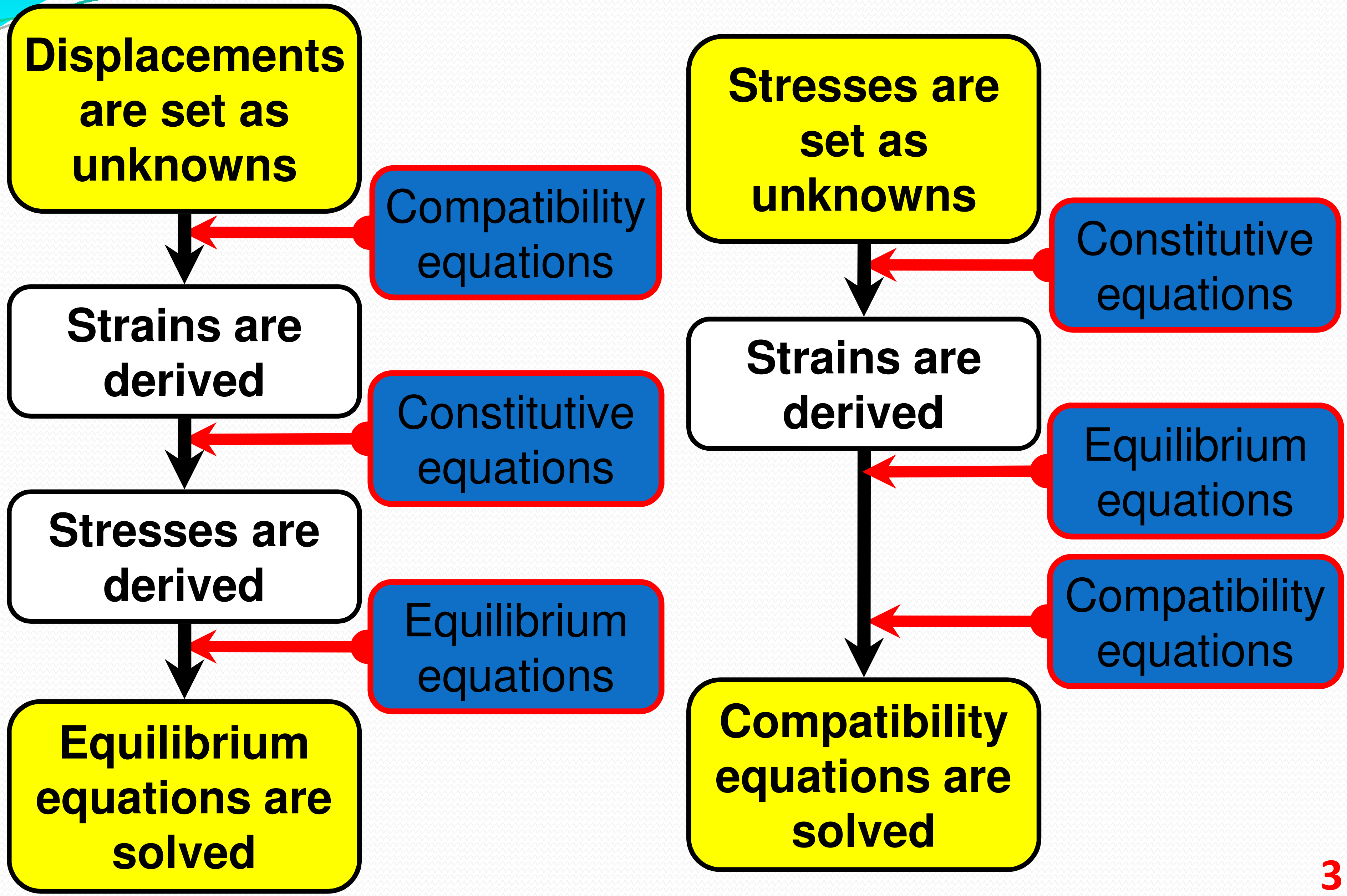
CE8602
STRUCTURAL ANALYSIS
II

UNIT – I

FLEXIBILITY METHOD

Equilibrium and compatibility – Determinate vs Indeterminate structures – Indeterminacy -Primary structure – Compatibility conditions – Analysis of indeterminate pin-jointed plane frames, continuous beams, rigid jointed plane frames (with redundancy restricted to two).

WAYS TO SOLVE A SOLID MECHANICS PROBLEM



Physical sense of equations (revision)

Equilibrium equations — This is not only the sum of forces or moments, but applies for elementary volume as well

Constitutive equations — Physical law, expresses the relation between stress and strain

Compatibility equations — Solid body should remain continuous while being deformed

METHODS TO SOLVE INDETERMINATE PROBLEM

Stiffness method (slope-deflection method)

Displacements are set as unknowns

↓

Equilibrium equations are solved

Flexibility method (force method)

Stresses are set as unknowns

↓

Compatibility equations are solved

FLOWCHART OF SOLUTION USING FORCE METHOD

Classification of the problem

Redundant constraints are removed

Basic system

In loaded state, external load is applied. In unit states, unit force is applied instead of constraint.

Loaded and unit states

Displacements corresponding to removed constraints are determined for each state

Canonical equations

Forces in removed constraints are determined

Total stress state

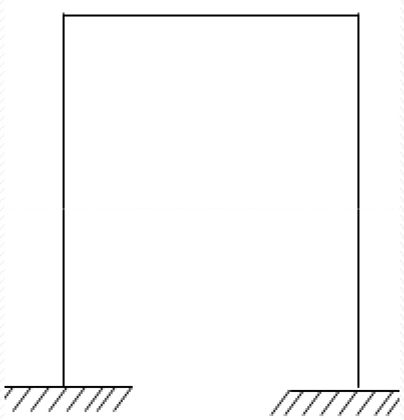
BASIC (PRIMARY) SYSTEM OF FORCE METHOD

Two major requirements exists:
- basic system should be stable;
- basic system should be statically determinate.

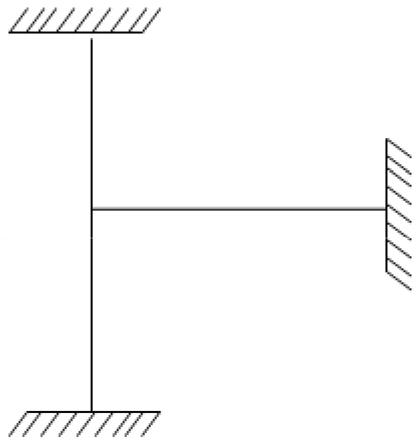
Finally, basic system should be chosen in such a way to simplify calculations as much as possible. For example, for symmetrical problem it is essential to choose a symmetrical basic system.

RIGID FRAMES-STATIC INDETERMINACY

- There are two types of frames-Free frames and constrained frames.
- A Free frame is constrained in only one end whereas constrained frame is constrained in anv line in both the ends.



