] = \int_0^{2\pi} X(t) \frac{1}{2\pi} d\theta$$

$$= \frac{1}{2\pi} \int_0^{2\pi} A\cos(\omega_c t + \theta)\, d\theta$$

$$= \frac{A}{2\pi} [\sin(\omega_c t + \theta)]_0^{2\pi} = 0$$

Therefore, mean is a constant.

(b) Autocorrelation

$$R_{XX}(t, t+\tau) = E[A\cos(\omega_c t + \theta)\, A\cos(\omega_c t + \omega_c \tau + \theta)]$$

$$= \frac{A^2}{2} \cos(\omega_c \tau) + A^2\, 4\pi(0)$$

$$= \frac{A^2}{2} \cos(\omega_c \tau).$$

So, autocorrelation function depends on the time difference τ

As, mean is constant and autocorrelation function depends only on τ, X(t) is a wide-sense stationary process.

7. *Show that the random process X(t) = A cos$(\omega_c t + \theta)$is a wide-sense stationary* process where, θ and ω_c are constant and A is a random variable.

Solution: For a random process to be WSS, it is necessary to show that,

- Mean is constant
- Autocorrelation function depends only on time difference.

(a) Mean

$$E[X(t)] = E[A\cos(\omega_c t+\theta)]$$
$$=\cos(\omega_c t+\theta)\,E[A]$$
$$E[X(t)] \neq 0$$

Therefore, mean is not a constant.

(b) Autocorrelation

$$R_{XX}(t,t+\tau)=E[A\cos(\omega_c t+\theta)\,A\cos(\omega_c t+\omega_c\tau+\theta)]$$
$$=\frac{1}{2}[\cos(\omega_c\tau)+\cos(2\omega_c t+2\theta+\omega_c\tau)]\,E[A^2]$$

Thus, the autocorrelation of X(t) is not a function of time difference τ only.So the given random process X(t) is not WSS.

8. Let X(t) = A cos ωt+ B sin ωt and Y(t) = B cos t - A sin t, where A and B are independent random variables both having zero mean and variance σ^2, and ω is constant. Find the cross-correlation of X(t) and Y(t).

Solution: The cross-correlation of X(t) and Y(t) is :

$$R_{XY}(t_1,t_2)=E[X(t_1)\,Y(t_2)]$$
$$=E[(A\cos\omega t_1+B\sin\omega t_1)(B\cos\omega t_2-A\sin\omega t_2)]$$
$$=E[A\,B](\cos\omega t_1\cos\omega t_2-\sin\omega t_1\sin\omega t_2)$$
$$-\,E[A^2]\cos\omega t_1\sin\omega t_2+E[B^2]\sin\omega t_1\cos\omega t_2$$

Since, $E[AB]=E[A]E[B]=0$ and $E[A^2]=E[B^2]=\sigma^2$

$$R_{XY}(t_1,t_2)=\sigma^2(\sin\omega t_1\cos\omega t_2-\cos\omega t_1\sin\omega t_2)$$
$$=\sigma^2\sin\omega(t_1-t_2)$$
$$R_{XY}(\tau)=-\sigma^2\sin\omega\tau$$

where, $\tau = t_2 - t_1$.

9. The input X(*t*) to a diode with a transfer characteristic $Y = X^2$ is a zero mean stationary Gaussian random process with an autocorrelation function $R_{XX}(\tau) = \exp(-|\tau|)$. Find the mean $\mu_Y(t)$ and $R_{YY}(t_1, t_2)$.

Solution:

$$\mu_Y = E[Y(t)] = E[X^2(t)] = R_{XX}(0) = 1$$

$$R_{YY}(t_1, t_2) = E[Y(t_1)Y(t_2)] = E[X^2(t_1)\, X^2(t_2)]$$

For zero mean Gaussian random variable,

$$E[X^2(t_1)\, X^2(t_2)] = E[X^2(t_1)]\, E[X^2(t_2)] + 2E[X(t_1)X(t_2)]^2$$

where, $$E[X^2(t_1)] = E[X^2(t_2)] = R_{XX}(0)$$

$$E[X(t_1)X(t_2)] = R_{XX}(|t_1 - t_2|)$$

Since X(*t*) is stationary,

$$R_{YY}(t_1, t_2) = [R_{XX}(0)]^2 + 2\,[R_{XX}(|t_1 - t_2|)]^2$$

or $$R_{YY}(\tau) = [R_{XX}(0)]^2 + 2\,[R_{XX}(\tau)]^2 = 1 + 2\exp(-2|\tau|)$$

10. A wide sense stationary random process X(*t*) is applied to the input of an LTI system with impulse response $h(t) = 3e^{-2t}\, u(t)$. Calculate the mean of the output Y(*t*) of the system if E[X(*t*)] = 2.

Solution: The frequency response of the system can be obtained by taking Fourier transform of the impulse response as:

$$H(\omega) = F[h(t)] = 3\frac{1}{j\omega + 2}$$

The mean value of the output Y(*t*) can be obtained as:

$$m_Y = m_X\, H(0)$$

$$H(0) = 3*(1/2) = 3/2$$

Therefore, mean of the output $Y(t) = 2 * (3/2) = 3$

11. Let X and Y be real random variables with finite second moments. Prove the Cauchy-Schwarz inequality $(E[XY])^2 \le E[X^2]\, E[Y^2]$. (April/May 2015)

Solution: The mean square value of a random variable can never be negative value, so

$$E[(X - aY)^2] \ge 0, \quad \text{for any value of } a$$

Expanding above equation we get,

$$E[X^2] - 2a\,E[XY] + a^2\,E[Y^2] \geq 0$$

Substituting $a = E[X\,Y]/E[Y^2]$ to get left hand side of this inequality as minimum, we get

$$E[X^2] - (E[EY]^2 / E[Y^2] \geq 0$$

(or) $$(E[X\,Y])^2 \leq E[X^2]\,E[Y^2]$$

12. Let X(*t*) and Y(*t*) be both zero-mean and WSS random processes. Consider the random process Z(*t*) = X(*t*) + Y(*t*). Determine the autocorrelation and power spectrum of Z(*t*) if X(*t*) and Y(*t*) are jointly WSS. (Apr/May 2015)

Solution: The autocorrelation of Z(t) is given by:

$$R_{ZZ}(t_1, t_2) = E[Z(t_1)Z(t_2)]$$

$$= E[[X(t_1) + Y(t_1)][X(t_2) + Y(t_2)]]$$

$$= E[X(t_1)X(t_2)] + E[X(t_1)Y(t_2)] + E[Y(t_1)X(t_2)] + E[Y(t_1)Y(t_2)]$$

$$= R_{XX}(t_1, t_2) + R_{XY}(t_1, t_2) + R_{YX}(t_1, t_2) + R_{YY}(t_1, t_2)$$

As X(t) and Y(t) are jointly WSS,

$$R_{ZZ}(\tau) = R_{XX}(\tau) + R_{XY}(\tau) + R_{YX}(\tau) + R_{YY}(\tau), \text{ where } \tau = t_2 - t_1$$

We know that the Fourier transform of the autocorrelation function gives power spectrum, taking Fourier transform on both sides, we get

$$S_{ZZ}(\omega) = S_{XX}(\omega) + S_{XY}(\omega) + S_{YX}(\omega) + S_{YY}(\omega)$$

13. Let $X(t) = A\cos(\omega t + \phi)$ and $Y(t) = A\sin(\omega t + \phi)$ where A and ω are constants and ϕ is a uniform random variable $[0, 2\pi]$. Find the cross correlation of X(*t*) and Y(*t*). (Apr/May 2015)(May/June 2016)

Solution: The cross-correlation of X(***t***) and Y(***t***) is

$$R_{XY}(t, t+\tau) = E[X(t)\,Y(t+\tau)]$$

$$= E[A^2\cos(\omega t + \phi)\sin(\omega t + \phi)]$$

$$= \frac{A^2}{2}E[\sin(2\omega t + \omega\tau + 2\phi) - \sin(-\omega\tau)]$$

$$= \frac{A^2}{2}\{E[\sin(2\omega t + \omega\tau + 2\phi)] + E[\sin(\omega\tau)]\}$$

$$= \frac{A^2}{2}\{0 + E[\sin(\omega\tau)]\}$$

$$R_{XY}(t, t+\tau) = R_{XY}(\tau) = \frac{A^2}{2}\sin(\omega\tau)$$

14. In a binary communication system, let the probability of sending a 0 and 1 be 0.3 and 0.7 respectively. Let us assume that a 0 being transmitted, the probability of it being received as 1 is 0.01 and the probability of error for a transmission of 1 is 0.1.

(i) What is the probability that the output of this channel is 1?

(ii) If a 1 is received then what is the probability that the input to the channel was 1?

(Nov/Dec 2015)

Solution: Let X and Y denote the input and output of the channel. Given that

$$P(X = 0) = 0.3, \quad P(X = 1) = 0.7$$

$$P(Y = 0 \mid X = 0) = 0.99, \quad P(Y = 1 \mid X = 0) = 0.01$$

$$P(Y = 0 \mid X = 1) = 0.1, \quad P(Y = 1 \mid X = 1) = 0.9$$

(a) We know that, from the total probability theorem.

$$P(Y = 1) = P(Y = 1 \mid X = 0) * P(X = 0) + P(Y = 1 \mid X = 1) * P(X = 1)$$

$$= 0.01 * 0.3 + 0.9 * 0.7$$

$$P(Y = 1) = 0.633$$

The probability that the output of this channel is 1 is 0.633.

(b) Using Baye's rule

$$P(X=1 \mid Y=1) = \frac{P(X=1)\,P(Y=1 \mid X=1)}{P(X=0)\,P(Y=1 \mid X=0) + P(X=1)\,P(Y=1 \mid X=1)}$$

$$= (0.7 * 0.9) / (0.3 * 0.01 + 0.7 * 0.9)$$

$$P(X = 1 \mid Y = 1) = 0.9953$$

If a 1 is received then the probability that the input to the channel was 1 is 0.9953.

15. Given a random process, $X(t)=A\cos(\omega t+\mu)$ where A and ω are constants and μ is a uniformly distributed random variable. Show that X(*t*) is ergodic in both mean and autocorrelation. (May/June 2016)

Solution: For X(*t*) to be ergodic in mean and autocorrelation

$$\overline{X}=<X(t)>=E[X(t)]=\mu_x$$

$$\overline{R}_{XX}(\tau)=<X(t)\,X(t+\tau)>=E[X(t)\,X(t+\tau)]=R_{XX}(\tau)$$

Ensemble Average: $E[X(t)]=\int_{-\infty}^{\infty}A\cos(\omega t+\mu)\,f_{\mu}(\mu)\,d\mu$

Assume μ is uniformly distributed over $-\pi$ to π

$$E[X(t)]=\frac{A}{2\pi}\int_{-\pi}^{\pi}\cos(\omega t+\mu)\,d\mu=0$$

$$R_{XX}(\tau)=E[X(t)\,X(t+\tau)]$$

$$=\frac{A^2}{2\pi}\int_{-\pi}^{\pi}\cos(\omega t+\mu)\cos(\omega(t+\tau)+\mu)\,d\mu$$

$$R_{XX}(\tau)=\frac{A^2}{2}\cos\omega\tau$$

Time Average:

$$\overline{X}=<X(t)>=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}A\cos(\omega t+\mu)\,dt$$

$$=\frac{A}{T_0}\int_{-T_0/2}^{T_0/2}A\cos(\omega t+\mu)\,dt$$

$$\overline{x}=0$$

$$\overline{R}_{XX}(\tau)=\langle X(t)\,X(t+\tau)\rangle$$

$$=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}A^2\cos(\omega t+\mu)\cos(\omega(t+\tau)+\mu)\,dt$$

$$=\frac{A^2}{T_0}\int_{-T_0/2}^{T_0/2}\frac{1}{2}[\cos\omega\tau+\cos(2\omega t+2\mu+\omega\tau)\,dt$$

$$\overline{R}_{XX}(\tau)=\frac{A^2}{2}\cos\omega\tau.$$

Thus time averaged mean and autocorrelation is equal to ensemble averaged mean and autocorrelation. So the given process X(t) is ergodic in both mean and autocorrelation.

16. Consider two linear filters connected in cascade as shown in Fig.1. Let $X(t)$ be a stationary process with auto correlation function $R_x(\tau)$, the random process appearing at the first input filter is $V(t)$ and the second filter output is Y(t).

(a) Find the autocorrelation function of $Y(t)$.

(b) Find the cross correlation function $R_{vy}(\tau)$ of $V(t)$ and $Y(t)$. (Apr/May 2017)

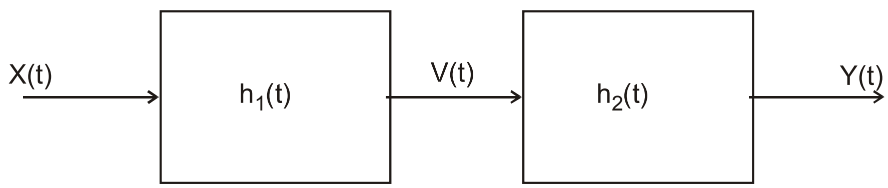


Fig. 1

Solution:

(a) The cascade connection of two filters is equivalent to a filter with the impulse response

$$h(t) = \int_{-\infty}^{\infty} h_1(\tau)\, h_2(t-\tau)\, d\tau$$

The autocorrelation function of Y(t) is given by:

$$R_Y(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1)\, h(\tau_2)\, R_X(\tau - \tau_1 + \tau_2)\, d\tau_1\, d\tau_2$$

(b) The cross correlation of V(t) and Y(t) is given by:

$$R_{VY}(\tau) = E[V(t+\tau)\, Y(t)]$$

The output Y(t) is given as $Y(t) = \int_{-\infty}^{\infty} V(\mu)\, h_2(t-\mu)\, d\mu$

So, $$R_{VY}(\tau) = E[V(t+\tau) \int_{-\infty}^{\infty} V(\mu)\, h_2(t-\mu)\, d\mu]$$

$$R_{VY}(\tau) = \int_{-\infty}^{\infty} h_2(t-\mu)\, R_V(t+\tau-\mu)\, d\mu$$

17. The amplitude modulated signal is defined as $X_{AM}(t) = A\,m(t)\cos(\omega_c t + \theta)$ where $m(t)$ is the baseband signal and $A\cos(\omega_c t + \theta)$ is the carrier. The baseband signal $m(t)$ is modeled as a zero mean stationary random process with the autocorrelation function $R_{xx}(\tau)$ and the PSD $G_x(f)$. The carrier amplitude A and frequency ω_c are assumed to be constant and the initial carrier phase θ is assumed to be random uniformly distributed in the interval $(-\pi, \pi)$. Furthermore, $m(t)$ and θ are assumed to be independent.

(i) Show that $X_{AM}(t)$ is Wide Sense Stationary

(ii) Find PSD of $X_{AM}(t)$. **(Apr/May 2017)**

Solution:

(i) For $X_{AM}(t)$ to be WSS, its :

- Mean $E[X_{AM}(t)] = \text{Constant}$
- Autocorrelation $E[X_{AM}(t)\, X_{AM}(t+\tau)]$ depends on τ

$$E[X_{AM}(t)] = E[A\,m(t)\cos(\omega_c t + \theta)]$$

$$= A\,E[m(t)]\,E[\cos(\omega_c t + \theta)]$$

Given that, $m(t)$ is zero mean stationary random process:

$$E[X_{AM}(t)] = 0 \text{ (Constant)}$$

$$R_{XAMXAM}(\tau) = E[X_{AM}(t)\, X_{AM}(t+\tau)]$$

$$= E[A\,m(t)\cos(\omega_c t + \theta)\, A\,m(t+\tau)\cos(\omega_c(t+\tau) + \theta)]$$

$$= A^2\, E[m(t)\,m(t+\tau)]\, E[\cos(\omega_c(t+\theta)\cos(\omega_c t + \omega_c\tau + \theta)]$$

$$= \frac{A^2}{2} R_{xx}(\tau)\, E[\cos\omega_c\tau + \cos(2\omega_c t + \omega_c\tau + 2\theta)]$$

$$R_{XAMXAM}(\tau) = \frac{A^2}{2} R_{xx}(\tau)\cos\omega_c\tau$$

Since mean of $X_{AM}(t)$ is constant and autocorrelation of $X_{AM}(t)$ depends on τ,

$X_{AM}(t)$ *is Wide Sense Stationary*

(ii) We know that the Fourier transform of the autocorrelation function gives power spectrum.

$$F[R_{XAMXAM}(\tau)] = F[\frac{A^2}{2} R_{xx}(\tau)\cos\omega_c\tau]$$

$$G_{XAMXAM}(\omega) = \frac{A^2}{2}\ F[R_{xx}(\tau)]\ F[\cos\omega_c\tau]$$

Given that PSD of $m(t)$ is $G_x(f)$.

We know that,

$$F[\cos\omega_c\tau] = \pi\delta(f - f_c) + \pi\delta(f + f_c)$$

Using frequency convolution theorem, we get

$$G_{XAMXAM}(f) = \frac{A^2}{2} G_x(f) * [\pi\delta(f - f_c) + \pi\delta(f + f_c)]$$

$$G_{XAMXAM}(f) = \frac{A^2\pi}{2}\ [G_x(f - f_c) + G_x(f + f_c)].$$

REVIEW QUESTIONS AND ANSWERS

PART- A

1. Define Random Variable. **(Nov/Dec 2015)**

A random variable is a function that assigns a real number X(S) to every element $s \in S$, where S is the sample space corresponding to a random experiment E.

Example: Tossing an unbiased coin twice. The outcomes of the experiment are HH, HT, TH,TT. Let X denote the number of heads turning up. Then X has the values 2,1,1,0.Here, X is a random variable which assigns a real number to every outcome of a random experiment.

2. **State Baye's rule.** **(Nov/Dec 2015)**

Baye's rule or Baye's theorem relates the conditional and marginal probabilities of stochasticevents A and B:

$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B)}$$

Where, P(A|B) is the conditional probability of A given B, P(B|A) is the conditional probability of B given A, P(A) is the marginal probability of A and P(B) is the marginal probability of B.

3. Define discrete random variable.

If X is a random variable which can take a finite number or countably infinite number of *p* values, X is called a discrete RV.Eg. Let X represent the sum of the numbers on the 2 dice, when two dice are thrown.

4. Define continuous random variable.

If X is a random variable which can take all values (i.e., infinite number of values) in an interval, then X is called a continuous RV. Eg. The time taken by a person who speaks over a telephone.

5. Define cumulative distribution function of a random variable.

The Cumulative Distribution Function (CDF) or distribution function of a random variable X is defined as, $F_X(X) = P\{\omega \in \Omega : X(\omega) \le x\}$, which can be simply written as

$$F_x(x) = P(X \le x)$$

6. List the properties of CDF.

1. $0 \leq F_x(x) \leq 1$
2. $F_x(x)$ is non-decreasing
3. $\lim_{x \to -\infty} F_x(x) = 0$ and $\lim_{x \to +\infty} F_x(x) = 1$
4. $F_x(x)$ is continuous from the right
5. $P(a < X \leq b) = F_x(b) - F_x(a)$
6. $P(X = a) = F_x(a) - F_x(a^-)$.

7. Define probability density function of a random variable.

The Probability Density Function or PDF of a continuous random variable X is defined as the derivative of its CDF. It is denoted by:

$$f_x(x) = \frac{d}{dx} F_x(x).$$

8. List the properties of PDF.

1. $f_x(x) \geq 0$
2. $\int_{-\infty}^{+\infty} f_x(x)\, dx = 1$
3. $\int_b^a f_x(x)\, dx = P(a < X \leq b)$
4. In general, $P(X \in A) = \int_A f_x(x)\, dx$
5. $F_x(x) = \int_{-\infty}^{x} f_x(u)\, du.$

9. Define mean of a random variable.

The mean of the random variable X is defined as:

$$E[X] = \int_{-\infty}^{\infty} x\, f_x(x)\, dx \quad \text{(Continuous)}$$

$$\mu_x = E[X] = \sum_x x\, P[X = x] \quad \text{(Discrete)}$$

10. Define variance of a random variable.

The variance of the random variable X is defined as:

$$\sigma_X^2 = \int_{-\infty}^{\infty} (x-\mu_X)^2 f_x(x)\, dx \quad \text{(Continuous)}$$

$$\sigma_X^2 = \sum_x (x-\mu_X)^2 \, P[X=x] \quad \text{(Discrete)}$$

11. Define covariance of a random variable.

The covariance of two random variables X and Y is defined as the expectation of the product of the two random variables given by:

$$\text{Cov}(X,Y) = E[XY] - \mu_X \mu_Y$$

12. Define correlation coefficient.

The correlation coefficient of two random variable is defined as:

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

where, the value of correlation coefficient ranges from –1 to 1.

13. When the two random variables are said to be uncorrelated?

The two random variables X and Y are said to be uncorrelated, if their covariance value is zero.

$$\text{Cov}(X,Y) = 0$$

14. Give the PMF of binomial random variable.

$$P(X=i) = \binom{n}{i} p^i (1-p)^{n-i}, \quad 0 \le i \le n$$

15. Give the mean and variance of binomial random variable.

Mean of binomial random variable is given by, $\mu = E(X) = np$

Variance of binomial random variable is given by, $\sigma^2 = np(1-p)$.

16. What is the importance of binomial random variable in the communication?

Binomial random variable can be used to model the total number of bits received in error, when sequence of *n* bits is transmitted over a channelwith a bit-error probability of *p*.

17. Give the PDF for uniform random variable.

This is a continuous random variable taking values between *a* and *b* with equal probabilities for intervals of equal length. The probability density function is given by:

$$f_X(x)=\begin{cases}\dfrac{1}{b-a}, & a<x<b\\ 0, & \text{Otherwise}\end{cases}$$

18. What is the importance of uniform random variable in the communication?

The phase of a received sinusoid carrier and quantization errors are usually modelled as a uniform random variable.

19. Give the PDF for Gaussian random variable.

The Gaussian or normal random variable is a continuous random variable described by the density function as :

$$f_x(x)=\frac{1}{\sqrt{2\pi}\,\sigma}e^{-\frac{(x-m)^2}{2\sigma^2}}$$

20. What is the significance of Gaussian random variable?

- The Gaussian random variable is the most important and frequently encountered random variable in communication systems. The reason is that thermal noise, which is the major source of noise in communication, has a Gaussian distribution.
- In robotics, Gaussian PDF is used to statistically characterize sensor measurements, robot locations and map representations.