FREE FRAME



CONSTRAINED FRAME

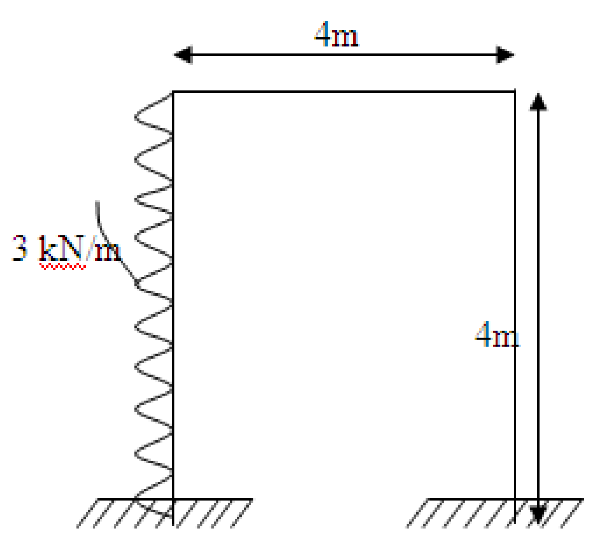
- Static indeterminacy of frames = 3m+r-3j

where j = total number of joints including supports

m = total number of members

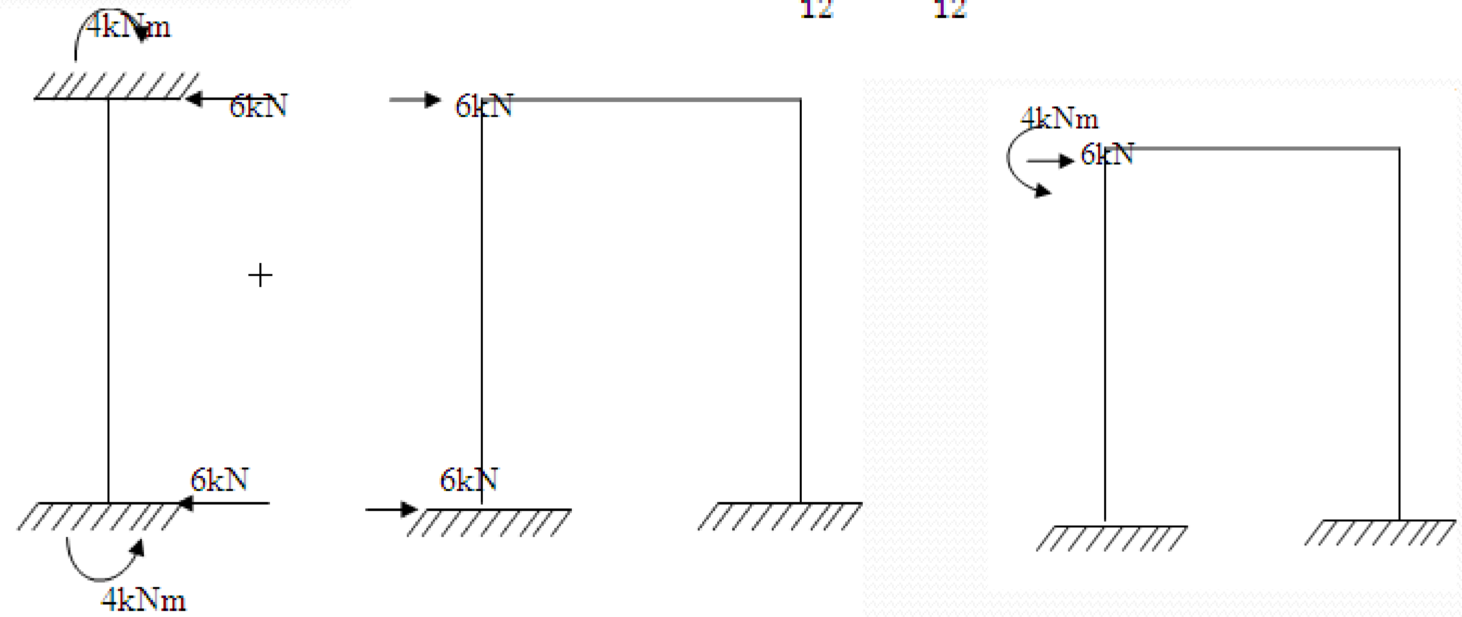
r = total reactions.

EQUIVALENT JOINT LOAD



- The joint loads that are determined from the intermediate loads on the members are called Equivalent Joint Loads.
- The equivalent joint loads are evaluated in such a manner that the resulting displacements of the structure are the same as the displacements produced by actual loads.

Fixed end moment $= \frac{wl^2}{12} = \frac{3 \times 4^2}{12} = 4kNm$ $R_A = R_B = 2 \times 3 = 6$ kN



FORCE TRANSFORMATION MATRIX

For a statically determinate system each of the member forces may be expressed in terms of the external joint (nodal) forces by using the equilibrium conditions of the system alone.

$$Q_1 = b_{11}R_1 + b_{12}R_2 + \ldots\ldots + b_{1n}R_n$$

$$Q_2 = b_{21}R_1 + b_{22}R_2 + \ldots\ldots + b_{2n}R_n$$

:

:

$$Q_m = b_{m1}R_1 + b_{m2}R_2 + \ldots\ldots + b_{mn}R_n$$

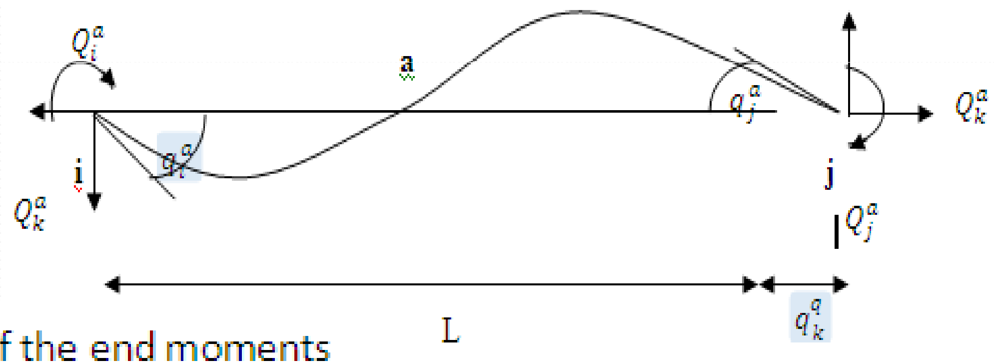
$R_1, R_2 \ldots\ldots R_n$ represents the total set of internal member forces. The matrix form for the set of equation can be represented as $Q = b_R R$ where $b_R = \begin{bmatrix} b_{11} & b_{12} \ldots\ldots b_{1n} \\ b_{21} & b_{22} \ldots\ldots b_{2n} \\ & \circ \\ & \circ \\ b_{m1} & b_{m2} \ldots\ldots b_{mn} \end{bmatrix}$

B_R matrix is called the force transformation matrix which relates the internal forces to the external nodal forces. The elements of the above matrix are called as influence coefficients of the force transformation matrix.

RELATIONSHIP BETWEEN JOINT DISPLACEMENTS AND EXTERNAL FORCES

The principle of virtual work

The principle of virtual work is used as a substitute for the equations of equilibrium or compatibility. It states that, " If a system in equilibrium under the action of a set of external forces is given a small virtual displacement compatible with the constraint imposed on the system, then the work done by the external forces equals the increase in strain energy stored in the system".