21. List the properties of Gaussian random variable.

- The sum of two independent Gaussian random variables is also a Gaussian randomvariable.
- The weighted sum of N independent Gaussian random variables is a Gaussian randomvariable.
- If two Gaussian random variables have zero covariance (uncorrelated), they are also independent.

22. State Central Limit Theorem. (May/June 2016)(Nov/Dec 2016)

Central limit theorem states that the normalized distribution of the sum of independent, identically distributed random variables approaches a Gaussian distribution as the number of random variables increases, regardless of the individual distributions.

23. List the applications of Central Limit Theorem.

- Signal processing
- Channel modelling
- Finance
- Population statistics
- Hypothesis testing
- Engineering research

24. Define Random Process.

A random process is defined as rule which assigns a function of time to each outcome‘s’ of a random experiment.

25. Differentiate random process from random variable.

Random variable is a mapping of event outcome to real numbers, whereas random process is the mapping of event outcome to signal waveforms. Random process is a function of time, but random variable is not a function of time.

26. What are the types of random process?

Based on the continuous or discrete nature of the state space S and parameter set T, a random process can be classified into discrete random sequence, continuous random sequence, discrete random process and continuous random process.

Based on the stationarity, a random process can be classified into stationary and non-stationary random process.

27. Define stationary random process.

A random process whose statistical characteristics do not change with time is classified as stationary random process or stationary process.

28. What is strict-sense stationary?

The random process X(t) is said to be Stationary in the Strict Sense (SSS) or strictly stationary if its statistics are invariant to a shift of origin:

$$F_{X(t_1+\tau),\ldots\ldots,X(t_k+\tau)}(x_1,\ldots\ldots.,x_k) = F_{X(t_1),\ldots\ldots X(t_k)}(x_1,\ldots\ldots.,x_k)$$

29. Define mean of the random process.

The mean of process X(t) is defined as the expectation of the random variable obtained by observing the process at some time t given by:

$$m_X(t) = E[X(t)] = \int_{-\infty}^{\infty} x\, f_{x(t)}(x)\, dx \cdot$$

30. Define autocorrelation of the random process. **(May/June 2016)**

Autocorrelation of the process X(t) is given by the expectation of the product of two random variables X(t_1) and X(t_2) obtained by observing the process X(t) at times t_1 and t_2 respectively.

$$R_{XX}(t_1, t_2) = E[X(t_1)X(t_2)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2\, f_{X(t_1),\, X(t_2)}(x_1, x_2)\, dx_1\, dx_2$$

31. List the properties of autocorrelation function.

- The mean square value of a process is equal to the value of autocorrelation at $\tau = 0$.

$$R_{XX}(0) = E[X^2(T)]$$

- The autocorrelation function is an even function of τ.

$$R_{XX}(\tau) = R_{XX}(-\tau)$$

- The autocorrelation function is maximum at $\tau = 0$.

$$|R_{XX}(\tau)| \leq R_{XX}(0)$$

32. Define autocovariance of the random process.

The autocovariance function of X(t) is defined as:

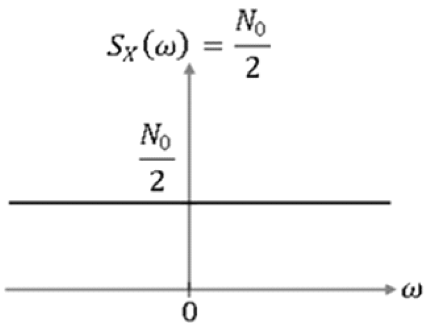
$$C_{XX}(t_1, t_2) = R_{XX}(t_2 - t_1) - m_X^2$$

33. Define white noise process.

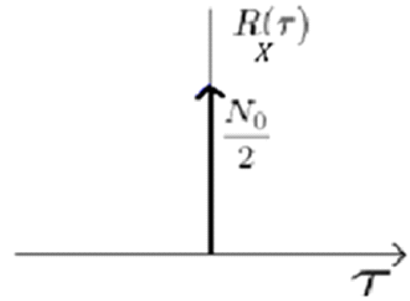
A random process is said to be white noise process X(t), if it is zero mean and power spectral density is defined as:

$$S_X(\omega) = N_0/2, \text{ for all frequencies}$$

34. Draw the PSD and autocorrelation function of white noise.



PSD of white noise



Autocorrelation of white noise

35. When a random process is said to be white Gaussian noise process?

The random process X(t) is said to be white Gaussian noise process, if X(t) is a stationary Gaussian random process with zero mean and flat power spectral density.

36. What is Wide-Sense Stationary? (Apr/May 2017)

A random process is called wide-sense stationary (WSS) if its

- Mean is constant.
- Autocorrelation depends only on the time difference.

37. Define Power Spectral Density of the random process.

The distribution of the power of the random processat different frequencies is the Power Spectral Density or Power Spectrum of the random process.

38. Give the power spectral density equation of a random process X.

$$S_{XX}(f) = \int_{-\infty}^{\infty} R_{XX}(\tau) \exp(-j2\pi f \tau_1)\, d\tau_1$$

39. List the properties of power spectral density.

- The zero-frequency value of the power spectral density of a stationary process equals the total area under the graph of the autocorrelation function.
- The mean square value of a stationary process equals the total area under the graph of the power spectral density.

40. What is ergodicity?

A random process is said to be ergodic if time averages are the same for all sample functions and equal to the corresponding ensemble averages.

41. When a process is ergodic in mean?

A stationary process is called ergodic in the mean if:

$$\lim_{T\to\infty} m_x(T) = m_x$$

$$\lim_{T\to\infty} \text{var}[m_x(T)] = 0$$

42. When a process is ergodic in autocorrelation?

A stationary process is calledergodic in the autocorrelation function if,

$$\lim_{T\to\infty} R_{xx}(\tau, T) = R_{xx}(\tau)$$

$$\lim_{T\to\infty} \text{Var}[R_{xx}(\tau, T)] = 0$$

43. Write Einstein-Wiener-Khintchine relations. (Nov/Dec 2016) (Apr/May 2017)

$$S_{XX}(f) = \int_{-\infty}^{\infty} R_{XX}(\tau) \exp(-j2\pi f\tau)\, d\tau$$

$$R_{XX}(\tau) = \int_{-\infty}^{\infty} S_{XX}(f) \exp(j2\pi f\tau)\, df$$

44. Give the importance of Wiener-Khintchine relations.

For a stationary process, the power spectral density can be obtained from the Fourier transform of the autocorrelation function.

45. What are the advantages of Gaussian process?

- Gaussian process has many properties that make ***analytic results possible***.
- Random processes produced by physical phenomena are often such that a Gaussian model is appropriate. Further the use of Gaussian model is confirmed by experiments.

46. List the properties of Gaussian process.

- If a Gaussian process is stationary, then the process is also strictly stationary.
- If a Gaussian process X(t) is passed through LTI filter, then the random process Y(t) at the output of the filter is also Gaussian process.

PART – B

1. Let X and Y be real random variables with finite second moments. Prove the Cauchy-Schwartz inequality $(E[XY])^2 \le E[X^2]E[Y^2]$. **(Apr/May 2015)**

2. Differentiate SSS with that of WSS process. **(Apr/May 2015)**

3. Let $X(t)$ and $Y(t)$ be both zero-mean and WSS random processes. Consider the random process $Z(t) = X(t) + Y(t)$. Determine the autocorrelation and power spectrum of $Z(t)$ if $X(t)$ and $Y(t)$ are jointly WSS. **(Apr/May 2015)**

4. Let $X(t) = A\cos(\omega t + \phi)$ and $Y(t) = A\sin(\omega t + \phi)$ where A and ω are constants and ϕ is a uniform random variable $[0, 2\pi]$. Find the cross correlation of $X(t)$ and $Y(t)$.

   (Apr/May 2015)(May/June 2016)

5. In a binary communication system, let the probability of sending a 0 and 1 be 0.3 and 0.7 respectively. Let us assume that a 0 being transmitted, the probability of it being received as 1 is 0.01 and the probability of error for a transmission of 1 is 0.1.

   (i) What is the probability that the output of this channel is 1?

   (ii) If a 1 is received then what is the probability that the input to the channel was 1?

   (Nov/Dec 2015)

6. What is CDF and PDF? State their properties. Also discuss them in detail by giving examples of CDF and PDF for different types of random variables. **(Nov/Dec 2015)**

7. Explain in detail about the transmission of a random process through a linear time invariant filter. **(May/June 2016)(Nov/Dec 2016)**

8. When a random process is said to be strict sense stationary (SSS), wide sense stationary (WSS) and ergodic process? **(May/June 2016)(Nov/Dec 2016)**

9. Given a random process, $X(t) = A\cos(\omega t + \mu)$ where A and ω are constants and μ is a uniformly distributed random variable. Show that $X(t)$ is ergodic in both mean and autocorrelation. **(May/June 2016)**

10. Define the following: Mean, Correlation, Covariance and Ergodicity. **(Nov/Dec 2016)**

11. What is a Gaussian random process and mention its properties. **(Nov/Dec 2016)**

12. Consider two linear filters connected in cascade as shown in fig.1. Let $X(t)$ be a stationary

process with auto correlation function $R_x(\tau)$, the random processappearing at the first input filter is $V(t)$ and the second filter output is $Y(t)$.

(a) Find the autocorrelation function of $Y(t)$.

(b) Find the cross correlation function $R_{vy}(\tau)$ of $V(t)$ and $Y(t)$. **(Apr/May 2017)**

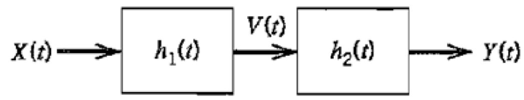


Fig. 1

13. The amplitude modulated signal is defined as $X_{AM}(t) = Am(t)\cos(\omega_c t + \theta)$ where $m(t)$ is the baseband signal and $A\cos(\omega_c t + \theta)$ is the carrier. The baseband signal $m(t)$ is modeled as a zero mean stationary random process with the autocorrelation function $R_{xx}(\tau)$ and the PSD $G_x(f)$. The carrier amplitude A and frequency ω_c areassumed to be constant and the initial carrier phase θ is assumed to be random uniformly distributed in the interval $(-\pi, \pi)$. Furthermore, m(t) and θ are assumed to be independent.

(i) Show that $X_{AM}(t)$ is Wide Sense Stationary

(ii) Find PSD of $X_{AM}(t)$. **(Apr/May 2017)**

❐ ❐ ❐

4.12 NOISE PERFORMANCE IN AM SYSTEMS

4.12.1 Noise in AM Receivers Using Envelope Detection. [AM with carrier]

✓ Block diagram – AM Receiver using Envelope Detector

✓ Filtered signal $x(t)$

✓ Phasor Diagram

✓ Receiver output signal $y(t)$

✓ $(SNR)_c$

✓ $(SNR)_o$

✓ Figure of merit (γ)

✓ Threshold Effect in AM.

Block Diagram:

The block diagram of noise in AM receiver using envelope detector is shown in figure.

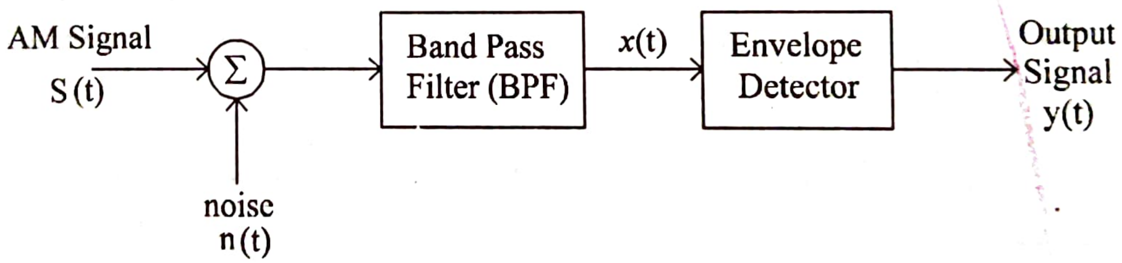


Figure 4.24 Block dia – AM receiver using Envelope detector

Channel Signal – to – Noise Ratio $(SNR)_c$:

$$(SNR)_c = \frac{\text{Average signal power } (P_{Sc})}{\text{Average noise power } (P_{Nc})}$$

Average Signal Power *(P_{Sc})*

To find the figure of merit (γ) the following components are discussed under.

The envelope detector consists of modulated message signal $s(t)$ + noise $n(t)$.

Filtered (BPF) signal $x(t)$:

$$x(t) = s(t) + n(t) \quad \text{...(1)}$$

$x(t) \rightarrow$ output of BPF

$s(t) \rightarrow$ AM [Modulated] signal

$n(t) \rightarrow$ Noise

Representing $n(t)$ in terms of inphase (I) & quadrature (Q) components.

$$(1) \Rightarrow x(t) = s(t) + n_I(t)\cos\omega_c t + n_Q(t)\sin\omega_c t \quad ...(2)$$

$$s(t) = V_c\,[1 + m_a V_m(t)]\cos\omega_c t \quad ...(3)$$

Substitute eqn (3) in (2) $\qquad \because \omega_c = 2\pi f_c$

$$(2) \Rightarrow x(t) = V_c\,[1 + m_a V_m(t)]\cos 2\pi f_c t + n_I(t)\cos 2\pi f_c t + n_Q(t)\sin 2\pi f_c t \quad ...(4)$$

$$\boxed{x(t) = [V_c + V_c m_a V_m(t) + n_I(t)]\cos 2\pi f_c t + n_Q(t)\sin 2\pi f_c t} \quad ...(5)$$

Phasor Diagram (Representation) of $x(t)$:

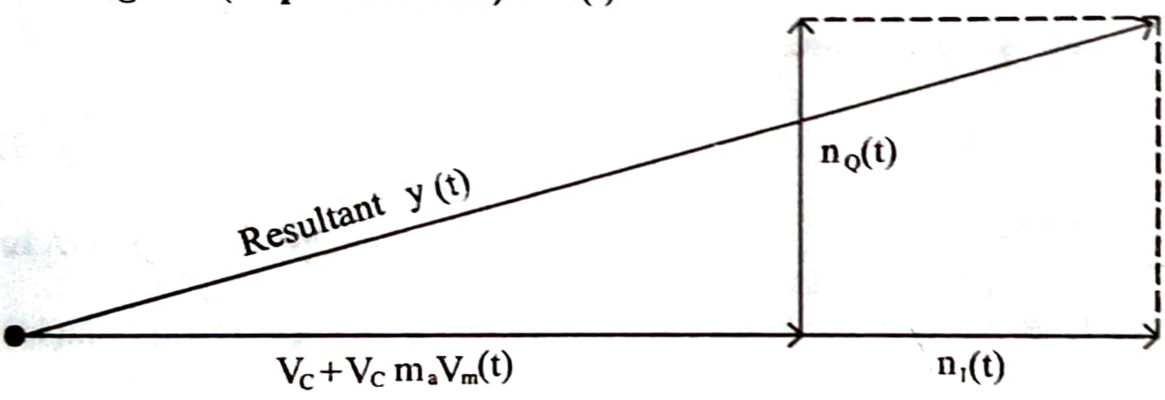


Figure 4.25 Phasor diagram of x (t)

Receiver Output Signal $y(t)$:

The output of the envelope detector is $y(t)$.

$$\boxed{y(t) = \sqrt{[V_c + V_c m_a V_m(t) + n_I(t)]^2 + [n_Q(t)]^2}} \quad ...(6)$$

When the signal is large ↑ compared to noise power, then $n_I(t)$ & $n_Q(t)$ will be very small compared to $V_c\,(1 + m_a V_m(t))$.

Then the approximated equation $y(t)$ is

$$y(t) = V_c + V_c m_a V_m(t) \quad ...(7)$$

The first term V_c in the above equation is the carrier amplitude (voltage) & it is removed with help of blocking capacitor after envelope detector.

$$(7) \Rightarrow y(t) = V_c\, m_a\, V_m(t) \qquad ...(8)$$

$y(t)$ → Receiver output signal

V_c → Carrier Amplitude.

m_a → Modulation Index for AM

$V_m(t)$ → Message signal.

Channel Average Signal Power (P_{Sc}):

The total Power P_t in the AM,

$$P_t = P_c\left(1 + \frac{m_a^2}{2}\right) \qquad ...(9)$$

P_c → carrier power

$$P_c = \frac{V_c^2}{2} \qquad ...(10)$$

Sub (10) in (9)

$$P_t = \frac{V_c^2}{2}\left(1 + \frac{m_a^2}{2}\right) \qquad ...(11)$$

$\frac{m_a^2}{2}$ is normalized power of message signal in eqn (11)

If 'P' is the average power of message signal,

$$\text{Average Signal Power} = \frac{V_c^2}{2}(1 + m_a^2\, P) \qquad ...(12)$$

$$\boxed{P_{Sc} = \frac{V_c^2\,(1 + m_a^2\, P)}{2}} \qquad ...(13)$$

Channel Average Noise Power (P_{Nc}):

$$P_{Nc} = P_n \qquad ...(14)$$

Where $$P_n = \int_{-w}^{+w} S_N(f)\cdot df \qquad ...(15)$$

$\because S_N(f) = \frac{N_o}{2} \rightarrow$ Power Spectral Density of white Gaussian noise. (PSD WGN)

$$P_{Nc} = P_n = \int_{-w}^{+w} \frac{N_o}{2} \cdot df \quad \text{...(16)}$$

$$P_{Nc} = \frac{N_o}{2}[2W] = N_o W$$

$$\boxed{P_{Nc} = N_o W} \quad \text{...(17)}$$

Channel Signal to Noise Ratio $(SNR)_c$

$$(SNR)_c = \frac{P_{Sc}}{P_{Nc}} \quad \text{...(18)}$$

Sub eqns (13) & (17) in eqn (18)

$$(SNR)_c = \frac{\frac{V_c^2(1+m_a^2 P)}{2}}{N_o W}$$

$$\boxed{(SNR)_c = \frac{V_c^2(1+m_a^2 P)}{2N_o W}} \quad \text{...(19)}$$

Output Average signal power (P_{So}):

$$\boxed{P_{So} = \frac{V_c^2 m_a^2 P}{2}} \quad \text{...(20)}$$

Output Average Noise Power (P_{No}):

Average Noise power at receiver output

$$\boxed{P_{No} = N_o W} \quad \text{...(21)}$$

Output Signal to Noise Ratio $(SNR)_o$:

$$\boxed{(SNR)_o = \frac{P_{So}}{P_{No}}} \quad \text{...(22)}$$

Sub eqns (20) & (21) in eqn (22)

$$(SNR)_o = \frac{\frac{V_c^2 m_a^2 P}{2}}{N_o W}$$

$$\boxed{(SNR)_o = \frac{V_c^2 m_a^2 P}{2N_o W}} \quad \text{...(23)}$$

Figure of merit (γ):

$$\gamma = \frac{(SNR)_o}{(SNR)_c} \quad ...(24)$$

sub equations (23) & (19) in eqn (24)

$$\gamma = \frac{\dfrac{V_c^2\, m_a^2\, P}{2N_o W}}{\dfrac{V_c^2 (1 + m_a^2 P)}{2N_o W}}$$

$$\boxed{\gamma = \frac{m_a^2\, P}{(1 + m_a^2\, P)}} \quad ...(25)$$

- ✓ The Figure of merit γ of an AM receiver using envelope detection is always **less than unity**.
- ✓ The noise performance of a full AM Receiver is always inferior to that of a DSB - SC Receiver.
- ✓ This is due to the wastage of transmitted power which results from transmitting the carrier as a component of AM wave.
- ✓ Let us assume that Receiver noise power is greater than the signal power.

$$\therefore y(t) = \sqrt{[\{V_c\,(1 + m_a V_m(t))\} + n_1(t)]^2 + n_Q^2(t)} \quad ...(26)$$

$$y(t) = \sqrt{V_c^2[1 + m_a V_m(t)]^2 + n_1^2(t) + 2V_c[1 + m_a V_m(t)]\,n_1(t) + n_Q^2(t)} \quad ...(27)$$

Consider $[1 + m_a V_m(t)]^2$ is very very small.

$$\therefore y(t) = \sqrt{n_1^2(t) + n_Q^2(t) + 2V_c\, n_1(t)\,[1 + m_a V_m(t)]}$$

$$y(t) = \sqrt{\left(n_1^2(t) + n_Q^2(t)\right)\left[1 + \frac{2V_c\, n_1(t)\,[1 + m_a V_m(t)]}{n_1^2(t) + n_Q^2(t)}\right]} \quad ...(28)$$

$$n_1^2(t) + n_Q^2(t) = r^2(t) \quad ...(29)$$

Sub eqn (29) in (28)

$$y(t) = \sqrt{r^2(t)\left[1 + \frac{2V_c n_I(t)[1 + m_a V_m(t)]}{r^2(t)}\right]}$$

$$y(t) = \sqrt{r^2(t)}\sqrt{1 + \frac{2V_c n_I(t)[1 + m_a V_m(t)]}{r^2(t)}}$$

$$y(t) = r(t)\left[1 + \frac{2V_c n_I(t)[1 + m_a V_m(t)]}{r^2(t)}\right]^{1/2}$$

$$y(t) \simeq r(t)\left[1 + \frac{2V_c n_I(t)[1 + m_a V_m(t)]}{2r^2(t)} + \cdots\right]$$

$$y(t) \simeq r(t)\left[1 + \frac{V_c n_I(t)[1 + m_a V_m(t)]}{r^2(t)}\right] \quad ...(30)$$

- ✓ From the above equation it is observed that at the demodulator output, the signal and the noise components are no longer additive and infact the signal component is multiplied by the noise and is no longer distinguishable.
- ✓ In this case, no meaningful SNR can be defined.
- ✓ Thus the system is operating below the **threshold**.
- ✓ The loss of message in an envelope detector that operates at low signal to noise ratio is called **threshold effect.**

Threshold Effect in AM:

Definition:

- ✓ When the carrier to noise ratio reduces below certain value, the message information is lost. The performance of envelope detector deteriorates rapidly and it has no proportion to carrier to noise ratio. This is called **threshold effect** in AM system.
- ✓ Every **non linear** receiver exhibits **threshold effect.**

- ✓ **Coherent receivers do not** have threshold effect.
- ✓ The detector output does not depend only on message signal $V_m(t)$, rather it is the function of noise also.
- ✓ As the value of the input signal to noise ratio below which output signal to noise ratio deteriorates much more rapidly than input signal to noise ratio is called as **threshold effect in AM.**

4.12.2 Noise in DSB – SC – AM using Coherent Detection

- ✓ Block diagram
- ✓ Average signal power (P_{sc})
- ✓ Average noise power (P_{Nc})
- ✓ Channel SNR $(SNR)_c$
- ✓ Output of BPF $x(t)$
- ✓ Output of product modulator $v(t)$
- ✓ Output of LPF $y(t)$
- ✓ Output SNR $(SNR)_o$
- ✓ Figure of merit (γ)

Block Diagram:

- ✓ The block diagram of the DSB – SC – AM noise performance system is figure.

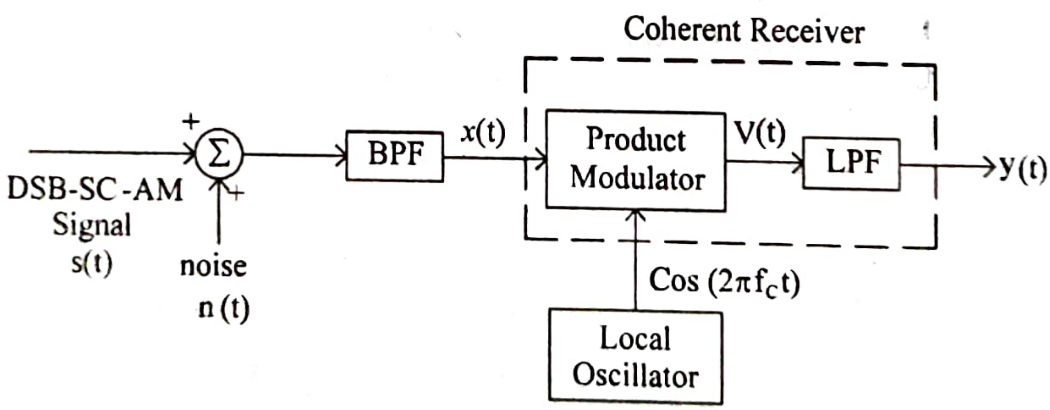


Figure 4.26 Noise performance of DSB – SC – AM receiver using coherent detection system

✓ The DSB – SC – AM signal $s(t)$ from the transmitter is passed thro' the channel to the receiver which is added with noise $n(t)$.

✓ The receiver signal is passed through BPF (Band pass filter) and the output of the BPF is $x(t)$.

✓ The product modulator receives signal from BPF as $x(t)$ and from a locally generated sinusoidal wave $\cos(2\pi f_c t)$.

✓ The $x(t)$ and $\cos(2\pi f_c t)$ are multiplied in the product modulator and then output of product modulator is V (t) which is low – pass filtered (LPF).

✓ The output of the LPF is given as $y(t)$.

✓ The noise $n(t)$ at the input of the receiver is **white** and **Gaussian** in nature **(WGN – White Gaussian Noise)**.

The DSB – SC – AM signal is given as,

$$s(t) = V_c V_m(t) \cos 2\pi f_c t \quad \text{...(1)}$$

$V_c \rightarrow$ carrier amplitude

$\cos 2\pi f_c t \rightarrow$ sinusoidal carrier wave

$V_m(t) \rightarrow$ message signal

The noise signal n (t) is given by,

$$n(t) = n_I(t) \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t) \quad \text{...(2)}$$

Average signal power (P_{Sc}):

✓ Assume that $V_m(t)$ is the sample function of a stationary process of zero mean.

✓ The PSD (power spectral Density) $S_N(f)$ is limited to a maximum frequency W (message bandwidth).

✓ The average power of the message sagnal is

$$P = \int_{-w}^{+w} S_N(f)\, df \quad \text{...(3)}$$

From equation (1) $[s(t)]$, the average signal power = $[s(t)]^2$

$$P_{Sc} = [V_c \cos(2\pi f_c t).V_m(t)]^2 \quad ...(4)$$

$$P_{Sc} = V_c^2 \cos^2(2\pi f_c t)\cdot V_m^2(t)$$

$$P_{Sc} = V_c^2\left(\frac{1+\cos 4\pi f_c t}{2}\right)\cdot P_m$$

Where, $P_m \rightarrow$ Average power of the original message signal $V_m(t)$.

The average power of the DSB - SC modulated signal is,

$$\boxed{P_{Sc} = \frac{V_c^2 P_m}{2}} \quad ...(5)$$

Average noise power (P_{Nc}):

The noise power at the channel given by,

$$\boxed{P_{Nc} = P_n} \quad ...(6)$$

Where,

$P_n \rightarrow$ Power of the noise signal having bandwidth W for DSB - SC - AM.

$$P_n = \int_{-\infty}^{+\infty} S_N(f)\,df \;;\; -W \le f \le W \quad ...(7)$$

For white noise, $S_N(f) = \frac{N_o}{2}$ ...(8)

$$P_n = \int_{-w}^{+w} \frac{N_o}{2}\,df$$

$$P_n = \frac{N_o}{2}\int_{-w}^{+w} df$$

$$P_n = \frac{N_o}{2}[f]_{-w}^{+w}$$

$$P_n = \frac{N_o}{2}[(W+W] = \frac{N_o(2W)}{2}$$

$$P_n = W N_o$$

$$\boxed{P_{Nc} = N_o W} \quad ...(9)$$

$(SNR)_c$ Channel Signal - to - Noise Ratio: $\left(\frac{S}{N}\right)_c$

$$(SNR)_c = \frac{P_{Sc}}{P_{Nc}} = \frac{\text{Average signal power}}{\text{Average Noise power}}$$

Sub eqns (5) & (9)

$$(SNR)_c = \frac{\frac{V_c^2 P_m}{2}}{W N_o}$$

$$\boxed{(SNR)_c = \frac{V_c^2 P_m}{2 W N_o}} \quad ...(10)$$

DSB - SB - AM Filtered (BPF) signal ($x(t)$):

$$\boxed{x(t) = s(t) + n(t)} \quad ...(11)$$

Substitute eqn (1) & (2) in eqn (11)

$$\boxed{x(t) = V_c V_m(t) \cos(2\pi f_c t) + n_I(t) \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t)} \quad ...(12)$$

$n_I(t) \rightarrow$ In - phase noise component.

$n_Q(t) \rightarrow$ Quadrature noise component.

Output of the product modulator V (t):

$$V(t) = x(t) \times \cos(2\pi f_c t) \quad ...(13)$$

Substitute equ (12) in (13)

$$V(t) = [V_c V_m(t) \cos(2\pi f_c t) + n_I(t) \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t)] \cos(2\pi f_c t)$$

$$V(t) = V_c V_m(t) \cos^2(2\pi f_c t) + n_I(t) \cos^2(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t) \cos(2\pi f_c t)$$

$$V(t) = [V_c V_m(t) + n_I(t)] \cos^2(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t) \cos(2\pi f_c t)$$

$$V(t) = (V_c V_m(t) + n_I(t)) \left(\frac{1 + \cos(4\pi f_c t)}{2}\right) - n_Q(t) \frac{\sin(4\pi f_c t)}{2}$$

$$\cos^2\theta = \frac{1 + \sin 2\theta}{2}$$

$$2 \sin\theta \cos\theta = \sin 2\theta$$

$$\sin\theta \cos\theta = \frac{\sin 2\theta}{2}$$

$$\boxed{V(t) = [V_c V_m(t) + n_I(t)] \left[\frac{1 + \cos 4\pi f_c t}{2}\right] - n_Q(t) \frac{\sin 4\pi f_c t}{2}} \quad ...(14)$$

Output of the LPF: [$y(t)$]:

The LPF (Low Pass Filter) allows only low frequency components & removes the high frequency components.

$$y(t) = \left[\frac{V_c V_m(t) + n_I(t)}{2}\right]$$

$$\boxed{y(t) = \frac{1}{2} V_c V_m(t) + \frac{1}{2} n_I(t)} \quad ...(15)$$

(1) The message signal $V_m(t)$ and In - phase noise component $n_I(t)$ of the filtered noise $n(t)$ appear additively at the receiver output.

(2) The quadrature component $n_Q(t)$ of the noise $n(t)$ is completely rejected by the coherent detector.

Signal Power at the Output (P_{S_o}):

The message signal component at the receiver output $= \dfrac{V_c V_m(t)}{2}$ ...(16)

The average RMS power of the message component at the receiver output

$$\boxed{P_{So} = \frac{V_c^2 P_m}{4}} \quad ...(17)$$

$P_m \rightarrow$ Average power of the original message signal $V_m(t)$.

Noise Power at the output (P_{N_o}):

$$P_n = \int_{-2w}^{+2w} S_N(f).df \quad ...(18)$$

$$P_n = \frac{N_o}{2}[2w + 2w]$$

$$= \left(\frac{N_o}{2}\right)[f]_{-2w}^{+2w}$$

$$P_n = \frac{N_o}{2}[2w + 2w]$$

$$P_n = \frac{4WN_o}{2}$$

$$\boxed{P_n = 2WN_o}$$

$$P_{N_o} = \frac{P_n}{4} = \frac{2WN_o}{4} = \frac{WN_o}{2}$$

$$\boxed{P_{N_o} = \frac{WN_o}{2}} \qquad \text{...(19)}$$

Output Signal - to - Noise Ratio: $(SNR)_o$ (or) $\left(\frac{S}{N}\right)_o$

$$(SNR)_o = \frac{P_{So}}{P_{No}} \qquad \text{...(20)}$$

Sub equations (17) & (19) in eqn (20)

$$(SNR)_o = \frac{\frac{V_c^2 P_m}{4}}{\frac{N_o W}{2}} = \frac{V_c^2 P_m}{4} \times \frac{2}{N_o W}$$

$$\boxed{(SNR)_o = \frac{V_c^2 P_m}{2WN_o}} \qquad \text{...(21)}$$

Figure of Merit (γ)

$$\gamma = \frac{(SNR)_o}{(SNR)_c} \qquad \text{...(22)}$$

Sub equations (10) & (21) in eqn (22)

$$\gamma = \frac{\frac{V_c^2 P_m}{2WN_o}}{\frac{V_c^2 P_m}{2WN_o}}$$

$$\boxed{\gamma = 1} \qquad \text{...(23)}$$

- ✓ Thus the figure of merit for a DSB - SC - AM using coherent detection is **unity**.
- ✓ There is no improvement in signal to noise ratio.

4.12.3 Noise Performance in SSB – SC – AM Using Coherent Detection

✓ The **SSB – SC** system is **similar** to that of **DSB – SC – AM** system except that the **bandwidth** of the **BPF** of SSB – SC – AM is **half** of that required for DSB – SC system.

Noise Performance in SSBSC AM using Coherent Detection.

Let the SSB – SC – AM signal is written as,

$$s(t) = V_c V_m(t)\cos 2\pi f_c t + V_c V_m(t)\sin 2\pi f_c t \quad ...(1)$$

And the noise be n (t)

$$n(t) = n_I(t)\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t \quad ...(2)$$

Average signal power: (P_{Sc})

From DSB - SC - AM, $P_{Sc} = \dfrac{V_c^2 P_m}{2}$

For SSB - SC - AM, $P_{Sc} = \dfrac{V_c^2 P_m}{2} + \dfrac{V_c^2 P_m}{2} = \dfrac{2V_c^2 P_m}{2}$

$$P_{Sc} = V_c^2 P_m \quad ...(3)$$

Average Noise Power: (P_{Nc})

$$P_{Nc} = P_n = \int_{-w}^{+w} S_N(f)\,df$$

$$P_{Nc} = \int_{-w}^{+w} \frac{N_o}{2}\,df = \frac{N_o}{2}[W + W] = \frac{2WN_o}{2}$$

$$P_{Nc} = N_o W \quad ...(4)$$

$(SNR)_c$ channel signal - to - Noise Ratio $\left(\dfrac{S}{N}\right)_c$:

$$(SNR)_c = \frac{P_{Sc}}{P_{Nc}}$$

$$(SNR)_c = \frac{V_c^2 P_m}{N_o W} \quad ...(5)$$

SSB - SC - AM filttered (BPF) signal $x(t)$:

Output of the BPF is $x(t)$

$$x(t) = s(t) + n(t) \quad \text{...(6)}$$

$$x(t) = [V_c V_m(t) \cos 2\pi f_c t + V_c V_m(t) \sin 2\pi f_c t] + n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$

$$x(t) = \left[V_c V_m(t) + n_I(t)\right] \cos 2\pi f_c t + \left[V_c V_m(t) - n_Q(t)\right] \sin 2\pi f_c t \quad \text{...(7)}$$

Block Diagram

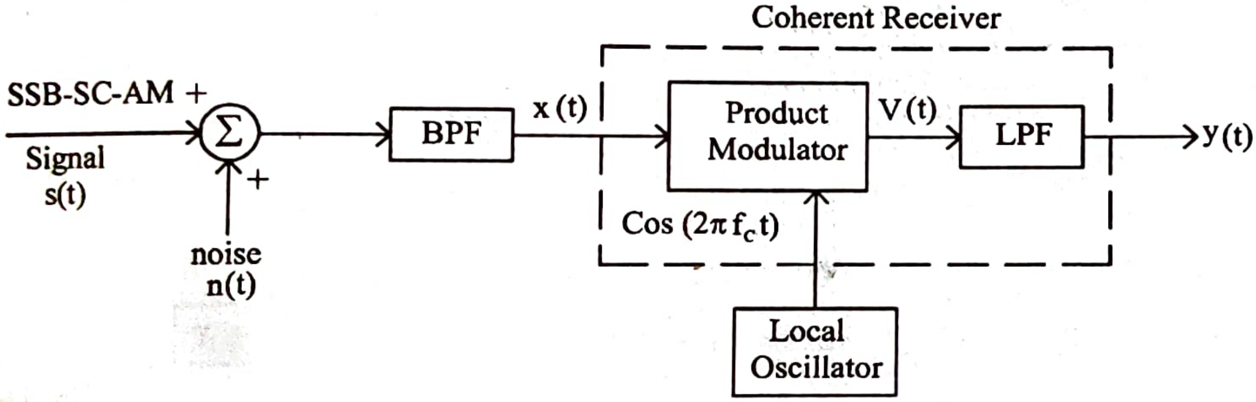


Figure 4.27 SSBSC AM noise performance using coherent detector

The output of the product modulator: V(t)

$$V(t) = x(t) \times \cos 2\pi f_c \quad \text{...(8)}$$

Sub eqn (7) in (8)

$$V(t) = [(V_c V_m(t) + n_I(t)) \cos 2\pi f_c t + (V_c V_m(t) - n_Q(t)) \sin 2\pi f_c t] \cos 2\pi f_c t$$

$$V(t) = [V_c V_m(t) + n_I(t)] \cos^2 2\pi f_c t + [V_c V_m(t) - n_Q(t)] \sin 2\pi f_c t \cos 2\pi f_c t$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sin\theta\cos\theta = \frac{\sin 2\theta}{2}$$

$$V(t) = [V_c V_m(t) + n_I(t)] \left[\frac{1+\cos 4\pi f_c t}{2}\right] + \left[V_c V_m(t) - n_Q(t)\right] \frac{\sin 4\pi f_c t}{2} \quad \text{...(9)}$$

The output of the LPF: $y(t)$ → Allows only low frequency components.

$$y(t) = \frac{V_c V_m(t) + n_1(t)}{2}$$

$$y(t) = \frac{1}{2} V_c V_m(t) + \frac{1}{2} n_1(t) \quad \text{...(10)}$$

Signal power at the output: (P_{So})

The average RMS power of the message component at the receiver output

$$\boxed{P_{So} = \frac{V_c^2 P_m}{4}} \quad \text{...(11)}$$

Noise power at the output: (P_{No})

$$P_{No} = \frac{P_n}{2} \quad \text{...(12)}$$

$$P_n = \int_{-\frac{W}{2}}^{+\frac{W}{2}} S_N(f)\, df = \int_{-\frac{W}{2}}^{+\frac{W}{2}} \frac{N_0}{2}\, df$$

$$P_n = \frac{N_0}{2}\left[\frac{W}{2} + \frac{W}{2}\right] = \frac{N_0}{2}[W]$$

$$P_n = \frac{N_0 W}{2} \quad \text{...(13)}$$

Sub eqn (13) in (12)

$$P_{N_0} = \frac{P_n}{2} = \frac{\frac{N_0 W}{2}}{2}$$

$$\boxed{P_{N_0} = \frac{N_0 W}{4}} \quad \text{...(14)}$$

Output signal - to - Noise Ratio: $(SNR)_o$

$$(SNR)_o = \left(\frac{S}{N}\right)_o = \frac{P_{So}}{P_{No}}$$

Sub eqns (11) & (14)

$$(SNR)_o = \frac{\frac{V_c^2 P_m}{4}}{\frac{N_0 W}{4}} = \frac{V_c^2 P_m}{N_0 W}$$

$$\boxed{(SNR)_o = \frac{V_c^2 P_m}{N_0 W}} \quad ...(15)$$

Figure of merit: (γ)

$$\gamma = \frac{(SNR)_o}{(SNR)_c}$$

Sub eqns (5) & (15)

$$\gamma = \frac{\frac{V_c^2 P_m}{N_0 W}}{\frac{V_c^2 P_m}{N_0 W}} = 1$$

$$\boxed{\gamma = 1} \quad ...(16)$$

Thus the figure of merit for a SSB-SC-AM using coherent detection is **unity**.

4.13 NOISE PERFORMANCE IN FM RECEIVER

✓ In a FM system, the message signal is transmitted by variations of the instantaneous frequency of the sinusoidal carrier wave and its amplitude is maintained constant.

✓ ∴ Any variations of the carrier amplitude at the receiver input must result from noise (or) interference.

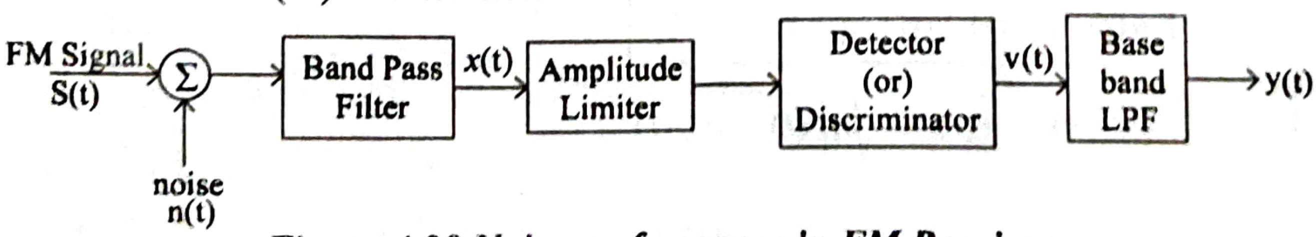


Figure 4.28 Noise performance in FM Receiver

Amplitude Limiter:

✓ The amplitude limiter following the Band pass Filter (BPF) is used to remove the amplitude variations by clipping the modulated wave at the filter output almost to the zero axis.

Discriminator (or) Detector:

✓ The output of the limiter is then fed to the discriminator which consists of two **components.**

(1) A **slope network** (or) **differentiator** produces a hybrid modulated wave in which both amplitude and frequency vary in accordance with the message signal.

(2) An **envelope detector** that recovers the amplitude variation and thus reproducing the message signal.

Base band Low pass filter (LPF):

✓ The output of the discriminator is then fed to the post detection filter (or) base band low pass filter.

✓ The baseband LPF is used to remove the out of band components of the noise of the discriminator output and thereby keeps the effect of the output noise to a minimum.

Noise $n(t)$ [Narrow Band Noise]

✓ The narrow band noise which consists of inphase component and quadrature component.

$$n(t) = n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t \quad ...(1)$$

$n(t)$ can be expressed interms of envelope & phase component.

$$n(t) = r(t) \cos [2\pi f_c t + \psi(t)] \quad ...(2)$$

Where, envelope $r(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$ ...(3)

Phase $\psi(t) = \tan^{-1}\left(\frac{n_I(t)}{n_Q(t)}\right)$ ...(4)

The envelope $r(t)$ is **Rayleigh distributed** and

The phase $\psi(t)$ **is Uniformly distributed** over 2π radians.

The noise power at the channel (P_{Nc}):

$$P_{Nc} = P_n$$

For white Gaussian noise,

$$S_N(f) = \frac{N_o}{2}$$

$$P_n = \int_{-w}^{+w} S_N(f)\, df$$

$$P_n = \int_{-w}^{+w} \frac{N_o}{2}\, df = \frac{N_o}{2}[W + W] = \frac{N_o}{2}(2W)$$

$$\boxed{P_{Nc} = P_n = N_o W} \quad ...(5)$$

Input signal [FM signal] $s(t)$:

The incoming FM signal $s(t)$ is given in simple form,

$$s(t) = V_c \cos[2\pi f_c t + \phi(t)] \quad ...(6)$$

$$\phi(t) = 2\pi k_f \int_0^t V_m(t)\, dt \quad ...(7)$$

Sub eqn (7) in (6)

$$s(t) = V_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t V_m(t)\, dt\right] \quad ...(8)$$

V_c → carrier amplitude

f_c → carrier frequency

k_f → Frequency sensitivity

$V_m(t)$ → message signal

$\phi(t)$ → Instantaneous Phase deviation.