Q_i^q *and* Q_j^q are the internal forces of the end moments

Q_k^a *are the axial forces*

q_i^a *and* q_j^a *are the end end rotations*

q_k^a is the axial elongation

External joint (nodal) loads are denoted by R = $\begin{bmatrix} R_1 \\ R_2 \\ \circ \\ \circ \\ \circ \\ R_n \end{bmatrix}$

Similarly the joint (nodal) displacements are denoted as r = $\begin{bmatrix} r_1 \\ r_2 \\ \circ \\ \circ \\ \circ \\ r_n \end{bmatrix}$

External work done W_E may be expressed as $W_E = R^T r = r^T R$ ------------(1)

Internal end forces are denoted as Q = $\begin{bmatrix} Q_i^a \\ Q_j^a \\ Q_i^b \\ Q_j^b \\ \circ \\ \circ \end{bmatrix}$

Internal displacements are denoted as $q = \begin{bmatrix} q_i^a \\ q_j^a \\ q_i^b \\ q_j^b \\ \circ \\ \circ \end{bmatrix}$

Internal work done or strain energy stored may be expressed as $W_r = Q^T q = q^T Q$ ------(2)

Equating W_E and W_r,

$R^T r = Q^T q$ -------------------(3)

(or)

$r^T R = q^T Q$ ------------------(4)

Eqn (3) and (4) is valid if R and Q are in equilibrium and r and q are compatible.

If virtual displacements are used, we have from eqn (4)

$\underline{r}^T R = \underline{q}^T Q$ ----------------------(5)

If virtual forces are used, we have from eqn (3)

$\underline{R}^T r = \underline{Q}^T q$ ---------------------(6)

$\underline{r}$, $\underline{q}$, $\underline{R}$ and $\underline{Q}$ represent real forces and displacements

From the Force transformation matrix we know that $Q = b_R R$ ----------------------(7)

If Q is the internal forces and q is the member deformations then $q = fQ$ ----------(8)

Where f is the element flexibility matrix.

$q = f\ b_R R$ ----------------(9)

Transposing Eqn (7), $\underline{Q}^T = \underline{R}^T b_R{}^T$ -----------------------(10)

We know from Eqn (6) $\underline{R}^T r = \underline{Q}^T q$

Substituting Eqn (9) and (10) in Eqn (6)

$\underline{R}^T r = \underline{R}^T b_R{}^T f\ b_R R$

$r = b_R{}^T f\ b_R R$ -----------------------------------(11)

If the total flexibility matrix is represented as $F_{RR} = b_R{}^T f\ b_R$

Then $r = F_{RR} R$ -----------------------------------(12)

Eqn (12) gives the direct solution of all the nodal displacements in terms of external nodal (joint) forces.

Relationship between nodal displacements and nodal loads considering the effects of the redundants

Any statically indeterminate structure can be made statically determinate and stable by removing the extra restraints called redundant forces. The statically determinate and stable structure that remains after the removal of the extra restraints is called the primary structure. These unknown redundant may be treated as part of the external loads of unknown magnitude. Internal member forces can be represented in terms of the original applied external forces R and unknown redundant X as

$Q = [\, b_R R + b_x X]$ ------------------------------(13)

$Q = [\, b_R \;\; b_x] \begin{bmatrix} R \\ X \end{bmatrix}$ ---(14)

Where b_R and b_x are force transformation matrices representing the external nodal loads R and unknown redundant X.

Let r_x be the displacements due to redundant force. (This will become 0)

R and r is related using the equation

$\underline{R}^T r + \underline{X}^T r_x = \underline{Q}^T q$ ----------------------------------(15)

Using Eqn (13) and virtual force

$\underline{Q} = b_R \underline{R} + b_x \underline{X}$

Transposing the above equation

$\underline{Q}^T = b_R^T \underline{R}^T + b_x^T \underline{X}^T$ --(16)

$\underline{Q}^T = b_R^T \underline{R}^T + b_x^T \underline{X}^T$ --(16)

From Eqn (8) $q = fQ$

From Eqn (13) $Q = b_R R + b_x X$

Substituting Eqn (13) in Eqn (8) we get

$q = f[b_R R + b_x X] = f b_R R + f b_x X$ ----------------------(17)

Substituting Eqn (16) and (17) in Eqn (15)

$\underline{R}^T r + \underline{X}^T r_x = (b_R^T \underline{R}^T + b_x^T \underline{X}^T)(f b_R R + f b_x X)$

$\underline{R}^T r + \underline{X}^T r_x = \underline{R}^T (b_R^T f b_R R + b_R^T f b_x X) + \underline{X}^T (b_x^T f b_R R + b_x^T f b_x X)$

Comparing the virtual forces on the left and the right sides of the Eqn, we have

$r = b_R^T f b_R R + b_R^T f b_x X$ ---------------------------(18)

$r_x = b_x^T f b_R R + b_x^T f b_x X$

Eqn (18) is known is Compatibility condition

$r = F_{RR} R + F_{RX} X$ ---------------------------(19)

$r_x = F_{XR} R + F_{XX} X$ ---------------------------(20)

where $F_{RR} = b_R^T f b_R$, $F_{RX} = b_R^T f b_X$, $F_{XR} = b_X^T f b_R$ and $F_{XX} = b_X^T f b_X$

F_{XX} is known as the Flexibility Influence Coefficient Matrix.

Eqn (18) is known is Compatibility condition

$r = F_{RR}R + F_{RX}\,X$ ---------------------------(19)

$r_x = F_{XR}R + F_{XX}\,X$ ---------------------------(20)

where $F_{RR} = b_R^T f\, b_R$, $F_{RX} = b_R^T f\, b_X$, $F_{XR} = b_X^T f\, b_R$ and $F_{XX} = b_X^T f\, b_X$

F_{XX} is known as the Flexibility Influence Coefficient Matrix.

Eqn (19) and (20) can be written in matrix form as $\left[\frac{r}{r_X}\right] = \begin{bmatrix} F_{RR} & F_{RX} \\ F_{XR} & F_{XX} \end{bmatrix}\left[\frac{R}{X}\right]$----------- (21)

For structures with rigid supports $r_x = 0$

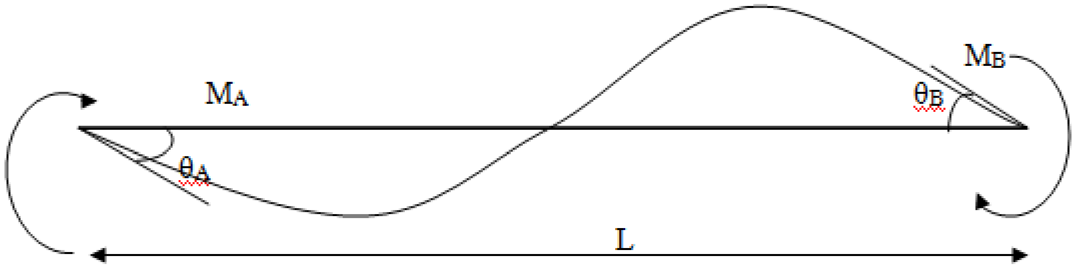
$$\left[\frac{r}{r_X=0}\right] = \begin{bmatrix} F_{RR} & F_{RX} \\ F_{XR} & F_{XX} \end{bmatrix}\left[\frac{R}{X}\right]$$

The compatibility condition is therefore $F_{XR}\,R + F_{XX}\,X = 0$ from which $F_{XX}\,X = -\,F_{XR}\,R$

Pre-multiplying F_{XX}^{-1} we get

$X = -\,F_{XX}^{-1}\,F_{XR}\,R$

FLEXIBILITY MATRIX FOR A BEAM ELEMENT



Using Slope Deflection Equation:

$$M_A = \frac{2EI}{l}\left(2\theta_A + \theta_B\right) \qquad M_B = \frac{2EI}{l}\left(2\theta_B + \theta_A\right)$$