The signal power at the channel: (P_{Sc})

$$\boxed{P_{Sc} = \frac{V_c^2}{2}} \quad ...(9)$$

$(SNR)_c$ Channel Signal - to - Noise Ratio.

$$(SNR)_c = \frac{P_{Sc}}{P_{Nc}}$$

Sub eqns (5) & (9)

$$(SNR)_c = \frac{\frac{V_c^2}{2}}{N_o W}$$

$$\boxed{(SNR)_c = \frac{V_c^2}{2N_o W}} \quad ...(10)$$

BPF Filtered Output: $x(t)$

$$x(t) = s(t) + n(t) \quad ...(11)$$

$$x(t) = V_c \cos[2\pi f_c t + \phi(t)] + r(t) \cos[2\pi f_c t + \psi(t)] \quad ...(12)$$

Phasor Diagram (or) Representation of $x(t)$:

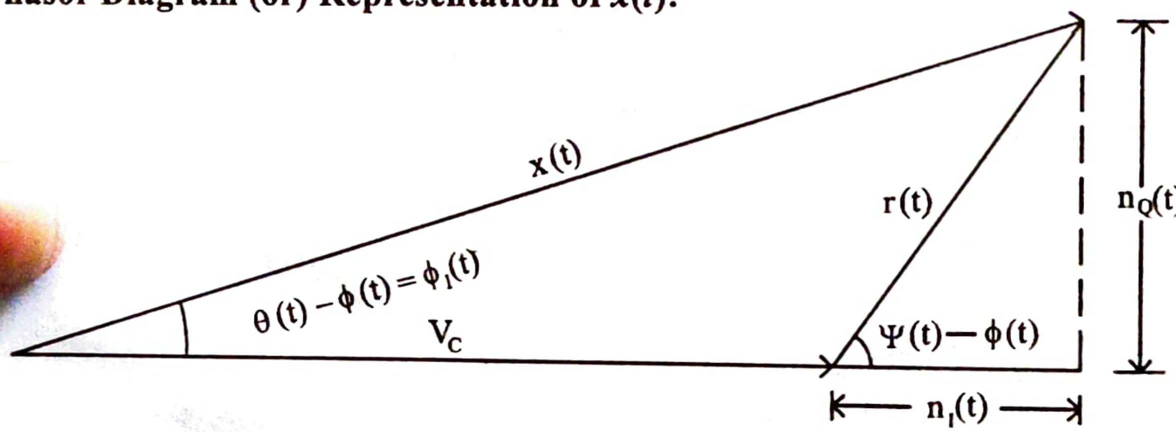


Figure 4.29 Phasor representation of x(t)

From the above phasor diagram,

$$\sin(\psi(t) - \phi(t)) = \frac{n_Q(t)}{r(t)}$$

$$n_Q(t) = r(t) \sin(\psi(t) - \phi(t)) \quad ...(13)$$

$$\cos(\psi(t) - \phi(t)) = \frac{n_I(t)}{r(t)}$$

$$n_1(t) = r(t)\cos(\psi(t) - \phi(t)) \quad \text{...(14)}$$

$$x(t) = V_c + n_1(t) \quad \text{...(15)}$$

Sub eqn (14) in (15)

$$x(t) = V_c + r(t)\cos(\psi(t) - \phi(t)) \quad \text{...(16)}$$

$$\phi_1(t) = \tan^{-1}\left[\frac{n_Q(t)}{V_c + n_1(t)}\right] \quad \text{...(17)}$$

sub eqns (13) & (14) in (17)

$$\phi_1(t) = \tan^{-1}\left[\frac{r(t)\sin[\psi(t) - \phi(t)]}{V_c + r(t)\cos[\psi(t) - \phi(t)]}\right] \quad \text{...(18)}$$

$$\phi_1(t) = \theta(t) - \phi(t) \quad \text{...(19)}$$

$$\theta(t) = \phi(t) + \phi_1(t) \quad \text{...(20)}$$

$$\theta(t) = \phi(t) + \tan^{-1}\left[\frac{r(t)\sin(\psi(t) - \phi(t))}{V_c + r(t)\cos(\psi(t) - \phi(t))}\right] \quad \text{...(21)}$$

✓ Let R be the random variable by observing the envelope r(t). Consider that the random variable R is small compared to the carrier amplitude V_c. So that the phase $\theta(t)$ is reduced to

$$\boxed{\theta(t) \simeq \phi(t) + \frac{r(t)}{V_c}\sin(\psi(t) - \phi(t))} \quad \text{...(22)}$$

Using the expression of $\phi(t)$,

Sub eqn (7) in (22)

$$\theta(t) \simeq 2\pi k_f \int_0^t V_m(t)\,dt + \frac{r(t)}{V_c}\sin[\psi(t) - \phi(t)] \quad \text{...(23)}$$

Discriminator output V(t):

The discriminator is proportional to $\frac{\theta'(t)}{2\pi}$

Where $\theta'(t)$ is the derivative of $\theta(t)$ with respect to time.

∴ The output of the discriminator is

$$V(t) = \frac{1}{2\pi}\theta'(t) \quad ...(24)$$

$$V(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t)$$

$$V(t) = \frac{1}{2\pi}\frac{d}{dt}\left[2\pi k_f \int_o^t V_m(t)\,dt + \frac{r(t)}{V_c}\sin[\psi(t)-\phi(t)]\right]$$

$$V(t) \simeq \frac{1}{2\pi}\frac{d}{dt}\left[2\pi k_f \int_o^t V_m(t)\,dt\right] + \frac{1}{2\pi V_c}\frac{d}{dt}[r(t)\sin(\psi(t)-\phi(t)]$$

$$V(t) \simeq k_f V_m(t) + n_d(t) \quad ...(25)$$

Where the noise term $n_d(t)$ is defined by,

$$\boxed{n_d(t) = \frac{1}{2\pi V_c}\frac{d}{dt}[r(t)\sin(\psi(t)-\phi(t)]} \quad ...(26)$$

- ✓ Since the phase $\psi(t)$ of the narrow band noise is uniformly distributed over **2π radians.**
- ✓ Then the phase difference $\psi(t)-\phi(t)$ is also uniformly distributed over **2π radians.**
- ✓ Therefore the noise $n_d(t)$ at the discriminator output would be independent of the modulating signal.

$$\boxed{n_d(t) \simeq \frac{1}{2\pi V_c}\frac{d}{dt}[r(t)\sin\psi(t)]} \quad ...(27)$$

From eqn (25), the message component in the discriminator output, given to the low - pass filter.

The signal power at the output: (P_{So})

$$\text{LPF output} = k_f V_m(t) \quad ...(28)$$

The average output signal power $\boxed{P_{So} = k_f^2 P}$ ...(29)

P → Average power of message signal $V_m(t)$.

Noise power at the output: (P_{No})

The power spectral density of $n_o(t)$ appearing at the receiver output is defined by,

$$S_{N_o}(f) = \begin{cases} \dfrac{N_o f^2}{V_c^2} & ; \ |f| \leq W \\ 0 & ; \ \text{otherwise} \end{cases} \quad ...(30)$$

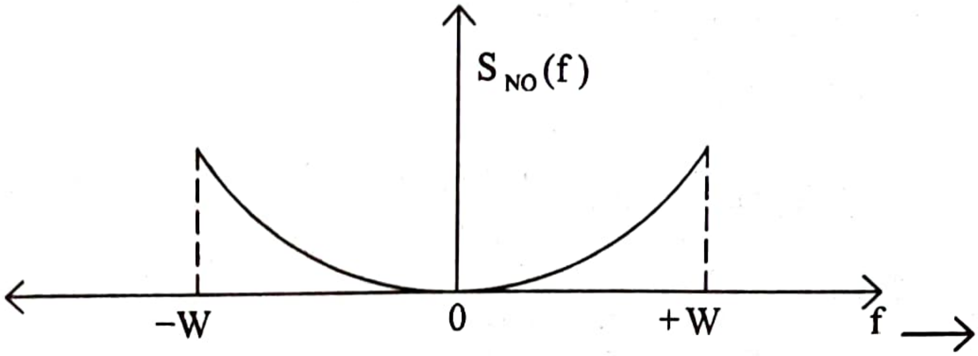


Figure 4.30 PSD of noise $n_o(t)$ at the receiver output

The average noise power at the output is defined by,

$$P_{No} = \int_{-w}^{+w} S_{N_o}(f).df \quad ...(31)$$

$$P_{No} = \int_{-w}^{+w} \frac{N_o f^2}{V_c^2}.df$$

$$P_{No} = \frac{N_o}{V_c^2}\left(\frac{f^3}{3}\right)_{-w}^{+w}$$

$$P_{No} = \frac{N_o}{3V_c^2}\left[W^3 - (-W)^3\right]$$

$$P_{No} = \frac{N_o}{3V_c^2}\left[W^3 + W^3\right] = \frac{2N_o W^3}{3V_c^2}$$

$$\boxed{P_{No} = \frac{2N_o W^3}{3V_c^2}} \quad ...(32)$$

✓ The average output noise power is inversely proportional to the average carrier power $\frac{V_c^2}{2}$.

✓ In FM system as the carrier power increases, the effect of noise at the receiver output is reduced. This is called **noise quieting effect.**

Signal to Noise Ratio at the Output: (SNR)$_o$

$$(SNR)_o = \frac{P_{So}}{P_{No}} \quad \text{...(33)}$$

Sub eqns (29) & (32) in (33)

$$\left(\frac{S}{N}\right)_o = \frac{k_f^2 P}{\frac{2N_0 W^3}{3V_c^2}}$$

$$\boxed{\left(\frac{S}{N}\right)_o = \frac{3V_c^2 k_f^2 P}{2N_0 W^3}} \quad \text{...(34)}$$

Figure of merit (γ)

$$\gamma = \frac{(SNR)_o}{(SNR)_c} \quad \text{...(35)}$$

Sub eqns (10) & (34) in (35)

$$\gamma = \frac{\frac{3V_c^2 k_f^2 P}{2N_0 W^3}}{\frac{V_c^2}{2N_0 W}}$$

$$\gamma = \frac{3V_c^2 k_f^2 P}{2N_0 W^3} \times \frac{2N_0 W}{V_c^2}$$

$$\boxed{\gamma = \frac{3k_f^2 P}{W^2}} \quad \text{...(36)}$$

For signal tone modulation,

✓ The modulated FM signal is defined by,

$$s(t) = V_c \cos\left[2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t\right] \quad ...(37)$$

Where $\Delta f \rightarrow$ peak frequency deviation.

Comparing the above equation with WBFM

$$s(t) = V_c \cos\left[2\pi f_c t + 2\pi k_f \int_o^t V_m(t)\, dt\right] \quad ...(38)$$

(*ie.,*) Compare eqn (37) & (38) & equating the eqns,

$$2\pi k_f \int_o^t V_m(t)\, dt = \frac{\Delta f}{f_m} \sin 2\pi f_m t \quad ...(39)$$

Differentiating eqn (39) on both sides with respect to t.

$$2\pi k_f V_m(t) = \frac{\Delta f}{f_m} [\cos(2\pi f_m t)] \cdot [2\pi f_m] \quad ...(40)$$

$$V_m(t) = \frac{\Delta f\, 2\pi f_m \cos(2\pi f_m t)}{f_m\, 2\pi k_f}$$

$$\boxed{V_m(t) = \frac{\Delta f}{k_f} \cos(2\pi f_m t)} \quad ...(41)$$

∴ The average power of the message signal is defined by,

$$\boxed{P = \frac{\Delta f^2}{2k_f^2}} \quad ...(42)$$

We know that,

The figure of merit of FM sign al is

$$(36) \Rightarrow \gamma = \frac{3k_f^2 P}{W^2} \quad ...(43)$$

Sub eqn (42) in (43)

$$\gamma = \frac{3k_f^2\left(\frac{\Delta f^2}{2k_f^2}\right)}{W^2}$$

$$\gamma = \frac{3}{2}\left(\frac{\Delta f^2}{W^2}\right) \quad \text{...(44)}$$

$$\gamma = \frac{3}{2}\left(\frac{\Delta f}{W}\right)^2 \quad \text{...(45)}$$

$$\boxed{\gamma = \frac{3}{2}\beta^2} \quad \text{...(46)}$$

$\beta \rightarrow$ Modulation Index for FM.

$$\boxed{\beta = \frac{\Delta f}{W}} \quad \text{...(47)}$$

4.14 CAPTURE EFFECT IN FM RECEIVER

Introduction:

✓ The phase deviation ($\Delta\theta$) produced by the noise signal in FM is much smaller than the phase deviation ($\Delta\theta$) produced by the message signal, provided that noise is smaller than carrier.

✓ Thus, in the low noise case, the distortion produced by the noise at the output of the FM Detector is negligible in comparison to the desired modulating signal.

✓ In other words, noise is almost suppressed by the signal.

✓ The noise suppression characteristics of FM is also be applied to any interfering signal of almost same frequency. This phenomena, known as **capture effect.**

Definition – Capture Effect

✓ When FM signals from two transmitters operated on the same or nearly same carrier frequency reach the receiver simultaneously, the signal of a weak magnitude is suppressed by a strong signal, and the FM receiver reproduced only the strong signal. This is called **capture effect in FM.**

- ✓ This effect is similar to noise – suppressing action with the weaker signal playing the role of the noise.
- ✓ The weak signals due to common channel and adjacent channel interferences are suppressed.
- ✓ This is a very useful feature of FM system.
- ✓ Let the desired signal have a carrier of peak amplitude A and frequency ω_c.
- ✓ Let the interfering signal have a frequency $(\omega_c + \Delta\omega)$ and a peak amplitude B.
- ✓ For our analysis here, the modulations of the desired and interfering signals may be totally ignored, as they do not play any part.

The received signal may be written as,

$$r(t) = A\cos\omega_c t + B\cos(\omega_c + \Delta\omega)t$$

$$r(t) = A\cos\omega_c t + B[\cos\omega_c t\,\cos\Delta\omega t - \sin\omega_c t\sin\Delta\omega t]$$

$$r(t) = A\cos\omega_c t + B\cos\omega_c t\,\cos\Delta\omega t - B\sin\omega_c t\sin\Delta\omega t$$

$$r(t) = (A + B\cos\Delta\omega t)\cos\omega_c t - (B\sin\Delta\omega t)\sin\omega_c t \quad ...(1)$$

Hence $r(t)$ may be written as,

$$r(t) = R(t)\cos[\omega_c t + \theta(t)]$$

$$R(t) = \sqrt{(A + B\cos\Delta\omega t)^2 + (B\sin\Delta\omega t)^2} \quad ...(2)$$

Where,

$$\theta(t) = \tan^{-1}\left(\frac{B\sin\Delta\omega t}{A + B\cos\Delta\omega t}\right) \quad ...(3)$$

and

Neglecting $B\cos(\Delta\omega t)$ in comparision with A as A >> B,

$$\boxed{\theta(t) \cong \tan^{-1}\left(\frac{B\sin\Delta\omega t}{A}\right)} \quad ...(4)$$

- ✓ In the case of FM, the amplitude $R(t)$ of the received signal $r(t)$ is of no consequence $\theta(t)$, the phase deviation of the desired carrier signal, caused by the interfering signal, is however, important as it produces an output in the receiver.

But

$$\theta(t) \cong \tan^{-1}\left[\left(\frac{B \sin \Delta\omega t}{A}\right)\right] \simeq 0 \; \textit{if}\, A >> B \qquad ...(5)$$

- ✓ So the stronger the desired signal, relative to the interfering signal, the better is the suppression of the interfering signal.
- ✓ It may be noted here that the interfering signal need not be only an undesired carrier or modulated signal.
- ✓ It may be made up of just noise frequency components closed to the desired carrier frequency. Thus, **capture effect suppresses noise too.**
- ✓ FM receiver have the ability to differentiate between 2 signals received at the same frequency.
- ✓ If 2 FM radio stations are received simultaneously at the same frequency, the receiver locks onto the stronger FM radio station while suppressing the weaker FM radio station.
- ✓ If the received signals levels of both FM radio stations are approximately same, the receiver cannot sufficiently differentiate between them and may switch back and forth.
- ✓ The capture effect is also observed when mobile FM receivers are moving from on FM transmitter to another one.
- ✓ There is no interference until the signal from the second FM transmitter is less than about half of the signal from the first one.
- ✓ But as the signal from the second FM transmitter becomes stronger than the first one, it becomes quite audible at the background of the first one.
- ✓ As the mobile FM receiver travels to closer and closer to the second FM transmitter the signal from the second transmitter becomes stronger.
- ✓ Ultimately it may performance the signal from the first transmitter eventually being captured by the second FM transmitter.
- ✓ If FM mobile receiver approximately at the centre of 2 FM transmitter then receiver signal would be alternating from 2 transmitters.
- ✓ Due to this the FM receiver will be captured alternately by 2 FM transmitters.

4.15 THRESHOLD EFFECT IN FM

Introduction:

- ✓ Threshold effect is the phenomenon that occurs when the signal to noise ratio (SNR) at the detector input decreases below a critical level.
- ✓ Below this level, the resulting output signal gets severely distorted by noise.
- ✓ FM system exhibits such **threshold behavior.**
- ✓ When a carrier – to – noise ratio becomes even slightly less than unity, an impulse of noise is generated. This noise impulse appears at the output of the FM discriminator in the form of a **'click sound'.**

Definition:

When the carrier – to – noise ratio is slightly less than unity, the frequency of spike generation is small, and each spike produces individual **'clicking sound'** in the receiver. But when the carrier – to – noise ratio is further decreased so that the ratio is moderately less than unity, the spikes are generated rapidly, and the clicks merge into a **'crackling'** or **'sputtering sound'**. This phenomenon is known as **threshold effect in FM.**

- ✓ The threshold is a minimum carrier to noise ratio yielding an **FM improvement** which is not significantly deteriorated from the value predicted by the usual signal to noise formula assuming small noise power.
- ✓ The threshold effect is **more severe** in **FM** than AM.
- ✓ FM demodulator consisting of an IF filter, discriminator and a base band filter.

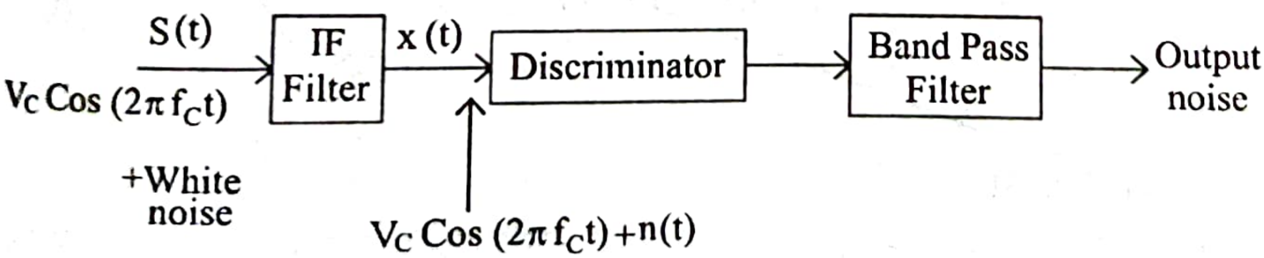


Figure 4.31 An FM discriminator and associated filter

✓ Assume that the input signal is an un modulated carrier, ie., message signal $V_m(t) = 0$, accompanied by **white noise**. The white noise is filtered and shaped by the IF filter.

$$s(t) = V_c \cos 2\pi f_c t \qquad ...(1)$$

$$x(t) = s(t) + n(t) \qquad ...(2)$$

$$x(t) = V_c \cos 2\pi f_c t + n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t \qquad ...(3)$$

$$\boxed{x(t) = [V_c + n_I(t)] \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t} \qquad ...(4)$$

Where,

$n_I(t) \rightarrow$ Inphase component

$n_Q(t) \rightarrow$ Quadrature component

$n(t) \rightarrow$ Narrow band noise with respect to the carrier wave

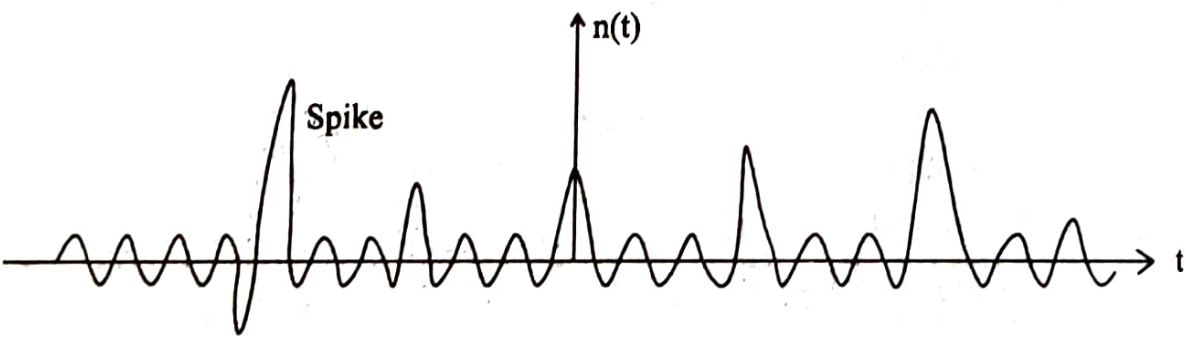


Figure 4.32 Ciruit noise at discriminator output

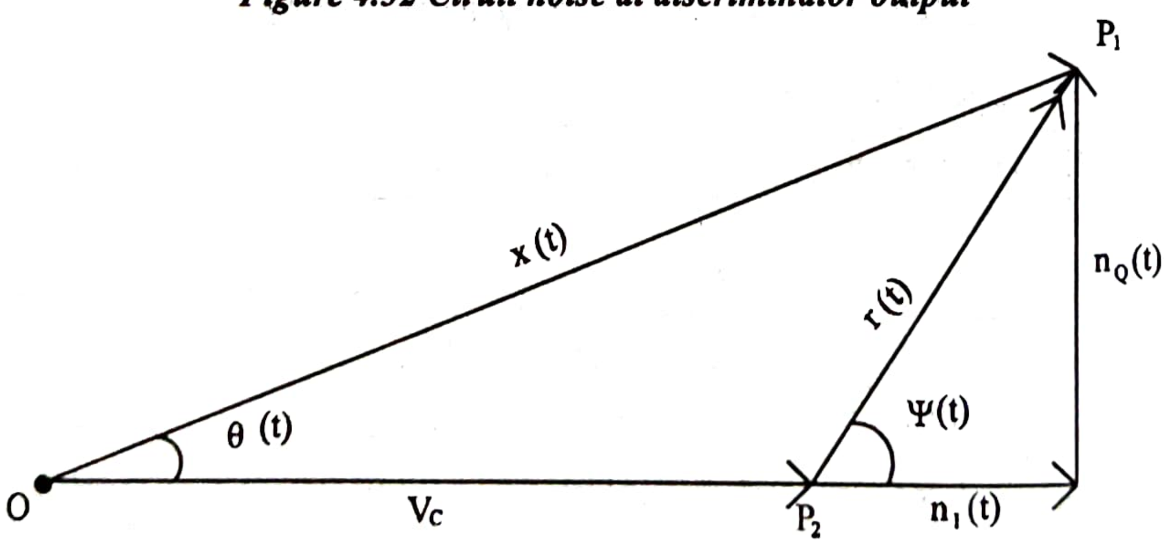


Figure 4.33 Phasor Diagram

✓ As the amplitude and phase of $n_I(t)$ & $n_Q(t)$ change with time in a random manner, the point P_1 wanders around the point P_2.

✓ When the carrier - to - noise ratio is large, $n_Q(t)$ & $n_I(t)$ as very much smaller than V_c, So P_1 spends most of its time near P_2. $[n(t) << V_c]$

Thus the angle $$\theta(t) \simeq \frac{n_Q(t)}{V_c} \quad ...(5)$$

with in a multiple of 2π.

Spike Generation:

✓ When the carrier to noise ratio is low, P_1 sweeps around the origin & $\theta(t)$ increases (or) decreases by 2π radians.

✓ The discriminator output is impulse like components,

$$(ie.,)\ V(t) = \frac{\theta'(t)}{2\pi} = \frac{1}{2\pi}\frac{d\theta(t)}{dt} \quad ...(6)$$

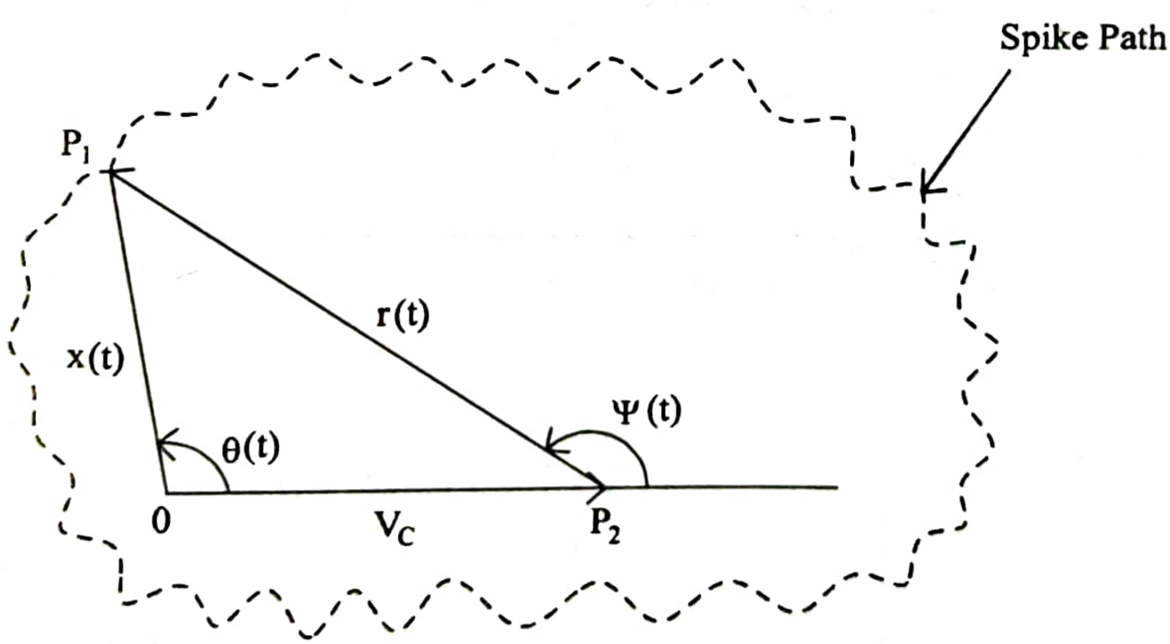


Figure 4.34 Phasor diagram for $r(t) >> V_c$

✓ When the noise phasor $r(t)$ is large as compared to carrier vector V_c $[r(t) >> V_c]$ the locus of the end point P_1 of the result x (t) moves away from point P_2 and may even rotate the origin.

✓ The locus encircles the origin and is referred to as spike path, since it generates a noise - spike.

✓ $\theta(t)$ changes by 2π radians, $\frac{d\theta}{dt}$ appears as a sharp spike (or) impulse with area of 2π.

✓ This area under each spike is **2π**.

✓ This can be seen by integrating the first impulse occuring for the interval (t_1, t_2)

$$\text{Area} = \int_{t_1}^{t_2} \frac{d\theta}{dt}\, dt = [\theta]_{t_1}^{t_2} = 2\pi \qquad \text{...(7)}$$

✓ Each spike components have different heights depending on how close the wandering point P_1 comes to the origin 0, but all have areas nearly equal to **±2π radians**. These spikes behave like **"shot noise"**.

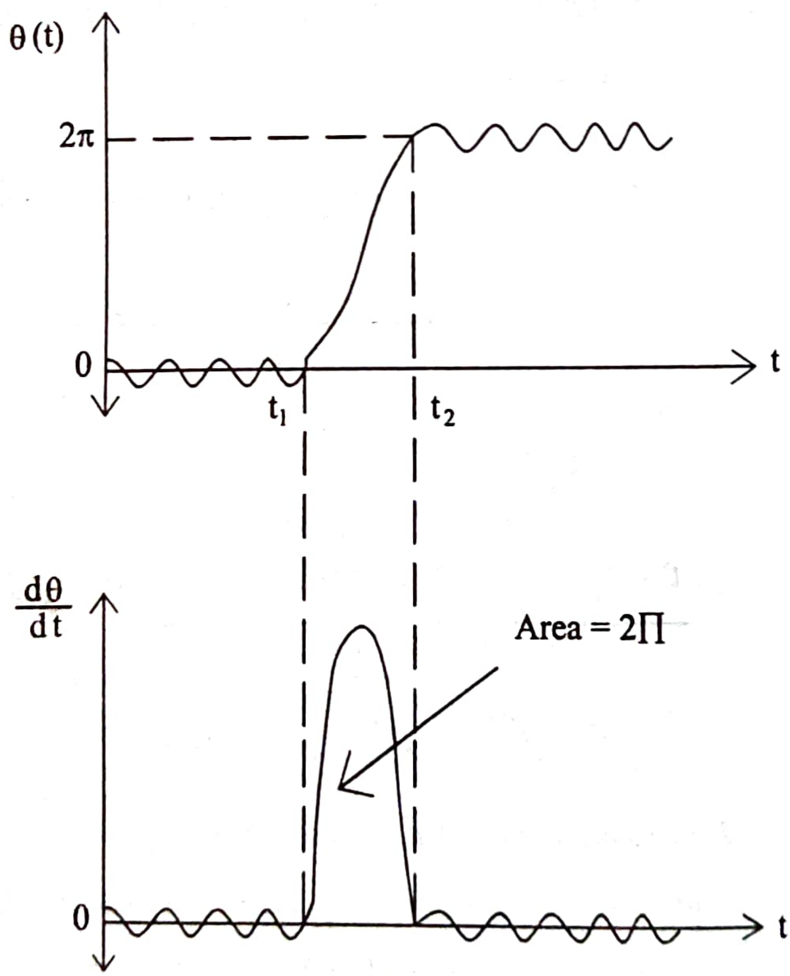


Figure 4.35 A Plot of $\frac{d\theta}{dt}$ as a function of time

Positive Spikes:

✓ A positive going clicks (or) spikes occurs when $r(t)$ & $\psi(t)$ of $n(t)$ satisfy the following conditions:

$$\begin{aligned} r(t) &> V_c \\ \psi(t) &< \pi \le \psi(t) + d\psi(t) \\ \frac{d\psi(t)}{dt} &> 0 \end{aligned} \quad \text{...(8)}$$

✓ These conditions ensure that the phase $\theta(t)$ of the resultant phasor ***x*(*t*) changes** by **2π** radians in the time increment **dt**, during which the phase of the narrow band noise increases by an incremental amount $d\psi(t)$.

When $x(t)$ rotates **counter clockwise**, the phase $\psi(t)$ changes by 2π and generates ***a* + *ve* spikes**. If $x(t)$ rotates **clockwise** the phase $\psi(t)$ changes by -2π and generates ***a* – *ve* spikes**.

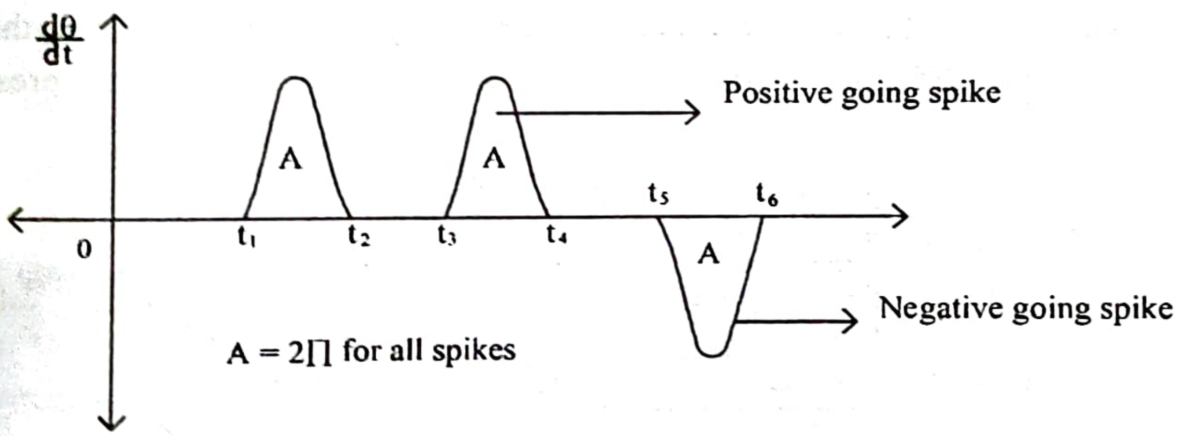


Figure 4.36 Noise output of discriminator

Negative spikes

A negative going clicks occur, when

$$\begin{aligned} r(t) &> V_c \\ \psi(t) &> -\pi > \psi(t) + d\psi(t) \\ \frac{d\psi(t)}{dt} &< 0 \end{aligned} \quad \text{...(9)}$$

Here $\theta(t)$ changes by -2π radians during the time increment dt.

Carrier - to - Noise Ratio: (ρ)

✓ The carrier signal to noise ratio $(\rho) = \frac{V_c^2}{2 N_o B_T}$ ...(10)

$B_T \rightarrow$ Bandwidth of IF filter

✓ As ρ is decreased, the average number of clicks/ unit time increases.

✓ When this number becomes appreciably large, **threshold** is said to occur.

✓ The average output noise power is calculated when there is no signal present, (*ie.,*) the carrier is un modulated, with no restriction imposed on the value of carrier to noise ratio (ρ).

✓ When the signal is present, the resulting modulation of the carrier tends to increase the average number of clicks/ second.

✓ Experimentally clicks are heard in the receiver output at a carrier to noise ratio of 13 dB.

✓ The increase in the average number of clicks/ seconds tends to cause the output SNR fall off more sharply just below the threshold level in the presence of modulation.

✓ The threshold effects can be avoided if $\rho \geq 20$

(*ie.,*) $$\frac{V_c^2}{2 N_o B_T} \geq 20 \quad ...(11)$$

(or) average transmitter power satisfies the condition

$$\frac{V_c^2}{2} \geq 20\, N_o B_T \quad ...(12)$$

$B_T \rightarrow$ Bandwidth of FM wave.

4.16 FM THRESHOLD REDUCTION (OR) EXTENSION METHODS

✓ There are two methods used for improvement of the FM threshold reduction (or) extension.

(1) Using Pre - emphasis & De - emphasis.

(2) FMFB [FM demodulator with negative Feed Back].

4.16.1 Threshold Improvement through De - emphasis

Pre - emphasis and De - emphasis in FM:

Introduction:

✓ The power spectral density of the noise at the output of an FM receiver has a square law dependence on the operating frequency which is shown in the figure.

✓ The power spectral density of a typical message source audio and video signals is shown in figure.

✓ From the figures, it is noted that the PSD of the message usually falls off appreciably at high frequencies.

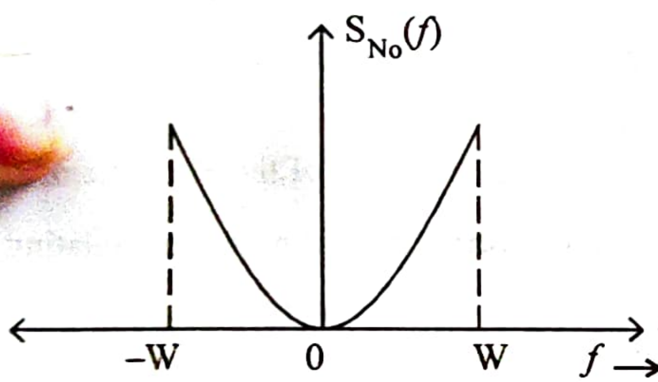


Figure 4.37 PSD of noise

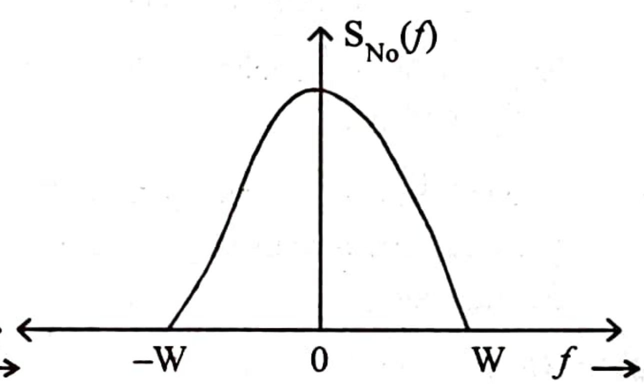


Figure 4.38 PSD of message

✓ On the other hand, the PSD of the output noise increases rapidly with frequency.

✓ Thus around $f = \pm W$ the relative spectral density of the **message** is quite **low**, where as that of output **noise** is quite **high** in comparison.

✓ Clearly, the message is not using the frequency band allotted to it in an efficient manner.

✓ Such more efficient use of frequency band and improved noise performance can be obtained with the help of pre - emphasis and de - emphasis circuits.

✓ The **pre - emphasis** in the **transmitter** and **De - emphasis** in the **receiver** is used to **improve the threshold.**

Block diagram with pre - emphasis and De - emphasis.

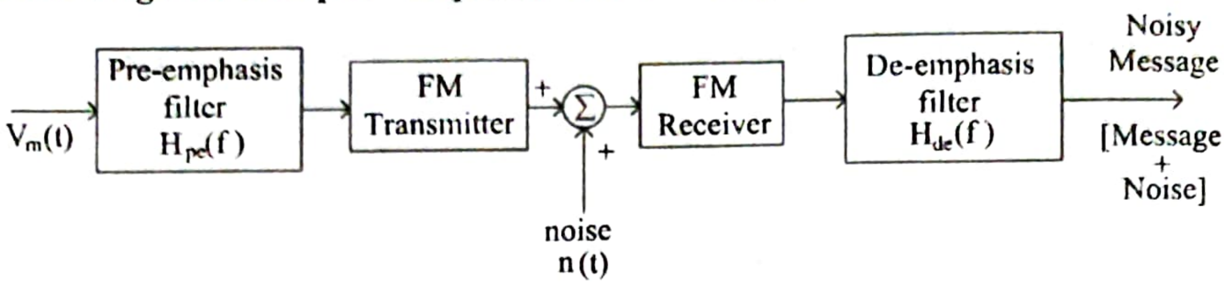


Figure 4.39 Pre - emphasis & De - emphasis in FM.

✓ In FM, the noise has greater effect on the higher modulating frequencies.

✓ This effect can be reduced by increasing the value of modulation index 'm_f' for higher modulating frequencies (f_m).

✓ This can be done by increasing the amplitude of message signal at higher modulating frequencies.

Pre – Emphasis:

✓ Pre – emphasis is a **circuit** used in the **transmitter** side of the FM system.

✓ Pre – emphasis is used to improve the **noise immunity** at higher modulating frequencies.

Definition:

✓ **Pre – emphasis** is a circuit which is used for **artificial boosting** the signal amplitude of higher modulating frequencies in the message band at the transmitter **before modulation.**

✓ As a result the Signal to Noise Ratio (SNR) is **improved.**

Circuit diagram: [RC network].

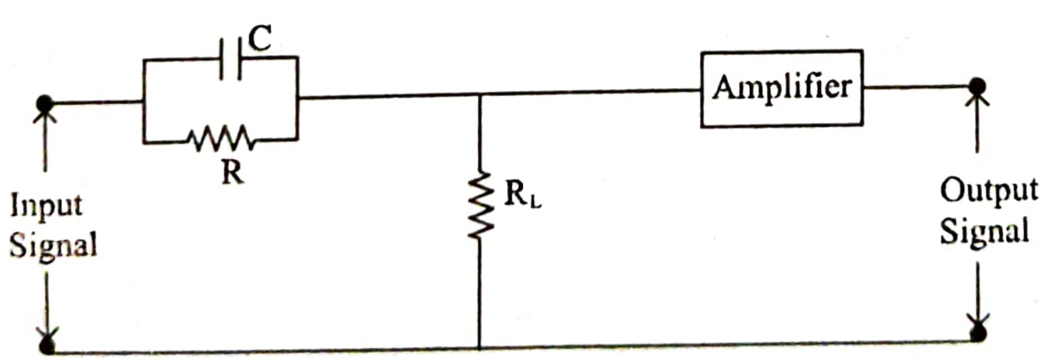


Figure 4.40 Pre – emphasis circuit diagram

- ✓ A simple pre – emphasis filter is shown in figure.
- ✓ This is a single pole filter whose frequency response is shown in figure.
- ✓ The break points ω_1 & ω_2 are given by,

$$\boxed{\omega_1 = \frac{1}{RC}} \quad \& \quad \boxed{\omega_2 = \frac{1}{RC}}$$

For broadcast systems f_1 is taken as 2.1 kHz, And $f_2 > 30\,kHz$.

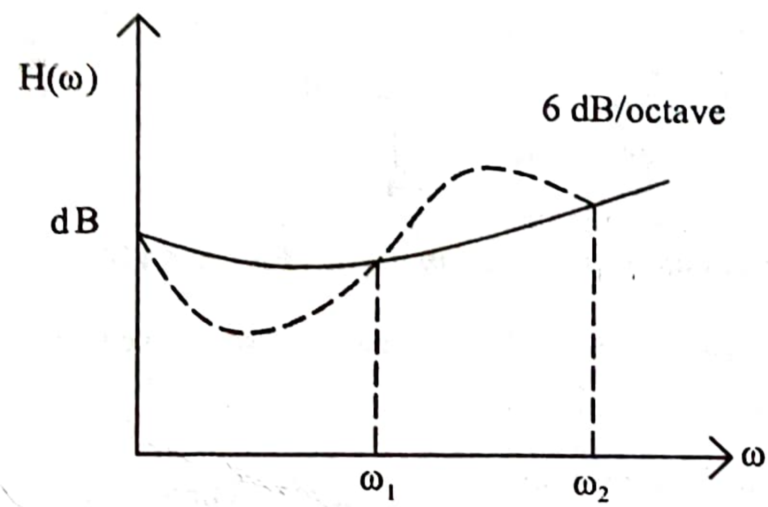


Figure 4.41 Frequency response: Pre-emphasis

- ✓ The signal to noise ratio $\left(\frac{S}{N}\right)_o$ becomes large enough to improve the threshold level over the entire message band.
- ✓ The modulating signal is passed through a high pass RC filter, before applying it to the FM modulator.
- ✓ As f_m (message signal frequency) **increases**, reactance of **C decreases** and **modulating voltage applied** to FM modulator goes on **increasing.**

De – emphasis:

Definition:

The artificial boosting gives to the higher modulating frequencies in the process of pre – emphasis is **nullified** (or) **compensated** at the receiver by a process called **'De – emphasis'**.

$H_{Pe}(f) \rightarrow$ Designates the frequency response of the pre – emphasis filter.

$H_{de}(f) \rightarrow$ frequency response of the de – emphasis filter.

$$H_{de}(f) = \frac{1}{H_{Pe}(f)}, \; -W \leq f \leq W \qquad ...(1)$$

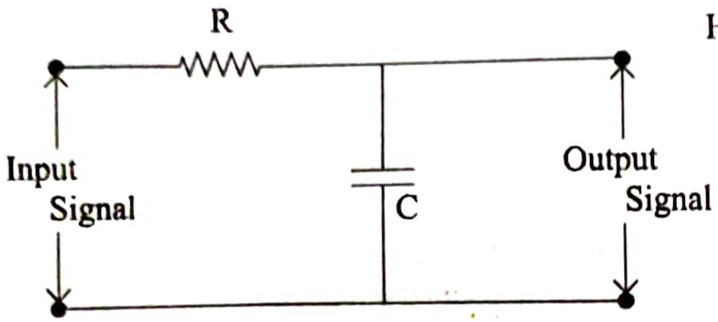


Figure 4.42(a) Circuit diagram of de-emphasis

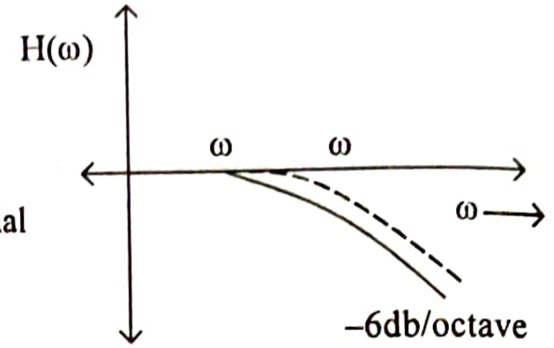


Figure 4.42(b) Frequency response

From the de - emphasis filter, The frequency response is given by,

$$H_{de}(f) = \frac{\left(\frac{1}{j2\pi f_c}\right)}{R + \frac{1}{j2\pi f_c}} \qquad \text{Where, } f_o = \frac{1}{2\pi RC}$$

$$H_{de}(f) = \frac{\left(\frac{1}{j2\pi f_c}\right)}{\left(\frac{Rj2\pi f_c + 1}{j2\pi f_c}\right)} = \frac{1}{1 + j2\pi f RC} = \frac{1}{1 + j\left(\frac{f}{f_o}\right)}$$

$$H_{de}(f) = \frac{1}{1 + j\left(\frac{f}{f_o}\right)} \qquad ...(2)$$

✓ The power spectral density of the noise component after the de-emphasis filter is given by,

$$S_{ND}(f) = S_{No}(f)\,|H_{de}(f)|^2 \qquad ...(3)$$

We know that,

$$S_{No}(f) = \begin{cases} \frac{N_o f^2}{V_c^2}, & -W \leq f \leq W \\ 0, & \text{otherwise} \end{cases} \qquad ...(4)$$

Sub eqns (4) & (2) in (3)

$$S_{ND}(f) = \left(\frac{N_0 f^2}{V_c^2}\right) \cdot \left(\frac{1}{1+\frac{f^2}{f_o^2}}\right)$$

$$\boxed{S_{ND}(f) = \frac{N_0 f^2}{V_c^2\left(1+\frac{f^2}{f_o^2}\right)}} \quad \text{...(5)}$$