Solving the above two equations:

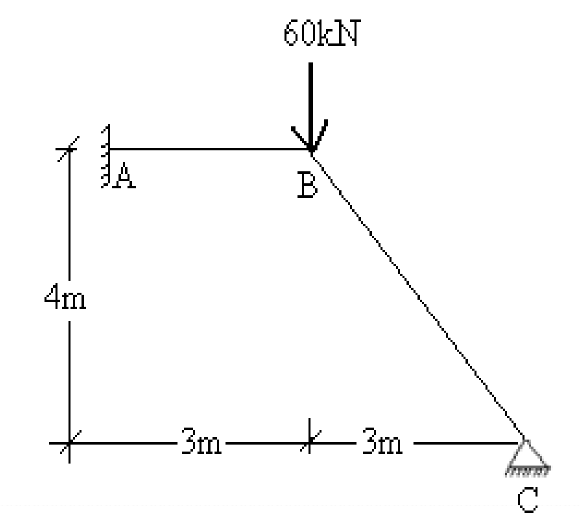
$$\theta_A = \frac{M_A l}{3EI} - \frac{M_B l}{6EI} \qquad \theta_B = \frac{-M_A l}{6EI} + \frac{M_B l}{3EI}$$

Rearranging in Matrix form

$$\begin{bmatrix}\theta_A \\ \theta_B\end{bmatrix} = \begin{bmatrix}\frac{l}{3EI} & -\frac{l}{6EI} \\ -\frac{l}{6EI} & \frac{l}{3EI}\end{bmatrix} \times \begin{bmatrix}M_A \\ M_B\end{bmatrix}$$

Flexibility matrix $\quad f = \frac{l}{6EI}\begin{bmatrix}2 & -1 \\ -1 & 2\end{bmatrix}$

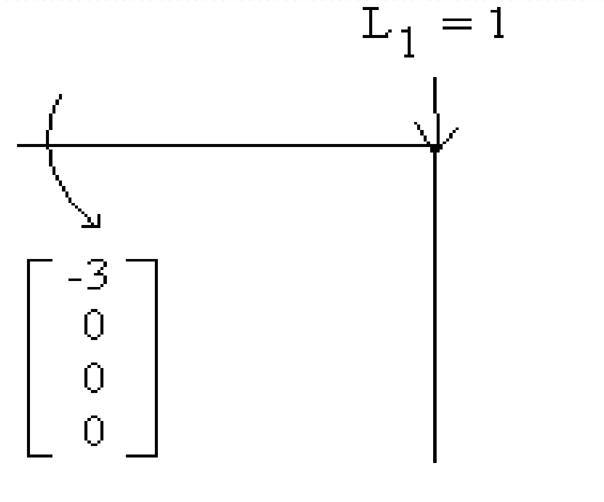
1. Solve the frame by flexibility method static indeterminacy of the frame =1. The reaction at support c is taken as redundant R.



$$Als = \begin{bmatrix} L \\ \overline{R} \end{bmatrix} = \begin{bmatrix} 60 \\ R \end{bmatrix}$$

Flexibility matrix F_m for members AB and BC

$$[F_m]_{AB} = \frac{1}{6EI}\begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix} [F_m]_{Bc} = \frac{1}{6EI}\begin{bmatrix} 10 & -5 \\ -5 & 10 \end{bmatrix}$$



Unassembled flexibility matrix

$$F_m = \frac{1}{6EI}\begin{bmatrix} 6 & -3 & 0 & 0 \\ -3 & 6 & 0 & 0 \\ 0 & 0 & 10 & -5 \\ 0 & 0 & -5 & 10 \end{bmatrix}$$

Transformation matrix corresponding to A_s

$$A_{ms} = \begin{bmatrix} -3 & 6 \\ 0 & -3 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}$$

Assembled flexibility matrix $F_s = A_{ms}^T f_m A_{ms}$

$$F_s = \begin{bmatrix} -3 & 0 & 0 & 0 \\ 6 & -3 & 3 & 0 \end{bmatrix} \frac{1}{6EI} \begin{bmatrix} 6 & -3 & 0 & 0 \\ -3 & 6 & 0 & 0 \\ 0 & 0 & 10 & -5 \\ 0 & 0 & -5 & 10 \end{bmatrix} \begin{bmatrix} -3 & 6 \\ 0 & -3 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}$$

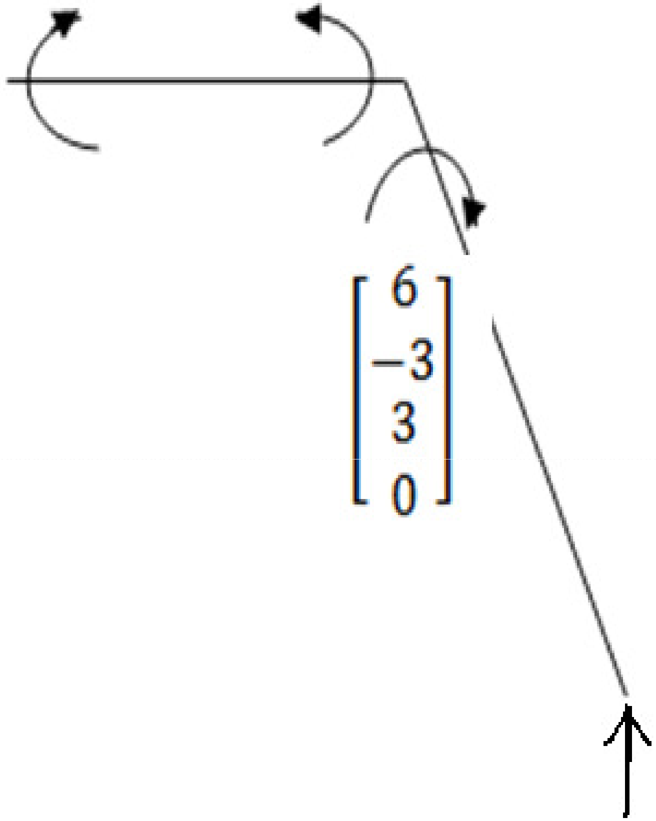
$$F_s = \frac{1}{6EI} \begin{bmatrix} 54 & : & -135 \\ \text{------} & & \text{------} \\ -135 & : & 468 \end{bmatrix}$$

Redundant Reaction $R = -F_{RR}^{-1} \times F_{RL} \times L$

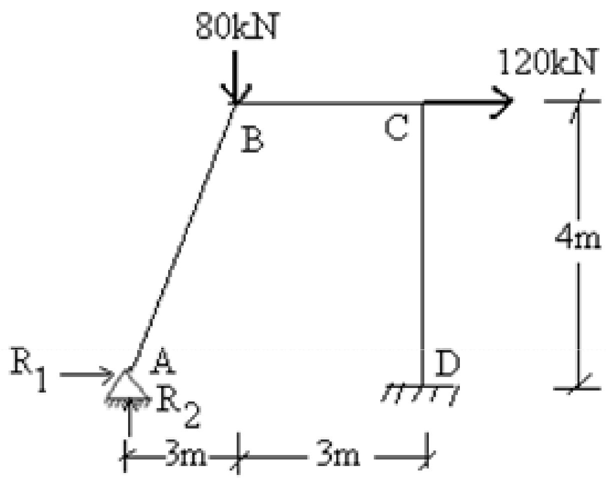
$$R = -\left[\frac{1}{6EI}468\right]^{-1} \times \frac{1}{6EI}\left[-135\right]\left[60\right] = 17.31KN$$

Member end forces $A_m = A_{ms}A_s$

$$A_m = \begin{bmatrix} -3 & 6 \\ 0 & -3 \\ 0 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 60 \\ 17.31 \end{bmatrix} = \begin{bmatrix} -76.15 \\ -51.92 \\ 51.92 \\ 0 \end{bmatrix} kN.m$$



2. Solve the given frame by flexibility method static indeterminacy of the frame =2.
The horizontal and vertical reactions at support A are taken as redundant R_1 and R_2.



$$A_s = \begin{bmatrix} L_1 \\ L_2 \\ R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} 80 \\ 120 \\ R_1 \\ R_2 \end{bmatrix}$$

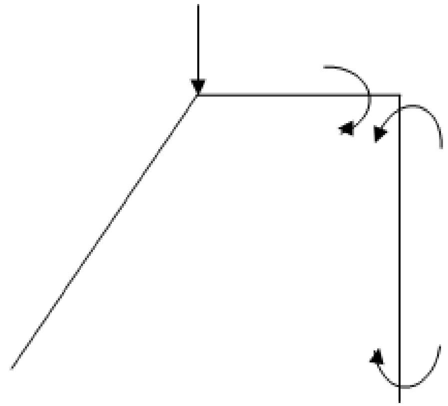
Flexibility matrix for member AB, BC and CD

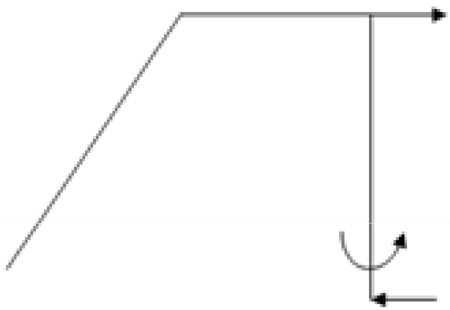
$$[F_m]\,AB = \frac{1}{6EI}\begin{bmatrix} 10 & -5 \\ -5 & 10 \end{bmatrix} [F_m]_{BC} = \frac{1}{6EI}\begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}$$

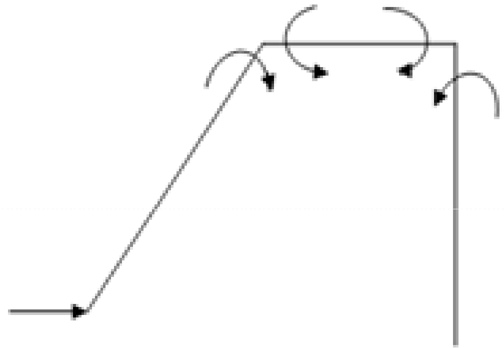
$$[F_m]_{CD} = \frac{1}{6EI}\begin{bmatrix} 8 & -4 \\ -4 & 8 \end{bmatrix}$$

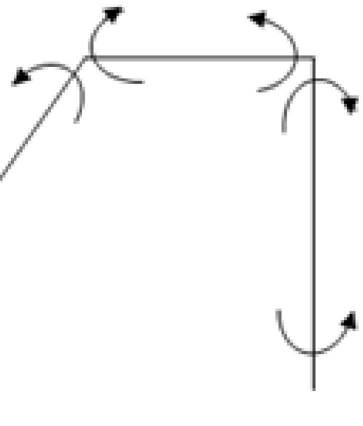
Unassembled flexibility matrix

$$F_m = \frac{1}{6EI}\begin{bmatrix} 10 & -5 & 0 & 0 & 0 & 0 \\ -5 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & -3 & 0 & 0 \\ 0 & 0 & -3 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & -4 \\ 0 & 0 & 0 & 0 & -4 & 8 \end{bmatrix}$$









Transformation matrix corresponding to A_s

$$A_{ms} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -3 \\ 0 & 0 & -4 & 3 \\ 3 & 0 & 4 & -6 \\ -3 & 0 & -4 & 6 \\ 3 & -4 & 0 & -6 \end{bmatrix}$$

Assembled flexibility matrix of the structure $F_s = F_s = A_{ms}{}^T F_m A_{ms}$

$$F_s = \begin{bmatrix} 0 & 0 & 0 & 3 & -3 & 3 \\ 0 & 0 & 0 & 0 & 0 & -4 \\ 0 & 4 & -4 & 4 & -4 & 0 \\ 0 & -3 & 3 & -6 & 6 & -6 \end{bmatrix} \frac{1}{6EI} \begin{bmatrix} 10 & -5 & 0 & 0 & 0 & 0 \\ -5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & -3 & 0 & 0 \\ 0 & 0 & -3 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & -4 \\ 0 & 0 & 0 & 0 & -4 & 8 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -4 \\ 3 & 0 & 4 \\ -3 & 0 & -4 \\ 3 & -4 & 0 \end{bmatrix}$$

$$F_s = \begin{bmatrix} 270 & -144 & : & 252 & -567 \\ -144 & 128 & : & -64 & 288 \\ ----- & ------ & & ------ & ------- \\ 252 & -64 & : & 576 & -732 \\ -567 & 288 & : & -732 & 1332 \end{bmatrix}$$

Redundant reactions

$$R = -F_{RR}^{-1} \times F_{RL} \times L$$

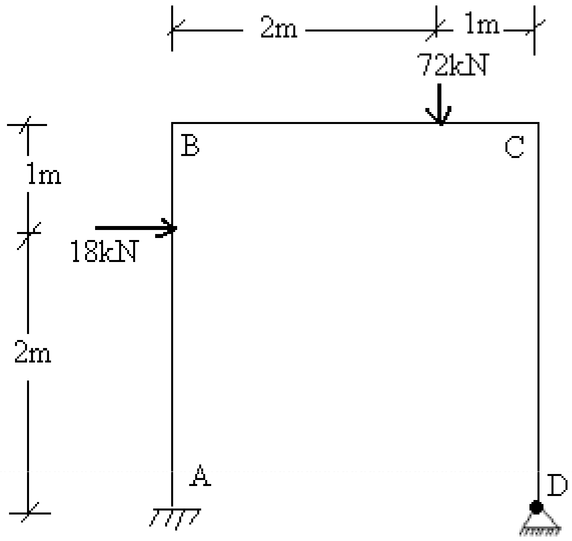
$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = -6EI \times \begin{bmatrix} 576 & -732 \\ -732 & 1332 \end{bmatrix}^{-1} \times \frac{1}{66EI} \begin{bmatrix} 252 & -64 \\ -567 & 288 \end{bmatrix}$$

Member end forces $A_m = A_{ms} \times A_s$

$$\begin{bmatrix} 37.69 \\ -12.59 \end{bmatrix}$$

$$A_m = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & -3 \\ 0 & 0 & -4 & 3 \\ 3 & 0 & 4 & -6 \\ -3 & 0 & -4 & 6 \\ 3 & -4 & 0 & -6 \end{bmatrix} \begin{bmatrix} 80 \\ 120 \\ 37.67 \\ -12.59 \end{bmatrix} = \begin{bmatrix} 0 \\ -112.91 \\ 112.91 \\ 164.86 \\ -164.86 \\ -164.46 \end{bmatrix} kN.m$$

3. Using the flexibility method analyze the frame as shown in figure. Static indeterminacy =2. The moment at B and C are taken as redundant R_1 and R_2 respectively.