✓ The average noise power with de - emphasis is given by,

$$P_{ND} = \int_{-w}^{+w} S_{ND}(f)\,df = \int_{-w}^{+w} \frac{N_0 f^2}{V_c^2\left(1+\frac{f^2}{f_o^2}\right)} \cdot df$$

$$P_{ND} = \frac{N_0}{V_c^2} \int_{-w}^{+w} \frac{f^2}{\frac{f_o^2+f^2}{f_o^2}} \cdot df = \frac{N_0 f_o^2}{V_c^2} \int_{-w}^{+w} \frac{f^2}{f_o^2+f^2} \cdot df$$

$$P_{ND} = \frac{N_0 f_o^2}{V_c^2}\left[f - f_o \tan^{-1}\left(\frac{f}{f_o}\right)\right]_{-w}^{w}$$

$$P_{ND} = \frac{N_0 f_o^2}{V_c^2}\left[W-(-W) - f_o \tan^{-1}\left(\frac{W-(-W)}{f_o}\right)\right]$$

$$P_{ND} = \frac{N_0 f_o^2}{V_c^2}\left[2W - f_o \tan^{-1}\left(\frac{2W}{f_o}\right)\right]$$

$$\boxed{P_{ND} = \frac{N_0 f_o^3}{V_c^2}\left[\left(\frac{2W}{f_o}\right) - \tan^{-1}\left(\frac{2W}{f_o}\right)\right]} \quad \text{...(6)}$$

Improvement Factor (I)

✓ The improvement factor I in output signal to noise ratio produced by the use of pre - emphasis in the transmitter and de - emphasis in the receiver is defined by,

$$I = \frac{\text{Average output noise power without pre-emphasis \& De-emphasis}}{\text{Average output noise power with pre-emphasis \& De-emphasis}}$$

$$I = \frac{P_{N0}}{P_{ND}} \quad ...(7)$$

We have that, $P_{N0} = \frac{2N_0 W^3}{3V_c^2}$ ...(8)

Sub eqns (8) and (6) in (7)

$$I = \frac{\frac{2N_0 W^3}{3V_c^2}}{\frac{N_0 f_o^3}{V_c^2}\left[\left(\frac{2W}{f_o}\right) - \tan^{-1}\left(\frac{2W}{f_o}\right)\right]} \quad ...(9)$$

$$I = \frac{2W^3}{3f_o^3\left[\left(\frac{2W}{f_o}\right) - \tan^{-1}\left(\frac{2W}{f_o}\right)\right]} \quad ...(10)$$

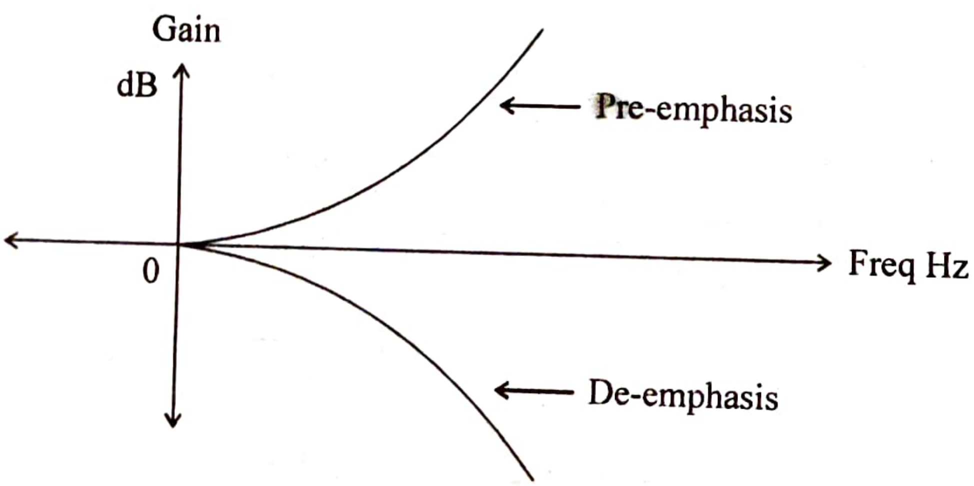


Figure 4.43 Frequency Response Curve

Applications:

1) Reduce Noise
2) Tape Recording

4.16.2 Threshold Extension by FMFB Technique

FMFB Demodulator [FM Demodulator with Negative FeedBack]

✓ The threshold reduction (or) extension is done by **FM** with **FeedBack** called **FMFB** technique, or by using a Phase Locked Loop (PLL) demodulator.

✓ Such devices are referred as **extended - threshold demodulators.**

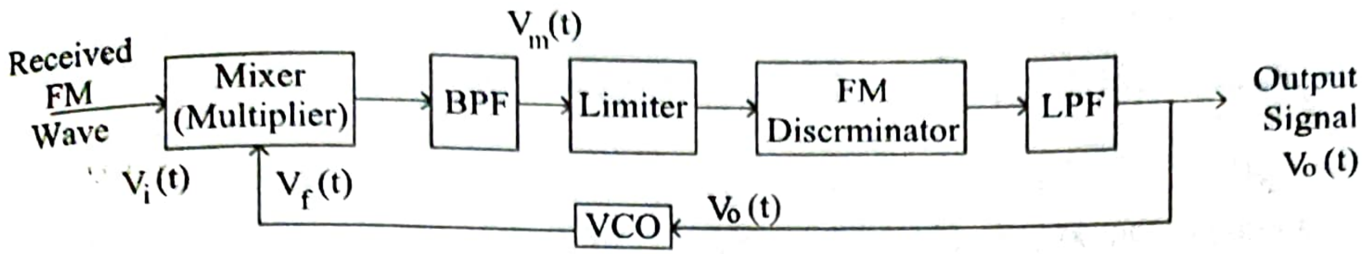


Figure 4.44 Block diagram – Threshold extension by FMFB

- ✓ The **local oscillator** of the conventional FM receiver has been replaced by a **Voltage Controlled Oscillator** (VCO) whose instantaneous output frequency is controlled by the demodulated signal.
- ✓ If the loop has sufficient gain & SNR of the receiver is reasonably large, then the VCO tracks the instantaneous phase charge.
- ✓ This tracking action reduces the frequency deviation from (Δf) to $\left(\frac{\Delta f}{1+k}\right)$.
- ✓ VCO tracking likewise reduces the noise frequency deviation.
- ✓ Mixer circuit output $= V_i(t)\, V_f(t)$ ...(1)

$$V_i(t) = A\cos\left[\omega_c t + \psi_n(t) + \psi_s(t)\right] \qquad ...(2)$$

Where,

A $\rightarrow$ Envelope of the carrier + Noise.

$\psi_n(t) \rightarrow$ Phase deviation produced in the FM wave by noise

$\psi_s(t) \rightarrow$ Phase deviation by message signal.

- ✓ VCO is an electrical oscillator, it changes its frequency for an external applied voltage $V_o(t)$.
- ✓ Thus the instantaneous frequency of VCO output becomes,

$$\omega_i = \omega_c + G_o V_o(t) \qquad ...(3)$$

$G_o \rightarrow$ Sensitivity of VCO

$\omega_c \rightarrow$ Carrier frequency.

✓ The corresponding phase angle of VCO output voltage is,

$$\phi(t) = \int \omega_i \, dt = \int \omega_c + G_o V_o(t) \, dt$$

$$\boxed{\phi(t) = \omega_c t + G_o \int V_o(t) \cdot dt} \quad ...(4)$$

Hence the VCO output becomes,

$$\boxed{V_{VCO} = C\cos\left[\omega_c t + G_o \int V_o(t) \, dt\right]} \quad ...(5)$$

C → Amplitude of VCO voltage.

✓ The demodulation process can be achieved with FeedBack (FMFB).

✓ The demodulated output voltage is similar to an open loop output, except that the output of FMFB is reduced.

✓ In the absence of a control voltage, the VCO is offset from ω_c by an amount ω_o ie., The VCO is adjusted to have a frequency $(\omega_c - \omega_o)$.

✓ The output of VCO is then given by,

$$\boxed{V_{VCO} = V_f(t) = C\cos\left[(\omega_c - \omega_o)t + G_o \int V_o(t) \, dt\right]} \quad ...(6)$$

✓ The output of the mixer is the sum and differences of input signals $V_i(t)$ & $V_f(t)$, but the BPF eliminates sum frequency and selects difference frequency.

Thus output of BPF is,

$$\boxed{V_m(t) = D\cos\left[\omega_o t + \psi_s(t) + \psi_n(t) - G_o \int V_o(t) \, dt\right]} \quad ...(7)$$

D → Amplitude of output of BPF.

✓ The resultant output with feedback,

$$V_{R_1}(t) = k\left[\frac{d\psi_s(t)}{dt} + \frac{d\psi_n(t)}{dt} - G_o V_o(t)\right] \quad ...(8)$$

k → Proportionality constant.

$$\boxed{V_{R_1}(t) = \frac{k}{1 + kG_o} \frac{d}{dt}\left[\psi_s(t) + \psi_n(t)\right]} \quad ...(9)$$

The output without feedback,

$$V_{R_2}(t) = k\frac{d}{dt}[\psi_s(t) + \psi_n(t)] \quad ...(10)$$

The eqns (9) & (10) are identical. i.e., output is similar to the output voltage of FMFB except, the amplitude of the FMFB output is reduced by a factor $(1 + kG_o)$.

$B_P \rightarrow$ Bandwidth of FMFB detector

$B_c \rightarrow$ Bandwidth of open loop FM detector.

$$B_P < B_c \quad ...(11)$$

We know that the output of BPF is,

$$(7) \Rightarrow V_m(t) = D\cos\left[\omega_o t + \psi_s(t) + \psi_n(t) - G_o\int V_o(t)\,dt\right]$$

Resultant output $\rightarrow V_{R_1}(t) = V_o(t)$

sub eqn (9) in eqn (7)

$$V_m(t) = D\cos\left[\omega_o t + \psi_s(t) + \psi_n(t) - G_o\int\frac{k}{1+kG_o}\frac{d}{dt}[\psi_s(t) + \psi_n(t)]\cdot dt\right]$$

$$V_m(t) = D\cos\left[\omega_o t + \psi_s(t) + \psi_n(t) - \frac{G_o k}{1+kG_o}[\psi_s(t) + \psi_n(t)]\right]$$

$$V_m(t) = D\cos\left[\omega_o t + \frac{(1+kG_o)[\psi_s(t) + \psi_n(t)] - kG_o[\psi_s(t) + \psi_n(t)]}{1+kG_o}\right]$$

$$V_m(t) = D\cos\left[\omega_o t + \frac{[\psi_s(t) + \psi_n(t)] + kG_o[\psi_s(t)+\psi_n(t)] - kG_o[\psi_s(t)+\psi_n(t)]}{1+kG_o}\right]$$

$$V_m(t) = D\cos\left[\omega_o t + \frac{1}{1+kG_o}[\psi_s(t) + \psi_n(t)]\right] \quad ...(12)$$

✓ The eqn (12) clearly shows that the frequency deviation in FMFB is reduced by a factor $(1+kG_o)$.

✓ The bandwidth of the FMFB is less than the bandwidth for a conventional discriminator, hence the frequency of spike generation is less in FMFB.

✓ Experimental studies confirm a threshold extension of **5 to 7 dB** for FMFB receiver which is a **significant factor** for minimum power design.

4.17 COMPARISON OF NOISE PERFORMANCE OF AM AND FM SYSTEMS

S.No	Parameter	AM	DSB - SB	SSB	FM
1.	$(SNR)_o$ and $(SNR)_c$	$(SNR)_o = \frac{m_a^2}{2 + m_a^2}(SNR)_c$	$(SNR)_o = (SNR)_c$	$(SNR)_o = (SNR)_c$	$(SNR)_o = \frac{3}{2} m_f^2 (SNR)_c$
2.	Bandwidth	$2 f_m$	$2 f_m$	f_m	$2 f_m \rightarrow$ NBFM $2(\Delta f + f_m) \rightarrow$ WBFM
3.	Threshold Effect	Present AM exhibits a threshold effect for low (S_i/N_i)	Absent	Absent	Present Threshold effect is more severe in FM than AM.
4.	Noise Performance	Poor	Better noise performance than AM	Better Noise Performance than AM	Good FM is superior to AM.
5.	Complexity	Minor	Major	Moderate	Moderate
6.	Linear and Non - linear operation	Linear	Linear	Linear	Non - Linear
7.	Figure of merit	$\gamma = \frac{(SNR)_o}{(SNR)_c} = \frac{1}{3}$	$\gamma = 1$	$\gamma = 1$	$\gamma = \frac{3}{2} m_f^2$
8.	$(SNR)_c$	$\frac{V_c^2 (1 + m_a^2 P)}{2 N_o W}$	$\frac{V_c^2 P_m}{2 W N_o}$	$\frac{V_c^2 P_m}{N_o W}$	$\frac{V_c^2}{2 N_o W}$
9.	$(SNR)_o$	$\frac{V_c^2 m_a^2 P}{2 N_o W}$	$\frac{V_c^2 P_m}{2 W N_o}$	$\frac{V_c^2 P_m}{N_o W}$	$\frac{3 V_c^2 k_f^2 P}{2 N_o W^3}$

Unit –V

Sampling & Quantization

Low pass sampling –Aliasing-Signal Reconstruction-Quantization-Uniform & non-uniform quantization -quantization noise-Logarithmic Companding of speech signal-PCM-TDM

1. **Introduction**

Why Digital Communication?

There are few reasons due to which people prefer digital communication over analog communication

1. Due to advancement in VLSI technology, it is possible to manufacture very high speed embedded circuits. Such circuits are used in digital communications.
2. High speeds computers are powerful software design tools are available. They make the development of digital communication systems feasible.
3. Internet is spread almost in every city and towns. The compatibility of digital communication systems with the internet has opened new areas of applications.

Block diagram of digital communication

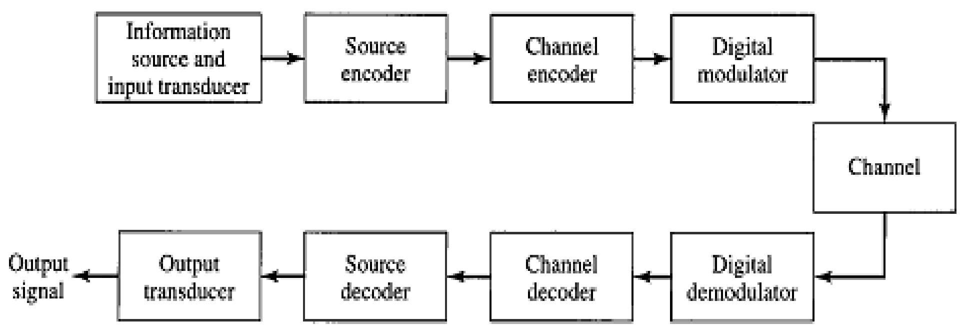


Fig.1.1 Digital Communication System

Information source: The information source generates the message signal to be transmitted. The example of discrete information sources are data from computers, teletypes etc. Even the message containing text is also discrete. The analog signal can be converted to discrete signal by sampling and quantization.

Source Encoder and Decoder: The symbols produced by the information source cannot be transmitted directly. They are first converted into digital form by source encoder. Every binary '1' and '0' is called a bit. The group of bits is called a codeword. The source encoder assigns codeword to the symbols. For every distinct symbol there is a unique codeword. The codeword can be of 4,8,16 or 32 bits length.

At the receiver, some decoder is used to perform the reverse operation to that of source encoder. It converts the binary output of the channel decoder into a symbol sequence. Both variable length and fixed length decoders and encoders can be synchronous or asynchronous.

Channel Encoder and Decoder: the communication channel adds noise and interference to the signal being transmitted. Therefore errors are introduced in the binary sequence received at the receiver. Hence errors are also introduced in the symbols generated from these binary codeword's. To avoid these errors, channel coding is done. The channel encoder adds some redundant binary bits to the input sequence.

The channel decoder at the receiver is thus able to detect error in the bit sequence, and reduce the effects of channel noise and distortion. The channel encoder and decoder thus serve to increase the reliability of the received signal.

Digital Modulators and Demodulators: the modulation allows the signals to be transmitted over a long distance. The carrier signal used by digital modulators is always continuous sinusoidal wave of high frequency. The digital modulators maps the input binary sequence of 1's and 0's to analog signal waveforms.

At the receiver, the digital demodulator converts the input modulated signal to the sequence of binary bits. Te most important parameter for the demodulator is the method of demodulation.

Communication Channel: The connection between transmitter and receiver is established through communication channel. The communication channel adds noise to the signal and attenuates it. The maximum powers that can be transmitted and signal bandwidth are limited by type of communication channel.

Advantages of Digital Communication

1. The digital communication has mostly common structure of encoding a signal so devices used are mostly similar.
2. The Digital Communication's main advantage is that it provides us added security to our information signal.
3. The digital Communication system has more immunity to noise and external interference.
4. Digital information can be saved and retrieved when necessary while it is not possible in analog.
5. Digital Communication is cheaper than Analog Communication.
6. The configuring process of digital communication system is simple as compared to analog communication system. Although, they are complex.

7. In Digital Communication System, the error correction and detection techniques can be implemented easily.

Disadvantages of Digital Communication:

1. Large System Bandwidth: Digital transmission requires a large system bandwidth to communicate the same information in a digital format as compared to analog format.

2. System Synchronization: Digital detection requires system synchronization whereas the analog signals generally have no such requirement.

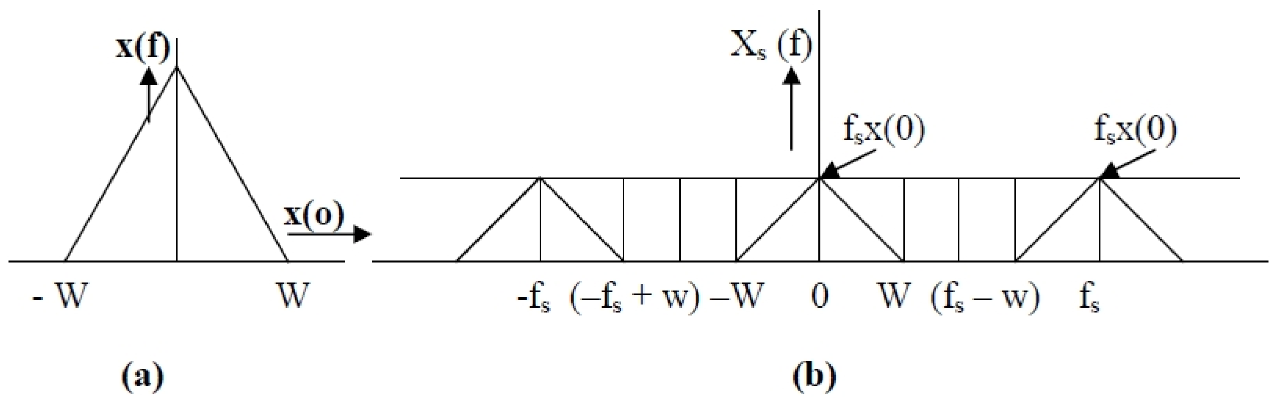


Fig 1.2 (a) Spectra of analog signal and its (b) sampled version

2 SAMPLING: A message signal may originate from a digital or analog source. If the message signal is analog in nature, then it has to be converted into digital form before it can transmit by digital means. The process by which the continuous-time signal is converted into a discrete-time signal is called Sampling. Sampling operation is performed in accordance with the sampling theorem.

LOW PASS SAMPLING

Statement of sampling theorem

1. A band limited signal of finite energy, which has no frequency components higher than W hertz, is completely described by specifying the values of the signal at instants of time separated by 1/2W seconds
2. A band limited signal of finite energy, which has no frequency components higher than W hertz, may be completely recovered from the knowledge of its samples taken at the rate of 2W samples per second.

3. The first part of the above statement tells about sampling of the signal and second part tells about reconstruction of the signal. Above statement can be combined and stated as follows.

A continuous time signal can be completely represented in its samples and recovered back if the sampling frequency is twice of the highest frequency content of the signal, ie. $f_s \geq 2W$

Here fs is the sampling frequency and W is the higher frequency content.

Proof of Sampling theorem:

There are two parts: I) Representation of x(t) interms of its Samples.

II) Reconstruction of x(t) from its samples.

Part I: Representation of x(t) interms of its Samples.

Step 1: Define $x_\delta(t)$

Step 2: Fourier transform of $x_\delta(t)$ $ie.$ $X_\delta(f)$

Step 3: Relation between X(f) and $X_\delta(f)$

Step 4: Relation between x(t) and x(nTs)

Step 1: Define $x_\delta(t)$:

The sampled signal $x_\delta(t)$ is given as

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t-nT_s) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t-nT_s)$$

$x(nT_s)$ is basically x(t) sampled at t=0 $\pm1, \pm2, \pm3, \dots$

Here observe that $x_\delta(t)$is the product of x(t) and impulse train $\delta(t)$ as shown in the figure.

In the above equation $\delta(t-nT_s)$ indicates that the samples placed at $\pm T_s$,$\pm 2T_s$,$3\pm T_s$ ... and so on.

Step 2: Fourier transform of $x_\delta(t)$ $ie.$ $X_\delta(f)$

Taking FT of equation

$$X_\delta(f) = FT\left\{\sum_{n=-\infty}^{\infty} x(t)\delta(t-nT_s)\right\}$$

=FT (Product of x(t) and impulse train)

We know that FT of product in time domain becomes convolution in frequency domain i.e.,

$$X_\delta(f) = FT\{x(t)\} * FT\{\delta(t-nT_s)\}$$

By definitions, x(t) $\overset{FT}{\longleftrightarrow}$ X(f) and

$\delta(t-nT_s)$ $\overset{FT}{\longleftrightarrow}$ $f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s)$

Hence equation becomes,

$$X_\delta(f) = X(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f - nf_s)$$

Since convolution is linear,

$$X_\delta(f) = f_s \sum_{n=-\infty}^{\infty} X(f) * \delta(f - nf_s)$$

$= f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$ by shifting property of impulse function

$$= \cdots f_s X(f - 2f_s) + f_s X(f - f_s) + f_s X(f_s) + \cdots$$

Comments

i) The R.H.S. of the above equation shows that X(f) is placed at $\pm f_s$,$\pm 2f_s$,$3 \pm f_s$ ...

ii) This means X(f) is periodic in f_s

iii) If sampling frequency is $f_s = 2W$, then the spectrums X(f) just touch each other.