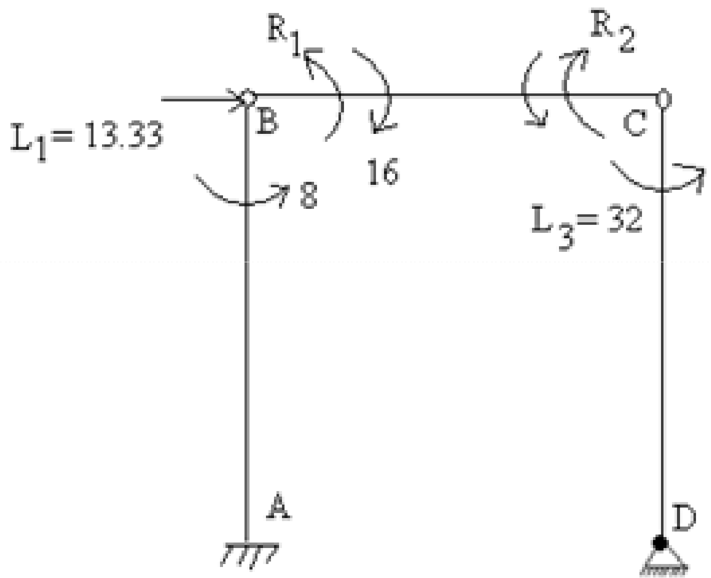


$$M_{FAB} = \frac{-wab^2}{l^2} = \frac{-18 \times 2 \times 1^2}{3^2} = -4\ kNm \quad M_{FBA} = \frac{wba^2}{l^2} = \frac{18 \times 1 \times 2^2}{3^2} = +8\ kNm$$

$$M_{FBC} = \frac{-wab^2}{l^2} = \frac{-72 \times 2 \times 1^2}{3^2} = -16\ kNm \quad M_{FCB} = \frac{wba^2}{l^2} = \frac{72 \times 1 \times 2^2}{3^2} = +32\ kNm$$

Fixed end beam reaction at B $= \frac{18\times2-4+8}{3} = 13.33$ kN. **system equivalent joint action.**

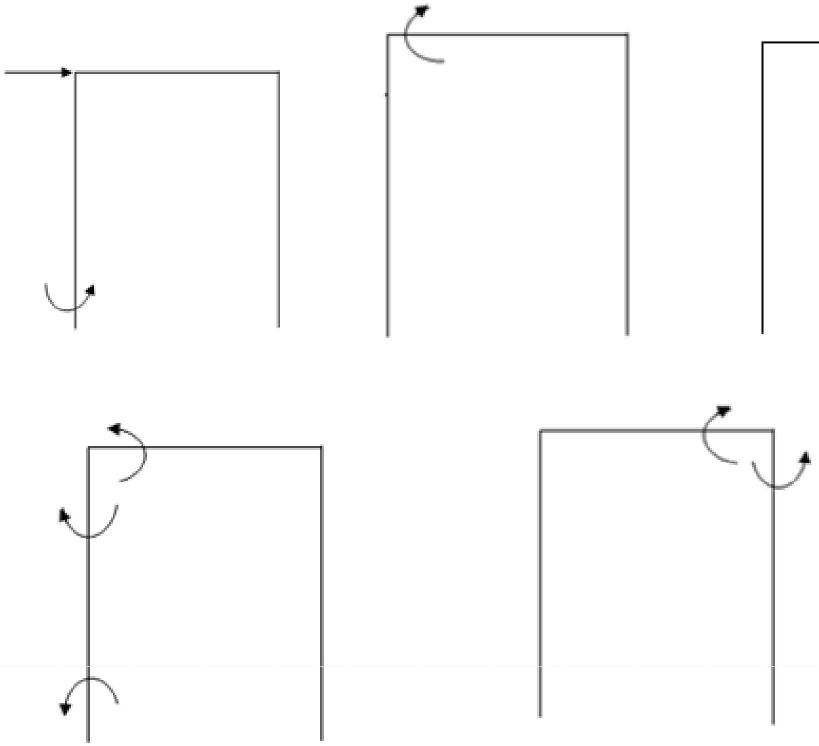
$$As = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ R_1 \\ R_2 \end{bmatrix} = \begin{bmatrix} 13.33 \\ 8 \\ 32 \\ R_1 \\ R_2 \end{bmatrix}$$



Member flexibility matrix

$$[F_m]\,AB, BC, CD = \frac{1}{6EI}\begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix}(OR) = \frac{3}{6EI}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Unassembled flexibility matrix $F_m = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$



Transformation matrix corresponding to As is

$$A_{ms}=\begin{array}{c} \begin{matrix} L_1 & L_2 & L_3 & R_1 & R_2 \end{matrix} \\ \begin{bmatrix} -3 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Assembled flexibility matrix $F_s = A^T_{ms} F_m A_{ms}$

$$F_s = \begin{bmatrix} -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \frac{1}{2EI} \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} -3 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F_s = \frac{1}{2EI} \left[\begin{array}{ccc:cc} 18 & 0 & 0 & -9 & -6 \\ 0 & 2 & 1 & -2 & -1 \\ 0 & 1 & 2 & -1 & -2 \\ \hdashline -9 & -2 & -1 & 8 & -2 \\ -6 & -1 & -2 & -2 & 6 \end{array} \right]$$

Redundant forces are given by R = $R = -F_{RR}^{-1} \times F_{RL} \times L$

$$R = -2EI \begin{bmatrix} 8 & -2 \\ -2 & 6 \end{bmatrix}^{-1} \times \frac{1}{6EI} \begin{bmatrix} -9 & -2 & -1 \\ -6 & -1 & -2 \end{bmatrix} \times \begin{bmatrix} 13.33 \\ 8 \\ 32 \end{bmatrix}$$

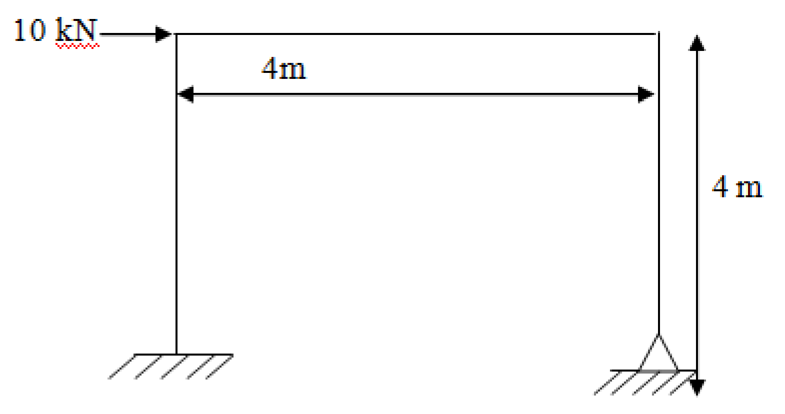
$$R = \begin{bmatrix} -2.91 \\ 24.66 \end{bmatrix} kN.m$$

Member end forces $A_m = A_{ms}\, A_s + A_{mf}$

$$A_m = \begin{bmatrix} -3 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 13.35 \\ 8 \\ 32 \\ -2.91 \\ 24.36 \end{bmatrix} + \begin{bmatrix} -4 \\ 8 \\ -16 \\ 32 \\ 0 \\ 0 \end{bmatrix}$$

$$A_m = \begin{bmatrix} -16.73 \\ 5.09 \\ -5.09 \\ 24.36 \\ -24.36 \\ 0 \end{bmatrix} kN.m$$

4. Analyse the frame shown in Fig. by Flexibility method.



Redundant force $\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} 5.452 \\ 2.276 \end{bmatrix}$

Element force $[Q] = \begin{bmatrix} -18.188 \\ -12.708 \\ +12.708 \\ +9.1032 \\ -9.1032 \\ 0 \end{bmatrix}$

Static indeterminacy = 3m+r-3j=3x3 + 5 -3x4 = 14-12=2.

Apply unit force at D both vertically and horizontaly , $[Q] = \begin{bmatrix} -4 & 4 & 0 \\ 0 & -4 & 4 \\ 0 & 4 & -4 \\ 0 & 0 & 4 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{bmatrix}$

Stiffness Matrix $[k] = \frac{4}{6EI} \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$

5. Analyse the frame shown in Fig. by Flexibility method

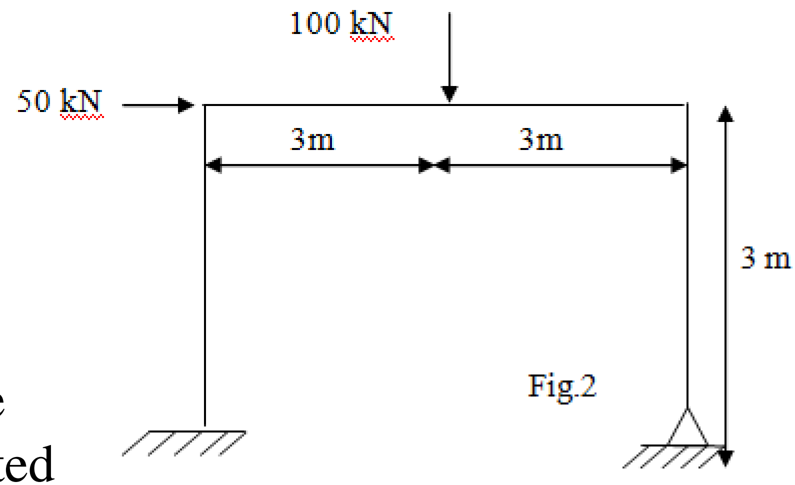
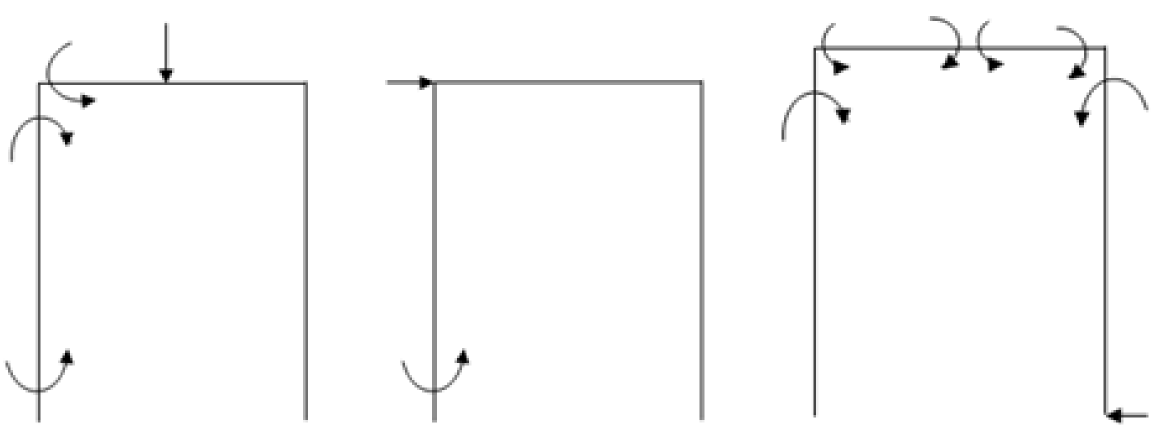


Fig.2

Static indeterminacy = 3m+r-3j=
3x3 + 5x4 -3x4 = 14-12=2.
The structure is made determinate by removing the hinged support at D. The two reaction components at D are treated as redundant.

Assembled element flexibility matrix

$$[\alpha]=\frac{1}{2}\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$



The transformation matrix [b]= $\begin{bmatrix} -3 & -3 & 0 & 6 \\ 3 & 0 & 3 & -6 \\ -3 & 0 & -3 & 6 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & -3 & 3 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$[\alpha_{00}] = [b_0]^T[\alpha]\,[b_0] = \begin{bmatrix} 0 & 3 & -3 & 3 & -3 & 3 & -3 & 0 \\ 6 & -6 & 6 & -3 & 3 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 0 & 6 \\ 3 & -6 \\ -3 & 6 \\ 3 & -3 \\ -3 & 3 \\ 3 & 0 \\ -3 & 0 \\ 0 & 0 \end{bmatrix}$$

$$[\alpha_{00}] = \frac{1}{2}\begin{bmatrix} 90 & -108 \\ -108 & 288 \end{bmatrix} = 9\begin{bmatrix} 5 & -6 \\ -6 & 16 \end{bmatrix}$$

$$[\alpha_{00}]^{-1} = \frac{9}{44}\begin{bmatrix} 16 & 6 \\ 6 & 5 \end{bmatrix}$$

$$[\alpha_0^*] = [b_0]^T[\alpha]\,[b^*] = \begin{bmatrix} 0 & 3 & -3 & 3 & -3 & 3 & -3 & 0 \\ 6 & -6 & 6 & -3 & 3 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} -3 & -3 \\ 3 & 0 \\ -3 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$[\alpha_0^*] = \frac{1}{2}\begin{bmatrix} 40.5 & 9 \\ -130.5 & -54 \end{bmatrix}$$

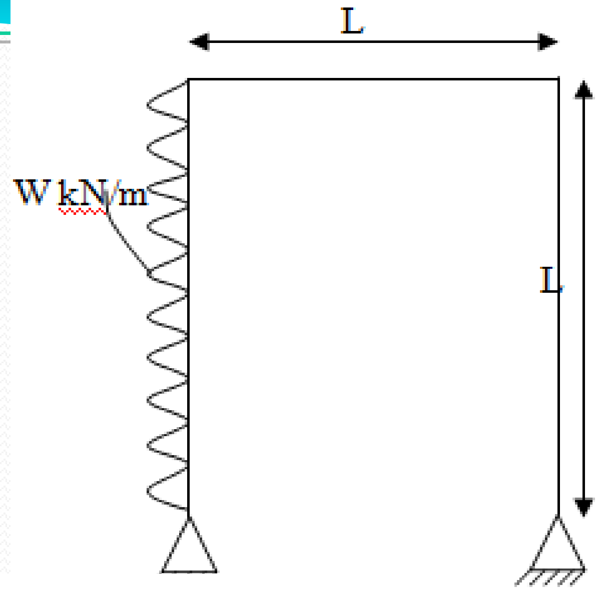
$$\{F^0\} = -[\alpha_{00}]^{-1}[\alpha_0^*]\{F^*\}$$

$$\{F^0\} = -\frac{9}{4}\begin{bmatrix} 16 & 6 \\ 6 & 5 \end{bmatrix} \times \frac{1}{2}\begin{bmatrix} 40.5 & 9 \\ -130.5 & -54 \end{bmatrix} = \begin{Bmatrix} 2301 \\ 5293 \end{Bmatrix}$$