Step 3: Relation between X(f) and $X_\delta(f)$

Important assumption: Let us assume that $f_s = 2W$, then as per above diagram,

$X_\delta(f) = f_s X(f)$ for -$W \leq f \geq W$ and $f_s = 2W$

X(f) = $\frac{1}{f_s} X_\delta(f)$

Step 4: Relation between x(t) and x(nTs)

DTFT is, X(Ω) =$\sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n}$

X(f) =$\sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi f n}$

In the above equation'f' is the frequency of DT signal, if we replace X(f) by $X_\delta(f)$, then 'f' becomes frequency of CT signal ie.,

$X_\delta(f) = \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi \frac{f}{f_s} n}$ in the above equation 'f ' is the frequency of CT signal and $\frac{f}{f_s}$ = Frequency of the DT signal. Since x(n)=x(nTs), i.e. samples of x(t), then we have,

$X_\delta(f) = \frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s) e^{-j2\pi f n T_s}$ since $\frac{1}{f_s}$=T_s

Putting above expression in equation

X(f) =$\frac{1}{f_s} \sum_{n=-\infty}^{\infty} x(nT_s) e^{-j2\pi f n T_s}$

Inverse Fourier Transform (IFT) of above equation x(t) i.e.,

x(t)= IFT$\left\{X(f) = \frac{1}{f_s}\sum_{n=-\infty}^{\infty} x(nT_s)e^{-j2\pi f n T_s}\right\}$

Part II: Reconstruction of x(t) from its samples.

Step 1: Take inverse Fourier transform of X(f) which is in terms of $X_\delta(f)$

Step 2: Show that x(t) is obtained back with the help of interpolation function.

Step 1: Take inverse Fourier transform of X(f) which is in terms of $X_\delta(f)$

x(t)= $\int_{-\infty}^{+\infty}\left\{\frac{1}{f_s}\sum_{n=-\infty}^{\infty} x(nT_s)e^{-j2\pi f n T_s}\right\}e^{-j2\pi f t}\, df$

Here the integration can be taken from$-W \le f \ge W$, Since X(f) = $\frac{1}{f_s}X_\delta(f)$ for

$-W \le f \ge W$

x(t)= $\int_{-W}^{W}\frac{1}{f_s}\left\{\sum_{n=-\infty}^{\infty} x(nT_s)e^{-j2\pi f n T_s}e^{-j2\pi f t}\right\} df$

Interchanging the order of summation and integration

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\frac{1}{f_s}\int_{-W}^{W} e^{-j2\pi f(t-nT_s)}\; df$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\frac{1}{f_s}\left[\frac{e^{-j2\pi f(t-nT_s)}}{j2\pi(t-nT_s)}\right]_{-W}^{W}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\frac{1}{f_s}\left\{\frac{e^{j2\pi W(t-nT_s)} - e^{-j2\pi W(t-nT_s)}}{j2\pi(t-nT_s)}\right\}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\frac{1}{f_s}\frac{\sin 2\pi W(t-nT_s)}{\pi(t-nT_s)}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\frac{\sin\pi(2Wt-2WnT_s)}{\pi(f_s t - f_s nT_s)}$$

Here $f_s = 2W$, hence $T_s = \frac{1}{f_s} = \frac{1}{2W}$, simplifying above equation,

X(t)= $\sum_{n=-\infty}^{\infty} x(nT_s)\frac{\sin\pi(2Wt-n)}{\pi(2Wt-n)}$

$= \sum_{n=-\infty}^{\infty} x(nT_s)\, sinc\,(2Wt-n) \qquad since\ \frac{\sin\pi\theta}{\pi\theta} = sinc\theta$

Step 2: Let us interpet the above equation. Expanding, we get,

x(t)=…+x(-2T_s) sinc(2Wt+2)+x(-T_s)sinc(2Wt+1)+x(0)sinc(2Wt)+x(T_s)sinc(2Wt-1)+…

Comments:

The samples x(nTs) are weighted by sinc functions.

The sinc function is the interpolating function.

Step 3: reconstruction of x(t) by low pass filter

When the interpolated signal is passed through the low pass filter of bandwidth $-W \leq f \geq W,$ then the reconstructed waveform is obtained. The individual sinc function are interpolated to get smooth x(t).

Effects of undersampling (Aliasing)

While proving sampling theorem we consider that fs ≥ 2W. Consider the case of fs < 2W. Then the spectrum of $X_\delta(f)$ will overlap.

Comments:

i) The spectrum located at X(f), X(f-fs), X(f-2fs),… overlap on each other.

ii) Consider the spectrum of X(f) and X(f-fs). The frequencies from (fs-W) to W are overlapping in these spectrums.

i) The high frequencies near 'ω' in X(f-fs) overlap with low frequencies (fs-W) in X(f).

3 **Definition of aliasing:** When the high frequency interferes with low frequency and appears as low frequency; then the phenomenon is called aliasing.

Effects of aliasing : i) since high and low frequencies interfere with each other, distortion is generated.

ii) The data is lost and it cannot be recovered.

Different ways to avoid aliasing

Aliasing can be avoided by two methods.

i) Sampling rate fs ≥ 2W.

ii) Strictly bandlimit the signal to 'W'

i) Sampling rate fs ≥ 2W

When the sampling rate is made higher than 2W, then the spectrum will not overlap and there will be sufficient gap between the individual spectrum.

Oversampling:

When the signal is sampled at a rate much higher than Nyquist rate, it is called oversampling. It is necessary to avoid aliasing error in the signal. But it increases transmission bandwidth,

ii) Bandlimiting the signal

The sampling rate is, fs=2W. Ideally speaking there should be no aliasing. But there can be few components higher than 2W. These components created aliasing. Hence low pass filter is used before sampling the signals. Thus the output of low pass filter is strictly band limited and there are no frequency components higher than 'W'. Then there will be no aliasing.

Nyqusit rate and Nyquist interval

Nyquist rate: when the sampling rate becomes exactly equal to 2W samples/sec for a given bandwidth of W hertz, then it is called Nyquist rate.

Nyquist interval: it is time interval between any two adjacent samples when sampling rate is Nyquist rate.

Sample and Hold Circuit for Signal Recovery.

In both the natural sampling and flat-top sampling methods, the spectrum of the signals are scaled by the ratio τ/Ts, where τ is the pulse duration and Ts is the sampling period. Since this ratio is very small, the signal power at the output of the reconstruction filter is correspondingly small. To overcome this problem a sample-and-hold circuit is used. The Sample-and-Hold circuit consists of an amplifier of unity gain and low output impedance, a switch and a capacitor; it is assumed that the load impedance is large. The switch is timed to close only for the small duration of each sampling pulse, during which time the capacitor charges up to a voltage level equal to that of the input sample. When the switch is open, the capacitor retains the voltage level until the next closure of the switch. Thus the sample-and-hold circuit produces an output waveform that represents a staircase interpolation of the original analog signal.

Natural Sampling:

In this method of sampling, an electronic switch is used to periodically shift between the two contacts at a rate of fs = (1/Ts) Hz, staying on the input contact for C seconds and on the grounded contact for the remainder of each sampling period. The output xs(t) of the sampler consists of segments of x(t) and hence xs(t) can be considered as the product of x(t) and sampling function s(t). $xs(t) = x(t) \cdot s(t)$

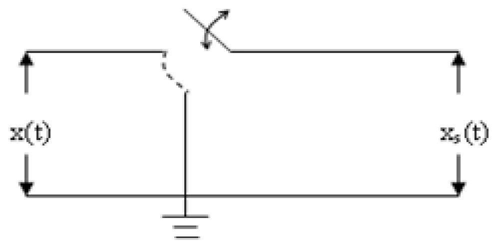


Fig.2.1 Circuit for Natural Sampling

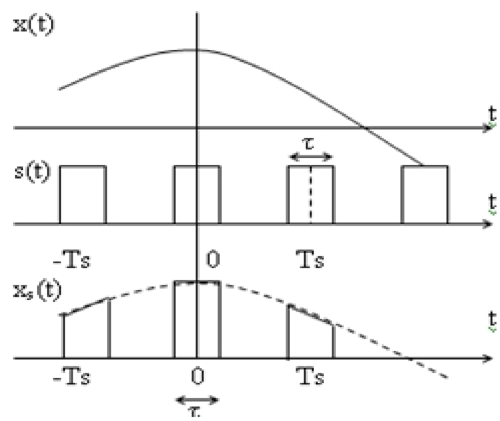


Fig:2.2 Natural Sampling Waveforms

Applying Fourier transform

Using $\begin{cases} x(t) \longleftrightarrow X(f) \\ x(t)\cos(2\pi f0t) \longleftrightarrow \frac{1}{2}[X(f-f0) + X(f+f0)] \end{cases}$

$Xs(f) = Co.X(f) + C1\,[X(f-f0) + X(f+f0)] + C2\,[X(f-f0) + X(f+f0)] + \ldots \ldots\; n \neq 0$

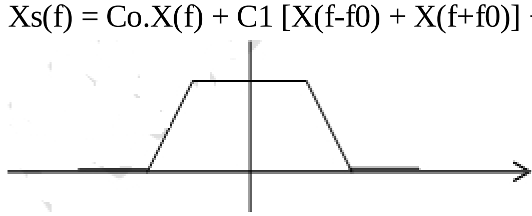


Fig 2.3 Message Signal Spectrum

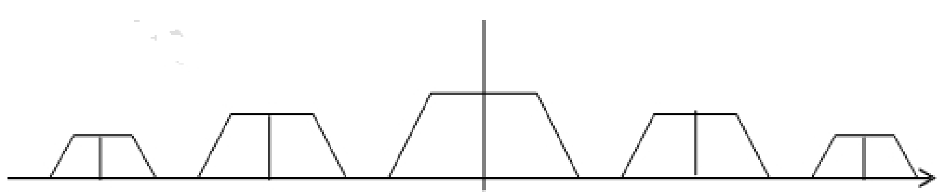


Fig 2.4 Sampled Signal Spectrum (fs > 2W)

The signal xs(t) has the spectrum which consists of message spectrum and repetition of message spectrum periodically in the frequency domain with a period of fs. But the message term is scaled by „Co". Since the spectrum is not distorted it is possible to reconstruct x(t) from the sampled waveform xs(t).

3 Quantization:

The process of transforming Sampled amplitude values of a message signal into a discrete amplitude value is referred to as Quantization (or)This is the process of setting the sample amplitude, which can be continuously variable to a discrete value

The quantization Process has a two-fold effect:

1. the peak-to-peak range of the input sample values is subdivided into a finite set of decision levels or decision thresholds that are aligned with the risers of the staircase, and

2. the output is assigned a discrete value selected from a finite set of representation levels that are aligned with the treads of the staircase..

A quantizer is memory less in that the quantizer output is determined only by the value of a corresponding input sample, independently of earlier analog samples applied to the input.

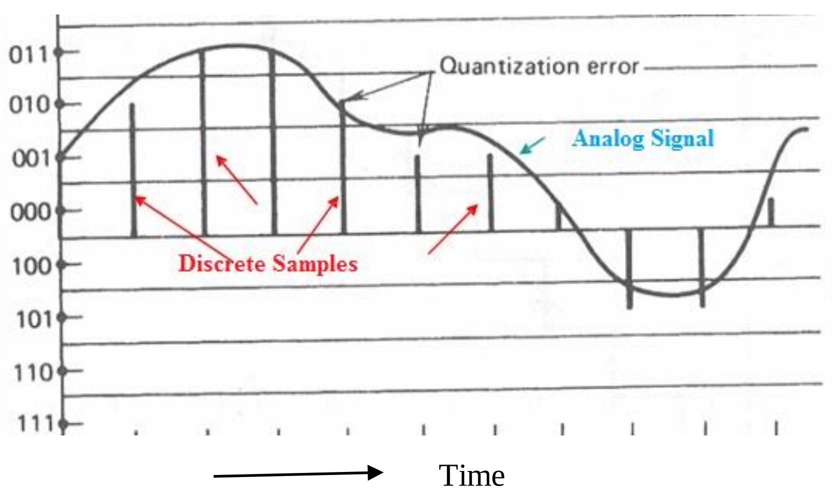


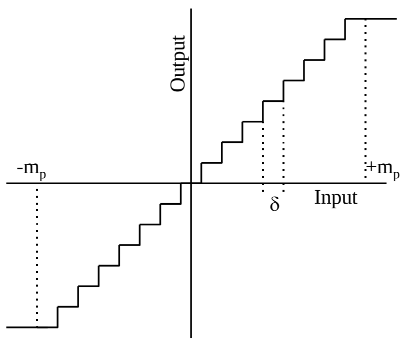
Fig:3.1 Typical Quantization process.

Types of Quantizers:

1. Uniform Quantizer
2. Non- Uniform Quantizer

. Look at Uniform Quantization first, where the discrete values are evenly spaced.

Uniform Quantization



We assume that the amplitude of the signal m(t) is confined to the range ($-m_p$, $+m_p$). This range ($2m_p$) is divided into L levels, each of step size δ, given by

$$\delta = 2\, m_p / L$$

A sample amplitude value is approximated by the midpoint of the interval in which it lies. The input/output characteristics of a uniform quantizer is shown in fig.

In Uniform type, the quantization levels are uniformly spaced, whereas in non-uniform type the spacing between the levels will be unequal and mostly the relation is logarithmic.

Types of Uniform Quantizers: (based on I/P - O/P Characteristics)

1. Mid-Rise type Quantizer
2. Mid-Tread type Quantizer

In the stair case like graph, the origin lies the middle of the tread portion in Mid -Tread type where as the origin lies in the middle of the rise portion in the Mid-Rise type.

Mid - tread type: Quantization levels - odd number.

Mid - Rise type: Quantization levels - even number.

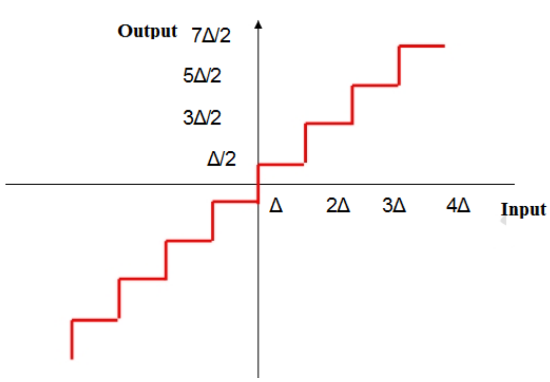


Fig 3.2 Input-Output Characteristics of a Mid-Tread type Quantizer

Derivation of Quantization Error/Noise or Noise Power for Uniform (linear) Quantization

Because of quantization, inherent errors are introduced in the signal. This error is called quantization error. We have defined quantization error as,

$\varepsilon = x_q(nT_s)-x(nT_s)$

Let an input $x(nT_s)$ be of continuous amplitude in the range $-x_{max}$ to $+x_{max}$.

We know that the total excursion of input x(nTs) is mapped into levels on vertical axix. That is when input is 4δ, output is $\frac{7}{2}\delta$ and when input is -4δ, output is $-\frac{7}{2}\delta$. That is $+x_{max}$ represents $\frac{7}{2}\delta$ and $-x_{max}$ represents $-\frac{7}{2}\delta$. Therefore the total amplitude range becomes,

Total amplitude range = $x_{max} - (-x_{max}) = 2x_{max}$

If this amplitude range is divided into 'q' levels of quantizer, then the step size $'\delta'$ is given as

$$\delta = \frac{x_{max} - (-x_{max})}{q}$$

$$= \frac{2x_{max}}{q}$$

If signal x(t) is normalized to minimum and maximum values equal to 1, then

$$x_{max} = 1$$

$$-x_{max} = -1$$

Therefore step size will be,

$\delta = \frac{2}{q}$ (for normalized signal)

If the step size $'\delta'$ is sufficiently small, then it is reasonable to assume that the quantization error, 'ε' will be uniformly distributed random variable. The maximum quantization error is given as,

$\varepsilon_{max} = \left|\frac{\delta}{2}\right|$ i.e,

$$-\frac{\delta}{2} \geq \varepsilon_{max} \geq \frac{\delta}{2}$$

Thus over the interval $\left[-\frac{\delta}{2}, \frac{\delta}{2}\right]$ quantization error is uniformly distributed random variable.

A random variable is said to be uniformly distributed over an interval(a,b). Then PDF of 'x' is given by,

$f_x = 0 \quad for \ x \leq a$

$= \frac{1}{b-a} \ for \ a < x > b$

$= 0 \quad for \ x > b$

Thus with the help of above equation we can define the probability density function for quantization error 'ε' as,

$f_\varepsilon(\varepsilon) = 0 \quad for \ \varepsilon \leq -\frac{\delta}{2}$

$= \frac{1}{\delta} \ for - \frac{\delta}{2} < \varepsilon \leq \frac{\delta}{2}$

$= 0 \quad for \ \varepsilon > \frac{\delta}{2}$

We can see that quantization error 'ε' has zero average value. That is mean 'm_ε' of the quantization error is zero.

The signal to quantization noise ratio of the quantizer is defined as

$$\frac{S}{N} = \frac{Signal\ power\ (normalized)}{Noise\ power\ (normalized)}$$

If type of signal at input i.e, x(t) is known, then it is possible to calculate signal power.

The noise power is given as,

Nosie power = $\frac{V_{noise}^2}{R}$

Here V_{noise}^2 is the mean square value of noise voltage. Since noise is defined by random variable 'ε' and PDF fε (ε), its mean square 'X' is given as,

$x^2 = E[X^2] = \int_{-\infty}^{\infty} x^2 f_x(x)dx$ by definition

Here $E[\varepsilon^2] = \int_{-\infty}^{\infty} \varepsilon^2 f_\varepsilon(\varepsilon)d\varepsilon$

We can write above equation as,

$$E[\varepsilon^2] = \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \varepsilon^2 \times \frac{1}{\delta} d\varepsilon = \frac{1}{\delta}\left[\frac{\varepsilon^3}{3}\right]_{-\frac{\delta}{2}}^{\frac{\delta}{2}} = \frac{1}{\delta}\left[\frac{(\delta/2)^3}{3} + \frac{(\delta/2)^3}{3}\right]$$

$$= \frac{1}{3\delta}\left[\frac{\delta^3}{8} + \frac{\delta^3}{8}\right] = \frac{\delta^2}{12}$$

The mean square value of noise voltage is,

V_{noise}^2 = Mean square value = $\frac{\delta^2}{12}$

When load resistance, R=1 ohm, then the noise power is normalized i.e,

Noise power (normalized) =$\frac{V_{noise}^2}{1}$

$$= \frac{\delta^2/12}{1} = \frac{\delta^2}{12}$$

Thus we have,

Normalized noise power or quantization noise power = $\frac{\delta^2}{12}$; for linear quantization.

Or Quantization error (in terms of power)

Derivation of Maximum signal to Quantization Noise ratio for Liner quantization

Signal to quantization noise ratio is given as

$$\frac{S}{N} = \frac{Signal\ power\ (normalized)}{Noise\ power\ (normalized)}$$

$$= \frac{Normalized\ power}{\frac{\delta^2}{12}}$$

The number of bits ‘v’ and quantization levels ‘q’ are related as,

Q=2^v

Putting this value in equation we have

$$\delta = \frac{2}{2^v} x_{max}$$

Putting this value in the equation we get,

$$\frac{S}{N} = \frac{P}{\frac{4x_{max}^2}{2^{2v}} \times \frac{1}{12}} = \frac{3P}{x_{max}^2} . 2^{2v}$$

This is the required relation for maximum signal to quantization noise ratio. Thus

Maximum signal to quantization noise ratio : $\frac{S}{N} = \frac{3P}{x_{max}^2} . 2^{2v}$

This equation shows that signal noise to noise power ratio of quantizer increases exponentially with increasing bits per sample.

If we assume that input x(t) is normalized, i.e.,

$x_{max} = 1$

then signal to quantization noise ratio will be,

$$\frac{S}{N} = 3 \times 2^{2v} \times P$$

If the destination signals power 'P' is normalized, i.e,

$P \leq 1$

Then the signal to noise ratio is given as,

$$\frac{S}{N} \leq 3 \times 2^{2v}$$

Since

$x_{max} = 1$ and ≤ 1 , the signal to noise ratio given by above equation is normalized.

Expressing the signal to noise ratio is decibels,

$\left(\frac{S}{N}\right) db = 10 log_{10} \left(\frac{S}{N}\right) db$ Since power ratio.

$\leq 10\ log_{10}[3 \times 2^{2v}]$

$\leq (4.8 + 6v) db$

Thus signal to quantization noise ratio for normalized values of power:

$\left(\frac{S}{N}\right) db \leq (4.8 + 6v) db$ 'P' and amplitude of input x(t)

Non - Uniform Quantizer:

In Non - Uniform Quantizer the step size varies. The use of a non - uniform quantizer is equivalent to passing the baseband signal through a compressor and then applying the compressed signal to a uniform quantizer. The resultant signal is then transmitted.

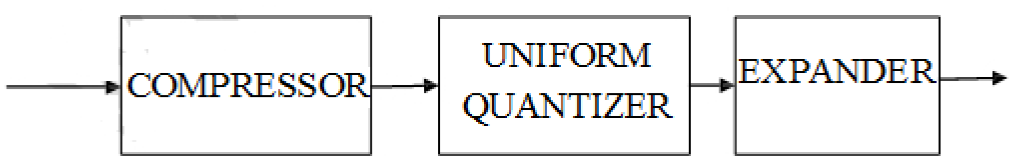


Fig: 3.3 MODEL OF NON UNIFORM QUANTIZER

At the receiver, a device with a characteristic complementary to the compressor called

Expander is used to restore the signal samples to their correct relative level. The Compressor and expander take together constitute a Compander.

Compander = Compressor + Expander

Advantages of Non - Uniform Quantization :

1. Higher average signal to quantization noise power ratio than the uniform quantizer when the signal pdf is non uniform which is the case in many practical situation.

2. RMS value of the quantizer noise power of a non - uniform quantizer is substantially proportional to the sampled value and hence the effect of the quantizer noise is reduced.

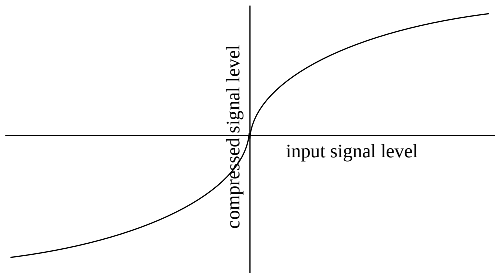
4. Companding

In a uniform or linear PCM system the size of every quantization interval is determined by the SQR requirement of the lowest signal to be encoded. This interval is also for the largest signal - which therefore has a much better SQR.

Example :A 26 dB SQR for small signals and a 30 dB dynamic range produces a 56 dB SQR for the maximum amplitude signal.

In this way a uniform PCM system provides unneeded quality for large signals. In speech the max amplitude signals are the least likely to occur. The code space in a uniform PCM system is very inefficiently utilised.