Element forces $\{P\} = [b]\ \{F\}$ where $\{F\} = \begin{Bmatrix} F^* \\ F^0 \end{Bmatrix}$

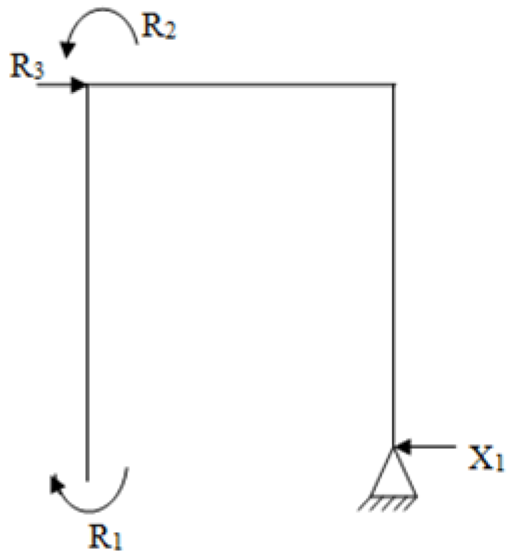
$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{Bmatrix} = \begin{bmatrix} -3 & -3 & 0 & 6 \\ 3 & 0 & 3 & -6 \\ -3 & 0 & -3 & 6 \\ 0 & 0 & 3 & -3 \\ 0 & 0 & -3 & 3 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} 100 \\ 50 \\ 2301 \\ 5293 \end{Bmatrix} = \begin{Bmatrix} 31308 \\ -24555 \\ 24555 \\ -8976 \\ 8976 \\ 6903 \\ -6903 \\ 0 \end{Bmatrix}$$

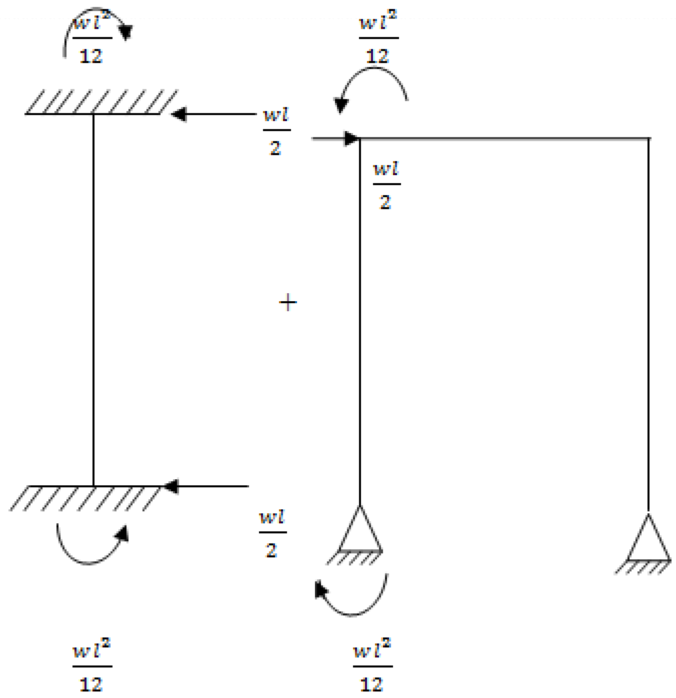
6. Analyse the two hinged frame by flexibility method.



Static indeterminacy is 1. The reaction at the bottom will directly go to the support whereas the reaction at the top will cause flexural deformation of the vertical member. This will cause end moments in the structure. The primary structure may be chosen subjected to nodal moments R_1, R_2 and equivalent horizontal reaction R_3 at the top and the redundant component X_1 at the right support.

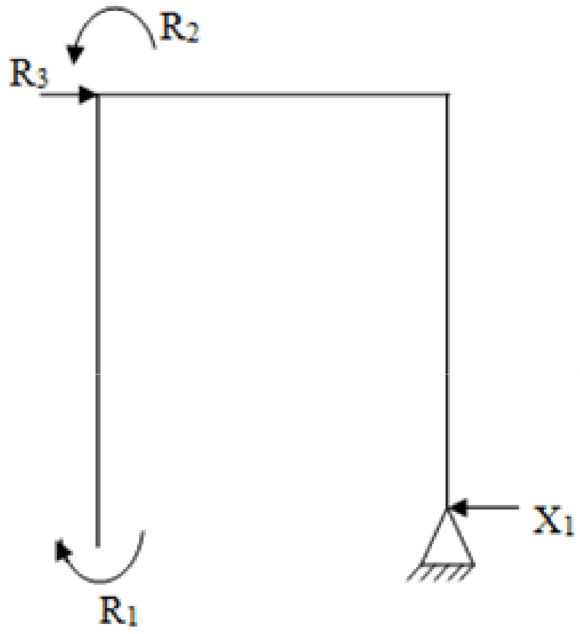
Primary Structure





Assembled element flexibility matrix $f = \frac{l}{6EI}\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$

Primary Structure



Transformation matrix [b] = $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 0 & -l & l \\ 1 & -1 & +l & -l \\ 0 & 0 & 0 & l \\ 0 & 0 & 0 & -l \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$[\alpha_{00}] = [b_0]^T[\alpha][b_0] = [0 \quad l \quad -l \quad l \quad -l \quad 0] \times \frac{l}{6EI} \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ l \\ -l \\ l \\ -l \\ 0 \end{bmatrix} = \frac{5}{3}\frac{l^3}{EI}$$

$$[\alpha_0^*] = [b_0]^T[\alpha][b^*] = [0 \quad l \quad -l \quad l \quad -l \quad 0] \times \frac{l}{6EI} \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & -l \\ 1 & -1 & l \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \frac{l}{6EI}[-6l \quad 3l \quad -5l^2]$$

Redundant force $\{F^0\}=-[\alpha_{00}]^{-1}[\alpha_0^*]\{F^*\} = -\frac{3}{5}\frac{EI}{l^3} \times \frac{l}{6EI}\begin{bmatrix}-6l & 3l & -5l^2\end{bmatrix} \times \begin{bmatrix}\frac{wl^2}{12} \\ \frac{wl^2}{12} \\ \frac{wl}{2}\end{bmatrix} = \frac{11}{40}wl$

Element forces $\{P\}=[b]\ \{F\} = \begin{bmatrix}1 & 0 & 0 & 0 \\ -1 & 0 & -l & l \\ 1 & -1 & +l & -l \\ 0 & 0 & 0 & l \\ 0 & 0 & 0 & -l \\ 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\frac{wl^2}{12} \\ \frac{wl^2}{12} \\ \frac{wl}{2} \\ -\frac{11wl^2}{40}\end{bmatrix} = \begin{bmatrix}\frac{wl^2}{12} \\ \frac{-148wl^2}{480} \\ \frac{9wl^2}{40} \\ \frac{11wl^2}{40} \\ \frac{-11wl^2}{40} \\ 0\end{bmatrix}$

The final moment is obtained by adding the fixed end moments to the end moments of member.

$$M_{AB} = \frac{wl^2}{12} - \frac{wl^2}{12} = 0 \qquad M_{BA} = \frac{wl^2}{12} - \frac{148wl^2}{40} = -\frac{9wl^2}{40}$$

$$M_{BC} = \frac{9wl^2}{40} \quad M_{CB} = \frac{11wl^2}{40} \quad M_{CD} = -\frac{11wl^2}{40} \quad M_{DC} = 0$$

THANK YOU