A more efficient coding is achieved if the quantization intervals increase with the sample value. When the quantization interval is directly proportional to the sample value (assign small quantization intervals to small signals and large intervals to large signals) the SQR is constant for all signal levels. With this technique fewer bits per sample are required to provide a specified SQR for small signals and an adequate dynamic range for large signals (but still with the SQR as for the small signals). The quantization intervals are not constant and there will be a non-linear relationship between the code words and the values they represent.



Originally to produce the non linear quantization the baseband signal was passed through a non-linear amplifier with input/output characteristics as shown before the samples were taken. Low level signals were amplified and high level signals were attenuated. The larger the sample value the more it is **comp**ressed before encoding. The PCM decoder ex**pands** the compressed value using an inverse compression characteristic to recover the original sample value. The two processes are called **companding**.

There are 2 companding schemes to describe the curve above:

1. **μ-LawCompanding**

   This is used in North America and Japan. It uses a logarithmic compression curve which is ideal in the sense that quantization intervals and hence quantization noise is directly proportional to signal level (and so a constant SQR).

2. **A-LawCompanding**

   This is the ITU-T standard. It is used in Europe and most of the rest of the world. It is very similar to the μ-Law coding. It is represented by straight line segments to facilitate digital companding.

Originally the non linear function was obtained using non linear devices such as special diodes. These days in a PCM system the A to D and D to A converters (ADC and DAC) include a companding function.

5 PULSE COMMUNICATION:

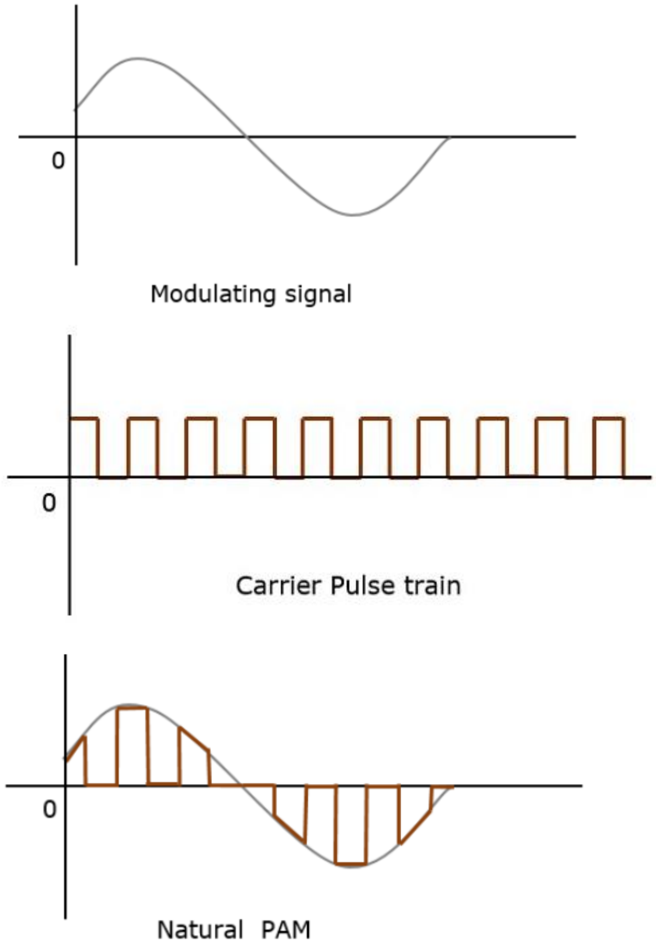
5.1 Pulse Amplitude Modulation

Pulse Amplitude Modulation (PAM) is an analog modulating scheme in which the amplitude of the pulse carrier varies proportional to the instantaneous amplitude of the message signal.

The pulse amplitude modulated signal, will follow the amplitude of the original signal, as the signal traces out the path of the whole wave. In natural PAM, a signal sampled at the Nyquist rate is reconstructed, by passing it through an efficient Low Pass Frequency (LPF) with exact cutoff frequency.

Flat-top sampling is the process in which sampled signal can be represented in pulses for which the amplitude of the signal cannot be changed with respect to the analog signal, to be sampled. The tops of amplitude remain flat. This process simplifies the circuit design.

Though the PAM signal is passed through an LPF, it cannot recover the signal without distortion. Hence to avoid this noise, flat-top sampling is done as shown in the following figure.
The following figures explain the Pulse Amplitude Modulation.



Though the PAM signal is passed through an LPF, it cannot recover the signal without distortion. Hence to avoid this noise, flat-top sampling is done as shown in the following figure.

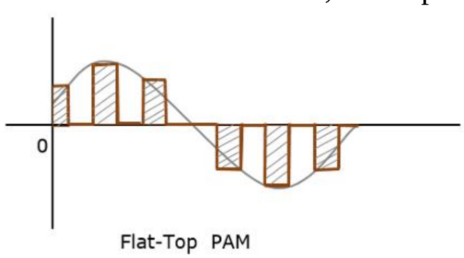


Fig 5.1 PAM Waveform

5.2 Pulse Width Modulation:

Pulse Width Modulation (PWM) or Pulse Duration Modulation (PDM) or Pulse Time Modulation (PTM) is an analog modulating scheme in which the duration or width or time of the pulse carrier varies proportional to the instantaneous amplitude of the message signal.

The width of the pulse varies in this method, but the amplitude of the signal remains constant. Amplitude limiters are used to make the amplitude of the signal constant. These circuits clip off the amplitude, to a desired level and hence the noise is limited.

The following figures explain the types of Pulse Width Modulations.

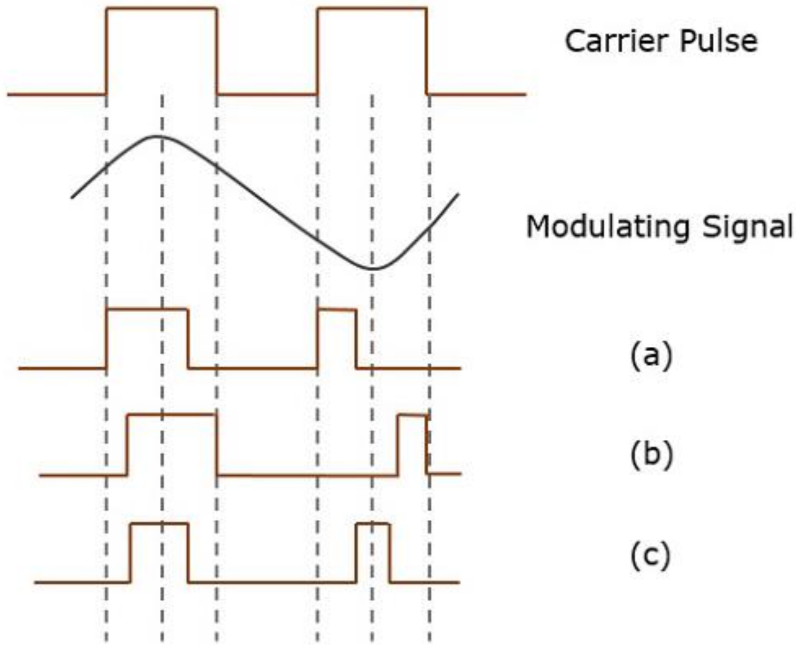


Fig 5.2 PWM waveform

There are three variations of PWM. They are −

- The leading edge of the pulse being constant, the trailing edge varies according to the message signal.
- The trailing edge of the pulse being constant, the leading edge varies according to the message signal.
- The center of the pulse being constant, the leading edge and the trailing edge varies according to the message signal.

5.3 Pulse Position Modulation (PPM) is an analog modulating scheme in which the amplitude and width of the pulses are kept constant, while the position of each pulse, with reference to the position of a reference pulse varies according to the instantaneous sampled value of the message signal.

The transmitter has to send synchronizing pulses (or simply sync pulses) to keep the transmitter and receiver in synchronism. These sync pulses help maintain the position of the pulses. The following figures explain the Pulse Position Modulation.

Pulse position modulation is done in accordance with the pulse width modulated signal. Each trailing of the pulse width modulated signal becomes the starting point for pulses in PPM signal. Hence, the position of these pulses is proportional to the width of the PWM pulses.

Advantage

As the amplitude and width are constant, the power handled is also constant.

Disadvantage

The synchronization between transmitter and receiver is a must.

Generation of ppm:

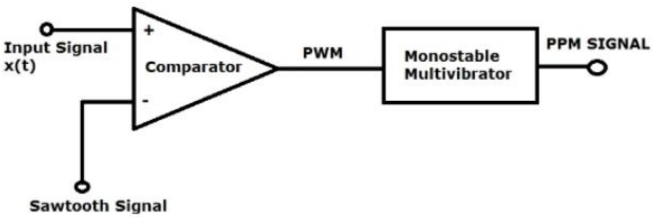


Fig 5.3 PPM Generation

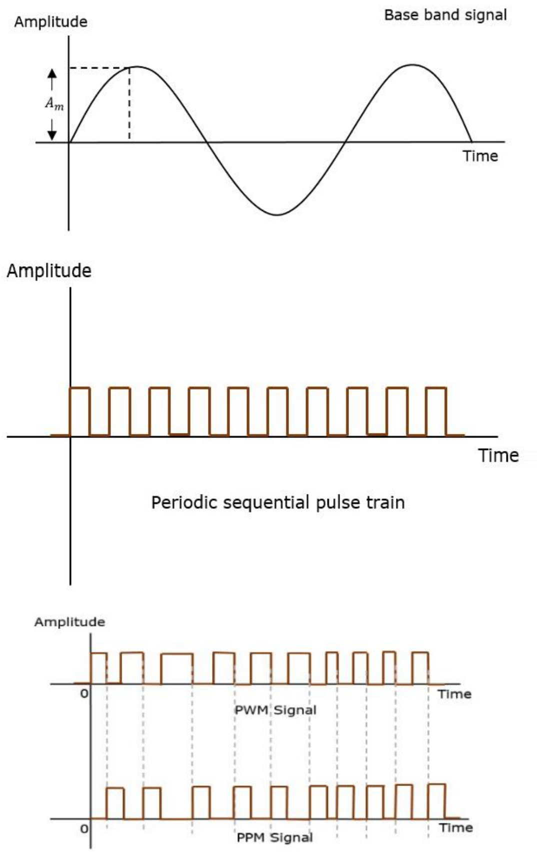


Fig 5.4 PWM and PPM Waveforms

6 COMPARISON BETWEEN PAM, PWM, AND PPM :

The comparison between the above modulation processes is presented in a single table.

PAM	PWM	PPM
Amplitude is varied	Width is varied	Position is varied
Bandwidth depends on the	Bandwidth depends on the	Bandwidth depends on the

width of the pulse	rise time of the pulse	rise time of the pulse
Instantaneous transmitter power varies with the amplitude of the pulses	Instantaneous transmitter power varies with the amplitude and width of the pulses	Instantaneous transmitter power remains constant with the width of the pulses
System complexity is high	System complexity is low	System complexity is low
Noise interference is high	Noise interference is low	Noise interference is low
It is similar to amplitude modulation	It is similar to frequency modulation	It is similar to phase modulation

7 PULSE CODE MODULATION:

Modulation is the process of varying one or more parameters of a carrier signal in accordance with the instantaneous values of the message signal.

The message signal is the signal which is being transmitted for communication and the carrier signal is a high frequency signal which has no data, but is used for long distance transmission.

There are many modulation techniques, which are classified according to the type of modulation employed. Of them all, the digital modulation technique used is **Pulse Code Modulation (PCM)**.

A signal is pulse code modulated to convert its analog information into a binary sequence, i.e., **1s** and **0s**. The output of a PCM will resemble a binary sequence. The following figure shows an example of PCM output with respect to instantaneous values of a given sine wave.

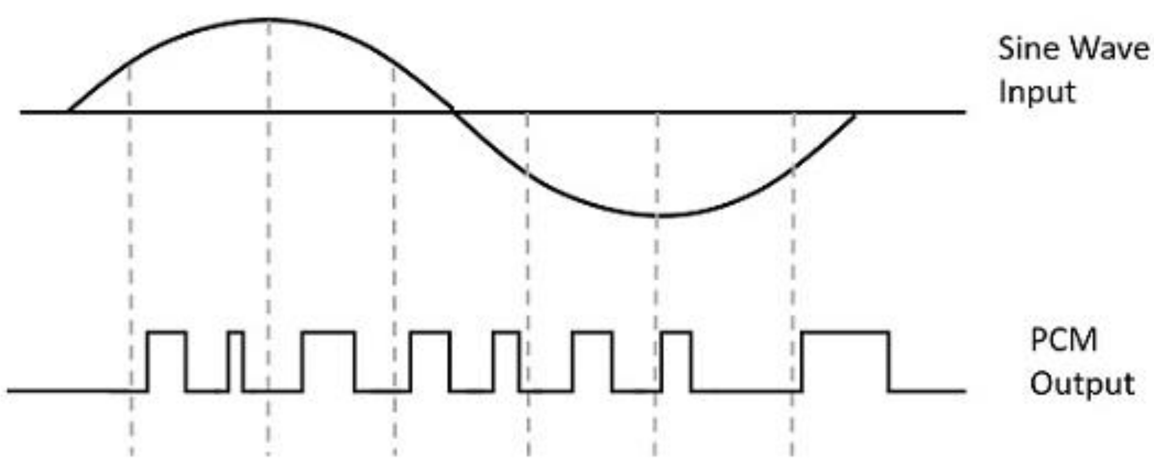


Fig 7.1 PCM Waveform

Instead of a pulse train, PCM produces a series of numbers or digits, and hence this process is called as **digital**. Each one of these digits, though in binary code, represent the approximate amplitude of the signal sample at that instant.

In Pulse Code Modulation, the message signal is represented by a sequence of coded pulses. This message signal is achieved by representing the signal in discrete form in both time and amplitude.

Basic Elements of PCM

The transmitter section of a Pulse Code Modulator circuit consists of **Sampling, Quantizing** and **Encoding**, which are performed in the analog-to-digital converter section. The low pass filter prior to sampling prevents aliasing of the message signal.

The basic operations in the receiver section are **regeneration of impaired signals, decoding,** and **reconstruction** of the quantized pulse train. Following is the block diagram of PCM which represents the basic elements of both the transmitter and the receiver sections.

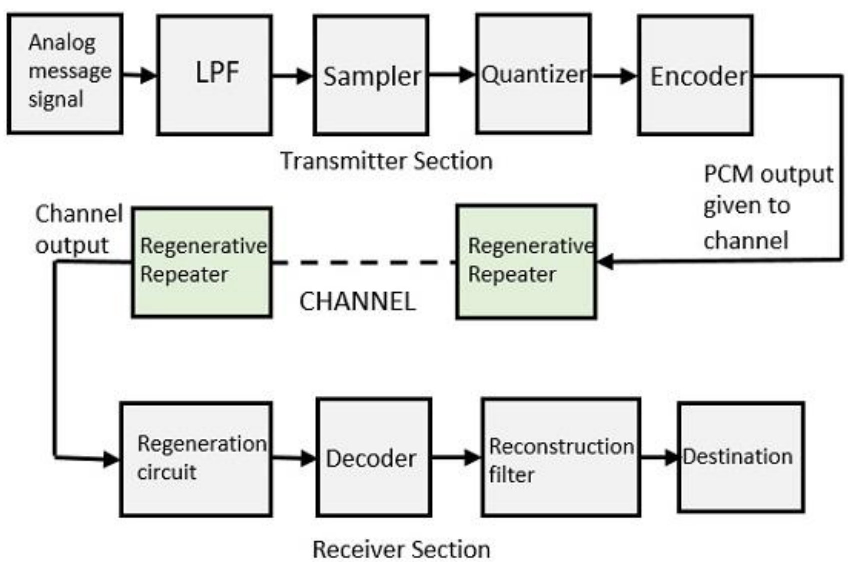


Fig 7.2 PCM Block Diagram

Low Pass Filter

This filter eliminates the high frequency components present in the input analog signal which is greater than the highest frequency of the message signal, to avoid aliasing of the message signal.

Sampler

This is the technique which helps to collect the sample data at instantaneous values of message signal, so as to reconstruct the original signal. The sampling rate must be greater than twice the highest frequency component **W** of the message signal, in accordance with the sampling theorem.

Quantizer

Quantizing is a process of reducing the excessive bits and confining the data. The sampled output when given to Quantizer, reduces the redundant bits and compresses the value.

Encoder

The digitization of analog signal is done by the encoder. It designates each quantized level by a binary code. The sampling done here is the sample-and-hold process. These three sections (LPF, Sampler, and Quantize) will act as an analog to digital converter. Encoding minimizes the bandwidth used.

Regenerative Repeater

This section increases the signal strength. The output of the channel also has one regenerative repeater circuit, to compensate the signal loss and reconstruct the signal, and also to increase its strength.

Decoder

The decoder circuit decodes the pulse coded waveform to reproduce the original signal. This circuit acts as the demodulator.

Reconstruction Filter

After the digital-to-analog conversion is done by the regenerative circuit and the decoder, a low-pass filter is employed, called as the reconstruction filter to get back the original signal.

Hence, the Pulse Code Modulator circuit digitizes the given analog signal, codes it and samples it, and then transmits it in an analog form. This whole process is repeated in a reverse pattern to obtain.

8. **Time-division multiplexing (TDM)** is considered to be a digital procedure which can be employed when the transmission medium data rate quantity is higher than the data rate requisite of the transmitting and receiving devices. In TDM, corresponding frames carry data to be transmitted from the different sources. Each frame consists of a set of time slots, and portions of each source is assigned a time slot per frame.

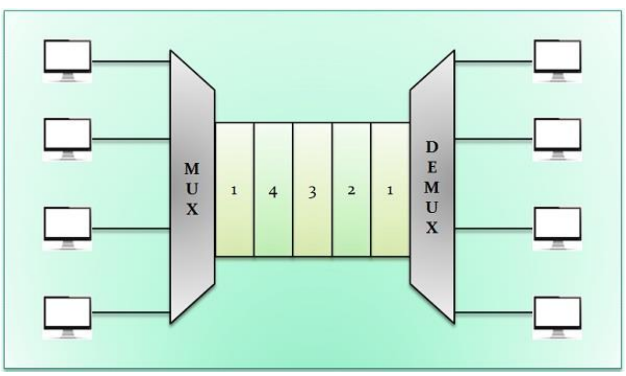


Fig 8.1 TDM system

Types of TDM :

- **Synchronous Time-Division Multiplexing** – In this type the synchronous term signifies that the multiplexer is going to assign precisely the same slot to each device at every time even if a device has anything to send or not. If it doesn't have something, the time slot would be empty. TDM uses **frames**to group time slots which covers a complete cycle of time slots. Synchronous TDM uses a concept, i.e., **interleaving** for building a frame in which a multiplexer can take one data unit at a time from each device, then another data unit from each device and so on. The order of the receipt notifies the demultiplexer where to direct each time slot, which eliminates the need of addressing. To recover from timing inconsistencies **Framing bits** are used which are usually appended to the beginning of each frame. **Bit stuffing** is used to force speed relationships to equalize the speed between several devices into an integer multiple of each other. In bit stuffing, the multiplexer appends additional bits to device's source stream.

- **Asynchronous Time-Division Multiplexing** – Synchronous TDM waste the unused space in the link hence it does not assure the efficient use of the full capacity of the link. This gave rise to Asynchronous TDM. Here Asynchronous means flexible not fixed. In Asynchronous TDM several low rate input lines are multiplexed to a single higher speed line. In Asynchronous TDM, the number of slots in a frame is less than the number of data lines. On the contrary, In Synchronous TDM the number of slots must be equal to the number of data lines. That's why it, avoids the wastage of the link capacity.

9. FDM

Frequency-division multiplexing (FDM) is an analog technique which is implemented only when the bandwidth of the link is higher than the merged bandwidth of the signals to be transmitted. Each sending device produces signals which modulate at distinct carrier frequencies. To hold the modulated signal, the carrier frequencies are isolated by adequate bandwidth.

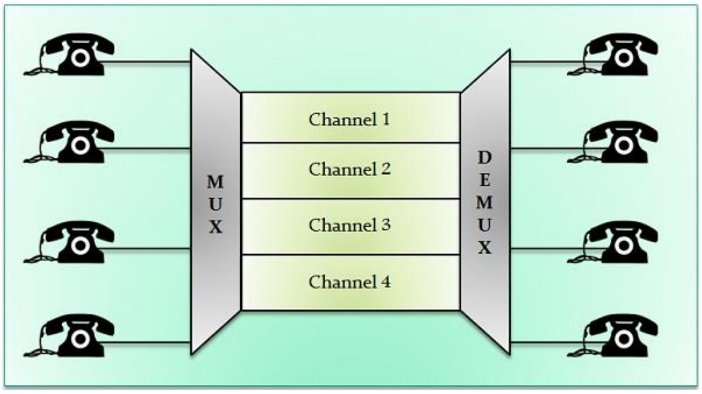


Fig 9.1 FDM System

The modulated signals are then merged into one compound signal that can be transferred by the link. The signals travel through the bandwidth ranges referred to as channels.

Signals overlapping can be controlled by using unutilized bandwidth strips for segregating the channels, these are known as **guard bands**. Also, carrier frequencies should not interrupt with the original data frequencies. If any condition fails to adhere, the original signals cannot be recovered.

Differences Between TDM and FDM

The time-division multiplexing (TDM) includes sharing of the time through utilizing time slots for the signals. On the other hand, frequency-division multiplexing (FDM) involves the distribution of the frequencies, where the channel is divided into various bandwidth ranges (channels).

Analog signal or Digital signal any could be utilized for the TDM while FDM works with Analog signals only.

Framing bits (Sync Pulses) are used in TDM at the start of a frame in order to enable synchronization. As against, FDM uses Guardbands to separate the signals and prevent its overlapping.

FDM system generates different carriers for the different channels, and also each occupies a distinct frequency band. In addition, different bandpass filters are required. Conversely, the TDM system requires identical circuits. As a result, the circuitry needed in FDM is more complex than needed in TDM.

The non–linear character of the various amplifier in the FDM system produces harmonic distortion, and this introduces the interference. In contrast, in TDM system time slots are allotted to various signals; as the multiple signals are not inserted simultaneously in a link. Although, the non-linear requirements of both the systems are same, but TDM is immune to interference (crosstalk).

The utilization of physical link in case of TDM is more efficient than in FDM. The reason behind this is that the FDM system divides the link in multiple channels which does not make use of full channel capacity.